

# SIMULATION, PARAMETER IDENTIFICATION AND CONTROL SYSTEM DESIGN OF AN AIRCRAFT USING UNIFIED MATHEMATICAL MODEL

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## Abstract

This paper discusses the role of mathematical model of an aircraft dynamics in flight mechanics related studies. A flight dynamics mathematical model represents the motion of a flight vehicle in six-degree-of-freedom. All forces and moments governing the flight motion are specifically represented in the equations, such as inertial force and moment, aerodynamics force and moment, propulsive force and moment, flight control force and moment, and external disturbances. In addition, the mathematical model also includes a set of kinematic and navigation equations, to provide a thorough representation of vehicle motions and movements in the air. Another way to obtain the mathematical representation of an aircraft dynamic is by employing system identification procedure, on which the measured input-output data of the system is analyzed and processed to obtain the parameters of the system. The mathematical model also can be used for simulating and analyzing the dynamic behavior of an aircraft. The simulation is usually conducted by implementing the model into numerical simulation software, such as MATLAB/Simulink. Another implementation of the unified mathematical model presented in this paper is for designing flight control system. In this paper, an aircraft such as Unmanned Aerial Vehicle (UAV) system will be used as a basis for demonstrating how mathematical model an aircraft can be obtained, and how it is further exploited for some analysis and studies in flight mechanics fields.

**Keywords:** Control system design, Flight simulation, Mathematical model, Parameter identification, UAV

## Introduction

Flight mechanics is one of aerospace fields which deals with the dynamic motions of aerial vehicles in the air, which defines their flight characteristics and performance. In the context of flight dynamics, the aircraft motion is defined as attitude and rate of change of the attitude measured from particular reference frame [1], [2], [3]. The dynamics motion of the aircraft can be represented quantitatively in the form of mathematical formulation. The mathematical model of an aircraft dynamic usually is governed from its equation of motion by considering its  $n$ -degree-of-freedom ( $n \leq 6$ ). The dynamic model, as mentioned above, can be obtained analytically by deriving the equation of motions of the aircraft. Alternatively, the mathematical representation can also be governed using parameters/system identification techniques. In this approach, the parameters of system are identified by examining and processing a set of data representing the relation between the input (control) signals fed into the aircraft and the corresponding output signals generated from aircraft response under particular conditions [4]. Some algorithms have also been developed on which the identification technique is exploited for correcting/improving a priori models obtained from analytical approach [5].

The mathematical model of aircraft dynamics plays key role in the engineering development of aircraft configurations and systems. The mathematical model of aircrafts describing the motions of aircraft in the air can be used for simulating, predicting, and analyzing the aircraft dynamics motions during its time in the air. Those functions are instrumental for evaluating the dynamic characteristics and performance of the aircraft and designing the required system to be implemented on the aircraft, such as the aircraft navigation, guidance, and control systems. For example, in designing the control system, information about the dynamic characteristics of the vehicle is usually required, so that a desired closed loop system can be defined and constructed [6], [7]. As has been previously mentioned, such information can be provided by the mathematical model of the vehicle.

This paper intends to show and elaborate the role of mathematical in relating different type of analysis usually conducted in the field of flight mechanics, where it can be seen as a common factor. In this paper it will be demonstrated that the mathematical representation of flight vehicle dynamics is obtained by analytically examining and relating the physical phenomena to the physics law. It will also be shown the process of augmenting the model parameters by processing and examining the input-output data of the flight vehicle system which are required for governing the vehicle dynamics representation. Further, the mathematical representation of the flight vehicle dynamics will be used for designing a flight control system such that an improved behavior of the vehicle can be obtained.

This paper is organized as follows: Section 2 will discuss the brief explanation of the derivation of flight vehicle mathematical model. Reference frames and forces and moments governing the mathematical model will be defined briefly to address the key elements which influence the dynamics of the aircraft. In Section 3, the implementation of the mathematical model for analyzing the dynamics behavior of the aircraft through dynamic simulations is elaborated. Following that, a procedure for obtaining the aircraft flight dynamic model through identification approach is demonstrated. Further, it will be presented in Section 5 how the mathematical model of the aircraft is exploited in the design of a flight control system. Finally, in the last section some comments and remarks will be presented to conclude this paper.

## Aircraft Flight Dynamics Model

A dynamical model of an aircraft (flight vehicle) usually consists of a set of mathematical formulations relating the dynamic response of the aircraft to all forces and moments working on the body of the aircraft. The dynamic response of an aircraft will be formed by the interaction between inertial force/moments and the aerodynamic, gravitation, propulsion, control, and disturbance forces/moments. It should be noted that forces and moments involved in the dynamics of an aircraft work in different frame of reference. Hence, the definition of reference frames for describing all the variables of motions and forces/moments are required. For conventional aircrafts 3 main reference frames are usually used, namely the body reference frame, the wind reference frame, the local horizon reference frame, the definitions of which can be found in [1] and [8].

### Aircraft Equation of Motion

The equation of motion of an aircraft can be derived by treating it as a solid body moving in the inertial frames. A formulation representing the force and moment equilibrium can be formed using Newton's Law, as described by the following equation which is referred to body-fixed reference frame [3].

$$\begin{aligned}\bar{F} &= m \left( \frac{\partial \bar{V}}{\partial t} + \bar{\omega} \times \bar{V} \right) \\ \bar{M} &= \frac{d}{dt} \int \bar{r} \times (\bar{\omega} \times \bar{r}) dm\end{aligned}\quad (1)$$

Expanding the right hand side term by taking into account the translational and rotational velocities vectors gives the translational equations:

$$\begin{aligned}\dot{u} &= rv - qw - g \sin\theta + \frac{F_x}{m} \\ \dot{v} &= -ru + pw + g \sin\phi \cos\theta + \frac{F_y}{m} \\ \dot{w} &= qu - pv + g \cos\phi \cos\theta + \frac{F_z}{m}\end{aligned}\quad (2)$$

and rotational equations:

$$\begin{aligned}\dot{p} &= (c_1 r + c_2 p)q + c_3 \dot{L} + c_4 N \\ \dot{q} &= c_5 pr - c_6 (p^2 - r^2) + c_7 M \\ \dot{r} &= (c_8 p - c_7 r)q + c_4 \dot{L} + c_9 N\end{aligned}\quad (3)$$

where;

$$\begin{aligned}c_1 &= \frac{(I_y - I_z)I_z - J_{xz}^2}{I_x I_z - J_{xz}^2}; & c_2 &= \frac{(I_x - I_y + I_z)J_{xz}}{I_x I_z - J_{xz}^2}; & c_3 &= \frac{I_z}{I_x I_z - J_{xz}^2}; \\ c_4 &= \frac{J_{xz}}{I_x I_z - J_{xz}^2}; & c_5 &= \frac{(I_z - I_x)}{I_y}; & c_6 &= \frac{J_{xz}}{I_y}; \\ c_7 &= \frac{1}{I_y}; & c_8 &= \frac{(I_x - I_y)I_x - J_{xz}^2}{I_x I_z - J_{xz}^2}; & c_9 &= \frac{1}{I_z};\end{aligned}$$

In Equation (2),  $m$  is the aircraft mass and the vectors  $(u,v,w)$  and  $(p,q,r)$  denote the translational and rotational velocity in and around body axes respectively. The total forces are projected on body axes and denoted as  $(F_x, F_y, F_z)$ . In Equation (3),  $I_{(\cdot)}$  and  $J_{(\cdot)}$  are the aircraft moments of inertias, and the vector  $(L,M,N)$  is the total moments acting around the aircraft body axes.

The total forces and moments in Equations (1) and (2) are generated from aerodynamic force and moment, gravity force, and propulsive forces and moment which can be described as follow:

$$\begin{aligned}\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix}^T &= \left\{ (A_x + W_x + T_x) \quad (A_y + W_y + T_y) \quad (A_z + W_z + T_z) \right\}^T \\ \begin{Bmatrix} \hat{L} \\ M \\ N \end{Bmatrix}^T &= \left\{ (L_A + L_T) \quad (M_A + M_T) \quad (N_A + N_T) \right\}^T\end{aligned}\quad (4)$$

The aerodynamics force and moment arise due to the interaction between aircraft body and the airflow which causes pressure difference over the aircraft body surface. The resultant aerodynamic force typically does not act in the center of gravity and this condition triggers the development of the aerodynamic moment. In aircraft body reference frame, the aerodynamics forces and moments are represented by the following equations:

$$\begin{aligned}\begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}^T &= f(D, L, Y, \alpha, \beta) \\ \begin{Bmatrix} L_A \\ M_A \\ N_A \end{Bmatrix}^T &= g(\hat{L}, M, N, \alpha, \beta)\end{aligned}\quad (5)$$

where  $\alpha$  and  $\beta$  are the angle of attack and side-slip, respectively, representing relative orientation between aircraft body reference frame and the wind reference frame. It can be seen that the forces and moments in Equation (5) basically are the projection of aerodynamic force  $(L, Y, D)$  and

moment ( $\hat{L}, M, N$ ) along the body axes. These forces/moments in wind reference frame are described by the following relations:

$$\begin{aligned} \{D \ Y \ L\}^T &= f(V, S, \rho, u, v, w, p, q, r, \delta_e, \delta_a, \delta_r) \\ \{\hat{L} \ M \ N\}^T &= f(V, S, \bar{b}, \rho, u, v, w, p, q, r, \delta_e, \delta_a, \delta_r) \end{aligned} \quad (6)$$

where the  $\rho$  is the air density,  $V$  is the airflow speed,  $\bar{b}$  is reference length (span), and  $S$  is the aerodynamic surface area. Some aerodynamic coefficients ( $C_D, C_Y, C_L, C_l, C_m, C_n$ ) are involved in the Equations (6), the definitions of which can be found in [1] and [8]. It should be noted that the Equations (6) also include the effects of control input, i.e. elevator, aileron, rudder deflections ( $\delta_e, \delta_a, \delta_r$ ).

