INITIAL APPROXIMATION FOR NEWTON-RAPHSON ITERATION TO CALCULATE THE NEUTRAL AXIS POSITION OF GLULAM BAMBOO

Inggar Septhia Irawati[1], Bambang Suhendro[2], Ashar Saputra[3], and Tibertius Agus Prayitno[4]

[1] Civil and Environmental Engineering Department, Gadjah Mada University, Yogyakarta, Indonesia,
Tel: +62 274545675, e-mail: inggar_irawati@yahoo.com
[2] Civil and Environmental Engineering Department, Gadjah Mada University, Yogyakarta, Indonesia,
Tel: +62 274545675, e-mail: bbsuhendro@ugm.ac.id
[3] Civil and Environmental Engineering Department, Gadjah Mada University, Yogyakarta, Indonesia,
Tel: +62 274545675, e-mail: ashar@tsipil.ugm.ac.id
[4] Wood Science and Technology Department, Gadjah Mada University, Yogyakarta, Indonesia,
Tel: +622746491428, e-mail: taprayitno@yahoo.com

Received Date: July 9, 2012

Abstract

Glued laminated (glulam) bamboo has possibility to be used as construction material. For structural purpose, an analytical calculation is needed to estimate the structure capacity. As bamboo typically has non linear material behavior under compression parallel to grain, it is essential to figure out the non linear behavior of glulam bamboo for structural component.

The research purpose is to figure out non linear behavior of glulam bamboo by determining maximum capacity under axial and lateral loadings according to actual properties of tension and compression parallel to grain. The failure load is numerically computed based on finite different method. A Newton-Raphson iterative procedure is employed to determine the deflection curve and neutral axis position that meet the equilibrium condition. Assumption used is that tensile or compressive stress is equal to zero when it is greater than its ultimate stress. This paper is focused to describe iterative procedure to obtain neutral axis position. The novelty of this study is the proposed initial approximation for iteration of neutral axis position which has never strongly been considered by previous study. Its application and detail discussion are presented. There are two reasons why initial approximation for obtaining neutral axis position is important to be understood. First, it influences the speed of convergence, and secondly, when the internal stress is close to the maximum strength, it might influence the convergence of calculation.

Keywords: Glulam bamboo, Initial approximation, Neutral axis position, Newton-raphson iteration, Non linear behavior

Introduction

Glued laminated (glulam) bamboo is made of several bamboo strips bonded together with adhesive (Figure 1). Based on bamboo strips layout, there are two type of glulam bamboo,

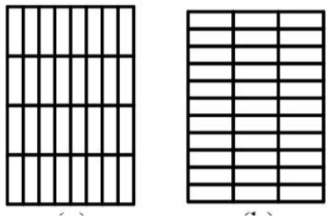

Figure 1. Cross Section: (a) vertically glulam bamboo; (b) horizontally glulam bamboo

