Invited paper

SCHEDULING OF ARRIVAL AIRCRAFT BASED ON MINIMUM FUEL BURN DESCENTS

Adriana Andreeva-Mori[1], Shinji Suzuki[2], and Eri Itoh[3]

[1]Department of Aeronautics and Astronautics, The University of Tokyo, Tokyo, Japan,
Tel: +81-3-5841-6568, e-mail: tt097088@mail.ecc.u-tokyo.ac.jp

[2]Department of Aeronautics and Astronautics, The University of Tokyo, Tokyo, Japan,
Tel: +81-3-5841-6566, e-mail: tshinji@mail.ecc.u-tokyo.ac.jp

[3]Air Traffic Management Department, Electronic Navigation Research Institute, Tokyo, Japan,
Tel: +81-422-41-3184, e-mail: eri@enri.go.jp

Received Date: August 23, 2011

Abstract

The main goal of this research is to investigate minimum fuel descent trajectories for various types of aircraft and consider appropriate arrival sequencing based on the acquired results. First, the validity of a point mass model to find optimized descent trajectories in order to obtain minimal fuel consumption is confirmed. Through single aircraft numerical simulations, the principle "higher for longer" is verified and the limitation induced by the 3 degree maximum value of the flight path angle is observed. An extension to multiple aircraft simulation generates results, which can be used effectively to determine the most advantageous order of arrival considering air traffic constraints and predefined flight time and time separation.

Keywords: Aircraft size, Descent trajectory, Minimum fuel burn, Sequence

Introduction

Aviation is one of the fastest developing industries and as such has been attracting attention for a long time. Global air-passenger traffic is forecasted to increase by up to 5% annually over the next 10 years [1]. Based on the recent economic developments and increase in GDP, it is safe to say that this number will be increasing dramatically [2]. Even though green leaders were the ones who started the stream for CO2 emissions abatement, their motives are not the only one at present. Unlike in the past years, the challenges for the aviation are likely to be environmental, rather than technological. It is said that "environmental issues are likely to impose fundamental limitation on air transportation growth in the 21 century" [3]. Furthermore, a major role plays the so-called "emission trading"- the administrative approach used to control pollution by providing economic incentives for achieving reductions in the emissions of pollutants. Unlike emissions from cars and households, emissions from aircraft are relatively easily estimated and are therefore subject to serious consideration in emissions trading. Newly-introduced caps on aviation emissions prove that [4]. Thus, even though the relative percentage of aviation emissions is small, these are very important. In addition, 98% of all civil aviationCO_2 emissions are due to the combustion of aircraft fuel. Therefore, the increase in oil prices makes the airlines aim for lower fuel consumption, which in turn means lower CO_2 emissions.

Numerous improvements to abate CO_2 emissions have already been made. These include advanced lightweight materials, more efficient propulsion systems, use of a simulator for pilot training and optimized flight trajectories. A major role in the trajectory optimization is played by the introduction of RNAV (aReaNAVigation). RNAV is a method of navigation that allows aircraft operation on any desired course within the coverage of station-referenced navigation signals or within the limits of a self-contained system capability, or a combination of these. RNAV was developed to provide more lateral freedom and thus more complete use of available airspace [5].

Every flight can be divided into several stages: taxing, take-off, climb, cruise, descent, final approach, landing and taxing. In general, climb is performed at close to maximum thrust so that the aircraft leaves the vicinity of the airport as soon as possible. This is done because the constantly increasing air traffic poses pressure to the airports. Cruising is performed at a flight speed close to the optimal. Descent, on the other hand, is thought to be the stage which still requires a lot of optimization as it allows for changes without jeopardizing the safety of the flight within the scope of the technologies available at present. Therefore, approach is of main interest for a lot of researchers in the field.

Before we elaborate on the new developments in descent research, let us consider the conventional approach. With the conventional aircraft approach, an airplane would be given clearance by air traffic control from the bottom level of the holding stack (normally an altitude of 6000 or 7000 feet) to descent to an altitude of typically 3000 feet. The aircraft would then fly level for several miles before intersecting the final3 degree glide path to the runway. During this period of level flight the pilot would need to apply additional engine power to maintain constant speed [6].

In contrast to the conventional staged approach, when a CDA procedure is flown the aircraft stays higher for longer, descending continuously form the level of the bottom of the stack and avoiding any level segments of flight prior to intercepting the final 3 degree glide path. Such a descent requires less engine thrust than prolonged level flight.

The keystone for the development of CDA is believed to be the paper [7] published in 2004. It considers and verifies CDA as a way to minimize noise pollution in the vicinity of an airport. It also proves reduction in the fuel consumption. However, even though there are a lot of papers focusing on environmentally friendly trajectories, most of them develop methods for single aircraft optimization. There are not as many on multiple aircraft optimization, though.

This research is aimed at developing optimized descent trajectories for multiple civil aircraft in order to reduce harmful CO_2 emissions by minimizing the fuel consumption. The authors consider the airlines and the air traffic controllers as shareholders and propose adequate flight management taking into account the requirements below.

The airlines desire a procedure which requires the least possible fuel consumption in order to save on fuel and increase their eco-friendly image among costumers. In other words, the aircraft thrust should be kept as low as possible for as long as possible. This would lead to a steep descent angle, but for practical considerations the flight path angle is limited to 3 deg. On the other hand, air traffic controllers need to separate and sequence aircraft easily and safely. To deal with this issue, currently they put a series of aircraft at the same speed with desired separation in time and space. However, this requires constant-speed segments which increase the fuel consumption.

In this research we consider the above perspectives and investigate plausible descent trajectories of medium and heavy aircraft. Based on the results of single aircraft optimization we propose the best sequencing of two aircraft in respect to the total fuel burn.