The propulsion force and moment acting on the aircraft depend on the power plant, the relative distance of thrust line to the aircraft center of gravity, and the thrust line angle relative to body axes. The equation of the propulsion force and moment in the body reference frame, assuming all engines work normally in a symmetric flight condition, is shown in the following equation:

$$\begin{aligned} \{T_x \ T_y \ T_z\}^T &= \{T \cos \tau_V \cos \tau_H \quad T \cos \tau_V \sin \tau_H \quad T \sin \tau_V\}^T \\ \{L_T \ M_T \ N_T\}^T &= \{0 \quad dx T_z - dz T_x \quad 0\}^T \end{aligned} \quad (7)$$

where ( $\tau_v, \tau_H$ ) are the engine setting angle relative to aircraft body axes, and ( $dx, dz$ ) are the distance between the engine thrust line to the aircraft center of gravity.

Lastly, the gravity is modeled to work at the center of gravity, hence it will not generate any moment, but its projections on the body axes depend on the relative angle between body frame and local horizon frames, as described by the following:

$$\{W_x \ W_y \ W_z\}^T = \{-W \sin \theta \quad W \sin \phi \cos \theta \quad W \cos \phi \cos \theta\}^T \quad (8)$$

where  $\theta$  and  $\phi$  are the pitch and roll angles, respectively.

In addition to the translational and rotational equations described in (1) and (2) above, a set of kinematic relations is also required to determine the dynamic motion of aircrafts, as expressed as follow.

$$\begin{aligned} \dot{\phi} &= p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \\ \dot{\theta} &= q \cos \phi + r \sin \phi \\ \dot{\psi} &= q \frac{\sin \phi}{\cos \theta} + r \frac{\cos \phi}{\cos \theta} \end{aligned} \quad (9)$$

Furthermore, for determining the movement of an aircraft during its flight, the following navigation equations can be employed.

$$\begin{aligned} \dot{x}_h &= u \cos \theta \cos \psi + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ \dot{y}_h &= u \cos \theta \sin \psi + v(\sin \phi \sin \theta \sin \psi - \cos \phi \cos \psi) + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ \dot{z}_h &= -u \sin \theta + v \sin \phi \cos \theta + w \cos \phi \cos \theta \end{aligned} \quad (10)$$

The navigation equation (10) above represents the velocity vectors of an aircraft ( $\dot{x}_h, \dot{y}_h, \dot{z}_h$ ) referred to a particular navigation frame, for example the NED (North, East, Down) frame.

As has been discussed previously the equations (1), (2), (9) and (10) represent the aircraft equations of motion in six degree of freedom (DOF), and includes some couplings between symmetric and asymmetric motions. For simplicity reason those equations can be decomposed into longitudinal (symmetric) and lateral/directional (asymmetric) modes as follows [1].

Longitudinal equations of motion:

$$\begin{aligned}\dot{u} &= -wq - g \sin \theta + \frac{F_x}{m} \\ \dot{w} &= uq + g \cos \theta + \frac{F_z}{m} \\ \dot{\theta} &= q \\ \dot{q} &= \frac{M}{I_y} \\ \Delta \dot{h} &= u \sin \theta - w \cos \theta\end{aligned}\tag{11}$$

Lateral/directional equations of motion:

$$\begin{aligned}\dot{v} &= wp - ur + \frac{F_y}{m} \\ \dot{\phi} &= p \\ \dot{\psi} &= r \cos \phi \\ \dot{p} &= \frac{L}{I_x} \\ \dot{r} &= \frac{N}{I_z}\end{aligned}\tag{12}$$

In principle, the so-called state equations (11) and (12) can be integrated numerically to obtain the state variables of the aircraft. The so-called output or observation variables can be derived, for example, such as true airspeed  $V$ , altitude  $h$ , angle of attack  $\alpha$ , angle of side slip  $\beta$ , etc. as follows:

$$\begin{aligned}\hat{V} &= \sqrt{u^2 + w^2} \\ \hat{h} &= h_0 + \Delta h \\ \hat{\alpha} &= \tan^{-1}\left(\frac{w}{u}\right) \\ \hat{\beta} &= \sin^{-1}\left(\frac{v}{V}\right)\end{aligned}\tag{13}$$

The measurement of the true airspeed, altitude, angle of attack, and side slip angle are usually corrupted with additive stochastic measurement error. This measurement error should be included in the observation model as well.

### **Linearized Equation**

It can be seen clearly that the state equations (11) and (12) above are representing non-linear dynamics in 6 coupled degrees of freedom. To simplify the solution, a small perturbation approach can be employed for linearizing the equations at particular trimmed conditions. To further simplify the equations, it can be assumed that the motions of the aircraft can be separated into two main

modes, namely the longitudinal mode representing the symmetric motions, and lateral-directional mode representing the asymmetric ones. Hence, the equations can be grouped into two decoupled sets of equations. The linearization and decoupling procedure of the equations are explained in detail in some literatures, such as in [1] and [8], and will be summarized in brief as follows. For the case of longitudinal equations of motion, Equation (11) can be expressed as:

$$\begin{aligned}
 m\dot{u} &= -W \cos \theta_0 + X_u u + X_w w + X_z z + X_{\delta e} \delta e \\
 m(\dot{w} - qV) &= -W \sin \theta_0 + Z_u u + Z_w w + Z_z z + Z_{\delta e} \delta e \\
 I_y \dot{q} &= M_u u + M_w w + M_{\dot{w}} \dot{w} + M_q q + M_z z + M_{\delta e} \delta e \\
 \dot{\theta} &= q \\
 \Delta \dot{z} &= -\Delta \dot{h} = V_o(\alpha - \theta)
 \end{aligned} \tag{14}$$

If the total force and moment in Equation (14) are made in non-dimensional form, then (14) becomes,

$$\begin{aligned}
 (C_{X_u} - 2\mu_c D_c) \hat{u} + C_{X_\alpha} \alpha + C_{Z_0} \theta + C_{X_z} \Delta z + C_{X_{\delta e}} \delta e_c &= 0 \\
 C_{Z_u} \hat{u} + (C_{Z_\alpha} - 2\mu_c D_c) \alpha - C_{X_0} \theta + 2\mu_c \hat{q} + C_{Z_z} \Delta z + C_{Z_{\delta e}} \delta e &= 0 \\
 C_{m_u} \hat{u} + (C_{m_\alpha} + C_{m_\alpha} D_c) \alpha + (C_{m_q} - 2\mu_c K_Y^2 D_c) \hat{q} + C_{m_z} \Delta z + C_{m_{\delta e}} \delta e &= 0 \\
 -D_c \theta + \hat{q} &= 0 \\
 \bar{c}(\theta - \alpha) + D_c \Delta z &= 0
 \end{aligned} \tag{15}$$

The linearized and decoupled equations then can be represented as a set of first order ordinary differential equations known as state space model as follow:

$$\begin{bmatrix} \dot{\hat{u}} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{\hat{q}} \end{bmatrix} = \begin{bmatrix} x_u & x_\alpha & x_\theta & 0 \\ z_u & z_\alpha & z_\theta & z_q \\ 0 & 0 & 0 & \frac{V}{\bar{c}} \\ m_u & m_\alpha & m_\theta & m_q \end{bmatrix} \begin{bmatrix} \hat{u} \\ \alpha \\ \theta \\ \hat{q} \end{bmatrix} + \begin{bmatrix} x_{\delta e} \\ z_{\delta e} \\ 0 \\ m_{\delta e} \end{bmatrix} \delta e \tag{16}$$

where the state variables are the x-axis velocity component  $u$ , angle of attack  $\alpha$ , pitch angle  $\theta$ , and the pitch rate  $q$ . While the control variable is elevator deflection  $\delta_e$ , assuming constant throttle setting. The definition of each of element matrix in Equation (16) is given as:

$$\begin{aligned}
 x_u &= \frac{V_0}{\bar{c}} \frac{C_{X_u}}{2\mu_c} & z_\theta &= -\frac{V_0}{\bar{c}} \frac{C_{X_0}}{2\mu_c} & m_q &= \frac{V_0}{\bar{c}} \left[ \frac{C_{m_\alpha} + C_{m_q}}{2\mu_c K_Y^2} \right] \\
 x_\alpha &= \frac{V_0}{\bar{c}} \frac{C_{X_\alpha}}{2\mu_c} & z_z &= \frac{V_0}{\bar{c}} \frac{C_{Z_z}}{2\mu_c} & m_z &= \frac{V_0}{\bar{c}} \left[ \frac{C_{m_z} + C_{Z_z} \frac{C_{m_\alpha}}{2\mu_c}}{2\mu_c K_Y^2} \right] \\
 x_\theta &= \frac{V_0}{\bar{c}} \frac{C_{Z_0}}{2\mu_c} & z_{\delta e} &= \frac{V_0}{\bar{c}} \frac{C_{Z_{\delta e}}}{2\mu_c} & m_{\delta e} &= \frac{V_0}{\bar{c}} \left[ \frac{C_{m_{\delta e}} + C_{Z_{\delta e}} \frac{C_{m_\alpha}}{2\mu_c}}{2\mu_c K_Y^2} \right] \\
 x_{\delta e} &= \frac{V_0}{\bar{c}} \frac{C_{X_{\delta e}}}{2\mu_c} & m_u &= \frac{V_0}{\bar{c}} \left[ \frac{C_{m_u} + C_{Z_u} \frac{C_{m_\alpha}}{2\mu_c}}{2\mu_c K_Y^2} \right] & \mu_c &= m / \rho S \bar{c}
 \end{aligned} \tag{17}$$

$$z_u = \frac{V_0}{\bar{c}} \frac{C_{z_u}}{2\mu_c} \quad m_\alpha = \frac{V_0}{\bar{c}} \left[ \frac{C_{m_\alpha} + C_{z_\alpha} \frac{C_{m_\alpha}}{2\mu_c}}{2\mu_c K_Y^2} \right] \quad \mu_c K_Y^2 = I_y / \rho S \bar{c}^3$$

$$z_\alpha = \frac{V_0}{\bar{c}} \frac{C_{z_\alpha}}{2\mu_c} \quad m_\theta = -\frac{V_0}{\bar{c}} \left[ \frac{C_{X_0} \frac{C_{m_\alpha}}{2\mu_c}}{2\mu_c K_Y^2} \right]$$