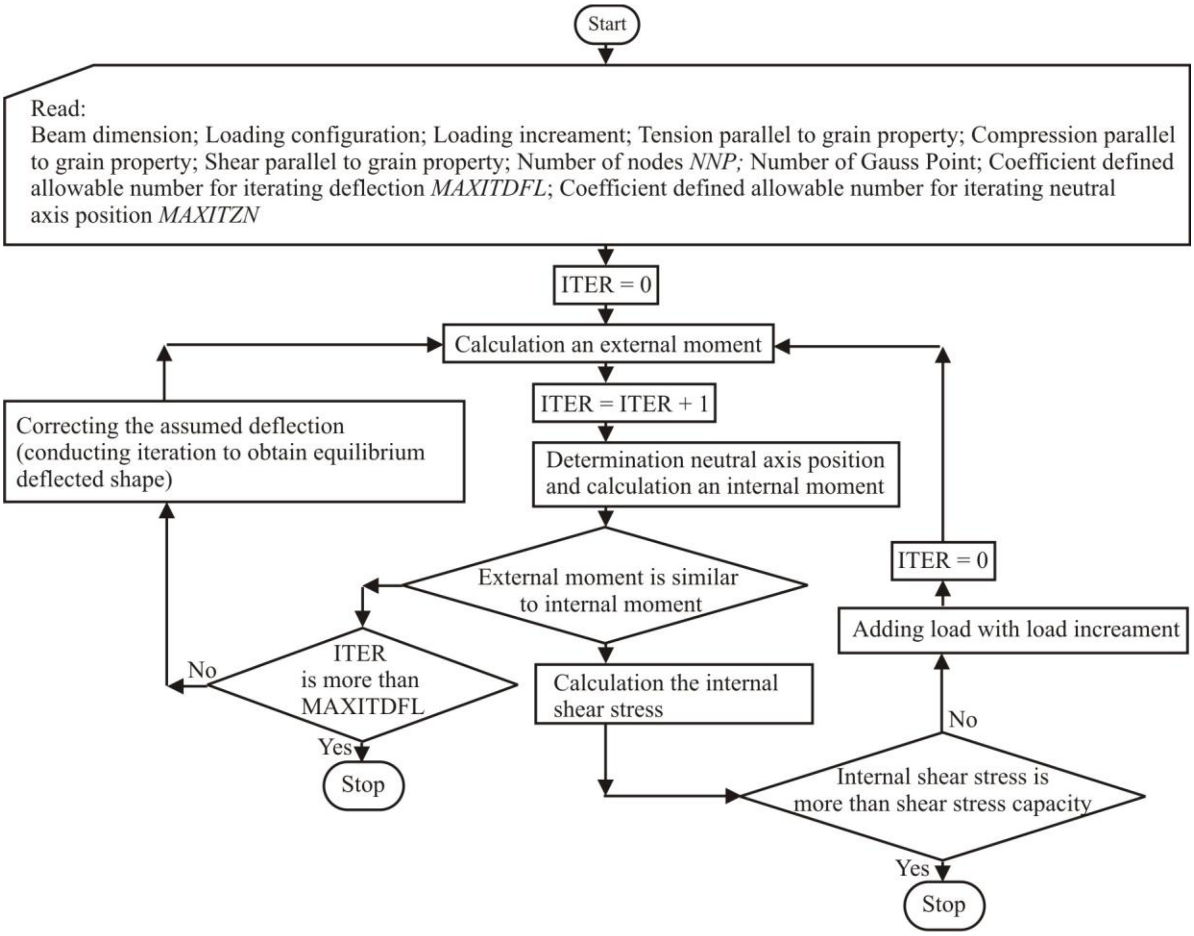

Figure 2. General numerical procedure

i.e., vertically and horizontally glulam bamboo. Glulam bamboo is comparable to other wood product [1]. It has possibility to be used as structural component, *i.e.*, column as well as structural beam [1]. Glulam bamboo also has similar problem to wood product [1]. It needs to be treated for fire and insects [1]. However, it is more eco-friendly material compared to other glulam timber. Bamboo as a raw material is more renewable material. It can grow quickly. It reaches structural grade material within 3 – 5 years whereas timber achieves structural grade material within 20 years [2]. Further, when glulam bamboo is used for structural purpose, an analytical calculation is needed to estimate the structure capacity.

Bamboo has non linear material behavior under compression parallel to grain. In spite of this, the structural capacity is usually calculated by using assumption that bamboo is a linear material. Accordingly, it is essential to figure out the non linear material behavior of glulam bamboo for structural component. For this reason, the research purpose is to understand non linear behavior of glulam bamboo beam-column by determining maximum capacity based on actual properties of tension and compression parallel to grain. Numerical program for calculating failure load of glulam bamboo subjected to axial and lateral load has being developed based on finite difference method. Structural member is divided into several nodal points that have equal span. Bending failure load is obtained by calculating the equilibrium deflected shape of the structure. Shear failure load is obtained by comparing the internal shear stress towards shear strength. In general, the developed numerical program is illustrated in Figure 2.

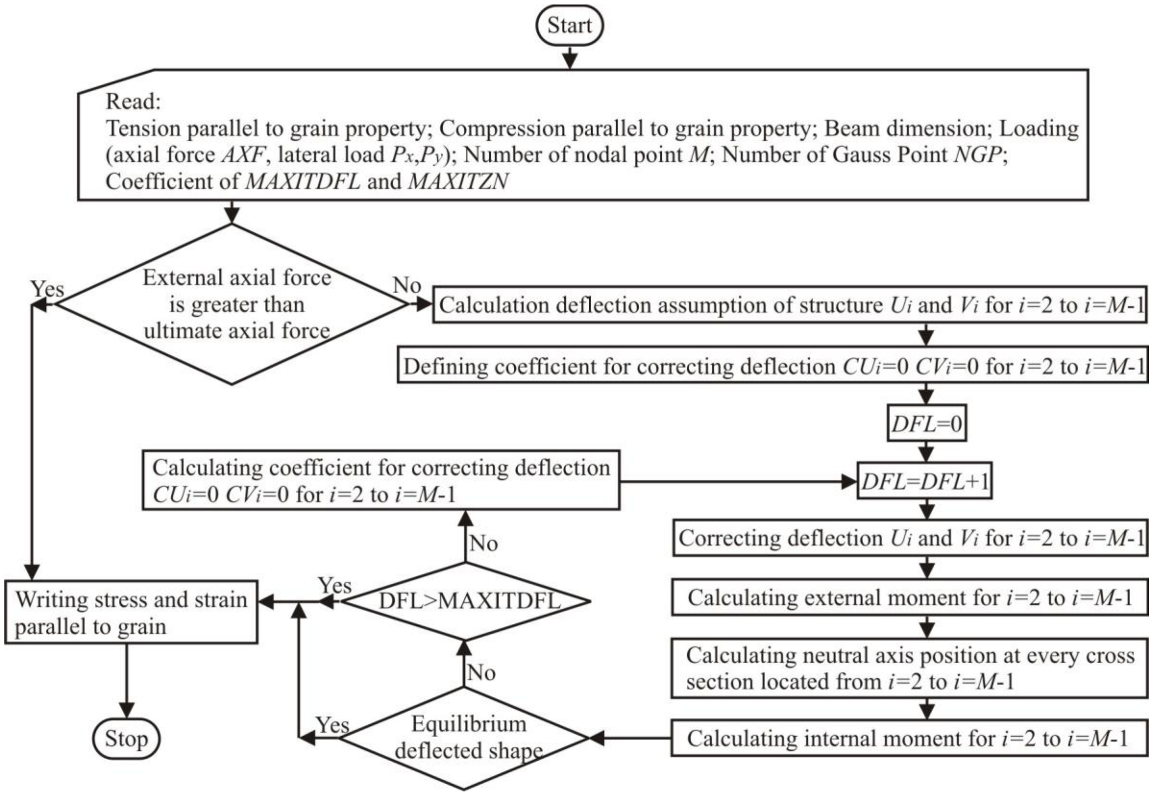

Figure 3. Procedure for obtaining equilibrium deflected shape

One of the important steps for calculating an equilibrium deflected shape is the determination of neutral axis position. The neutral axis position is defined by obtaining an equilibrium condition for axial forces at each nodal point. Concerning glulam bamboo axially and laterally loaded, the neutral axis position will vary along the beam depending on deflection, internal axial force as well as internal moment at each cross section. An iterative procedure is needed to determine neutral axis position. Accordingly, Newton-Raphson method is employed for this iteration. It is useful technique for finding zeros of function [3]. However, a question arises with respect to choosing initial approximation. Previous procedure for calculating neutral axis position has been published [4; 5; 6] but initial approximation has not been clearly stated. It is important to be known for the reason that it will greatly influence the speed of convergence [7]. Besides that, by using improper initial approximation, non convergent result may be occurred during calculation [8]. In case of glulam beam subjected to axial and lateral load, non convergent result may be occurred when the internal stress is close to the maximum strength. Thus, this paper presents observation process as well as its result to identify the proper initial value to calculate neutral axis position of glulam bamboo by using Newton-Raphson method.