For easy comprehension and simplicity, the rest of this thesis will be divided into five sections. Section 2 focuses on the problem formulation. It starts off my introducing an aircraft model which is to be used through the entire research. It continues to discuss the choice of aircraft and the constraints considered. Section 3 is mainly about single aircraft trajectory simulations. Besides, the effects of the constraints and their influence on the optimal value of the objective function are examined. Section 4 introduces two-aircraft numerical simulations and for the first time in this paper discusses the importance of the order of arrival. It ends with a very important conclusion which is the building stone of the entire research. The paper is summarized in Section 5 with some discussions and conclusions.

Aircraft Model

Aerodynamics

From the perspective of air traffic control an aircraft can be modeled adequately using a point mass model.

The four main forces acting on an airplane are lift, drag, gravity, and thrust Figure 1. All of these forces have effects on the performance of the airplane.

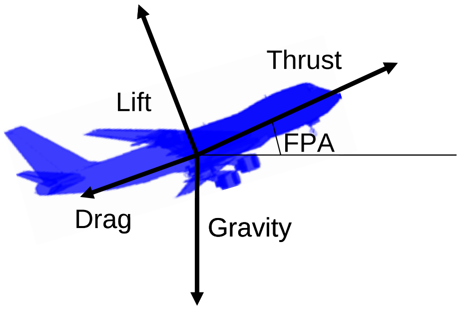


Figure 1.Forces acting on an airplane

$$L = \frac{1}{2}\rho V^2 S C_L \tag{1}$$

$$D = \frac{1}{2}\rho V^2 S C_D \tag{2}$$

where

ρ is the air density (kg/m^3)

S is the wing reference area (m^2)

V is the true airspeed (m/s).

True airspeed is the actual speed of the aircraft relative to the surrounding air.

Note that the air density is a function of altitude as described in subsection 2.2.

Here, $C_L$is the so-called lift coefficient and it relates the total lift generated by an aircraft to the total area of the wing of the aircraft. Under nominal conditions, the drag coefficient, $C_D$is specified as a function of the lift coefficient C_L as follows:

$$C_D = C_{D0} + K{C_L}^2 \tag{3}$$

where

C_{D0} is the parasite drag coefficient

K is the induced drag coefficient

C_L is the lift coefficient

Equation3 is valid for all situations except for the final approach and landing where other drag coefficients are to be used.

Let the angle between the velocity vector and the local horizon, i.e. the flight path angle be Γ. As we are dealing mainly with commercial airliners we can assume that the aircraft always operates near trimmed flight conditions, i.e. the angle of attack is close to zero.

Therefore, from the equilibrium in the direction of the thrust vector:

$$T = \frac{1}{2}\rho V^2 S\left(C_{D0} + K{C_L}^2\right) + Mg\sin\Gamma \tag{4}$$

$$0 = -\frac{1}{2}\rho V^2 S C_L + Mg\cos\Gamma \tag{5}$$

Furthermore, using empty weight, thrust can be written as:

$$T = C_T M_0 g \tag{6}$$

Here C_T depends on the engine and is called specific fuel consumption.

The next equation shows how the weight of the aircraft decreases as time proceeds.

$$\frac{dM}{dt} = -\frac{b}{g}T \tag{7}$$

Actually, it is exactly this relationship that has allowed us to define our objective function, which will be discussed in detail later on.

Atmospheric Model

Most commercial aircraft are flown in the tropopause, which is at approximately 11000 m altitude for standard atmospheric conditions. Calculations for lift, drag, and requited thrust require the determination of several atmospheric properties as a function of altitude. The equation for air density used by BADA for the standard atmosphere is shown below [8].

$$\rho = \rho_0\left[1 - \frac{6.5 * h}{1000 * T_0}\right]^{-\frac{g}{K_T R}-1} \tag{8}$$

where

ρ_0 is the air density at sea level, $\rho_0 = 1.225$ kg/m^3

K_T is the ISA temperature gradient with altitude below the tropopause,
K_T = -0.0065 K/m

R is the real gas constant for air, R = 287.04 m^3/Ks^2

g is the gravitational acceleration, g = 9.81 m/s^2

T_0 = 216.65 K

h is the altitude, specified in meters.

This model of calculating the air density does not take into account changes in humidity and water vapor contents. In reality, the air density changes even faster with altitude.

Variables

In this research the variables used to capture the state of the aircraft are divided into two categories- state variables and control variables. The state variables are the lateral coordinates x and y, the altitude z, the weight of the aircraft M, the speed V and the flight path angle Γ. On the other hand, the control variables are set to be the lift and thrust coefficients, C_L and C_T.

Stage Division

In this paper a three-dimensional trajectory is considered, i.e. the flight path is described by the lateral coordinates x and y, and the altitude z. Furthermore, the flight path is divided into N stages, each of which is characterized by a certain flight time. In the first part of our research we consider stages of equal flight time, whereas in the latter one we have dealt with the time as a variable within a certain range.

