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MATHEMATICAL MODELING WITH PARAMETER IDENTIFICATION FOR HEXAROTOR SYSTEM: A HAMILTONIAN APPROACH

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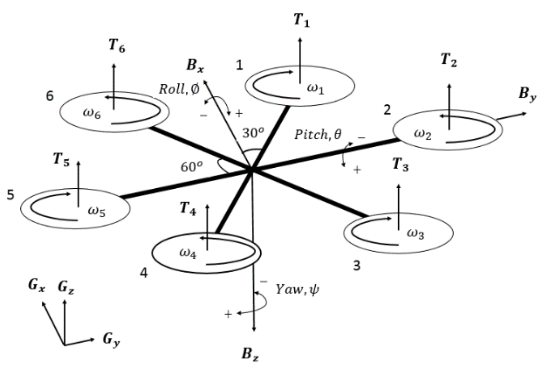
Graphical abstract



Abstract

This paper presents a mathematical modeling with parameters identification of Unmanned Aerial Vehicle (UAV) system or hexarotor system using the Hamiltonian approach. The mathematical model of the hexarotor is derived from the Hamiltonian approach which involved the storage, dissipation, and routing of energy elements from the UAV. This UAV model parameters identification method is proposed as an alternative to the commonly used wind tunnel testing, which is complex and tedious. This Hamiltonian model is made of a fully actuated subsystem with roll, pitch, and yaw angles as output, as well as an under-actuated subsystem with position coordinates as its output. Thrust constant, drag constant and speed of hexarotor are determined through the experimental setup while moment of inertia is determined by physical measurement and calculation. The outcome from this research works demonstrates an undemanding, yet effective method of modeling an UAV, and is useful for designing nonlinear controller to perform the important UAV tasks such as taking off, hovering, and landing.
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Keywords: Hamiltonian Approach, Hexarotor System, Mathematical Modeling, Parameter Identification, Unmanned Aerial Vehicle (UAV)

1.0 INTRODUCTION

Nowadays, there has been surge of interest in multirotor or unmanned aerial vehicles (UAV) in both research and business areas [1]. In addition, UAVs are also commonly used for environmental monitoring, aerial photography, search and rescue mission, military, meteorological purposes and many more. Hexarotor is a type of rotary UAV or multirotor with characteristics such as mechanically simple, has capacity for vertical take-off, landing and hovering, which gives it advantages over other aircraft types. Hexarotor consists of six motors and propellers which are connected to become rotors and attached to a rigid body frame. Compared to quadrotor, two additional rotors on a hexarotor makes it capable of carrying higher payload. Furthermore, it provides greater maneuverability as the dynamics of each angular rotation are attributed by at least four rotors of the hexarotor. These two criteria made hexarotors a preferred choice of UAV as compared to quadrotors [1-4]

However, its advantages come at cost and still faces some challenges, as the hexarotor has a highly nonlinear dynamics, multivariable system, and an under-actuated system with only four actuators having six degree of freedom. Under-actuated systems are characterized as a system that having fewer of control inputs than its degree of freedom. They are difficult to control due to nonlinear coupling between the actuator and the degree of freedom [1, 2]. In addition, research on multirotor previously focus mainly on the multirotor control issue. Due to the two extra rotors, the torque of the hexarotor around each axis differs from the quadrotors, and consequently affects the dynamical reaction differently [6, 7]. As a result, multirotor mathematical modeling is critical for both mechanical and electronic systems to address its stability analysis and controller design issues.

To design and implement a UAV control system, a precise multirotor parameter values, such as mass, moments of inertia, and aerodynamic parameters are critical to develop a correct mathematical model. This mathematical modeling can be derived by using the Newtonian, Lagrangian, and Hamiltonian approaches [4, 5, 10]. From these three approaches, Newtonian deals with force and acceleration, while both Hamiltonian and Lagrangian deal with energy. But, the physical concept of energy is more closely associated with Hamiltonian mechanics than with Lagrangian mechanics.

One of the multirotor parameters, rotary inertia varies with the rotation of the multirotor in the inertial frame and stored in the generalized momentum. And as the former states of Hamiltonian consist of generalized momentum, it could simplify the method of model construction and therefore this makes the model more concise as compare with the Newtonian and Lagrangian models [1]. In addition, the Hamiltonian model has variety of applications in the field of control, such as turbo-generators and power systems. As the port-Hamiltonian approach is closer to physical modeling and is capable to capture more information than just the energy-balance of passivity, it has recently been proven that it could be applied to the control design for quadrotor systems [8,9].