Equilibrium Deflected Shape

Iterative procedure to obtain equilibrium deflected shape is illustrated in Figure 3, where *MAXITDFL* is the maximum number of iteration for obtaining equilibrium deflected shape; *U* and *V* are deflection in *X* and *Y* direction, respectively; *CU* and *CV* are the correction of deflection in *X* and *Y* direction, respectively; *M* is number of nodal point. Material properties that consist of tension and compression parallel to grain, beam dimension, loading configuration, number of nodal point, number of Gauss point and coefficients needed are defined. A number of assumptions are employed in the calculation, as follow:

- Plane sections before bending remain plane upon flexure.

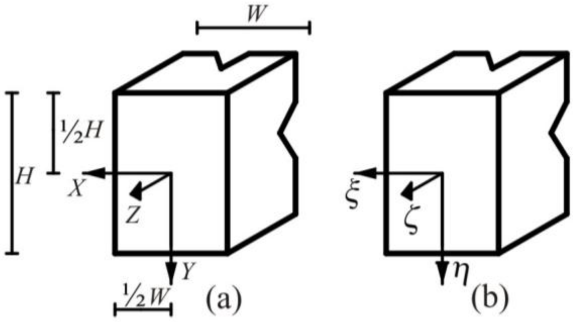

Figure 4. Coordinates system: (a) cartesian coordinate system; (b) natural coordinate system

- The study is focused on the non linear material of glulam bamboo. The relation between curvature Φ and deflection *V* that is written in Equation (1) is simplified into Equation (2) [9]. It can be rewritten that curvature is presented by the second derivatives of deflection.

$$\Phi = \frac{-V''}{(1+V'^2)^{3/2}} \quad (1)$$

$$\Phi = -V'' \quad (2)$$

- Tensile as well as compressive stresses parallel to grain are equal to zero when they are greater than the ultimate stress. Glulam bamboo is assumed as homogeneous material.
- Lateral loads act through the centroid of the cross section so that the twist moment is not considered.
- Beam is supported by simple bearings.

The initial assumption of horizontal deflection *U* and vertical deflection *V* for nodal *i*=2 to *i*=*M*-1 are obtained by using Equation (3) and Equation (4).

$$U_i = 0 \quad (3)$$

$$V_i = \frac{L}{2000} \times \sin\left(\frac{z_{mi}}{L} \times 3.1416\right) \quad (4)$$

where z_{mi} is the distance of nodal point *i* measured from point 0, and *L* is the beam length. Coordinate system can be seen in Figure 4(a). External moments for nodal point *i*=2 to *i*=*M*-1 are calculated by using the Equation (5) and Equation (6).

$$M_{xi} = M_{x0} - \frac{z_{mi}}{L} M_{x0} + \frac{z_{mi}}{L} M_{xL} - AXF \times V_i - \sum_{k=1}^{k=NL} P_{y_k} \left(z_{mi} - z_{p_k}\right)$$

$$+\frac{z_{mi}}{L}\sum_{j=1}^{j=NP}P_{y\,j}\left(L-z_{p\,j}\right) \quad (5)$$

$$M_{y_i}=M_{y_0}-\frac{z_{mi}}{L}M_{y_0}+\frac{z_{mi}}{L}M_{y_L}-AXF\times U_i-\sum_{k=1}^{k=NL}P_{xk}\left(z_{mi}-z_{p_k}\right)$$

$$+\frac{z_{mi}}{L}\sum_{j=1}^{j=NP}P_{x\,j}\left(L-z_{p\,j}\right) \quad (6)$$

where M_{xi}, M_{x0} and M_{xL} are external moments in X direction at nodal point i, 0 and L, respectively; M_{yi}, M_{y0} and M_{yL} are external moments in Y direction at nodal point i, 0 and L, respectively; z_{pj} is the distance of lateral load j that is located at the left side from nodal

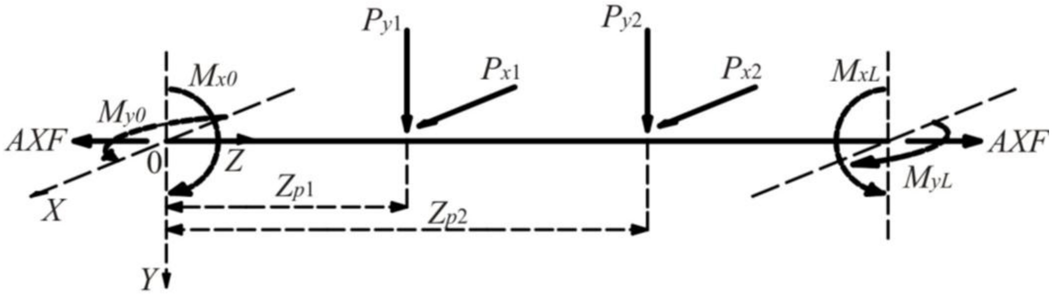

Figure 5. Figure denotation

point i and measured from point 0; P_{yj} and P_{xj} are lateral loads in Y and X direction, respectively; NP is the number of lateral load; NL is the number of lateral load located on the left side from nodal point i; AXF is external axial force. Zero point is located on the left bearing, as shown in Figure 5.

Internal moments of cross sections located at nodal point i=2 to i=M-1 are calculated by using Equation (7) and Equation (8).

$$M_{\text{int}x_i}=M_{\xi}=\int_{-1}^{1}\int_{-1}^{1}\sigma(x,y)\eta|\mathbf{J}|\mathrm{d}\xi\mathrm{d}\eta \quad (7)$$

$$M_{\text{int}y_i}=M_{\eta}=\int_{-1}^{1}\int_{-1}^{1}\sigma(x,y)\xi|\mathbf{J}|\mathrm{d}\xi\mathrm{d}\eta \quad (8)$$

where M_{intxi} and M_{intyi} are internal moment at nodal point i in X and Y directions, respectively; σ is tensile or compressive stress parallel to grain; **J** is Jacobian matrix; ξ and η are the distance between certain Gauss point and neutral axis position in ξ and η directions, respectively. Gaussian quadrature method is employed to simplify integral calculation of the internal stress. To adopt Gaussian quadrature method, Cartesian coordinates (X,Y) is changed into natural coordinate (ξ,η), as illustrated in Figure 4(b).