Let Δt_i be the flight time of stage i. Equation 7 can be rewritten as:

$$M_{i+1} = M_i - \frac{\Delta t_i \cdot b \cdot T_i}{g} \tag{9}$$

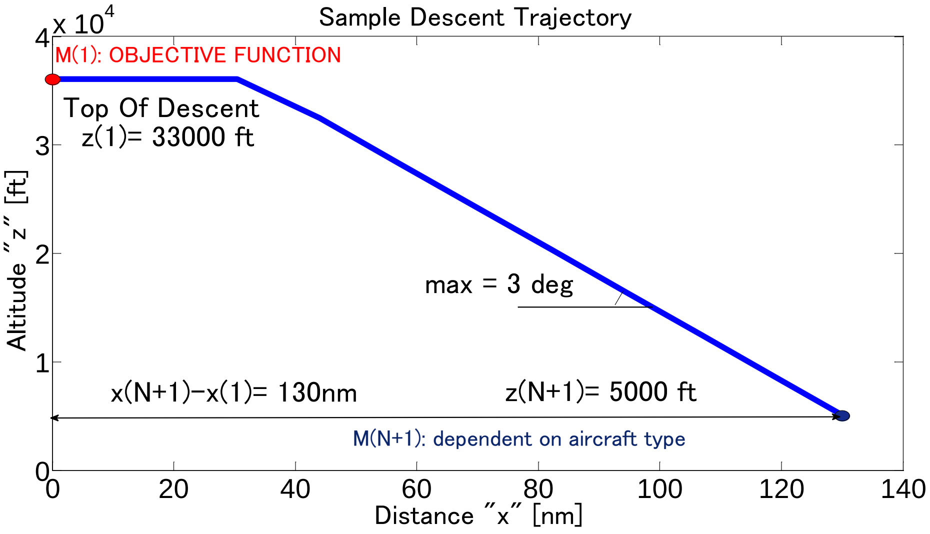
Choice of Aircraft

Based on the maximum take-off weight aircraft are divided into three main categories- light (less than 15 400 lb), medium (less than 300 000 lb but more than 15400 lb) and heavy (more than 300 000 lb)[9]. In our numerical calculations we consider heavy and medium civil aircraft. The medium aircraft is chosen to be the Boeing 737, as this is one of the most-widely used aircraft in its category in the civil aviation. Boeing 737 is a short-to-medium range airplane with a maximum take-off weight of 180 000 lb and a standard seating capacity of 137. From the heavy aircraft group we have chosen the Boeing 747, a long-range airplane, which has come to symbolize its class. Its maximum take-off weight is 875 000 lb and the standard seating capacity is 366.

Dynamic Constraints

To analyze the problem discussed earlier, the aircraft is approximated as a point mass. The position of the aircraft at waypoint i is given by ($x(i), y(i), z(i)$). The mass of the vehicle, its velocity and flight path angle are subject to magnitude constraints. The final mass $M(N+1)$ is fixed and depends on the type of aircraft. On the other hand, the initial mass $M(1)$ is actually the objective function. The constraints on the altitude are imposed based on air traffic considerations. The initial altitude $z(1)$ is set at 33000 ft to equal the cruising altitude, which is also the top of descent (TOD). Determining the TOD is crucial to optimal descent. The final altitude $z(N+1)$ is chosen to be 5000 ft since depending on each airport this is the approximate altitude at which the aircraft’s control is switched to approach control. Furthermore, the upper constraint on the flight path angle is 3°. Also, the lateral distance is 130 nm. These constraints are summarized and shown in Figure 2.

All optimizations were performed in MATLAB using the Sequential Quadratic Programming method.



.

Figure 2. Constraints

Single Aircraft Trajectory Optimization

Sample Trajectory

Figure 3 shows the optimal flight trajectory for a single aircraft B747 under the dynamic constraints described above. The markers show the waypoints, i.e. the end of each stage. The optimal trajectory resulted in fuel consumption:

$$M_{fuel} = M_{initial} - M_{final} = M(1) - M(N+1) \tag{10}$$

$$M_{fuel} = 12065 \quad \text{lb} \tag{11}$$

The total flight time is

$$t_{total} = 1530.2 \quad \text{s} \tag{12}$$

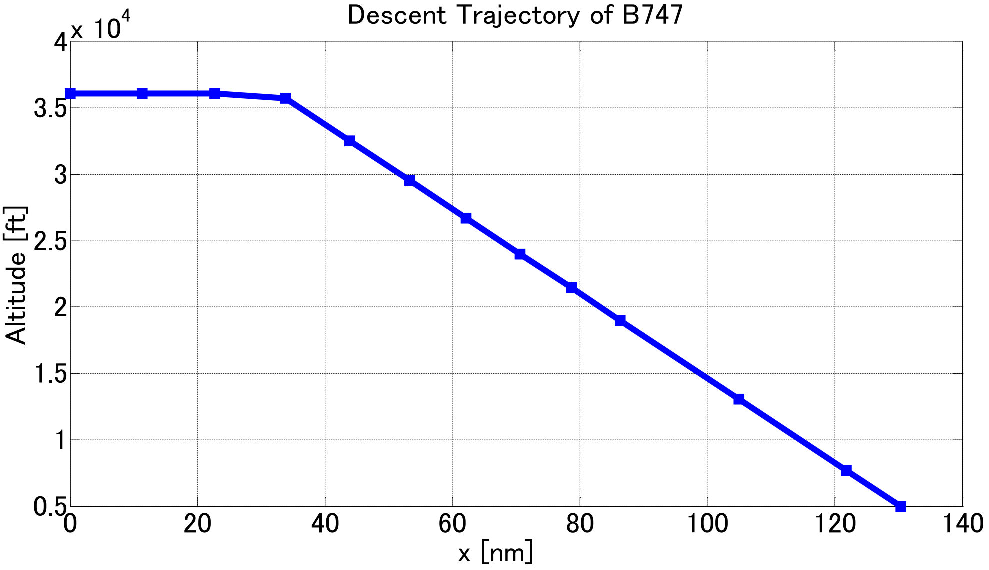


Figure 3. Descent trajectory of B747

Higher for Longer

Several main conclusions can be made from the results obtained. First, the trajectory confirmed the fact that flying higher for longer results in lower fuel consumption. Below the tropopause, where most commercial aircraft are flown, the air density decreases as the altitude increases. As seen from Equation 2, the lower the air density, the lower the drag. Consequently, as shown in Equation 4, the lower drag results in lower trust necessary. Therefore, even though the length of the flight path is the same in both cases shown in Figure 4, the trajectory shown in (a) is more efficient in terms of fuel consumption.

