STUDY ON ROBOT MANIPULATOR USING (REAL-TIME) NONLINEAR CONTROL - SIMULATION OF ROBOT MANIPULATOR UNDER DISTURBANCE

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Abstract

Robot manipulator is currently essential as human assistant. While performing their tasks, the robot leaved its well structured environment and confronted with unexpected disturbances. The conventional linear control system is insufficient to guarantee the prescribe performance of the robot manipulator when unknown external disturbance is considered. Thus, implementing a nonlinear control system is essential to guarantee the performance of the desired tasks. Firstly, the proportional, integral and derivative (PID) controller will be tested and the restriction of the linear control system is studied. Then, two type of nonlinear controller the PID computed torque control (PIDCTC) and Sliding mode computed torque control (SMCTC) are implemented. From the result, the optimal variable value is determined and performances of three controllers are compared.

Keywords: Computed torque control, External disturbance, Proportional integral and derivative, Robot manipulator, Sliding mode control

Introduction

Robot manipulator is a nonlinear system and currently essential as human assistant including industrial automation, medical robot, home appliance, etc. While performing their tasks, the robot leaved its well structured environment and confronted with large degree of uncertainty and unexpected disturbances. Thus the study on capability of position tracking performance is one of important control problem applied for robot manipulator. Proportional, derivative and integral (PID) is one of simplest controller due to its clear physical meaning, easily and separately adjustable parameter especially in the absence of robot knowledge [1]. However tuning the PID gain will be tedious when nonlinear system is considered. Computed torque control (CTC) is one of controller applied for nonlinear system where the advantages is its effectiveness for trajectory control of robot manipulator [2]. Another nonlinear control is the sliding mode control (SMC) which is well-known for its robustness toward model uncertainty and external disturbance [3]. For this study, we will focus on controlling the robot manipulator in the consideration of external disturbance. PID, CTC and SMC will be implemented and the optimal variable gain for the system will be determined.

This paper is organized into 4 sections. Section 2 discusses the methodology including the dynamic of robot manipulator, design PID, CTC and SMC. Next, section 3 provides the result and discussion followed by conclusion in section 4.

Methodology

Dynamics Model of Robot Manipulator

Based on Figure 1 and Table 1, the dynamic equation of motion for robot manipulator joint 1 and 2 in Equation 1 are derived using Lagrange method. Detail derivation of such equation can be referred to many available text book [4].

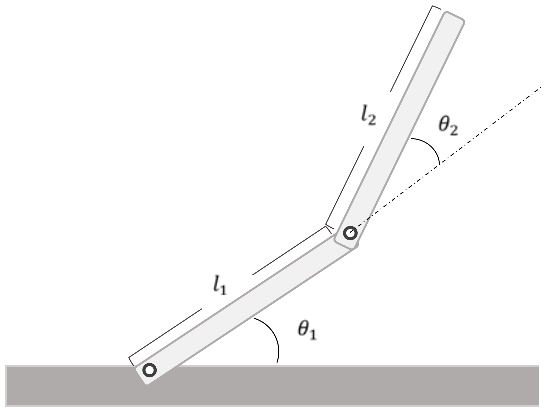

Figure 1. Dynamic model of robot manipulator

Table 1. Specification of Robot Manipulator

Symbol	Definition
l_1	Length of link 1
l_2	Length of link 2
θ_1	Angular position of link 1
θ_2	Angular position of link 2
m_1	Mass of link 1
m_2	Mass of link 2
g	Gravitational acceleration

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = [A]\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + [B]\begin{bmatrix}\dot{\theta}_1\dot{\theta}_2\\ \dot{\theta}_2\dot{\theta}_1\end{bmatrix} + [C]\begin{bmatrix}\dot{\theta}_1{}^2\\ \dot{\theta}_2{}^2\end{bmatrix} + [D] \quad (1)$$