The neutral axis position is obtained when the equilibrium condition for axial force is attained, as written in Equation (9) and Equation (10).

$$AXF=FTOT_i \quad (9)$$

$$FTOT_i=\int_{-1}^{1}\int_{-1}^{1}\sigma(x,y)|\mathbf{J}|\mathrm{d}\xi\mathrm{d}\eta \quad (10)$$

where $FTOT_i$ is internal axial force at nodal point i. An equilibrium deflected shape is obtained if the requirements in Equation (11) and Equation (12) are satisfied.

$$M_{xi}=M_{\text{int}xi} \quad (11)$$

$$M_{y_i} = M_{\mathrm{int}\,y_i} \tag{12}$$

Newton-Raphson Method for Obtaining Neutral Axis Position

Newton-Raphson method that uses the first term of Taylor is written in Equation (13). For iterating neutral axis position, Equation (13) can be rewritten as Equation (14). *FTOT*' can be computed by using Equation (15). Iteration is started by approximating neutral axis position ZN_j. When equilibrium condition for axial force has not been obtained, *ZNj* is corrected by using Equation (16). The procedure is repeated until the the internal forces is similar to the external forces.

$$\mathrm{f}(x_i + \Delta x_i) \approx \mathrm{f}(x_i) + \mathrm{f}'(x_i)\Delta x_i \tag{13}$$

$$AXF \approx FTOT(ZN_j) + FTOT'(ZN_j)CZN \tag{14}$$

$$FTOT'(ZN_j) = \frac{FTOT(ZN_j + \Delta ZN) - FTOT(ZN_j)}{\Delta ZN} \tag{15}$$

$$ZN_{j+1} = ZN_j + CZN \tag{16}$$

where ZN_j and ZN_{j+1} are neutral axis position measured from centroid of the cross section for *j* and *j*+1 iteration, respectively; ΔZN is the increment of neutral axis *ZN*; *CZN* is the correction of neutral axis position.

Observation Method

Computational Procedure

The application of Newton-Raphson method for obtaining neutral axis position is illustrated in Figure 6.(a), where ε is maximum allowable relative error and *MAXITZN* is the maximum number of neutral axis position iteration. Figure 6.(a) also depicts the break-

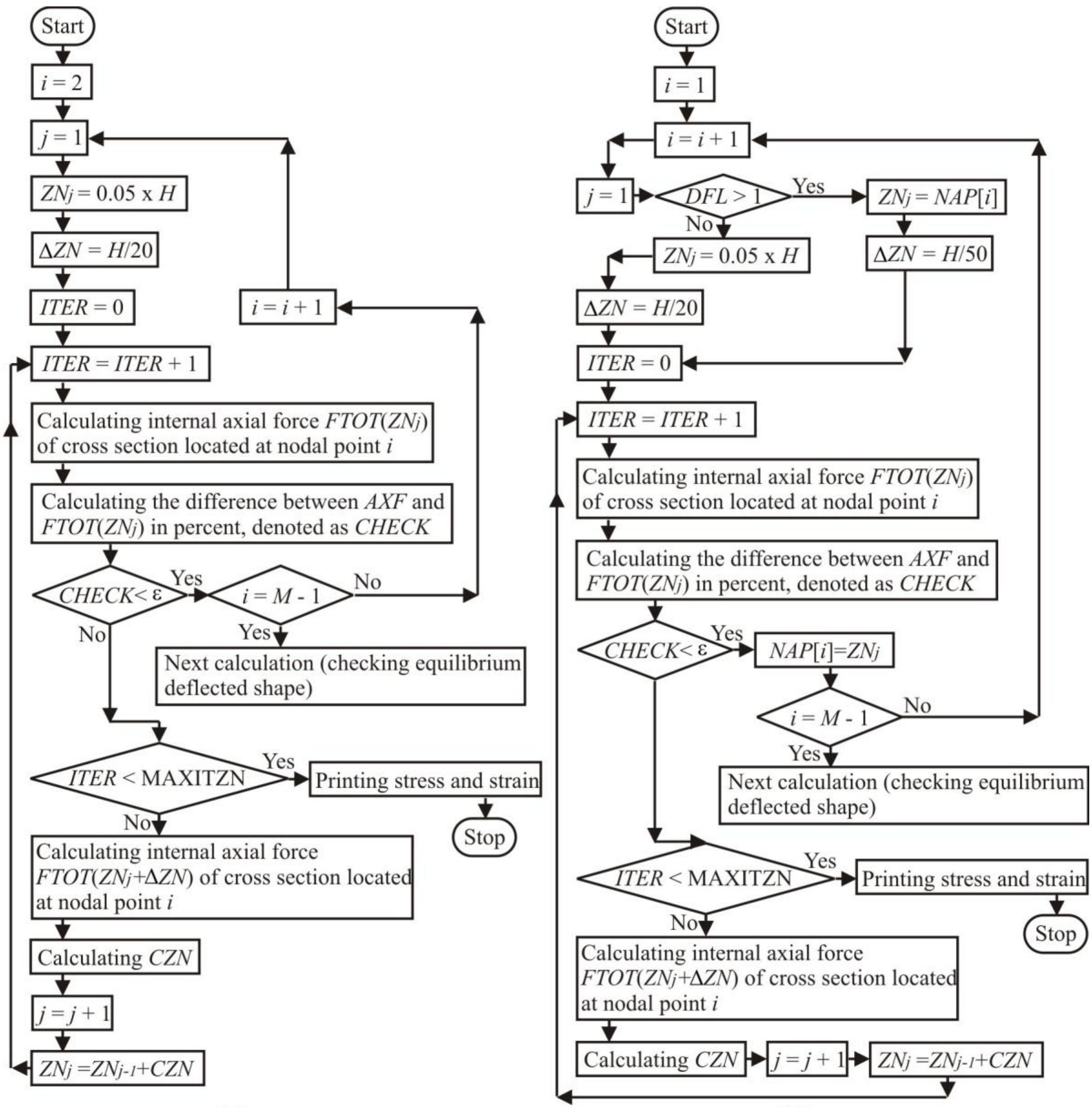

Figure 6. (a) Procedure for obtaining neutral axis position; (b) modification of neutral axis iteration procedure