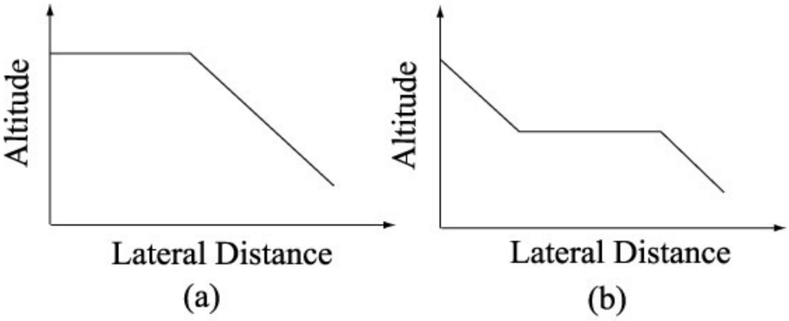


Figure 4. Illustration of “higher for longer” principal

Thrust Coefficient Analysis

Second, as seen in Figure 5, the thrust coefficient C_T never reaches its minimal allowed value zero. Idle thrust is not achieved, even though theoretically speaking such a value would result in near zero fuel consumption for that stretch. In a lot of papers, it is said that the idle thrust is a desirable state for sustain able descent. Looking at the simulation results showing the flight path angle during the descent, the author has concluded that when the 3°constraint on the flight path angle becomes active, C_T is limited to a value higher than zero and idle-power descent is not executed. When the constraints on the flight path angle are weakened, the C_T can reach a lower value and thus the fuel consumption can be decreased.

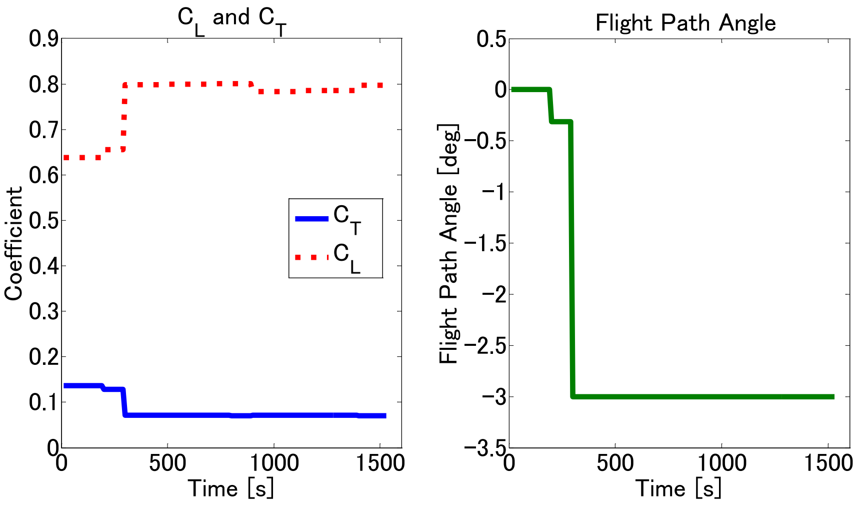


Figure 5. C_L, C_T and flight path angle vs. time

To further examine the effect of maximum allowed flight path angle, a series of numerical simulations was conducted. In these numerical simulations, only the value of the allowed flight path angle was altered while all the other parameters and constraints remained unchanged. The results are summarized in Figure 6.

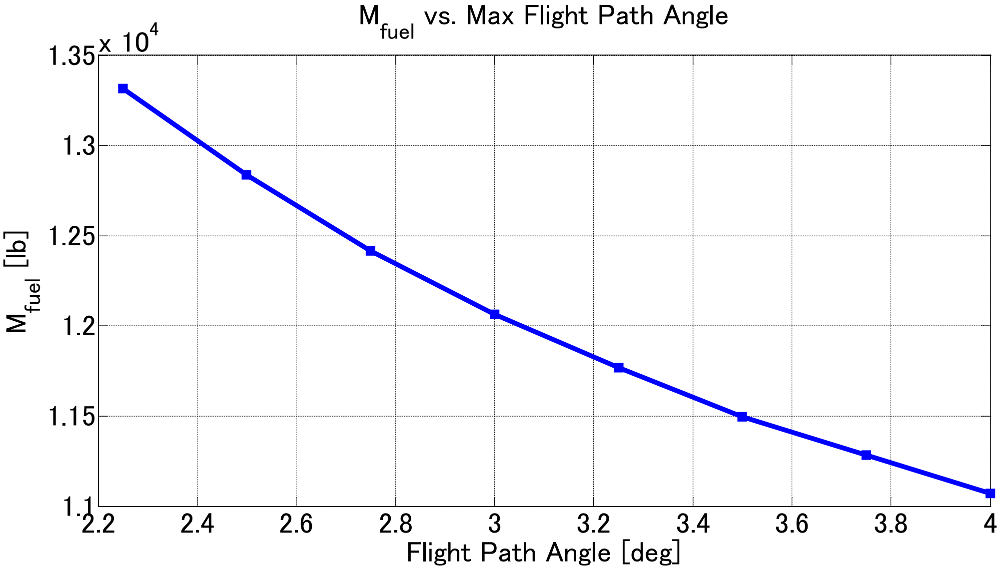


Figure 6. Fuel consumption vs maximum flight path angle

Let us consider some of the figures shown in Table 1. Based on some discussions with pilots, it was concluded that a descent angle of 5 deg is too steep and unreasonable, at least in the present air traffic control situation. It seems that currently, a value of 2.5 to 3 deg is standard. However, if we could let the aircraft descent at a slightly steeper flight path angle of 4 deg, we would be able to safe as much as 8.2% fuel. This would be a very significant reduction which should be taken into account.