Where the A is the $n \times n$ generalized inertia matrix as describe below:

$$[A] = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} \quad (2)$$

$$A_{11} = \frac{1}{3}m_1 l_1{}^2 + m_2 l_1{}^2 + \frac{1}{3}m_2 l_2{}^2 + m_2 l_1 l_2 cos\theta_2 \quad (3)$$

$$A_{12} = \frac{1}{3}m_2 l_2{}^2 + \frac{1}{2}m_2 l_1 l_2 cos\theta_2 \quad (4)$$

$$A_{21} = \frac{1}{3}m_2 l_2{}^2 + \frac{1}{2}m_2 l_1 l_2 cos\theta_2 \quad (5)$$

$$A_{22} = \frac{1}{3} m_2 {l_2}^2 \tag{6}$$

B is a $n \times n$ matrix that represents the Coriolis force at the first link due to the velocity at the second link. It is happened due to the first link act as the rotating frame for the second link (Niku, 2011).

$$[B] = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \tag{7}$$

$$B_{11} = -m_2 l_1 l_2 sin\theta_2 \tag{8}$$

$$B_{12} = B_{21} = B_{22} = 0 \tag{9}$$

C is described below as a $n \times n$ matrix is a centripetal term caused by the centrifugal effect.

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{10}$$

$$C_{11} = C_{21} = 0 \tag{11}$$

$$C_{12} = -\frac{1}{2} m_2 l_1 l_2 sin\theta_2 \tag{12}$$

$$C_{22} = -C_{12} \tag{13}$$

D is a $n \times m$ matrix related to the gravitational acceleration,

$$[D] = \begin{bmatrix} D_{11} \\ D_{12} \end{bmatrix} \tag{14}$$

$$D_{11} = \left(\frac{1}{2} m_1 + m_2\right) g l_1 cos\theta_1 + \frac{1}{2} m_2 g l_2 cos\theta_{12} \tag{15}$$

$$D_{12} = \frac{1}{2} m_2 g l_2 cos\theta_{12} \tag{16}$$

Linear PID Control System

The PID control law in its standard form is in Equation 17 [5].

$$\tau(t) = K_P e(t) + K_D \frac{de(t)}{dt} + K_I \int_0^t e(t) dt \tag{17}$$

K_P,$K_I$and $K_D$is the proportional, integrator and derivative gain respectively, $e(t) = \theta_d - \theta$ is the tracking error of the system.

The performance of PID control system strictly relied on the value of gain K_P, $K_I$and K_D. The gain of PID controller is tuned based on the individual effect of the three terms in closed loop performance and can be referred to[6]. However, for this case a combination of PID gain modification will be considerate. The K_P gain is vary from 5, 10 and 15,$K_I$gain vary from 0 to 5 and K_D gain is vary from 5, 10 and 15.

Computed Torque Control (CTC) System

Computed torque control (CTC) which consist of two main parts the feed-forward and the feedback component. The feed-forward component is a nonlinear compensation provides the amount of torque required to drive the system along its nominal path. On the other hand, the feedback component provided a corrective torque to reduce any error along the trajectory of manipulator. PID computed torque control (PIDCTC) is one of the controllers which implemented the PID as the feedback component.

The overall PIDCTC control input is expressed in Equation 18.

$$\tau = A(\theta)\left[\ddot{\theta}_d + K_P e + K_D \frac{de}{dt} + K_I \int e\, dt\right] + B\big(\theta,\dot{\theta}\big)\dot{\theta} + D(\theta) \tag{18}$$

The K_P,$K_I$and K_D gain are tuned to obtain the best performance of position control for the robot system. The gain is chosen to vary from 0, 20, 60 to 100.