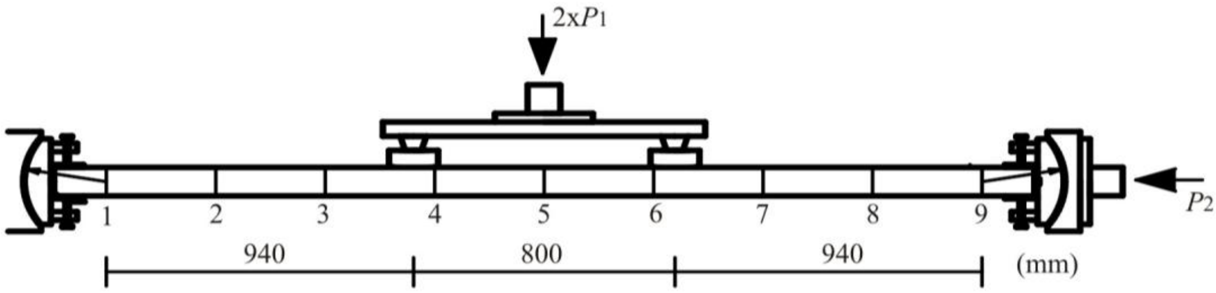

Figure 7. Applied load and nodal point

Table 1. Iteration Result for Obtaining Equilibrium Deflected Shape

Loading Step	Lateral Load (N)	Axial Load (N)	Iteration Result
1	1000	-16000	Equilibrium deflected shape were obtained
2	2000	-16000	Equilibrium deflected shape were obtained
3	3000	-16000	Equilibrium deflected shape were obtained
4	4000	-16000	Equilibrium deflected shape were obtained
5	5000	-16000	Equilibrium deflected shape were obtained
6	6000	-16000	Equilibrium deflected shape were obtained
7	6200	-16000	Equilibrium deflected shape were not obtained
8	6500	-16000	Equilibrium deflected shape were not obtained
9	6700	-16000	Equilibrium deflected shape were not obtained

down of Figure 3 related to the calculation step of neutral axis position.

The initial value of neutral axis position ZN_j and the neutral axis increment ΔZN are determined using Equation (17) and Equation (18).

$$ZN_j = 0.05 \times H \tag{17}$$

$$\Delta ZN = \frac{H}{20} \tag{18}$$

$FTOT(ZN_j)$ and $FTOT(ZN_j+\Delta ZN)$ are calculated by using Equation (10). The correction of neutral axis position CZN is determined by using Equation (14). Maximum allowable relative error ε used in iterative procedure is 0.05%. After the material property, beam dimension, loading configuration, number of nodal point and some coefficients are defined, program is run. Then, the identification of proper initial value for determining neutral axis position was carried out by observing the stress distribution on cross sections that are resulted from numerical program calculation.

Beam Dimension, Load Configuration and Other Data for Calculation

Beam dimension, load configuration, material properties and some coefficients are required for obtaining equilibrium deflected shape and neutral axis position. Beam dimension was 70 mm in width, 100 mm in height and 2680 mm in length. Loading configuration and loading step are illustrated in Figure 7 and Table 1, respectively. Lateral loads P_1 were increased gradually for each calculation and the axial load was constant. The ultimate tension and compression stress parallel to grain are 194.29 MPa and 51.22 MPa,

Table 2. Number Iteration for Obtaining Equilibrium Deflected Shape and Neutral Axis Position

Loading Step	*DFL*	*ITER* at *M*=5
1	7	3
2	7	3
3	8	2
4	8	3
5	9	4
6	10	5
7	7	*ITER* > *MAXITZN*
8	5	*ITER* > *MAXITZN*
9	4	*ITER* > *MAXITZN*

respectively. The elasticity modulus of tension parallel to grain is 17,200 MPa. The non linearity of compressive stress-strain curve is represented by Equation (19).

$$\sigma_{c//} = -2.10^{6}\varepsilon_{c//}^{\ \ 2} + 22048\varepsilon_{c//} - 0.150 \quad (19)$$

where $\sigma_{c//}$ and $\varepsilon_{c//}$ are the compressive stress and strain parallel to grain, respectively. Number of nodal point *M* defined was 9 (see Figure 5). Number of gauss point in *X* direction was 5. Number of gauss point in *Y* direction was 5. Maximum number of equilibrium deflected shape iteration *MAXITDFL* is 30. Maximum number of neutral axis position iteration *MAXITZN* is 25.

Result and Discussion

The Iteration Result of Equilibrium Deflected Shape and Neutral Axis Position

In general, the iteration result of equilibrium deflected shape can be seen in Table 1. Iteration number needed to get equilibrium deflected shape *DFL* can be seen in Table 2. Further, Table 2 also depicts iteration number needed to get neutral axis position *ITER* at nodal point 5 for each loading step.

Referring to Table 1, the equilibrium deflected shape can be obtained for loading step 1 up to 6. It also means that the iteration for defining neutral axis position get convergent result for loading step 1 up to 6. The neutral axis position is achieved when the equilibrium condition for axial force is attained. Then, when internal moments are equal to external moments, it will lead to achievement of equilibrium deflected shape.