Table 1. C_T Analysis

Max Flight Path Angle	**MinC_T**	**M_{fuel}**
3 deg	0.0705	12 065 lb
4 deg	0.0502	11 072 lb
5 deg	0.0311	10467 lb

Fuel Consumption and Flight Time

In reality, an aircraft does not always descent in its optimal time. An aircraft might have to wait before it reaches a certain check point. This could be because the pilot has been told by the air traffic controllers to hold the airplane due to busy traffic. It could also occur due to weather considerations, after-landing management issues (such as passenger services and management of luggage), etc. Unlike on the ground, an airplane cannot just stop and thus cut its fuel consumption. It has to keep flying which makes the problem of waiting much more serious and complicated. On the other hand, an aircraft might be asked to arrive slightly earlier, too. Consider other flight connections, for example. Therefore, the flight time is crucial when determining an optimal descent trajectory and it poses additional constraints. Numerical simulations with fixed flight time were conducted and the results for B747 and B737 are shown in Figure 7 and Figure 8. These are also used when performing the multiple aircraft optimization. The markers indicate each numerical simulation.

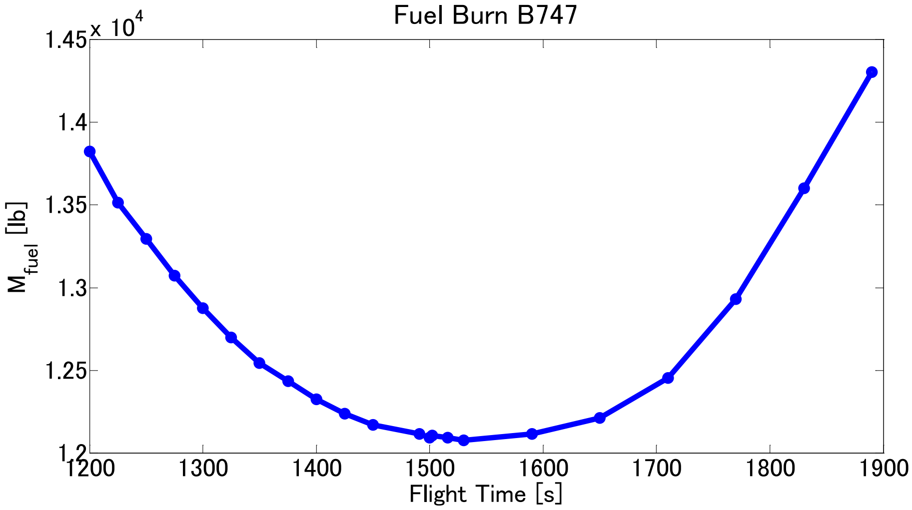


Figure 7. B747 Fuel consumption vs. flight time

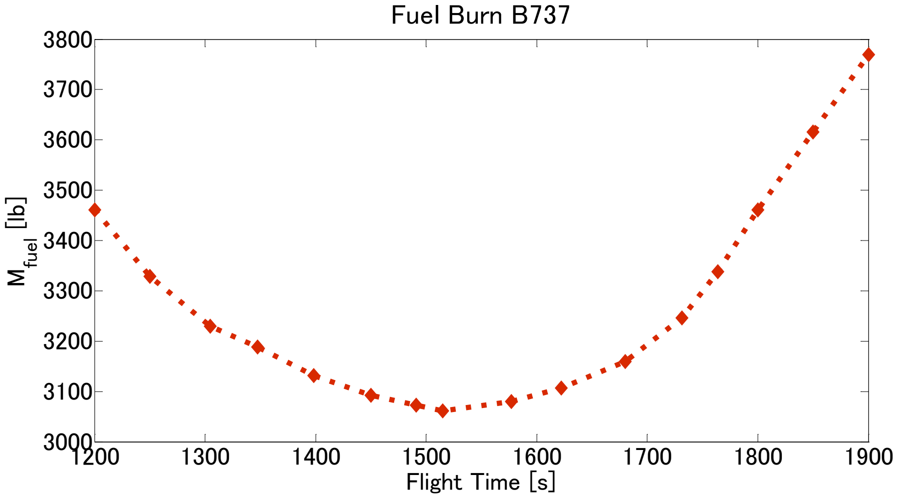


Figure 8. B737 Fuel consumption vs. flight time

Multiple Aircraft Descent Optimization

Time Separation

Because only one aircraft can land or depart from a runway at the same time, and because aircraft must be separated by a certain time interval to avoid collisions, every airport has a finite capacity; it can only safely handle so many aircraft per hour. This capacity depends on many factors, such as the number of runways available, layout of taxi tracks, availability of air traffic control, but also on current or anticipated weather. Especially the weather can cause large variations in capacity because strong winds may limit the number of runways available, and poor visibility may necessitate increases in separation between aircraft. Air traffic control can also be limiting, there are only so many aircraft an air traffic control unit can safely handle. Staff shortages, radar maintenance or equipment faults can lower the capacity of a unit. This can affect both airport air traffic control as well as en-route air traffic control centers.

In order to cope with this issue, we propose an arrival at a certain intermittent point (the final point of our descent trajectory) with a time separation. In such a way the workload of the air traffic controllers can be significantly reduced while in the same time the fuel consumption, hence the costs and environmental impacts, are limited to their minimum.