Sliding Mode Control (SMC) System

The feedback component in CTC can be designed using many different approached. The sliding mode control (SMC) is one of it and known as a sliding mode computed torque control (SMCTC). The objective of the SMC part is to drive the system trajectory from the initial condition to the sliding surface and remain at the surface by keeping the switching function near zero. In designing the SMC law, first of all, there is a need to design a sliding surface where the state trajectory of the robot manipulator is restricted to such a surface in order to obtain the desired response.The second step is to construct a control input that drives the state of the robot manipulator to the sliding surface, and keeps it there [7], [8].The control input is expressed in Equation 19. Thus, such controller exhibit robustness characteristic toward unknown external disturbance and is modeled as periodic sine wave in Equation 20.

$$\tau = A(\theta)\big[\ddot{\theta}_d + c\dot{e} + ksign(s)\big] + B\big(\theta,\dot{\theta}\big)\dot{\theta} + D(\theta) \tag{19}$$

$$25 \sin 0.1t \tag{20}$$

However, due to discontinuous control action of $\dot{s} = ksign(s)$ lead to high frequency oscillation called chattering. Fortunately, such problem can be eliminated by introducing the boundary layer approach in which replacing the signum function with saturation function [3],

The term in Equation 21 is known as a constant rate reaching law, where the k value determined the reaching performance of the robot manipulator and also influenced the tracking performance.

$$\dot{s} = ksign(s) \qquad\qquad k > 0 \tag{21}$$

The sliding condition, c and reaching gain, k is tuned to obtain the best performance for robot manipulator. The gain value is varied from 20, 60 and 100.

Result and Discussion

Step Response of PID, PIDCTC and SMCTC

From the result, shows that PID performance satisfied the condition in[6]. The effect of varying $_P$, $_D$ and $_I$ can be referred in Table 2.

Table 2. **Effect of Varying PID Gains**

Variable			Percent Overshoot, PO (%)		Steady State Error, sse (%)	
K_P	K_I	K_D				
5	0	5	2.73	0.01	1.47	10.28
10	0	5	10.15	0.73	0.82	6.42
15	0	5	63.51	58.19	0.72	5.08
15	0	10	0.99	0.00	0.47	4.56
15	0	15	0.58	0.00	0.45	4.53
15	5	15	9.34	17.69	1.58	3.46

Based on Figure 2, increasing K_P, improved the steady state error. However, it caused overshoot to increase. Since the overshoot can be compensated by derivative term, as will be discussed in the next paragraph, we will choose higher gain of K_P as 15.