Moreover, Table 1 and Table 2 show that the equilibrium deflected shape cannot be obtained when the beam was subjected to loading step of 7, 8, and 9. It is caused by the iteration number for obtaining neutral axis position *ITER* exceeds the allowable maximum number of neutral axis position iteration *MAXITZN*. Further, the question is why the neutral axis position cannot be resulted. It is needed to know whether the iteration has correctly stopped or not. Thus, the observation was carried out on a cross section at nodal point 5 (see Figure 7) which has the maximum tensile and compressive stress.

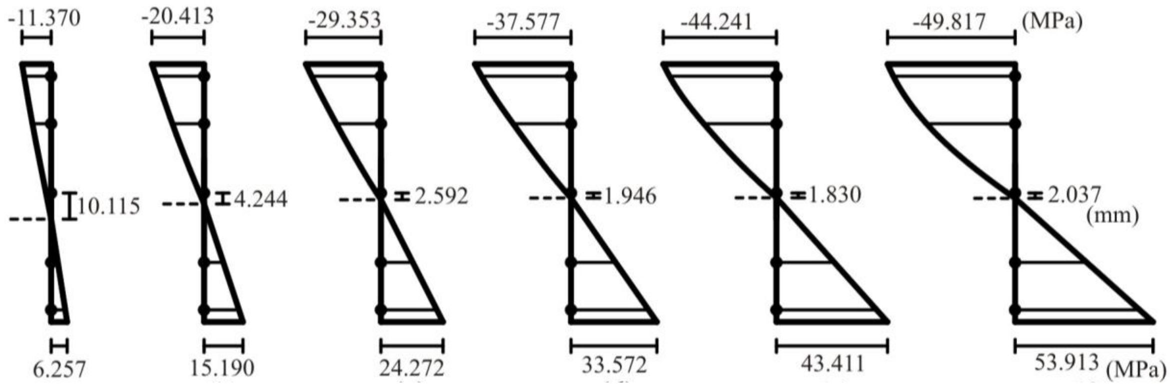

Figure 8. Stress distribution on a cross section located at nodal point *M* = 5 for load number: (a) 1; (b) 2; (c) 3; (d) 4; (e) 5; (f) 6

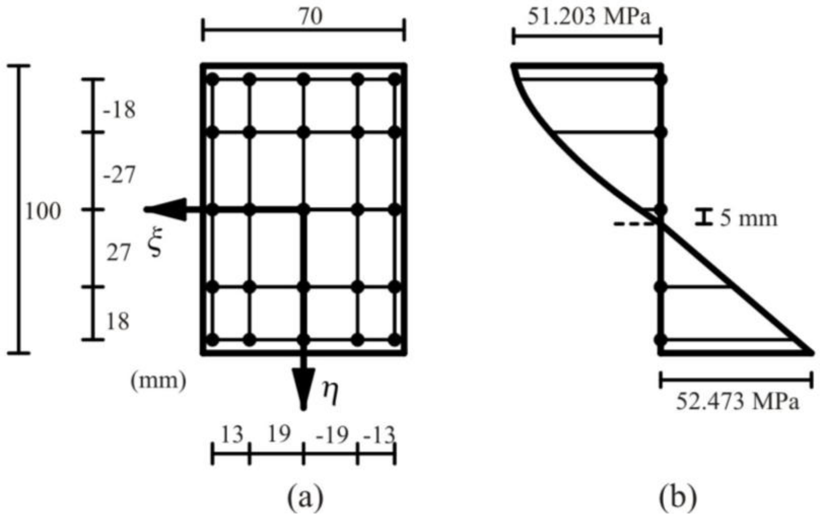

Figure 9. (a) Gauss point position on a cross section; (b) stress distribution at nodal point 5, *DFL* = 7, *ITER* = 1

The Result of Stress Distribution Observation

Figure 8 depicts the stress occurred at Gauss point, uppermost side, as well as bottommost side of a cross section that is located at nodal point 5. The location of Gauss point on a cross section can be seen in Figure 9 (a).

As illustrated in Figure 8, the neutral axis position at nodal point 5 is 10.115 mm, 4.244 mm, 2.592 mm, 1.946 mm, 1.830 mm and 2.307 mm when the structure was accordingly subjected to loading step 1 to 6. It depicts that the neutral axis position tends to move upward when the lateral load is increased while the axial load is constant. In case of loading step 6, maximum tension and compression parallel to grain are 53.913 MPa and 49.817 MPa, respectively. According to the mechanical properties of glulam bamboo, the ultimate tension and compression stress are 194.29 MPa and 51.22 MPa, respectively. It means that the structure had not failed yet, neither in tension or compression. When the load was increased, as can be seen in Table 2, equilibrium deflected shape cannot be obtained because iteration number of neutral axis position *ITER* is greater than the allowable maximum number of iteration for neutral axis position *MAXITZN*. This reason causes the iteration of neutral axis position cannot achieve convergent result. Regarding structure subjected to axial compression and lateral load as well as the assumption that the stress is zero when it is greater than the ultimate stress, non convergent

Table 3. Neutral Axis Position *ZN* at Nodal Point *M* = 5 for Every *DFL* When Structure is Loaded with Load Number of 7

DFL	***ZN* (mm)**	***DFL***	***ZN* (mm)**	***DFL***	***ZN* (mm)**
1	59.272	4	2.004	7	Stop at *M*=5
2	1.817	5	2.067	-	-
3	1.915	6	2.105	-	-

Table 4. Neutral Axis Position *ZN* for Every Iteration of Neutral Axis Position *ITER* When Structure is Loaded with Load Number of 7 at Nodal Point *M* = 5, Iteration Number of Equilibrium Deflected Shape *DFL* = 12

ITER	***ZN* (mm)**	***ITER***	***ZN* (mm)**	***ITER***	***ZN* (mm)**
1	5	11	7.258	21	17.291
2	20.128	12	28.488	22	7.256
3	6.523	13	20.47	23	28.479
4	25.738	14	6.422	24	20.471
5	35.323	15	25.407	25	6.422
6	18.05	16	34.851	26	25.406
7	7.128	17	18.127		
8	28.002	18	7.115		
9	38.448	19	27.95		
10	17.276	20	38.376		

result will take place if compressive stress is greater than ultimate stress of material.