Having verified the model in the previous section, this example applies it to a multiple aircraft problem, in particular to a double aircraft problem. The objective function in this case is analogous to the one in the case of single aircraft- here we consider the total initial weight of both aircraft. Furthermore, in order to ensure simultaneous optimization of the trajectories, a separation is to be maintained between the aircraft at all times during the descent.

Multiple Aircraft Simulation Constraints

Along with the constraints from the single aircraft simulations, several new constraints to accommodate the condition of multiple aircraft environment have been introduced.

First, the spacing minimum, i.e. separation, is set. It is known that all aircraft generate vortex wake turbulence, which can cause problems for following aircraft. Hence, a cuboid exclusion region in which no other vehicle can enter is introduced. This separation guarantees a safe trajectory planning and lack of collisions between the aircraft. The vertical separation above 29000 ft should be no less than 2000 ft and below 29000 ft it should be at least 1000 ft. In our numerical simulation, due to the initial conditions, a danger of the two aircraft coming too close to each other exists after initializing the descent, so a vertical separation of 1000 ft is mainly considered.

Second, new initial conditions on the lateral coordinate y are set.

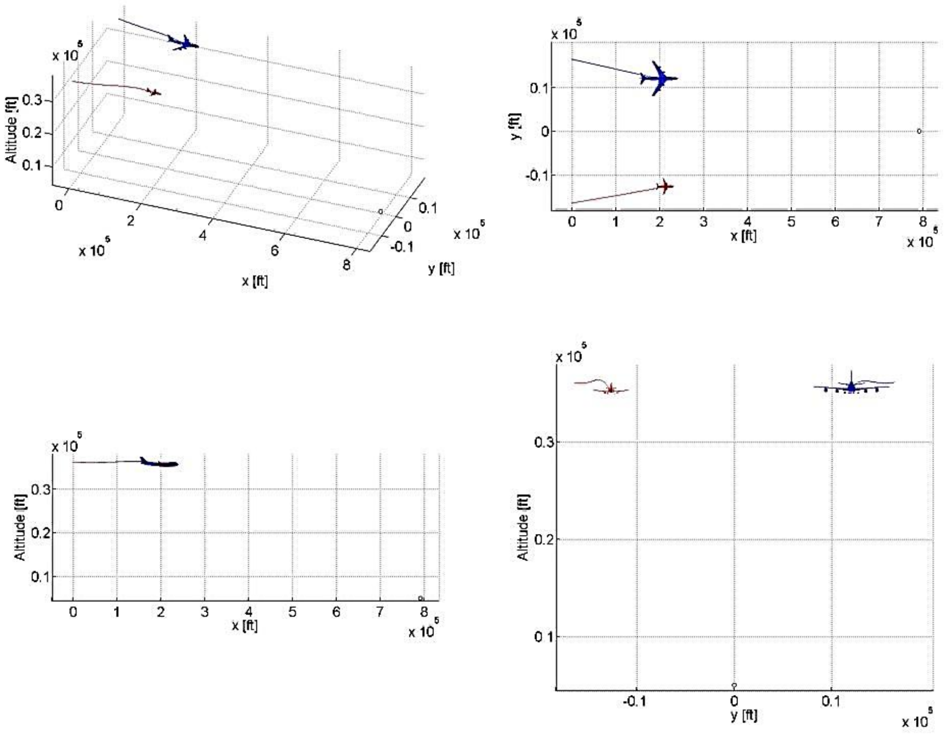
$$\begin{cases} y_{B737}(1) = y_{initial} \\ y_{B747}(1) = -y_{initial} \end{cases} \tag{13}$$

Note that the final coordinate y for both aircraft is zero.

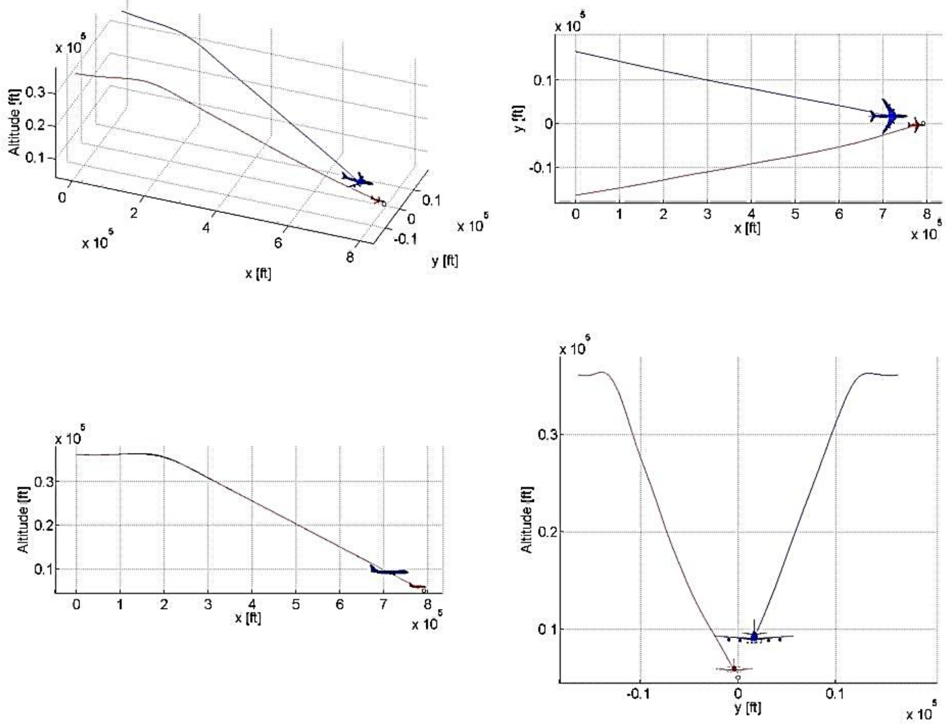
Third, time separation at the final waypoint is introduced. One of the aircraft is to reach the final point a certain time interval prior to the other one. In other words, one of the aircraft flies shorter than the other. In such a case, the aircraft would be in an order for landing before their final approach, which would help air traffic controllers deal with busy traffic.