The output of the observed variables here is derived for the case where true airspeed  $V$ , angle of attack  $\alpha$ , and altitude variation  $\Delta h$  are observed. Noise or measurement error may be added in the observation model.

$$\begin{Bmatrix} V \\ \alpha \\ \Delta h \end{Bmatrix} = \begin{bmatrix} V & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -V & V & 1 \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{\alpha} \\ \hat{\theta} \\ \hat{q} \end{Bmatrix} + \begin{Bmatrix} n_V \\ n_\alpha \\ n_h \end{Bmatrix} \quad (18)$$

For representing the lateral-directional (asymmetric) motions the following state spacemodel can also be derived in the same principle for the case of longitudinal mode.

$$\begin{Bmatrix} \dot{\hat{\beta}} \\ \dot{\hat{\phi}} \\ \dot{\hat{p}} \\ \dot{\hat{r}} \end{Bmatrix} = \begin{bmatrix} y_\beta & y_\phi & y_p & y_r \\ 0 & 0 & 2\frac{V}{b} & 0 \\ l_\beta & 0 & l_p & l_r \\ n_\beta & 0 & n_p & n_r \end{bmatrix} \begin{Bmatrix} \hat{\beta} \\ \hat{\phi} \\ \hat{p} \\ \hat{r} \end{Bmatrix} + \begin{bmatrix} 0 & y_{\delta_r} \\ 0 & 0 \\ l_{\delta_a} & l_{\delta_r} \\ n_{\delta_a} & n_{\delta_r} \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix} \quad (19)$$

The state variables in Equation (19) are sideslip angle  $\beta$ , roll angle  $\phi$ , roll rate  $p$ , and yaw rate  $r$ . While the control variables are the aileron deflection  $\delta_a$ , and rudder deflection  $\delta_r$ . The definitions of coefficients in both state space matrices above can be found in some flight dynamic literatures, such as [1] and [8].

As has been explained earlier, the linearized form of the aircraft equation of motions is obtained by assuming small perturbation around particular trimmed point condition. Hence, the obtained state space can be used for simulating and analyzing the dynamics characteristics and behavior of an aircraft only within a limited region around a predefined flight condition.

## Flight Simulation

Using the non-linear as well as linearized equations already explained in the previous sections, flight dynamics model of a hypothetical aircraft such as UAV will be formed, and then simulated and analyzed. The hypothetical UAV is a fixed wing small UAV the specification of which is listed in Table 1, while the aerodynamics coefficients related to longitudinal mode when flying at altitude of 1000 m, speed of 80 knots is presented in Table 2. The scheme for flight simulation is depicted in Figure 1.

Table 1. Configuration Data		
Parameter	Value	Unit
MTOW :	115.00	[kg]
c.g. position :	14.00	[% mac]
Wing area :	4.0	[m <sup>2</sup> ]
mac :	0.63	[m]
span :	6.34	[m]
Engine power :	20.00	[hp]
Diameter		
propeller:	34.00	[inch]

Table 2. Aerodynamic Coefficients		
Parameter	Value	Unit
$C_L$ :	0,3002	[-]
$C_D$ :	0,0264	[-]
$C_{L_\alpha}$ :	5,6719	[per rad]
$C_{m_\alpha}$ :	-2,6412	[per rad]
$C_{m_q}$ :	-26,0776	[per rad/s]
$C_{L_{\dot{\alpha}}}$ :	5,6719	[per rad/s]
$C_{m_{\dot{\alpha}}}$ :	-2,6412	[per rad/s]

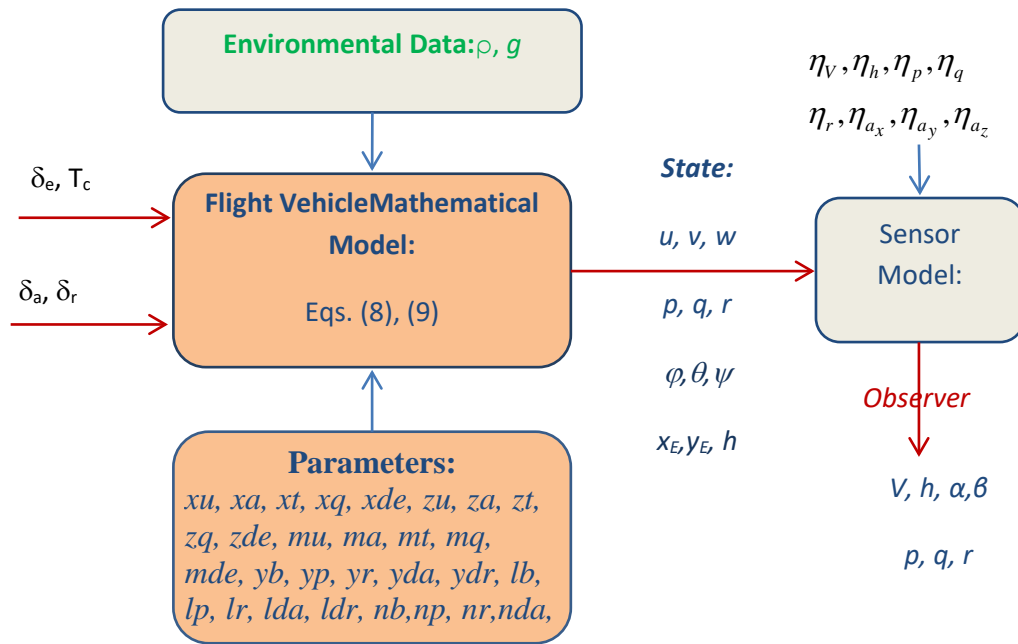


Figure 1. The schematic diagram of flight simulation of the aircraft.

The aerodynamic coefficients listed on Table 2 are computed using DATCOM software [9] or by simple handbook method, based on the configuration and dimension of the UAV, and also the chosen flight condition.

For the case when **non-linear mathematical model** expressed in Equation (11) is used, the flight simulation results are given in Figures 2 and 3. In Figure 2, the state variables of the UAV due to elevator input are presented, while in Figure 3, the output variables, i.e. airspeed, altitude and angle of attack are given.

In the case when linear mathematical model as given in Equation (16) is used, the state space model of the longitudinal dynamics of the UAV is expressed as,

$$\begin{Bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{\theta} \\ \dot{q} \\ \dot{h} \end{Bmatrix} = \begin{bmatrix} -0.0093 & 0.138 & -0.239 & 0 & 0 \\ -0.472 & -4.479 & 0.0046 & 59.913 & 0 \\ 0 & 0 & 0 & 65.27 & 0 \\ 0.0149 & -1.389 & -0.0001 & -16.998 & 0 \\ 0 & -41.12 & 41.12 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ \alpha \\ \theta \\ q \\ h \end{Bmatrix} + \begin{bmatrix} 0 & 0.795 \\ -0.291 & 0 \\ 0 & 0 \\ -0.849 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \delta_e \\ \delta_T \end{Bmatrix} \quad (20)$$

and the output relation:

$$\bar{y} = \begin{bmatrix} 41.12 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 65.27 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u \\ \alpha \\ \theta \\ q \\ h \end{Bmatrix} \quad (21)$$

where the output vector consists of x-axis velocity change  $u$  in (m/s), angle of attack in (rad), pitch angle in (rad), pitch rate in (rad/s), and altitude change  $h$  in (m). Note that altitude variable  $h$  is added into the state variable vectors, hence the size of the model is increased. Also, in the model (10), throttle input  $\delta_T$  is included, as represented by the second column of the B matrix. Based on



this linear longitudinal model, some open loop simulation and analysis are conducted. The dynamic characteristics of the UAV can be represented by its Eigenvalues, which are listed in the Table 3.

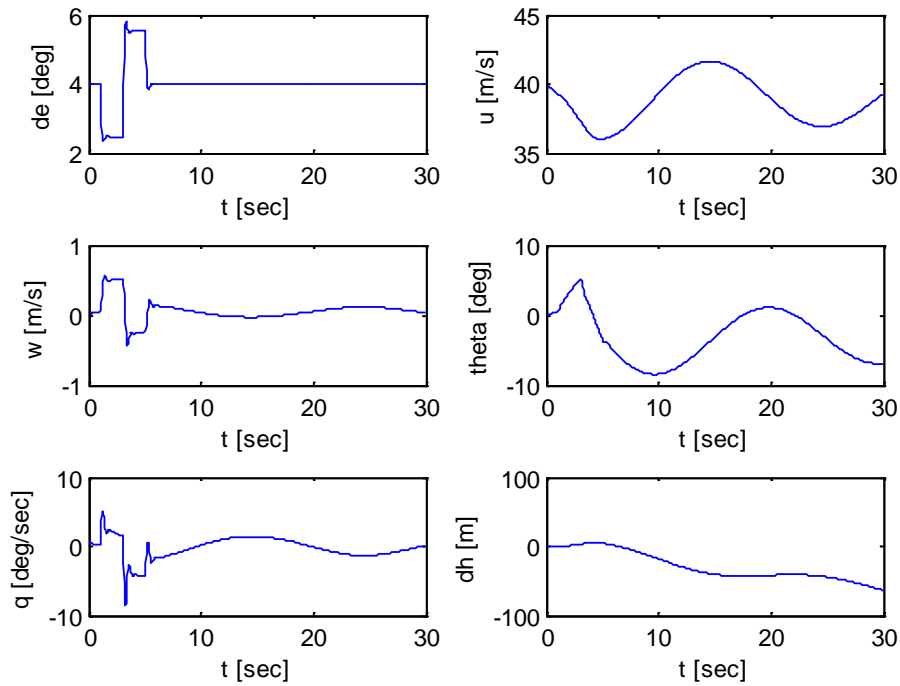


Figure 2. The state variables due to elevator input  $\delta_e$ : component of airspeed  $u$ , component of airspeed  $w$ , angle of pitch  $\theta$ , pitch rate  $q$  and altitude variation  $\Delta h$