Table 2 shows that iterative calculation for obtaining equilibrium deflected shape of structure that was subjected to loading step 7 stopped when the iteration number of equilibrium deflected shape DFL is equal to 7. In detail, the iteration can be seen in Table 3. It informs that, the neutral axis position tends to be located around 2 mm from the centroid of cross section. Referring to Figure 8, this location comes close to the neutral axis position for the previous loading when equilibrium deflected shape was obtained. However, in case of loading step 7, the iteration stopped before the neutral axis position was determined. Accordingly, further observation was needed to figure out whether the internal compression stress is greater than the ultimate compression stress of the material.

The observation was conducted by investigating neutral axis position resulted from every iteration *ITER*, as shown in Table 4. It is shown that the uppermost position of neutral axis is 5 mm, resulted from iteration number one. This stress distribution can be seen in Figure 9.(b). It depicts that the maximum compressive and tensile stress are 51.203 MPa and 52.473 MPa, respectively. It implies that the both stress are less than the ultimate stress of material. Referring to Figure 8 and Table 3, if the equilibrium deflected shape of the structure subjected to loading step 7 can be obtained, the neutral axis will be located around 2 mm from the centroid of a cross section. Concerning Figure 9.(b), the compressive stress in which the neutral axis is located around 2 mm is less than that in which the neutral axis is located at 5 mm. It can be concluded that the ultimate compression stress of the material has not been achieved when the structure is subjected to

Table 5. Iteration Result for Obtaining Equilibrium Deflected Shape After Modification on Neutral Axis Iteration Procedure

Loading Step	Lateral Load (N)	Axial Load (N)	Iteration Result
1	1000	-16000	Equilibrium deflected shape were obtained
2	2000	-16000	Equilibrium deflected shape were obtained
3	3000	-16000	Equilibrium deflected shape were obtained
4	4000	-16000	Equilibrium deflected shape were obtained
5	5000	-16000	Equilibrium deflected shape were obtained
6	6000	-16000	Equilibrium deflected shape were obtained
7	6200	-16000	Equilibrium deflected shape were obtained
8	6500	-16000	Equilibrium deflected shape were obtained
9	6700	-16000	Equilibrium deflected shape were not obtained

loading step 7. In line with this conclusion, the iteration for neutral axis position should achieve convergent result. The non convergent result might be caused by inappropriate initial value for neutral axis position.

Modifying Initial Value for Neutral Axis Iteration

According to previous observation result, initial approximation of neutral axis position presented in Equation (17) is modified to Equation (20).

$$ZN_j = \begin{cases} 0.05 \times H, & DFL = 1 \\ NAP[i] \quad \text{for } i = 2 \text{ to } (M-1), & DFL > 1 \end{cases} \tag{20}$$

where *NAP* is array of neutral axis position that is defined from previous iteration for each nodal point.

In principle, the iteration results of neutral axis position are stored in variable array *NAP*. Then, if the equilibrium deflected shape has not been achieved, the calculation is continued by applying previous neutral axis position that is stored in variable array NAP as the next initial value. Its application on numerical program can be seen in Figure 6.(b). Besides the improvement of initial approximation, the increment of neutral axis ΔZN is modified as follows.

$$\Delta ZN = \begin{cases} H/20, & DFL = 1 \\ H/50, & DFL > 1 \end{cases} \tag{21}$$

The modification aims to increase the accuracy for each step of neutral axis iteration.

Using Modified Initial Neutral Axis Position and Its Increment

Table 5 demonstrates the calculation results after initial value of neutral axis position and its increment have been modified. It shows that equilibrium deflected shape can be obtained for loading step 1 up to 8 when the initial value of neutral axis position is defined as Equation (20). As previously described, it also means that neutral axis position can be obtained when the initial neutral axis position written in Equation (20) is used. Further, besides getting convergent result on the iteration of equilibrium deflected shape and neutral axis position, the number of iteration *ITER* needed to achieve neutral axis position after