In this paper, we derived a novel mathematical modeling using the Hamiltonian method which then can be applied to develop proper methods for hovering, stabilization and trajectory control of the hexarotor. Then, parameter identifications of hexarotor such as mass moment of inertia, thrust constant and torque constant are identified through the laboratory experiments and standard formula calculations. F550 hexarotor frame kit model was used in the experiments.

2.0 MATHEMATICAL MODELING

The mathematical model of Unmanned Aerial Vehicle (UAV) or multirotor system namely hexarotor system can be derived from three well-known mathematical modeling, such as from a classical mechanics approach of *Newton-Euler method*, a conservation of energy approach of *Euler-Lagrange method* and a total energy approach of *Hamiltonian method*. This paper will focus on modeling of UAV via the Hamiltonian approach.

Hexarotor Hamiltonian Dynamics Model

The dynamics model of hexarotor is derived based on Hamiltonian approach. Hamiltonian formalism is similar to Lagrangian formalism and both formulations are convertible by Legendre transformation. In addition, Hamiltonian mechanics derivation also possible to achieve by Legendre transformation.

Figure 1 shows the six rotors attached to a hexarotor rigid body frame in the "X" configuration. Let *{G} = {G_x, G_y, G_z}* denote an inertial frame with *{B} = {B_x, B_y, B_z}* be a body-fixed frame for the hexarotor airframe. The body-fixed frame {*B*} has its positive z-axis downward following the standard aerospace convention. All rotors (motor + propeller) are labeled 1 to 6 respectively where rotors with odd number rotate counterclockwise (ω_1, ω_3, ω_5) and rotors with even number rotate clockwise (ω_2, ω_4, ω_6). This opposite direction of propeller is important for hovering of the UAV since the three of the propellers will push air upward while the remaining will push air downward. *T* denotes the thrust generates by each rotor and *d* is the distance from center of rotor to center of mass. A rotation matrix R in the special orthogonal group SO(3) can define the orientation of the rigid body frame {*B*} relative to the inertial frame {*G*} [6, 11].

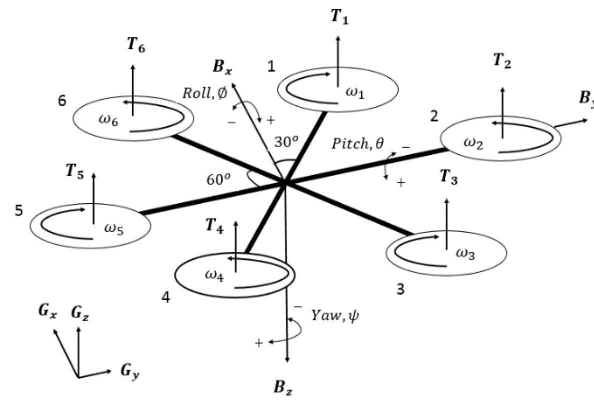

Figure 1 Notation for hexarotor equations of motion in "X" configuration

The dynamic models of hexarotor are the combination of translational and rotational coordinates. Let the generalized coordinates be the vector $\boldsymbol{q} = [\boldsymbol{\xi}^T\ \boldsymbol{\eta}^T]^T \in \mathbb{R}^6$, where $\boldsymbol{\xi} = [x\ y\ z]^T \in \mathbb{R}^3$ denotes the position represent x-position, y-position and z-position of the hexarotor. Z-position is also known as the altitude or height of the hexarotor. While $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T \in \mathbb{R}^3$are Tait-Bryan Euler angles and represent the attitude of hexarotor where roll angle ϕ, pitch angle θ and yaw angle ψ determine the rotation of hexarotor around x-axis, y-axis and z-axis, respectively. The Euler angles are assumed bounded as follows:

$$\phi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \psi \in (-\pi, \pi) \qquad (1)$$

In Lagrange mechanics, the total kinetic energy minus with total potential energy is defined as the Lagrangian value. So, the total kinetic energy on a hexarotor is mostly the thrust force created by motors in translational, KE_T and rotational dynamics, KE_R. Translational kinetic energy of hexarotor is $\boldsymbol{KE_T} = 1/2(\dot{\boldsymbol{\xi}}^{\boldsymbol{T}}\boldsymbol{M}\dot{\boldsymbol{\xi}})$ where $\boldsymbol{M} = m\,\boldsymbol{I_{3\times3}}$, with $I_{3\times3}$ is the identity matrix and m is the mass of hexarotor. The potential energy is $V = -mgz$ where g is the gravitational acceleration and $\boldsymbol{KE_R} = 1/2(\boldsymbol{\zeta^T M \zeta})$ is the rotational kinetic energy where $I = diag(I_{xx}, I_{yy}, I_{zz}) \in \mathbb{R}^{3\times3}$ is the inertia matrix [1, 4, 5, 12].