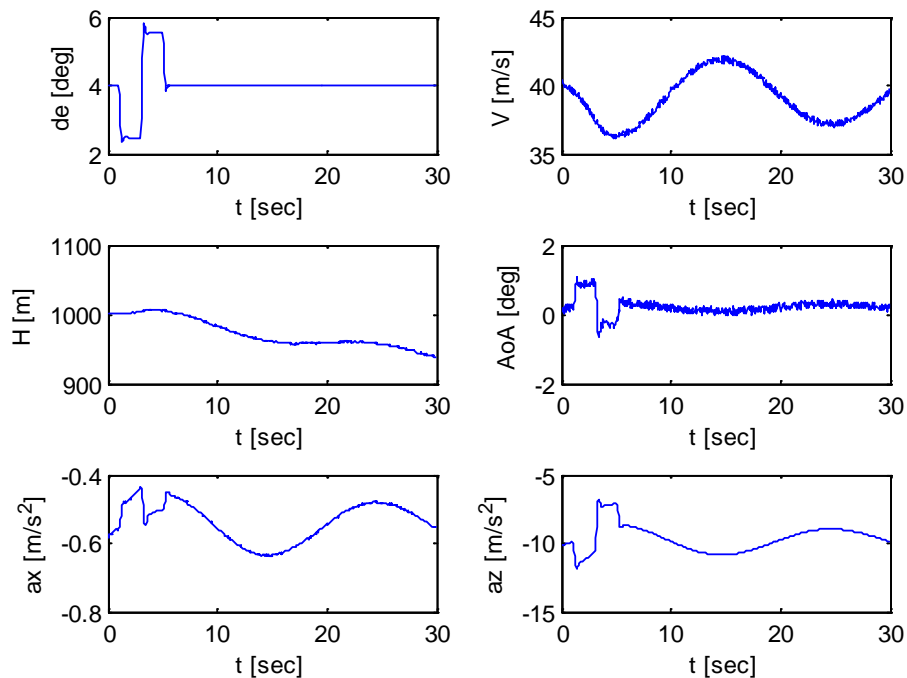


Figure 3. The output variables due to elevator input  $\delta_e$ : airspeed  $V$ , altitude  $H$ , angle of attack  $\alpha$ , accelerations  $a_x$  and  $a_z$

**Table 3. Dynamic Characteristic – Longitudinal Mode**

Eigenvalues	Damping	Natural Freq (rad/s)	Remarks
$-10.7 \pm 6.63i$	0.851	12.6	Pitch oscillation
$-0.00513 \pm 0.266i$	0.0193	0.266	Phugoid
0			Altitude var. (integrator)

Given a particular elevator deflection input, the open loop response of the UAV variables are depicted in Figures 4 and 5. It can be seen from the results that the phugoid mode of the UAV dominates the response, as indicated by the low frequency and lightly damped responses of almost all variables.

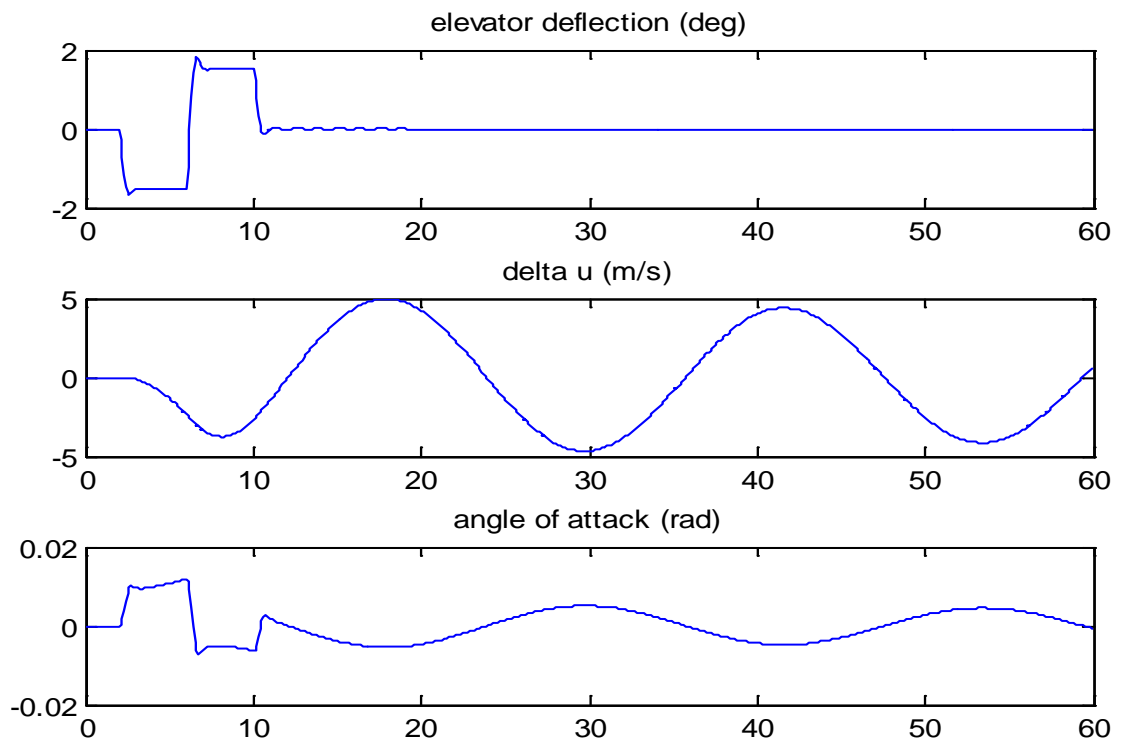


Figure 4. Linear Model Open Loop Response

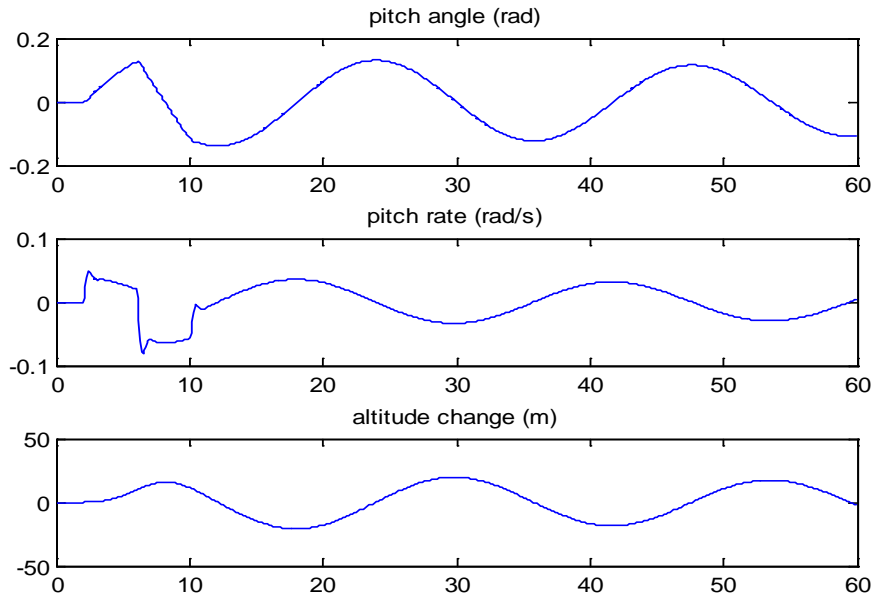


Figure 5. Linear Model Open Loop Response

## Parameter Identification

The objective of parameter identification is estimate the parameters of the flight vehicle from flight data. To estimate the parameter of the aircraft, a mathematical model of the aircraft under test must be postulated. This mathematical model can be derived from equation of motions the aircraft. Figure 6 shows the schematic diagram of parameter identification, in which the simulated output from the postulated mathematical model and sensor model is compared to the measured flight data. The difference between the measured and the simulated outputs will be minimized by adjusting the parameters in the model.