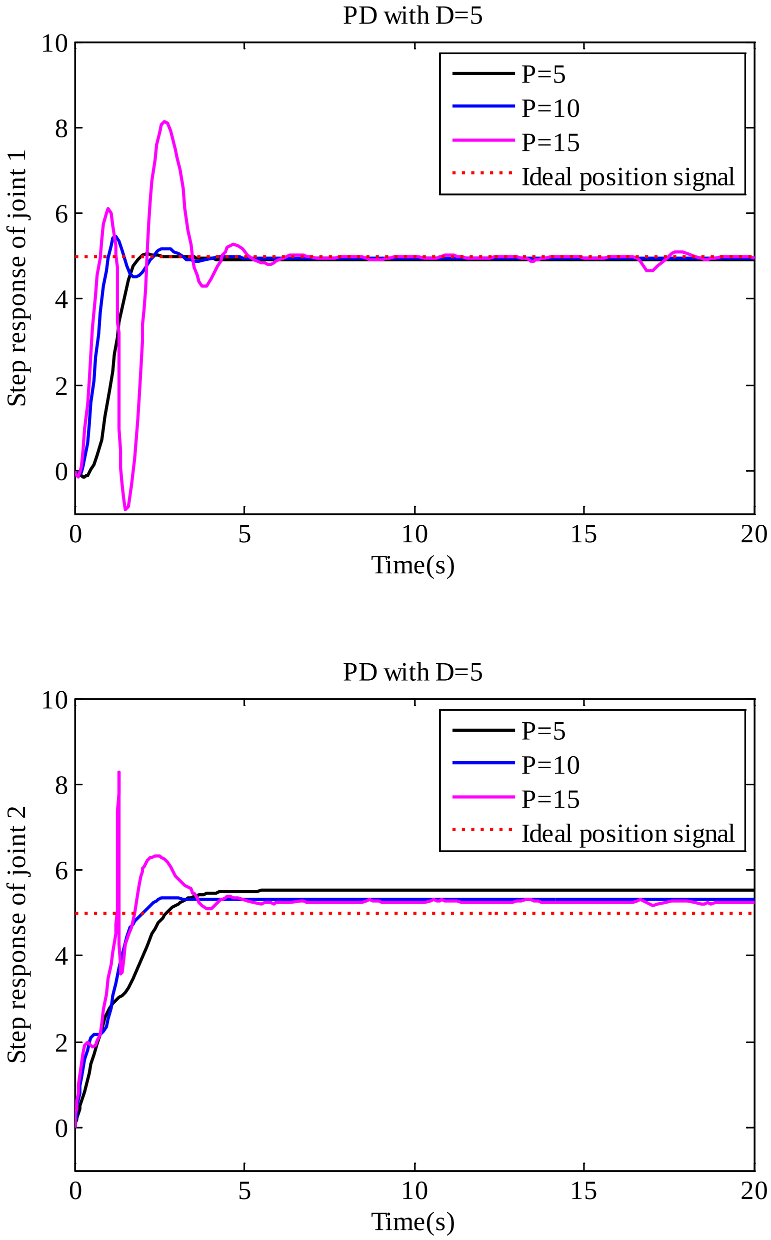

Figure 2. Step response PD for varyingK_P

Referring to Figure 3, the effect of increasing $K_D$when K_P is 15, caused reductions in overshoot and steady state error compare to K_D = 5 and 10. Thus, K_D is chosen as 15.