Optimized Trajectories

A snapshot of the optimized trajectories of B737 and B747 is shown in Figure 9. In this case, B737 arrives prior to B747.



a) Shortly After the Initialization



b) Shortly Before Reaching the Final Point

Figure 9. Trajectory snapshots

Minimum Total Fuel Burn

Air traffic controllers often have to require aircraft to arrive at the final point at a certain fixed time in order to manage the overall traffic and assure safe descent. Therefore, the descent time of an aircraft is not necessarily its ideal flight time. In the next series of numerical simulations, we considered the case of a fixed time of arrival and a predefined time separation. The separation at the final waypoint was decided based on ICAO separation standards, shown in Table 2 [10]. Also, from the results of single aircraft descent optimization the airspeed at the final point is obtained and so the equivalent time separation can be calculated. When the heavy aircraft follows the medium one, the necessary separation is considered to be 60 s, whereas when the leading aircraft is the heavy one, 90 s separation is needed.

Table 2. ICAO Separation Standards

Lead Follower	Heavy	Medium	Light
Heavy W > 136t	4 nm	5 nm	6 nm
Medium 7t < W ≤ 136	3 nm	3 nm	5 nm
Light W ≥ 7t	3nm	3nm	3 nm

To examine the effects of any delays or early arrivals, the fuel increase in respect to the flight time shift is plotted on Figure 10. As expected, the heavy aircraft shows a higher increase in fuel burn, e.g. the fuel increase when B737 is 100 s ahead of its optimal flight time is the same as that when B747 is just 30 s early. Therefore, in the next series of simulation, the flight time of B747 was chosen as a reference time.

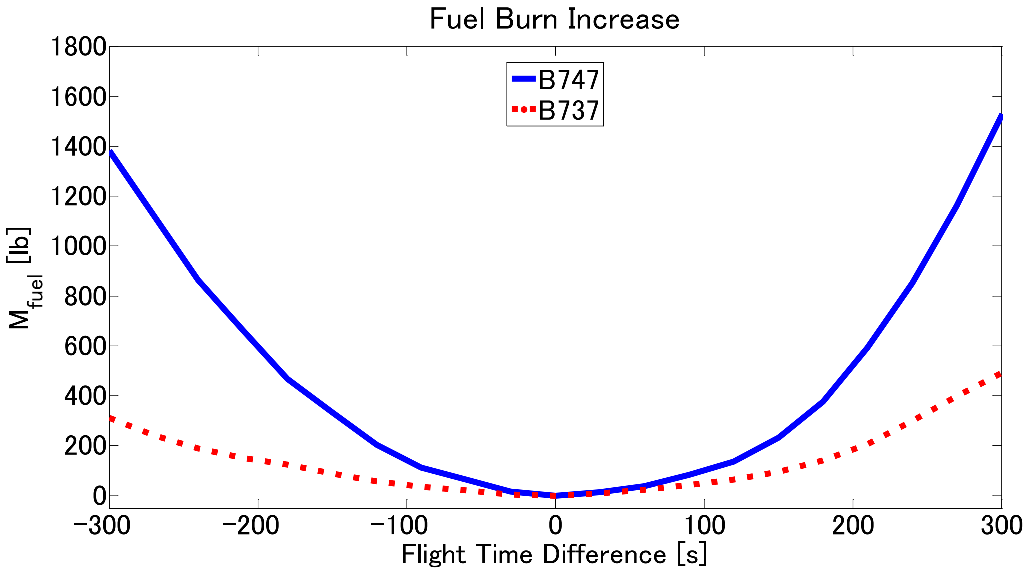


Figure 10. Fuel burn increase vs flight time shift

The results from the optimization of two aircraft are shown in Figure 11. The solid line shown the total fuel burn when the heavy aircraft arrives before the medium one, thus requiring a separation of 90s, while the dotted line shows the combined fuel burn of both aircraft when B737 precedes B747 (separation of 60 s). The main horizontal axis shows the flight time of B747. The two sub horizontal axes show the flight time of B737 in both cases. The fuel increase compared can be seen on the vertical axis.

Based on the arrival time important observations have been made. When the flight time of B747 is less than 1520 s (the shaded zone of the Figure 11), less fuel is consumed when B737 arrives at the last waypoint after B747. However, when the flight time of B747 is more than 1520 s, B737 should be the first to arrive. Were there no constraints on the arrival time window, the sequence should be B737 followed by B747, with the flight time of B747 being 1550 s. This would result in absolute minimum total fuel burn.