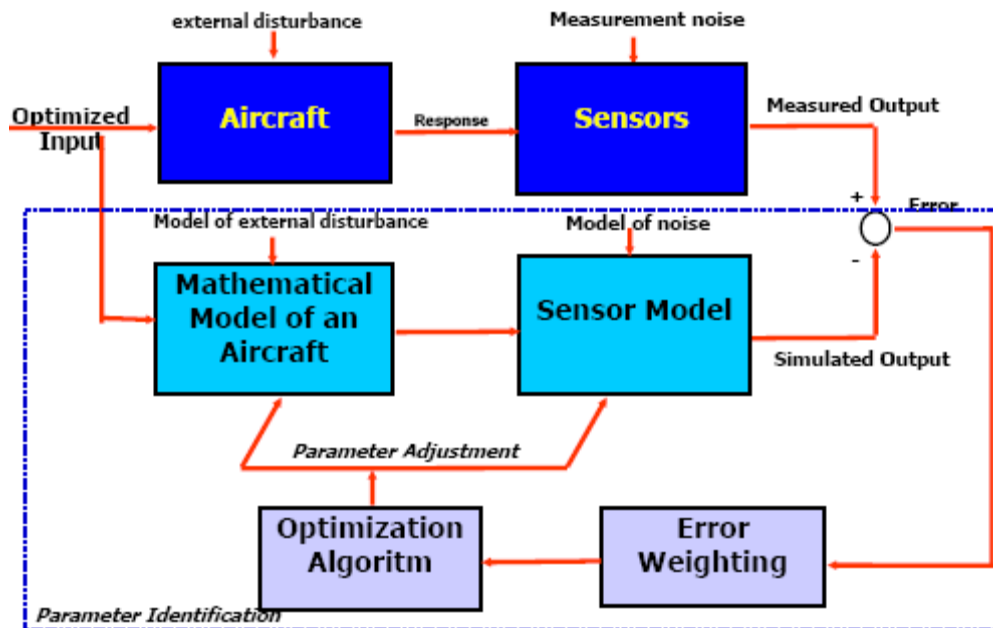


Figure 6. Schematic Diagram of Parameter Identification [9]

## Mathematical Model of Aircraft

The mathematical of the aircraft can be expressed by the non-linear equations of motion of as given in Equation (2), (3), (9) and (10). The linearized forms of the equations of motion are presented in Equations (16) and (19). If only the longitudinal equation has to be considered, Equation (11) can be used. If the total force and moment are expressed as follows:

$$\begin{aligned} F_x &= \frac{1}{2} \rho V^2 S C_X \\ F_z &= \frac{1}{2} \rho V^2 S C_Z \\ M &= \frac{1}{2} \rho V^2 S \bar{c} C_m \end{aligned} \quad (22)$$

Then, Equation (11) can be written as,

$$\begin{aligned} \dot{u} &= -wq - g \sin \theta + \frac{\frac{1}{2} \rho V^2 S C_X}{m} \\ \dot{w} &= uq + g \cos \theta + \frac{\frac{1}{2} \rho V^2 S C_Z}{m} \\ \dot{\theta} &= q \\ \dot{q} &= \frac{M}{I_y} = \frac{\frac{1}{2} \rho V^2 S \bar{c} C_m}{I_y} \\ \dot{\Delta h} &= u \sin \theta - w \cos \theta \end{aligned} \quad (23)$$

The aerodynamic forces and moment coefficients can be expressed in terms of several state and control variables of the aircraft, for example angle of attack  $\alpha$ , pitch rate  $q$ , deflection of the elevator control surface  $\delta_e$ , and the thrust coefficient  $T_c$ . The dependency of those coefficients on these variables can be expressed as follows:

$$\{C_x \ C_z \ C_m\}^T = f(\alpha, q, \delta_e, T_c) \quad (24)$$

Each coefficient in Equation (24) can be expressed in a Taylor series expansion as a function of the state and control variables as well as thrust coefficient. If terms up to the first order are included, these coefficients are expressed as follows:

$$\begin{aligned} C_x &= C_{x_0} + C_{x_\alpha} \alpha + C_{x_q} \frac{q\bar{c}}{V} + C_{x_{\delta_e}} \delta_e + C_{x_{T_c}} T_c \\ C_z &= C_{z_0} + C_{z_\alpha} \alpha + C_{z_q} \frac{q\bar{c}}{V} + C_{z_{\delta_e}} \delta_e + C_{z_{T_c}} T_c \\ C_m &= C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} \frac{q\bar{c}}{V} + C_{m_{\delta_e}} \delta_e + C_{m_{T_c}} T_c \end{aligned} \quad (25)$$

In Equation (25), the pitch rate  $q$  is made dimensionless. The derivatives in Equation (25), i.e.  $C_{x_\alpha}$ ,  $C_{x_q}$ ,  $C_{x_{\delta_e}}$ ,  $C_{z_\alpha}$ , ...,  $C_{m_q}$ , and  $C_{m_{\delta_e}}$  indicate the aerodynamic parameters, also known as stability derivatives, and  $C_{x_{T_c}}$ ,  $C_{z_{T_c}}$ , and  $C_{m_{T_c}}$  are the thrust parameters expressing the effect of the thrust on the total aerodynamic forces and moment.

Finally, the longitudinal mathematical model of the aircraft can be obtained by substituting Equation (25) into (23). From numerical integration of Equation (23), the output variables such as

true airspeed  $V$ , the angle of attack  $\alpha$ , and the pressure altitude  $h$  can be predicted. Also, noise can be added to these output variables.

### State and Parameter Estimation

The mathematical model of the aircraft discussed in the previous section can be written in non linear stochastic differential equations as follows :

$$\begin{aligned}\dot{x}(t) &= f[x(t), u(t), \theta] + w(t); & x(0) &= x_0 \\ y(t) &= g[x(t), u(t), \theta]\end{aligned}\quad (26)$$

and the discrete form of the observations as:

$$z(i) = y(i) + v(i); \quad i = 1, 2, 3, \dots, N \quad (27)$$

where  $x(t)$ ,  $u(t)$ ,  $y(t)$  and  $z(i)$  are the state, input, observations and measurement vectors respectively,  $\theta$  represents the vector of the unknown parameters, i.e. the aerodynamic parameters,  $x_0$  is the vector of the (unknown) initial conditions of the state,  $w(t)$  and  $v(i)$  are the process and measurement noise vectors respectively, and  $N$  is the number of data points.

For a linearised mathematical model, Equation (26) can be expressed as:

$$\begin{aligned}\dot{x}(t) &= A x(t) + B u(t) \\ y(t) &= C x(t) + D u(t)\end{aligned}\quad (28)$$

where matrices  $A$  and  $B$  in equation (28) are known as the **discretised state** and **input** matrices for the linearised model,  $C$  and  $D$  are known as the **discretized observation** and **input** of the linearised observation model. Note that the formulation in Equation (28) is similar to those in Equation (16) and (19).

In general, the problem of estimating the state trajectory and the aerodynamic parameters of the aircraft can be solved using the so-called **maximum likelihood method**. In this method, both the state and the parameters are estimated simultaneously by maximization of the so-called likelihood function. The likelihood function can be defined in terms of the difference between the measured and the simulated value from the sensor model.

If certain conditions concerning the accuracy and type of the variables measured in flight are met, for example the total force in  $X$ ,  $Y$  and  $Z$  directions are measured using accurate accelerometers and the rotational rate  $\mathbf{p}$ ,  $\mathbf{q}$ , and  $\mathbf{r}$  are measured using rate gyro, then the maximum likelihood estimation problem can be decomposed into two separate estimation problems, i.e. a state trajectory estimation problem followed by a parameter estimation problem [14,15]. In the case of longitudinal motion, the state equation (23) can be re-written as follow:

$$\begin{aligned}\dot{u} &= -wq - g \sin \theta + a_x \\ \dot{w} &= uq + g \cos \theta + a_z \\ \dot{\theta} &= q \\ \Delta \dot{h} &= u \sin \theta - w \cos \theta\end{aligned}\quad (29)$$

where accelerations  $a_x$  and  $a_z$  and pitch rate  $\mathbf{q}$  serve as input in state equation (29). These two separate problems are much easier to be solved than the original estimation problem.

## State Estimation

The state trajectory estimation is a recursive prediction of the aircraft state and its covariance matrix from a non linear mathematical model, see Equation (11) or (23), using a state estimation technique such as the Kalman filter.

The Kalman filter equations used in the estimation of the state of the aircraft are given in three steps as follows:

**Step 1 :** prediction of the state  $\hat{x}(i|i-1)$ , observation  $\hat{y}(i|i-1)$  and covariance matrix  $\hat{P}(i|i-1)$  at time (i) based on the estimate of the state, observation and covariance matrix at time (i-1), where  $i=1,2,3 \dots N$ .

$$\begin{aligned}\hat{x}(i|i-1) &= \hat{x}(i-1|i-1) + f[\hat{x}(i-1|i-1), u(i)] \Delta t \\ \hat{y}(i|i-1) &= g[\hat{x}(i|i-1), u(i)] \\ \hat{P}(i|i-1) &= \Phi(i|i-1) \hat{P}(i-1|i-1) \Phi^T(i|i-1) + \Gamma_w(i|i-1) V_w \Gamma_w^T(i|i-1)\end{aligned}\quad (30)$$

with initial condition of the predicted state and covariance matrix as follows:

$$\hat{x}(0|0) = x_0 \quad \text{and} \quad \hat{P}(0|0) = P_0$$

**Step 2:** calculation of the Kalman filter gain  $K_f(i)$

$$K_f(i) = \hat{P}(i|i-1) C^T(i-1) [C(i-1) \hat{P}(i-1|i-1) C^T(i-1) + V_v]^{-1}\quad (31)$$

**Step 3:** estimation of the state  $\hat{x}(i|i)$  and the covariance matrix  $\hat{P}(i|i)$  at time (i)

$$\begin{aligned}\hat{x}(i|i) &= \hat{x}(i|i-1) + K_f(i) \{z(i) - \hat{y}(i|i-1)\} \\ \hat{P}(i|i) &= \hat{P}(i|i-1) - K_f(i) C(i-1) \hat{P}(i|i-1)\end{aligned}\quad (32)$$

The matrices  $V_w$  and  $V_v$  are the variance of the system and measurement noises respectively. The matrices  $\Phi$  and  $\Gamma_w$  denote the discretised system and noise matrices of the linearised mathematical model, see Equation (28). These matrices are calculated as follows:

$$\begin{aligned}\Phi(i|i-1) &\approx I + A(i-1) \Delta t + \frac{1}{2!} A^2(i-1) \Delta t^2 + \dots \\ \Gamma_w(i|i-1) &\approx B(i-1) \Delta t + \frac{1}{2!} A(i-1) B(i-1) \Delta t^2 + \dots\end{aligned}\quad (33)$$

The Kalman filtering solution to the state estimation problem presented in Equations (30) to (32), is recursive in the sense that the state estimate is estimated sequentially from previous measurements of  $Z=[z(1), z(2), \dots, z(N)]$ .