$$L(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{T}(q, \dot{q}) - \boldsymbol{V}(q) = \boldsymbol{KE_T} + \boldsymbol{KE_R} - \boldsymbol{V} \qquad (2)$$

$$= 1/2(\dot{\boldsymbol{\xi}}^{\boldsymbol{T}}\boldsymbol{M}\dot{\boldsymbol{\xi}}) + 1/2(\boldsymbol{\zeta^T M \zeta}) + mgz \qquad (3)$$

The Euler-Lagrange formalism with external generalized force, u ∈ R^6 can be used to define the dynamic equation of the hexarotor as follows:

$$\frac{d}{dt}\frac{\partial L(\boldsymbol{q}, \dot{\boldsymbol{q}})}{\partial \dot{\boldsymbol{q}}} - \frac{\partial L(\boldsymbol{q}, \dot{\boldsymbol{q}})}{\partial \boldsymbol{q}} = \boldsymbol{u} \qquad (4)$$

Let p denote the generalized momentum, $\boldsymbol{p} = [p_x \;\; p_y \;\; p_z \;\; p_\phi \;\; p_\theta \;\; p_\psi]^T \in \mathbb{R}^6$. The Hamiltonian mechanics approach explain that the Hamiltonian as the summation of the total kinetic energy, $T(\boldsymbol{q},\boldsymbol{p})$ with total potential energy, $V(\boldsymbol{q})$ and focus on generalized position, $\boldsymbol{q}$ and generalized momenta, $\boldsymbol{p}$ variables. Thus, the Hamiltonian for hexarotor can be obtained as follows:

$$H(\boldsymbol{q},\boldsymbol{p}) = T(\boldsymbol{q},\boldsymbol{p}) + V(\boldsymbol{q}) \tag{5}$$

By using Legendre transformation to obtain Hamiltonian equation,

$$\boldsymbol{p} = \frac{\partial L(\boldsymbol{q},\dot{\boldsymbol{q}})}{\partial \dot{\boldsymbol{q}}} \tag{6}$$

Then, the controlled Hamiltonian model for the full hexarotor dynamics with generalized coordinates, $\boldsymbol{q}$ generalized momenta, $\boldsymbol{p}$ and external generalized forces, $\boldsymbol{u}$ can be obtained as follows:

$$\dot{\boldsymbol{q}} = \frac{\partial H(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{p}}; \tag{7}$$

$$\dot{\boldsymbol{p}} = -\frac{\partial H(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{q}} + \boldsymbol{u} \tag{8}$$

Where $\boldsymbol{u} = (\boldsymbol{F},\boldsymbol{\tau})$ with $\boldsymbol{F_b} = (0 \;\; 0 \;\; F_t) \in \{\boldsymbol{B}\}$ is the translational force and the throttle control input in the hexarotor frame and $\boldsymbol{\tau} = (\tau_x \;\; \tau_y \;\; \tau_z) \in \{\boldsymbol{B}\}$ is the total torque applied to the hexarotor airframe with respect to the roll, pitch and yaw moments . The translational force is $\boldsymbol{F} = \boldsymbol{R}_b^i \boldsymbol{F_b}$ where $\boldsymbol{R}_b^i$ is the rotation matrix from the body fixed frame to the inertia frame given by:

$$\boldsymbol{R}_b^i = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \tag{9}$$

The translational force in the body-fixed frame is $\boldsymbol{F_b} = [0 \;\; 0 \;\; T_t]^T$ and T_t is the main thrust and T_i is the thrust moment generated by each motor. The total thrust force, T_t in hovering is the summation of the individual thrust of each rotor and can be expressed as, [6], [13], [14].

$$T_t = \sum_{i=1}^{6} T_i \tag{10}$$

Momentum theory is used to model the steady state thrust generated by hovering motor in free air as,

$$T_i = C_T \omega_i^{\,2} \tag{11}$$

where constant parameter C_T denotes the positive thrust constant of propeller and ω_i is the angular velocity of the motor i for $i = 1,2,3 \dots ,6$ in a hexarotor case. The reaction torque, Q_i due to the drag force acting on the hexarotor airframe generated by hovering rotor can be modelled as,

$$Q_i = C_Q \omega_i^{\,2} \tag{12}$$

with C_Q is a positive torque constant. Then, the generalized torque, $\boldsymbol{\tau}$ for the generalized coordinates of hexarotor is given by:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} d(T_5 - T_2) + (T_6 + T_4 - T_3 - T_1)d/2 \\ (T_1 + T_6 - T_4 - T_3)d\sqrt{3}/2 \\ Q_1 - Q_2 + Q_3 - Q_4 + Q_5 - Q_6 \end{bmatrix} \tag{13}$$

where *d* is the distance from the center of rotor to the center of mass as shown in Figure 1. Thus, the altitude of control input can be defined as

$$u_T = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 + \omega_5^2 + \omega_6^2 \tag{14}$$