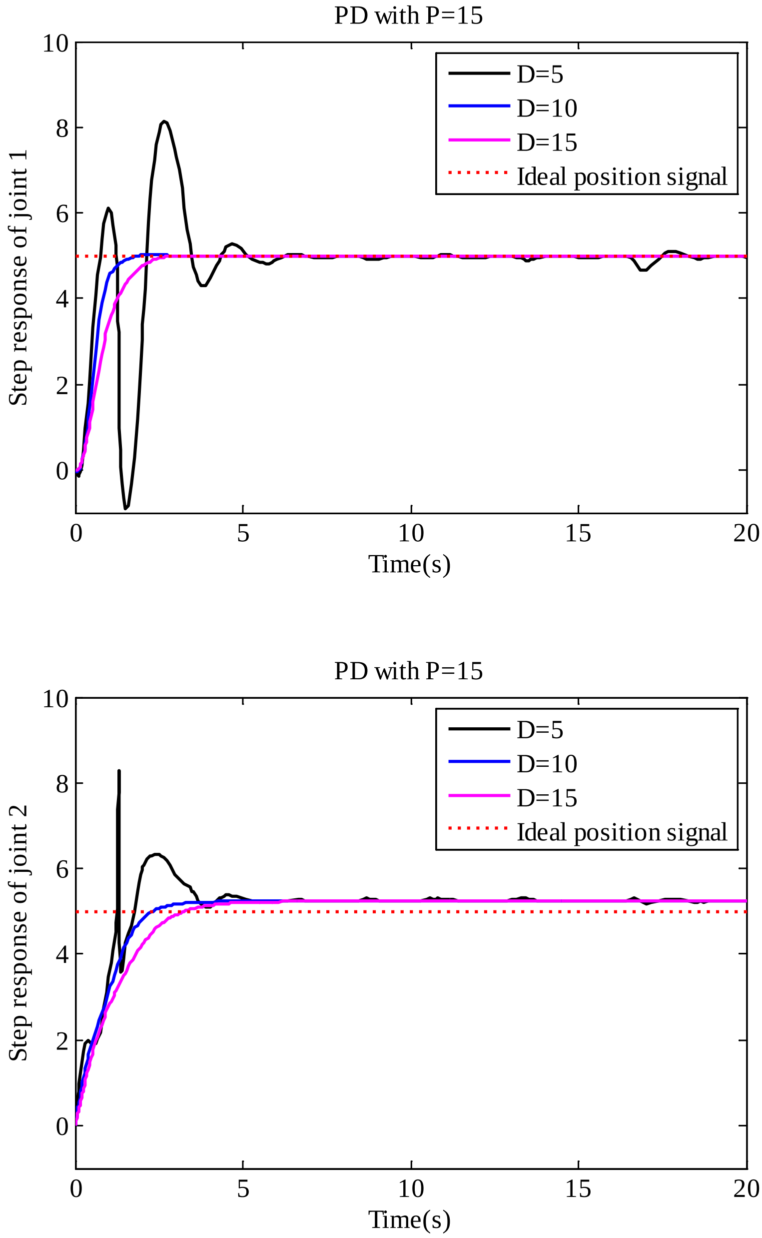

Figure 3. Step response PD for varying K_D

Based on the above discussion, setting K_P and K_D as 15, the effect of installing integral term, $K_I = 5$, improved the steady state error especially for joint 1. However it negatively affects the overshoot of the system as illustrated in Figure 4.

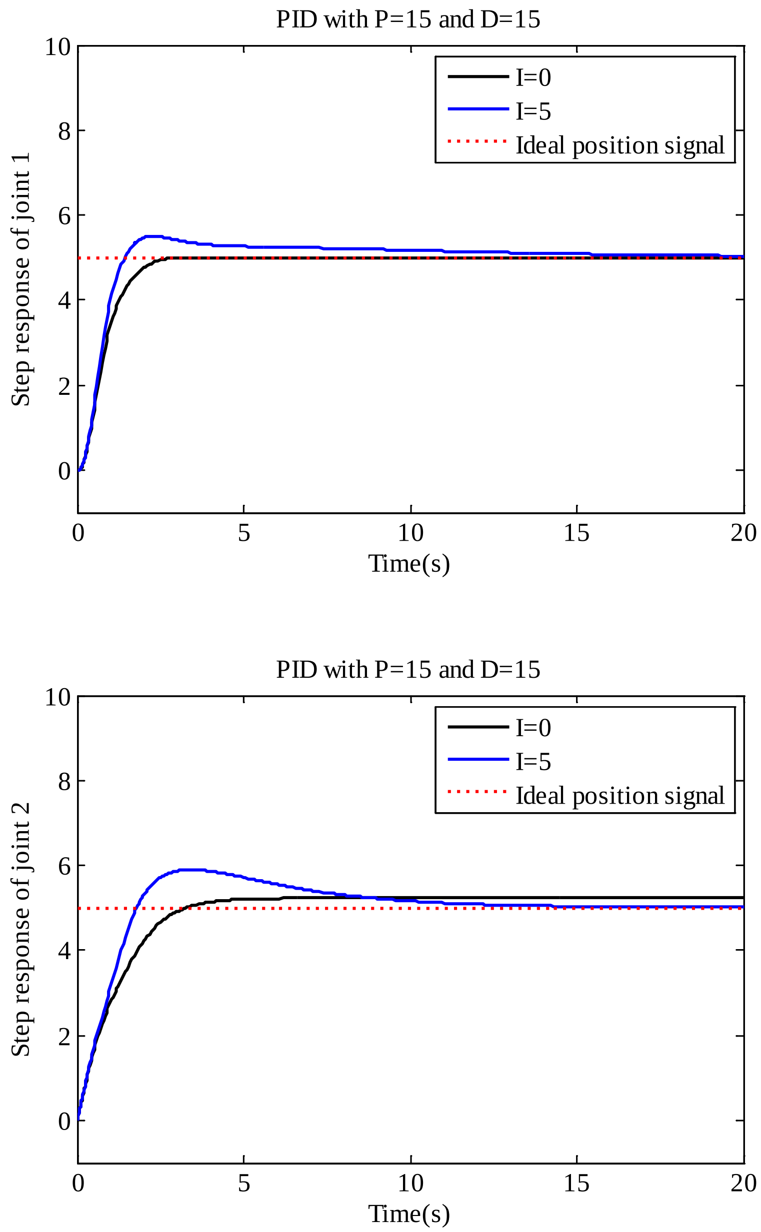

Figure 4. Step response PID for varying K_I

Thus the combination of PID gain is chosen as, $K_P = 15$, $K_I = 5$ and $K_D = 15$ or $K_P = 15$, $K_I = 0$ and $K_D = 15$. The selection between such gains depends on the requirement of the system. For the system which can tolerate the overshoot but require high accuracy, the PID is a better choice. However, if the accuracy is not the matter to be concern, PD is better choice. The result obtained from PID controller is a trade-off between steady state error and overshoot.