Figure X shows an example of time history of the estimated aircraft state trajectory using Kalman filter algorithm. The flight test data was obtained from simulated flight maneuver of the elevator doublet at altitude of 1000 m and nominal airspeed 42 m/sec, see Figures 7. The filtered estimates of the airspeed component along the longitudinal axis  $\hat{u}$ , the airspeed component along the vertical axis  $\hat{w}$ , the pitch angle  $\hat{\theta}$ , and the altitude variation  $\Delta \hat{h}$  are presented in this figure by

solidline.

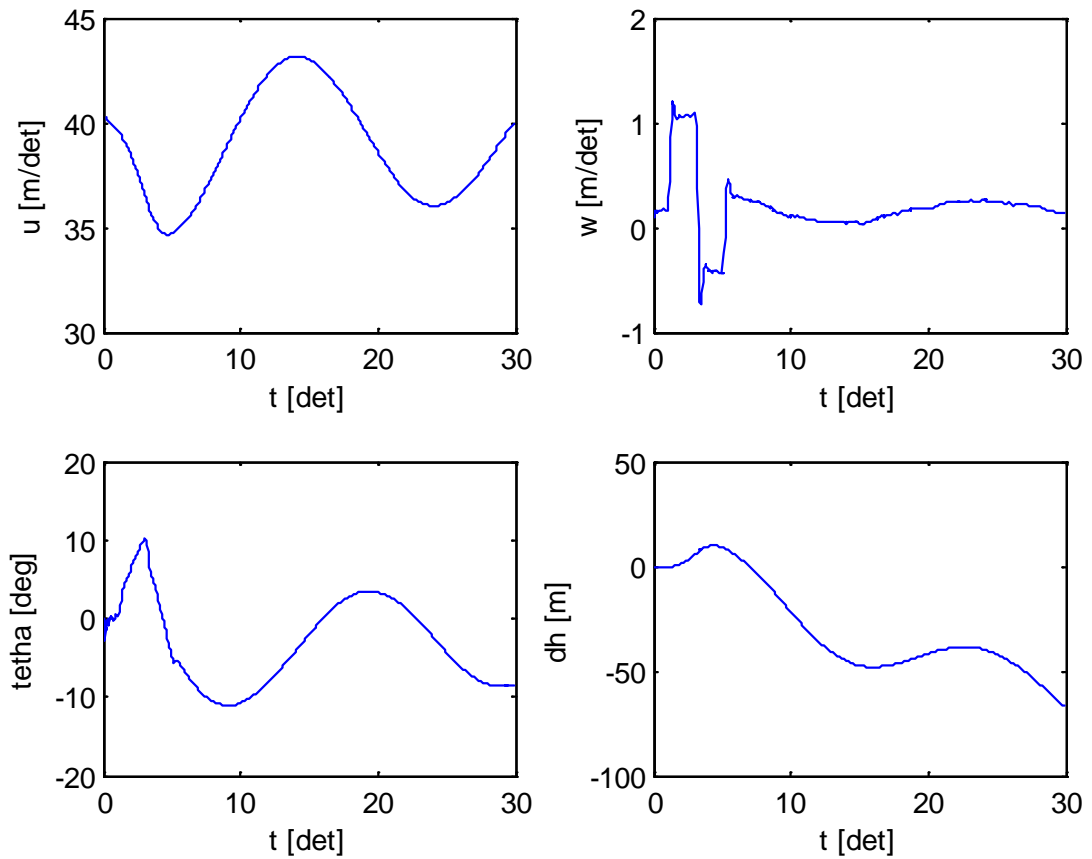


Figure 7. The estimated aircraft state trajectory from dynamic flight test maneuver

Figure 8 shows the Kalman filter estimate of true airspeed, vane-angle of attack, pitch angle, and altitude variation (dotted line). The measured true airspeed, vane-angle of attack, pitch angle and altitude variation are also given in this figure. The difference between the estimated and the measured (residual) are also presented in this figure.

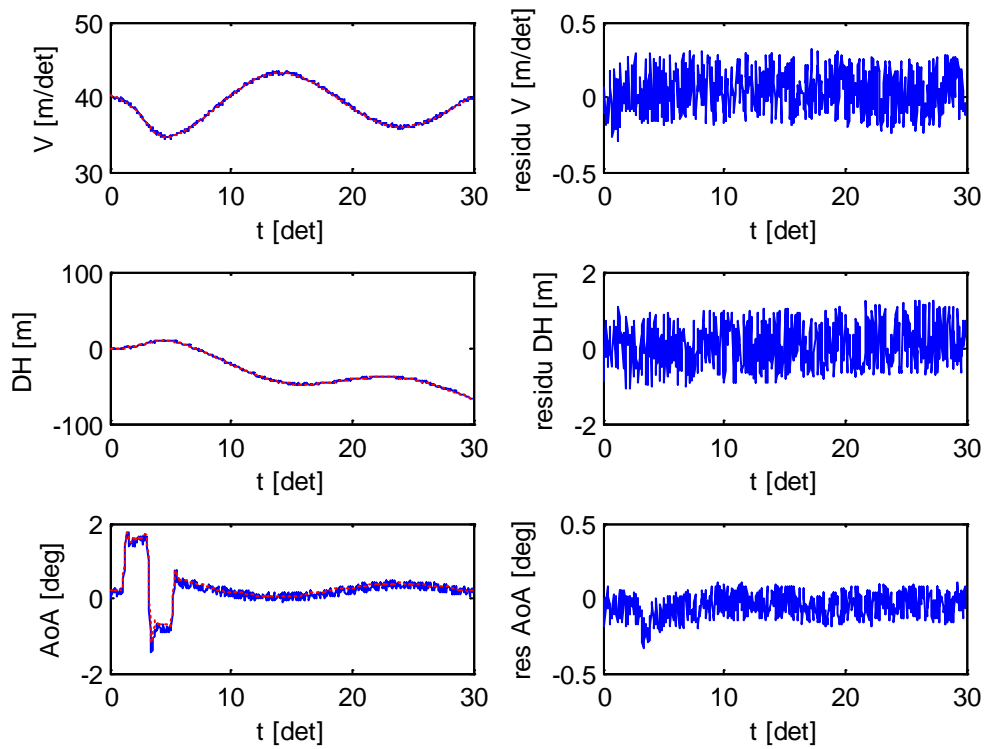


Figure 8. The estimated and measured of the true airspeed, vane-angle of attack, angle of pitch and altitude variation. Their residuals between the measured and the estimated are also given

### Parameter Estimation

Assuming that the airspeed  $V$ , altitude  $h$  (consequently the air density  $\rho$  can be estimated) and the angle of attack  $\alpha$  are estimated using Kalman filter, the coefficients of the total forces and moment in Equation (25) can be expressed as follows:

$$\begin{aligned}
 C_x &= \frac{X}{0.5\rho V^2 S} = \frac{m a_x}{0.5\rho V^2 S} = C_{x_0} + C_{x_\alpha} \alpha + C_{x_q} \frac{q\bar{c}}{V} + C_{x_{\delta_e}} \delta_e + C_{x_{T_c}} T_c + \varepsilon_x \\
 C_z &= \frac{Z}{0.5\rho V^2 S} = \frac{m a_z}{0.5\rho V^2 S} = C_{z_0} + C_{z_\alpha} \alpha + C_{z_q} \frac{q\bar{c}}{V} + C_{z_{\delta_e}} \delta_e + C_{z_{T_c}} T_c + \varepsilon_z \\
 C_m &= \frac{M}{0.5\rho V^2 S c} = \frac{I_y \dot{q}}{0.5\rho V^2 S c} = C_{m_0} + C_{m_\alpha} \alpha + C_{m_q} \frac{q\bar{c}}{V} + C_{m_{\delta_e}} \delta_e + C_{m_{T_c}} T_c + \varepsilon_m
 \end{aligned} \tag{34}$$

In Equation (34), modeling error and measurement error are included in the last term of the right hand side of that equation. In a regression equation form, Equation (34) can be expressed as

$$y(i) = \theta_0 + \theta_1 x_1(i) + \theta_2 x_2(i) + \dots + \theta_p x_p(i) + \varepsilon(i) \tag{35}$$

where  $y(i)$  is the dependent variable, i.e. the aerodynamic force and moment coefficients,  $x_p(i)$  denote the independent variables, i.e. the (estimated) state and control variables,  $\theta_p$  is the vector of the aerodynamic parameters, and  $\varepsilon(i)$  is the stochastic equation error, accounting for measurement error on the dependent variable.

Equation (35) can be written in the matrix form as follows:



$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} = \begin{bmatrix} 1 & x_{12} & x_{13} & \dots & x_{1p} \\ 1 & x_{22} & x_{23} & \dots & x_{2p} \\ 1 & x_{32} & x_{33} & \dots & x_{3p} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ 1 & x_{N2} & x_{N3} & \dots & x_{Np} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \\ \theta_p \end{Bmatrix} + \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_N \end{Bmatrix} \quad (36)$$

or, in the more compact form as:

$$Y = X\theta + \varepsilon \quad (37)$$

The matrix of independent variables  $X$  is assumed known exactly from the state estimation, i.e. using Kalman filter, or from direct measurement of state variable using instrumentation system with highest accuracy.

Using the Least Squares (LS) technique, the parameter vector  $\theta$  can be estimated from:

$$\hat{\theta} = [X^T X]^{-1} [X^T Y] \quad (38)$$

Following the estimated parameter  $\hat{\theta}$ , the estimated dependent variable and the estimated error model can be calculated from:

$$\begin{aligned} \hat{Y} &= X \hat{\theta} \\ \hat{\varepsilon} &= Y - \hat{Y} = Y - X \hat{\theta} \end{aligned} \quad (39)$$

Finally, the goodness of fit the model can be expressed in terms of a correlation coefficient between  $Y$  and  $\hat{Y}$ . This coefficient is usually referred to as the multiple correlation coefficient whose square is :

$$R^2 = \frac{[\hat{Y} - \bar{Y}]^T [\hat{Y} - \bar{Y}]}{[Y - \bar{Y}]^T [Y - \bar{Y}]}; \quad \bar{Y} = \frac{1}{N} \sum_{i=1}^N \hat{Y}(i) \quad (40)$$

The value of the multiple correlation coefficient  $R$  can be as high as one when the model fit is perfect, or zero when no correlation exists between the model and the measured dependent variable  $Y$ .