Let the attitude of the control input be as $u_\eta = (u_1 \;\; u_2 \;\; u_3)^T \in \mathbb{R}^3$ and it can be described as

$$u_\eta = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} (\omega_5^2 - \omega_2^2) + (\omega_6^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)/2 \\ (\omega_1^2 + \omega_6^2 - \omega_3^2 - \omega_4^2)\sqrt{3}/2 \\ \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2 \end{bmatrix} \tag{15}$$

The equations (7) and (8) can be partitioned into the dynamics of the ξ coordinates and the η coordinates respectively. From equations (7) and (8), we can obtain

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{M^{-1}} & \boldsymbol{0_{3\times3}} \\ \boldsymbol{0_{3\times3}} & \boldsymbol{I^{-1}} \end{bmatrix} \boldsymbol{p}, \tag{16}$$

$$\dot{\boldsymbol{p}} = [0 \; 0 \; mg \; 0 \; 0 \; 0]^T + (\boldsymbol{F},\boldsymbol{\tau}) \tag{17}$$

Finally, by combining equations (7) and (8) with equations (16) and (17), the dynamic model of the hexarotor can be derived as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} p_x/m \\ p_y/m \\ p_z/m \end{bmatrix}; \begin{bmatrix} \dot{p_x} \\ \dot{p_y} \\ \dot{p_z} \end{bmatrix} = \begin{bmatrix} (c\phi s\theta c\psi + s\phi s\psi)T_t \\ (c\phi s\theta c\psi - s\phi s\psi)T_t \\ mg + c\phi c\theta T_t \end{bmatrix} \tag{18}$$

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}p_{\phi}/I_{xx}\\ p_{\theta}/I_{yy}\\ p_{\psi}/I_{zz}\end{bmatrix}, \begin{bmatrix}\dot{p_{\phi}}\\ \dot{p_{\theta}}\\ \dot{p_{\psi}}\end{bmatrix} = \begin{bmatrix}\tau_{\phi}\\ \tau_{\theta}\\ \tau_{\psi}\end{bmatrix} \tag{19}$$

This mathematical model can be divided into two subsystems. First subsystem is a fully-actuated subsystem with three outputs (ϕ, θ, ψ) as in (19) and three inputs ($\tau_{\phi}, \tau_{\theta}, \tau_{\psi}$). The second subsystem is an under-actuated subsystem (18) with three output (x, y, z) and one input, T_t. Thus the whole model of the hexarotor is an under-actuated system.

3.0 PARAMETER IDENTIFICATION

In general, constant values for hexarotor parameters can be identified by several methods. The first method is first principle modeling approach where nominal values of m, d, *I = diag* (I_{xx}, I_{yy}, I_{zz}), c_T, c_Q, and g are identified by the standard formula and experiments. Let m, denotes the mass of hexarotor, d is the distance from center of rotor to center of mass, *I* is the mass moment of inertia, c_T, denotes the torque constant, c_Q is the drag constant and *g* is the gravitational force. The second method is system identification approach by using software or system identification tool in Matlab based on time-domain flight data during the hovering mission [6, 11, 15, 16].

In this research, the first method is chosen to identify the parameters. The mass of hexarotor *m*, was measured by digital weight scale, the distance from center of rotor to center of mass *d* was measured by ruler. The arm length of hexarotor, radius and height of motor were also measured by ruler. While, the gravity force, *g* is assumed constant.

Mass Moment of Inertia

There are several methods to find mass moment of inertia which include physical measurement and calculations, experimental test (bifiliar test or rope suspension approach), technical drawing (CATIA drawing) and system identification in Matlab using black box method. In this paper, first method is selected which are involve the physical measurement of the hexarotor components and then substituted into the specific formula of moment of inertia of hexarotor [1].