The PID gain in PIDCTC does not follow completely the condition in [6]. Table 3 shows the performance of varying the PIDCTC gains. Varying K_P gain is shown in row 1, 2 and 3. Using the lowest K_P = 20, yield the highest PO for joint 1. Increasing K_P to 60 reduced the PO to 0.15. Further increase to 100, slightly increase the PO to 0.16. However, using the highest K_P= 100, produced the lowest SSE. Thus higher proportional gain is better. $K_P$is chosen as 100.

Table 3. Effect of Varying PIDCTC Gains

Row	Variable			Percent Overshoot, PO (%)		Steady State Error, SSE (%)	
	K_P	K_I	K_D				
1	20	0	20	1.33	0.00	1.58	14.06
2	60	0	20	0.15	0.01	1.06	5.00
3	100	0	20	0.16	0.03	0.81	2.82
4	100	0	60	0.01	0.00	0.78	2.77
5	100	0	100	0.00	0.00	0.76	2.71
6	100	0	20	0.16	0.03	0.81	2.82
7	100	20	20	3.80	6.95	0.78	0.53
8	100	60	20	9.06	13.37	0.21	0.12
9	100	100	20	13.73	18.07	0.11	0.07

The effect of varying K_D gain is shown in row 3, 4 and 5. Increasing K_D give a similar pattern as in [6]. PO reduced as increased the K_D gain. There is slightly no change in steady state error for both joints. However, considering the settling time of the system, as increasing the K_D gain, increased the time taken for the system to reach the steady state from 0.53s to 3.42s and 0.72s to 4.50s for joint 1 and joint 2 respectively. Thus, K_D gain is chosen as 20.

Row 6, 7, 8 and 9 shows the effect of varying K_I from 20, 60 to 100. The obvious effect is increasing the percent overshoot as increasing the K_I gain. However, it improved the SSE for joint 1.

Thus the combination of PIDCTC gain can be chosen as $K_P = 100$,$K_I = 100$and $K_D = 20$ or $K_P = 100$, $K_I = 0$ and $K_D = 20$. The selection between such gains shared the similar concept as PID controller.

The result SMCTC for varying c and k shows the SMC exhibit virtually no steady state error and overshoot. Thus for choosing the best gain for the system, the fast reaching mode is considered. Lower$c$ value and higher k provided the fastest reaching mode. Thus, c and k is chosen as 20 and 100 respectively.

Tracking Performance of Controllers

Table 4 shows the comparison of tracking performance between the three controllers without considering the presence of disturbance and in the presence of disturbance. PIDCTC is better tracker than PID controller. However, when the system imposed to an external disturbance, the performance of PIDCTC is degraded as the tracking error increase drastically. While, SMCTC performance is not affected by disturbance.

Table 4. Comparison of Tracking Error

Control	Joint	Average Tracking Error	
		No Disturbance	With Disturbance
PID	1	0.29	1.05
	2	0.14	0.91
CTC	1	0.05	0.13
	2	0.04	0.53
SMC	1	0.00	0.00
	2	0.00	0.00

As shown in Figure 5, the result for SMCTC is overlapped with the ideal position track. That means the system is able to track the ideal position perfectly compared to PIDCTC and PID controller with tracking error explicitly lowest among all controllers.