The estimation of the aerodynamic parameters is performed with Least Squares method after estimation of the state trajectory of the aircraft. Figure 9 shows the measured (solid line) and estimated (dotted line) values of the longitudinal force coefficient  $C_x$ , the vertical force  $C_z$ , and the pitching moment coefficient  $C_m$ . The residuals between the measured and the estimated are also presented in this figure. The values of the total correlation coefficient  $R$  of the forces and moment coefficients are 0.995, 0.997 and 0.91 respectively.

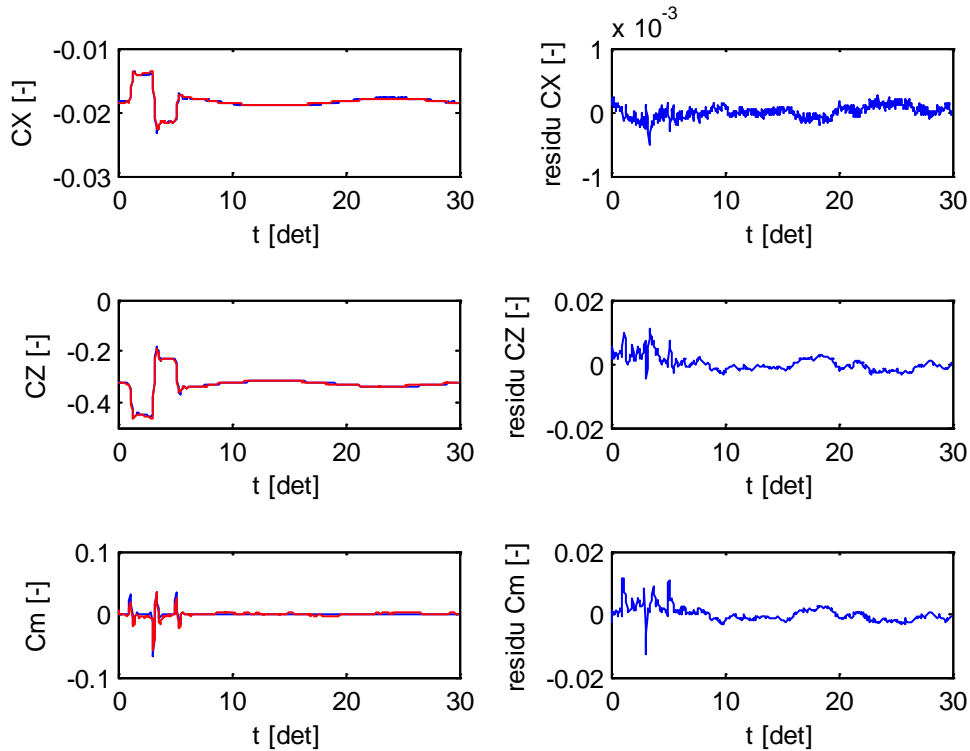


Figure 9. The estimated and measured and their residuals of the longitudinal force coefficient  $C_X$ , vertical force coefficient  $C_Z$ , and the pitching moment coefficient  $C_m$ .

## Design Flight Control System

In this section, it will be demonstrated the role of aircraft dynamic model in the flight control design. As has been explained earlier, the dynamic model contains important information about the dynamics characteristic of the aircraft under consideration. Hence, it plays key role in the design process, since one of the main purposes of the flight control system is to improve and regulate the dynamics variables of the aircraft.

In this section, an altitude hold controller will be designed to improve the performance of the UAV in maintaining and regulating its flight altitude. As has been revealed by the open loop response results, it is already known that the UAV longitudinal dynamics is dominated by the low frequency and lightly damped behavior, which is not appropriate in the view of achieving good closed loop performance. Hence, in addition to achieve a good tracking behavior, the designed controller must be able to stabilize the dominant characteristics of the system so that the UAV can follow the altitude reference command. A scheme to obtain a tracking controller with good disturbance rejection property will be employed in this case.

### Tracking Controller

Consider a system which is described as follows [10]:

$$\begin{aligned}\dot{\bar{x}} &= A\bar{x} + B_1\bar{w} + B_2\bar{u} \\ \bar{z} &= C\bar{x} + D_1\bar{w} + D_2\bar{u}\end{aligned}\quad (41)$$

where  $\bar{x} \in \mathfrak{R}^n$  is the system state,  $\bar{w} \in \mathfrak{R}^{m_1}$  is an unknown and bounded disturbance input,  $\bar{u} \in \mathfrak{R}^{m_2}$  is the control input, and  $\bar{z} \in \mathfrak{R}^p$  is the performance output. Assuming that the state  $\bar{x}$  is available, and that the performance output  $\bar{z}$  can be measured, then the problem is to design a controller which can follow a reference input  $\bar{r} \in \mathfrak{R}^p$  from arbitrary initial condition  $\bar{x}(0) = \bar{x}_0$ , while keeping the system state  $\bar{x}$  bounded for  $t \geq 0$ .

Assuming that the system is stabilizable and defining new state variable  $\bar{x}_a$ , performance output  $\bar{z}_a$ , and the disturbance input  $\bar{w}_a$  as follows:

$$\begin{aligned}\bar{x}_a &= \begin{bmatrix} \bar{x}^T & \bar{x}_e^T \end{bmatrix}^T \\ \bar{z}_a &= \bar{z} - \bar{r} \\ \bar{w}_a &= \begin{bmatrix} \bar{w}^T & \bar{r}^T \end{bmatrix}\end{aligned}\quad (42)$$

where the state  $\bar{x}_e$  is defined as the integration of the tracking error:

$$\dot{\bar{x}}_e = \bar{z} - \bar{r}\quad (43)$$

Hence we obtain the augmented plant :

$$\begin{aligned}\dot{\bar{x}}_a &= \tilde{A}\bar{x}_a + \tilde{B}_1\bar{w}_a + \tilde{B}_2\bar{u} \\ \bar{z}_a &= \tilde{C}\bar{x}_a + \tilde{D}_1\bar{w}_a + \tilde{D}_2\bar{u}\end{aligned}\quad (44)$$

where

$$\begin{aligned}\tilde{A} &= \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} & \tilde{B}_1 &= \begin{bmatrix} B_1 & 0 \\ D_1 & -I \end{bmatrix} & \tilde{B}_2 &= \begin{bmatrix} B_2 \\ D_2 \end{bmatrix} \\ \tilde{C} &= [C \quad 0] & \tilde{D}_1 &= [D_1 \quad -I] & \tilde{D}_2 &= D_2\end{aligned}\quad (45)$$

Having the pair  $(\tilde{A}, \tilde{B}_2)$  stabilizable, we can choose a state feedback matrix  $K$  such that the closed loop  $(\tilde{A} + \tilde{B}_2 K)$  is asymptotically stable. The matrix  $K$  can be partitioned as  $K = [K_p \quad K_I]$  where  $K_p \in \mathfrak{R}^n$  is the first  $n$  columns of  $K$  and  $K_I \in \mathfrak{R}^p$  is the remain  $p$  columns of  $K$  which related to variable  $\bar{x}_e$  as the integration of  $\bar{z} - \bar{r}$ . Hence the controller for the system (41) can be obtained as:

$$\begin{aligned}\dot{\bar{x}}_e &= \bar{z} - \bar{r} \\ \bar{u} &= K_p \bar{x} + K_I \bar{x}_e\end{aligned}\quad (46)$$

such that the controller may be expressed as a PI-like controller:

$$u = K_p \bar{x} + K_I \int (\bar{z} - \bar{r}) dt\quad (47)$$

Implementing the gain feedback  $K$ , the closed loop system becomes:

$$\begin{aligned}\dot{\bar{x}}_a &= [\tilde{A} + \tilde{B}_2 K] \bar{x}_a + \tilde{B}_1 \bar{w}_a \\ \bar{z}_a &= [\tilde{C} + \tilde{D}_2 K] \bar{x}_a + \tilde{D}_1 \bar{w}_a\end{aligned}\quad (48)$$

If a gain  $K$  can be obtained such that the closed loop (48) is asymptotically stable, and if the disturbance  $\bar{w}$  is bounded, then  $\bar{x}(t)$  is bounded for  $\forall t \geq 0$ , and  $\bar{x}_e \rightarrow 0$  meaning that  $\bar{z} \rightarrow \bar{r}$  ( $\bar{z}$  tracks  $\bar{r}$ ).

### LQR Controller

An optimal heading hold LQR controller can be computed by taking into account the augmented system defined in Equation (44). LQR controller is computed by minimizing the following cost function [10], [11]:

$$J = \int (\bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u}) dt \quad (49)$$

where  $Q$  and  $R$  are symmetric matrices for weighting the states and control variables, and the relation of the system states is as described in Equation (42). A state feedback controller then is computed, such that the control signal can be described as follows:

$$\bar{u} = -K\bar{x} \quad (50)$$

where

$$K = R^{-1} B^T P \quad (51)$$

and  $P$  is a positive definite matrix obtained by solving Riccati equation:

$$-\dot{P} = A^T P + P A - P B R^{-1} B^T P + Q \quad (52)$$

Having  $K$  computed to produce an acceptable reference tracking behaviour, then the altitude hold controller can be implemented in a structure depicted in Figure 10.