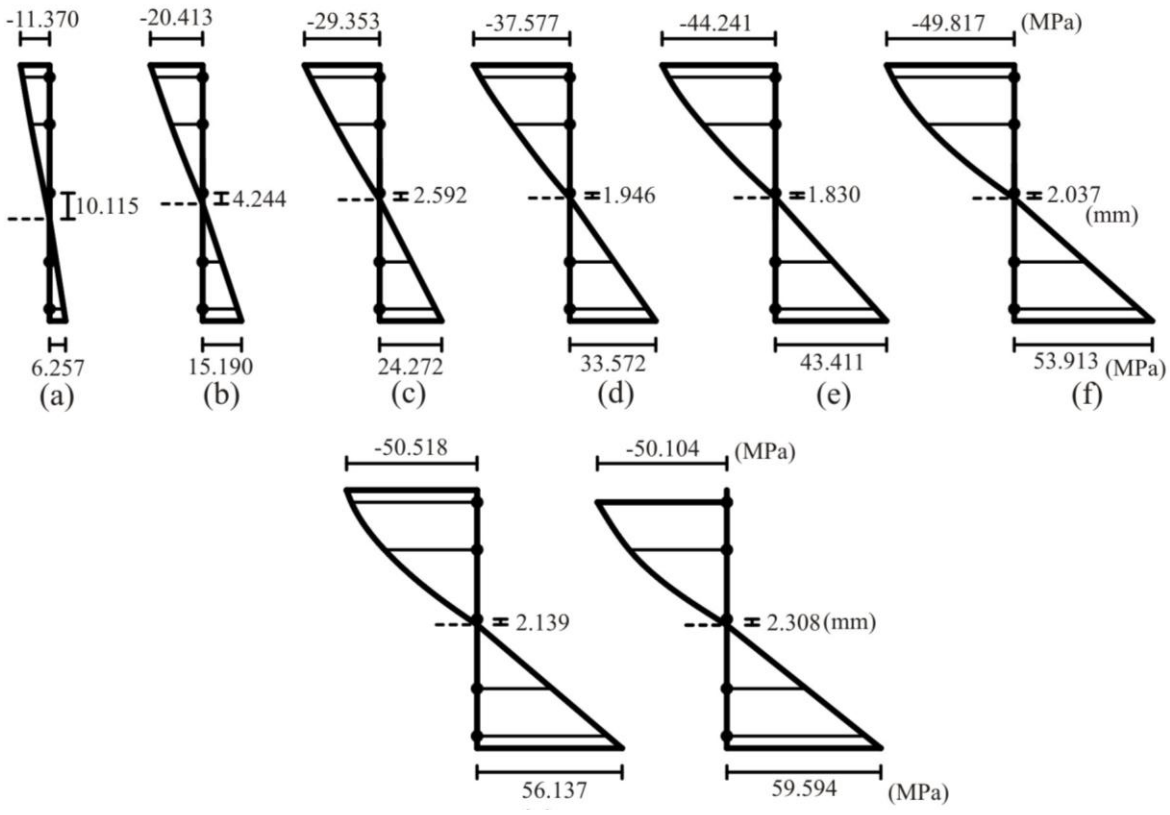

Figure 10. Stress distribution on a cross section located at nodal point $M = 5$ after modification of initial neutral axis position for load number:
(a) 1; (b) 2; (c) 3; (d) 4; (e) 5; (f) 6; (g) 7; (h) 8

Table 6. Number Iteration for Obtaining Equilibrium Deflected Shape and Neutral Axis Position After Modification on Neutral Axis Iteration Procedure

Loading Step	DFL	ITER at M=5	Load Number	DFL	ITER at M=5
1	7	2	6	10	2
2	7	2	7	10	2
3	8	2	8	11	2
4	8	2	9	25	ITER > MAXITZN
5	9	2			

modification is less than that before modification, as depicted in Table 6. It indicates that the modification also can speed up calculation.

The stress distribution and neutral axis position on a cross section located at nodal point 5 can be seen in Figure 10. In case of loading step 7, the compressive and tensile stresses are 50.518 MPa and 56.137 MPa, respectively. It proves the previous conclusion stating that the maximum compressive strength of the material has not been achieved

Figure 10 illustrates that when structure is loaded with loading step 8, the stress on uppermost side has exceeded the ultimate stress of material. As load is increased to load number of 9, internal stress is greater. It can cause the equilibrium condition for axial force cannot be obtained. Hence, the equilibrium deflected shape cannot be resulted, as depicted in Table 5. In line with this reason, the iterative program is already correct. It means that Equation (20) can be considered as appropriate initial approximation for Newton-Raphson iteration to define neutral axis position.

Conclusions

In this study, the appropriate initial approximation for Newton-Raphson method to obtain the neutral axis position of glulam bamboo has been observed. The assumption used for calculating internal forces is that the tension and compression parallel to grain are equal to zero when they are greater than its ultimate stress. Glulam bamboo is assumed as homogeneous material. Based on the observation, Equation (20) can be considered as an appropriate initial value for neutral axis position while Equation (21) is applied for calculating neutral axis increment.

Acknowledgments

The authors would like to acknowledge to Prof. Morisco and Dr. Fitri Mardjono (Department of Civil and Environmental Engineering, Gadjah Mada University, Indonesia) for great supervision during the research. The authors also would like to thank to Prof. Akinori Tani and Prof. Yuichiro Yamabe (Department of Architecture Engineering, Kobe University, Japan) for the facility given during accomplishing this article.

References

[1] S.M. Rittironk, and M. Elnieri, "Investigating laminated bamboo lumber as an alternate to wood lumber in residential construction in The United States," In *Modern Bamboo Structures: Proceedings of the First International Conference*, 1st Edition, X. Yan , M. Inoue, and S.K. Paudel, eds.: Taylor and Francis Group, London, United Kingdom, pp. 83-97, 2008.

[2] Morisco, *Rekayasa Bambu*, Nafiri, Yogyakarta, Indonesia, 1999.

[3] P. Kornerup, and J.M. Muller, "Choosing starting values for certain Newton-Raphson iteration," *Theoretical Computer Science*, Vol. 351, No. 1, pp. 101-110, 2006.

[4] P.V.R. Murthy, and K.P. Rao, "Finite element analysis of laminated anisotropic beams of bimodulus materials," *Computers and Structures*, Vol. 18, No. 5, pp. 779-787, 1984.

[5] M. Brunner, and M. Schnueriger, "Timber beams strengthened by attaching prestressed carbon FRP laminates with a gradiented anchoring device," In: *Proceedings of the International Symposium on Bond Behaviour of FRP in Structures*, International Institute for FRP in Construction, pp. 465-471, 2005.

[6] R. Mu, J.P. Forth, A.W. Beeby, and R. Scott, "Modelling of shrinkage induced curvature of cracked concrete beams," In *Tailor Made Concrete Structure: New Solutions for our Society*, J.C. Walraven, and D. Stoelhorst, eds.: Taylor and Francis Group, London, United Kingdom, pp. 573-578, 2008.

[7] T.E. Shoup, *Numerical Methods for the Personal Computer*, Prentice-Hall, New Jersey, United States, 1983.

[8] W.J. Gilbert, "Generalization of Newton's Method," *Fractals, World Scientific Publishing Company*, Vol. 9, No. 3, pp. 251-262, 2001.

[9] W.K. Chen, and T. Atsuta, *Theory of Beam-Columns: In-Plane Behaviour and Design*, McGraw-Hill, United States of America, 1976.