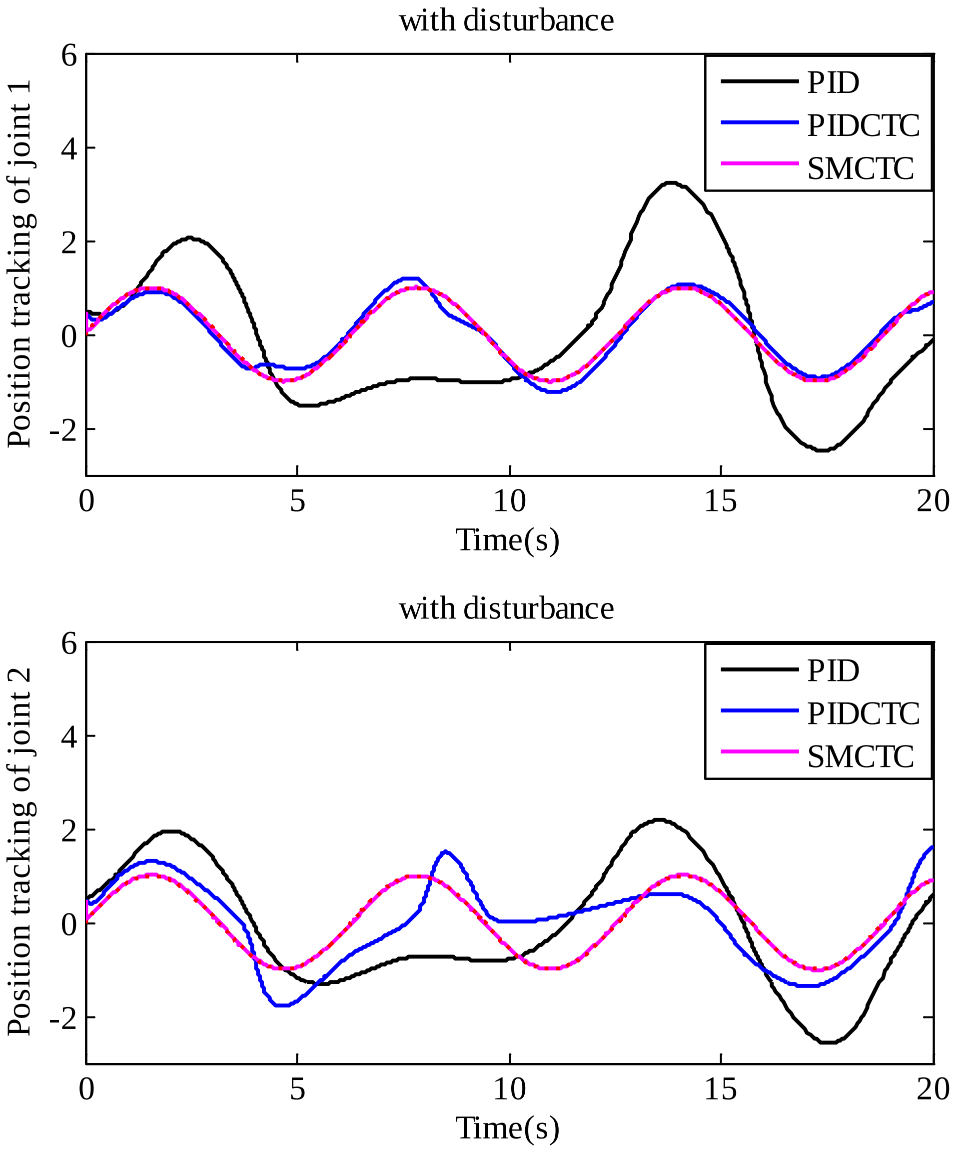

Figure 5. Comparison position tracking for PID, CTC and SMC with disturbance

Conclusions

For the proposed robot manipulator system, the PID gain can be chosen as $K_P = 15, K_I = 5$ and $K_D = 15$ or $K_P = 15, K_I = 0$ and $K_D = 15$ and the PIDCTC gain are $K_P = 100, K_I = 100$ and $K_D = 20$ or $K_P = 100, K_I = 0$ and $K_D = 20$. The result shows that both controllers unable to performed the lowest overshoot and steady state error for a single gain configuration. However, PIDCTC shows a better tracking performance than PID. But in real application, the robot experienced unknown external disturbance. When installing disturbance in the system, SMCTC is superior in tracking control than PIDCTC and PID. It is proven that the SMC is a robust control system.

For further improvement, model estimation should be considered especially for the controller system required precise knowledge of system modeling. This is essential for controller verification that will be tested on servo motor robot manipulator.

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