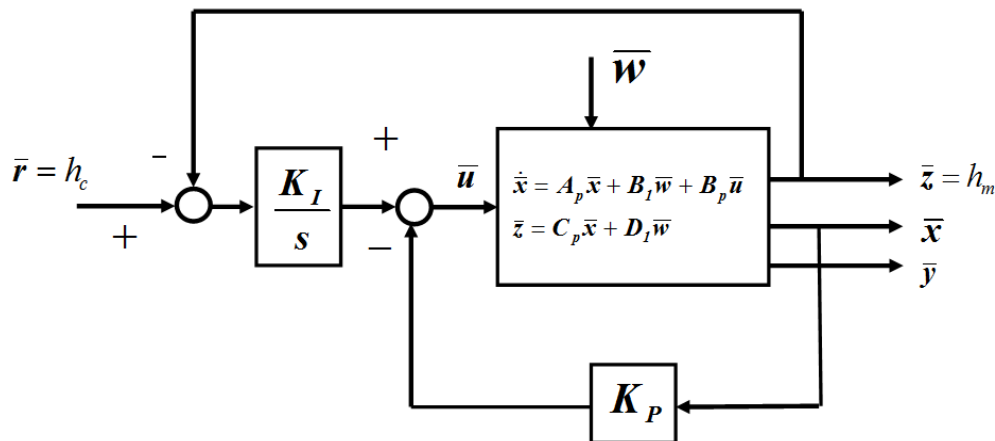


Figure 10. Tracking Controller for Altitude Hold System

### Controller Design and Results

Based on the state model described in Equation (20) and (21), an altitude hold controller is designed using the approach already explained in the previous subsection. It is obvious that the

variable required as the performance variable  $z$  is the measured altitude, hence it is assumed that a barometric altitude sensor is used, which is modeled as a first order system. In addition to that, a first order system is also used for modeling the elevator actuator. An augmented system then is obtained following the procedure in the previous subsection, which is modeled as in Equation (44). Using this augmented model, then an LQR controller is computed by choosing appropriate  $Q$  and  $R$  matrices. In this case,  $Q$  matrix is chosen such as it can produce more weighing on the measured altitude variable and the error variable, while  $R$  matrix is chosen to relatively limit the magnitude of the generated control signal (the elevator deflection). Solving the optimal control problem, a gain controller  $K$  is obtained, which can be partitioned into a gain feedback vector  $K_p$  and an integral gain  $K_i$ . The gain vector  $K_p$  then is used as a state feedback controller for forming an inner closed loop system, and the gain  $K_i$  will form an outer loop system, by feeding the measured altitude variable back into the controller, comparing to the reference value, and integrating the error, as illustrated in Figure 10.

The characteristics of the inner loop system which has  $K_p$  as its state feedback controller is summarized in Table 4.

**Table 4. Inner Closed Loop Dynamic Characteristic – Longitudinal Mode**

Eigenvalues	Damping	Natural Freq (rad/s)	Remarks
$-10.7 \pm 6.63i$	0.851	12.6	Pitch oscillation
$-1.97 \pm 3.09i$	0.537	3.67	Phugoid
$-0.304 \pm 1.43i$	0.208	1.47	
-4.2			

The performance of the closed loop system to follow particular altitude reference command can be elaborated from some of the simulation results presented below. In Figure 11, it can be seen that the controller can provide a quite good tracking performance, although further adjustment in the selection of  $Q$  and  $R$  matrices still can be carried out to obtain more improved results.

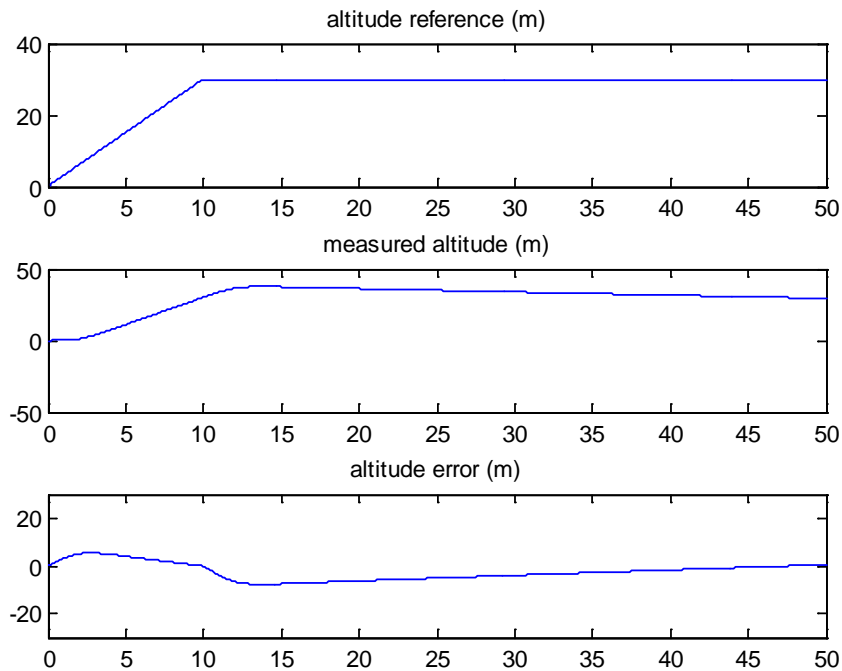


Figure 11. Tracking Performance of the closed loop system

In addition to that, the simulation also shows that the developed system, while can provide an acceptable tracking behavior, may also produce realistic response of the controlled system with bounded control input, as showed in Figures 12 and 13. It can be seen that, as indicated by the elevator deflection response, the controller can manage the task with realistically bounded control action.

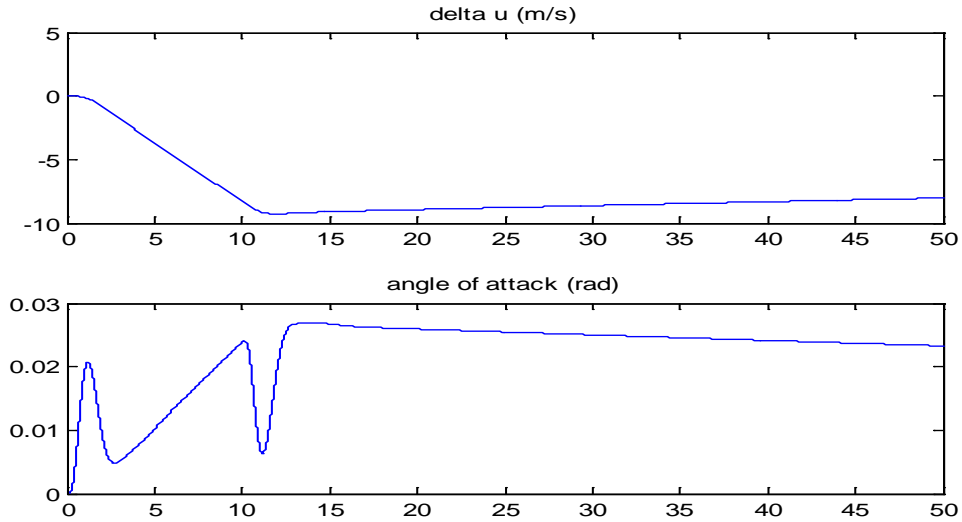


Figure12. System and control variables response (a)

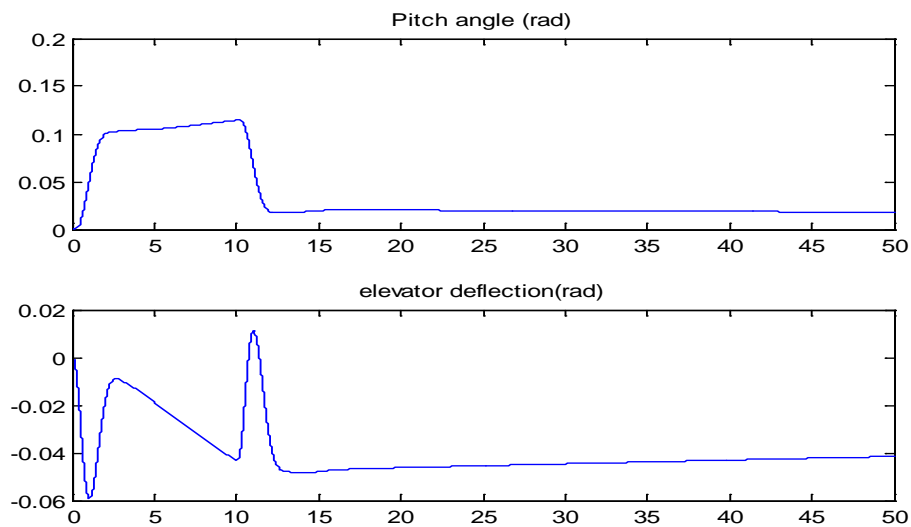


Figure13. System and control variables response (b)

## Conclusions

Some aspects related to flight dynamics model of aircrafts have been discussed in this paper. Aircraft flight dynamic model can be viewed as a unifying factor, which relates most of different analysis and synthesis activities in flight mechanics fields. It has been showed how a dynamic mathematical model of aircraft motion can be derived by evaluating all forces and moments that may affect the aircraft motions. The model can be complicated by the fact that the motion of

aircraft involves translational and rotational motions in 6 degree of freedom. In addition to that, any forces and moments working on aircraft body have their own characteristics, for instance they may have different mechanism that determines their magnitude and direction. These forces and moments also have their dependencies to the motions of the aircraft body, hence the amount and direction of the forces/moments will change as functions of aircraft motions. This kind of interaction is depicted mathematically in the model, and can be used for determining the characteristic and behavior of the aircraft. It has also been described the role of aircraft dynamic model in flight control development. Flight control system is usually used for manipulating the characteristics of an aircraft so that it can generate better response. Hence, the dynamic model will play key role in the design of a control system, since it will be the basis for governing the controller equations. One point that must be noted is that the compatibility of the model with the real characteristic of the aircraft must be ensured, so that the designed control system will perform as required in the real condition.

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