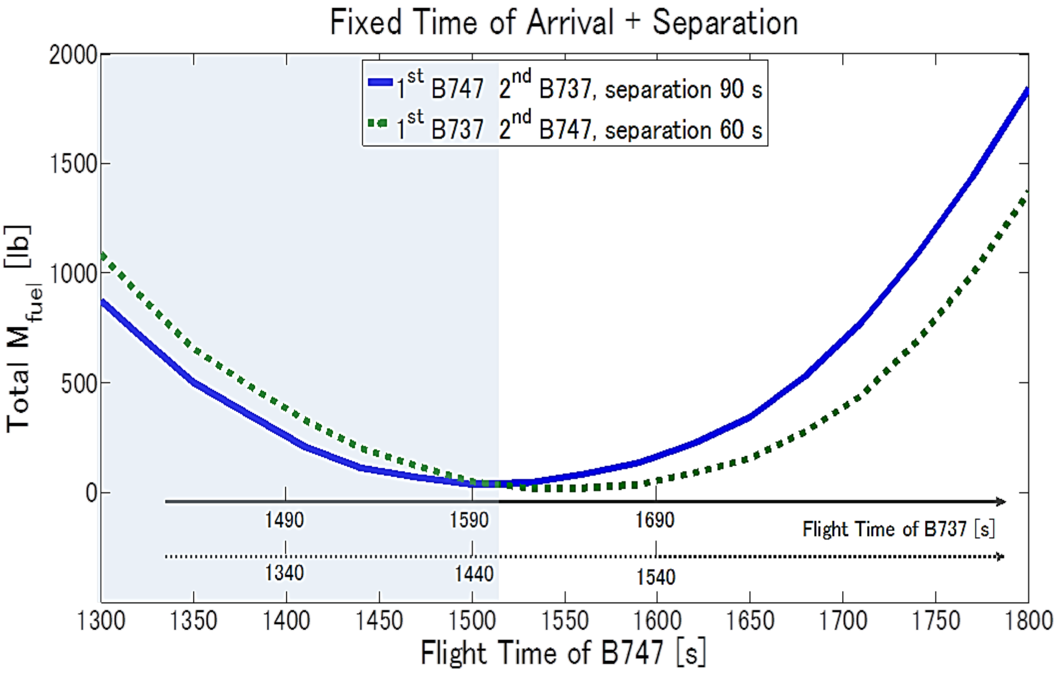


Figure 11. Arrival sequence

Two important conclusions can be made. First, because of the differences in the required separation, absolute minimum fuel burn occurs when the heavy aircraft follow the medium one. Second, the heavy aircraft is not necessarily the one to fly shorter. In fact, an optimal solution is acquired when in the two aircraft case the flight time of the B747 is closer to its own optimal flight time (1530 s) when the B737 is not considered. When the flight time of B747 is shorter than its optimal flight time, more thrust is required so that the aircraft can fit in the time limit. On the other hand, when the flight time of B747 is longer than its optimal flight time, more fuel is consumed as a heavy aircraft consumes more fuel than a light one over unit time. These results agree with Figure 12345, too. Besides, if the flight time of B747 is within 60 s of its optimal flight time, regardless of the sequencing the increase in the total fuel burn can be limited to 1%. Therefore, air traffic controllers should choose the order of arrival accounting for the optimal flight time of the heavier aircraft mainly.

Summary and Conclusions

In this research optimal descent paths with their correspondent fuel burn were investigated. First, single aircraft descents were simulated. The results agreed with the generally well-known principle “higher for longer”. It was shown that an increase in the maximum allowed flight path angle could result in substantial fuel savings. Furthermore, flight time constraints were introduced and the fuel burn in each case was determined. Next, optimizations of two arrival aircraft were performed. Constraints on the required separation and safety reflecting the present air traffic control rules were also implemented. The results showed that despite the commonly accepted practice, the heavy aircraft should not necessarily be the one to arrive first at an airport when congestion occurs. As long as the heavy aircraft is flown as close as possible to its own optimal light time, the total fuel burn will be reduced. However, if for some reason the allowed time window in which the aircraft could arrive is shifted from the optimal flight time of the heavy aircraft, the sequence should be determined as follows: the heavy aircraft should precede the medium one for “early arrivals” and vice versa for “late arrivals.”

Further research on numerous aircraft and increased number of scenarios is in progress. The present results, however, can be used as a basis for guidelines to help air traffic controllers sequence arrival aircraft in eco-friendly manner.

References

[1] International Civil Aviation Organization, *ICAO (2000) Air Traffic Forecasts*, ICAO, Geneva, [Online]. Available: http://www.icao.int [Accessed: Mar, 2009]

[2] P. Upham, J. Maughan, D. Raper, and C. Thomas, *Towards Sustainable Aviation*, Earthscan Publications Ltd, 2003

[3] National Science & Technology Council, *National Research and Development Plan For Aviation Safety, Security, Efficiency and Environmental Compatibility*, Cambridge, MA, 1999

[4] Committee on Climate Change, Retrieved from http://www.theccc.org.uk/news/press-releases

[5] S. Nagaoka, “ENRI’s R&D long-term vision,” In: *ENRI International Workshop on ATM/CNS*, Tokyo, pp.13-17, 2009

[6] Environmental Research and Consultancy Department Civil Aviation Authority, “*Basic Principles of the Continuous Descent Approach (CDA) for the Non-Aviation Community*,” [Online]. Available: http://www.caa.co.uk/docs/68/Basic_Principles_CDA.pdf [Accessed: Feb, 2010]

[7] J.B. Clarke, N.T. Ho, L. Ren, J.A. Brown, K.R. Elmer, K. Zou, C. Hunting, D.L. McGregor, B.N. Shivashankara, K.O. Tong, A.W. Warren, and J.K. Wat, “Continuous descent approach: Design and flight test for Louisville International Airport,” *Journal of Aircraft*, Vol.41, No.5, pp. 1054-1066, 2004.

[8] Eurocontrol, User Manual for the Base of Aircraft Data (BADA)-Revision 3.5, EEC Note No. 11/03, ACE-C-E2, Eurocontrol Experimental Center, France, 2003.

[9] *Air Traffic Controller Manual*, (n.d.) [Online]. Available: http://www.vateud-td.org/handbooks/manual.asp [Accessed: Nov, 2008]

[10] International Civil Aviation Organization, *Procedures for Navigation Services: Air Traffic Management*, 14th Edition, Doc 4444 ATM/501, 2001.