Physical Measurement and Calculations

Mass moment inertia of the hexarotor can be determined by using experimental and calculation method. Here mass moment of inertia toward the x-, y-, and z-axis were determined by using the calculation method. Physical measurement of the components of the hexarotor were carried out individually with suitable equipment. Body frame used in this project is a commercial Remote-Control (RC) model of F550 hexarotor platform as shown in Figure 2.

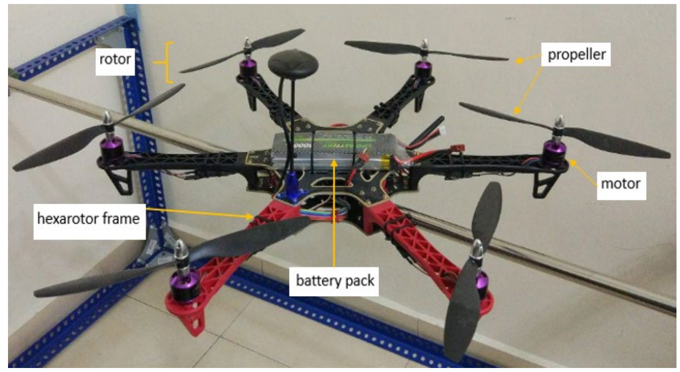

Figure 2. F550 hexarotor on the static platform

The mass moment inertia for the hexarotor, I_{xx}, I_{yy}, I_{zz} are explained as follows (Derawi, D., 2014):

$$I_{xx} = m\left(\left(h_{cg}^{\ 2} + 3r_{cg}^{\ 2}\right)/12\right) + 2m_m l^2 + \left(m_m/6\right)\left(3r_m^2 + 4h_m^2\right) + 2m_r l^2 + \left(m_r r^2/3\right) \tag{20}$$

$$I_{yy} = m\left(\left(h_{cg}^{\ 2} + 3r_{cg}^{\ 2}\right)/12\right) + 2m_m l^2 + \left(m_m/6\right)\left(3r_m^2 + 4h_m^2\right) + 2m_r l^2 + \left(m_r c^2/3\right) \tag{21}$$

$$I_{zz} = \left(m r_{cg}^{\ 2}/2\right) + 4m_m l^2 + 2m_m r_m^2 + \left(m_r\left((2r)^2 + c^2\right)/3\right) + 4m_r l^2 \tag{22}$$

Assumption: The mass of hexarotor *m* is centered at the center of gravity with cylindrical about B_z of radius, r_{cg} and height, h_{cg}. Let m_r denotes the mass of each rotor with radius of blades, *r* and chord length, *c*. Let *l* signifies the arm length and m_m signifies the mass of each rotor with radius r_m and height h_m.

After physical measurement and calibration of the individual components of motors, rotors, body frame and blades of the hexarotor, the mass moment of inertia with respect to x-,y-,z-axis are then calculated based on the measurement values in Table 1 and from formula given in Equations (20), (21), and (22) respectively. The calculated moment of inertia is shown in Table 2.

Table 1. Hexarotor Physical Measurement Values

Names	Symbol	Value	Unit
Mass of Hexarotor	*M*	0.890	*Kg*
Mass of Motor	m_m	0.054	*Kg*
Mass of Rotor	m_r	0.070	*Kg*
Height of Hexarotor	h_{cg}	0.04	*M*
Height of Motor	h_m	0.03	*M*
Radius of Hexarotor	r_{cg}	0.11	*M*
Radius of Motor	r_m	0.0135	*M*
Radius of Blades	*R*	0.125	*M*
Chord Length	*C*	0.027	*M*
Arm Length	*l*	0.275	*M*

Table 2. Hexarotor Mass Moment of Inertia

Names	Symbol	Value	Unit
Mass Moment of Inertia (x-axis)	I_{xx}	0.02197	kgm^2
Mass Moment of Inertia (y-axis)	I_{yy}	0.02162	kgm^2
Mass Moment of Inertia (z-axis)	I_{zz}	0.04366	kgm^2

Static Thrust Test

The thrust coefficient c_T and torque constant $\boldsymbol{c_Q}$ can be obtained by the static thrust test or also known as force lift test. The experiment setup is shown in Figure 3. Figure 3 shows that a rotor system which is consist of motor and propeller assembly is placed on top of the digital weightage. The digital weightage is set to zero and the motor is given a triggering signal from program speed between 0 to 180 with increments of 10. The propeller starts to rotate at a triggering signal at 40 and reach maximum value at 130. This corresponds to 0% to 100% of the propeller full rotational speed. At the same time, the weight generated by the propeller are recorded. The experiment procedures are then repeated for other motors. Figure 4 shows the variation of thrust force with different rotor speed.

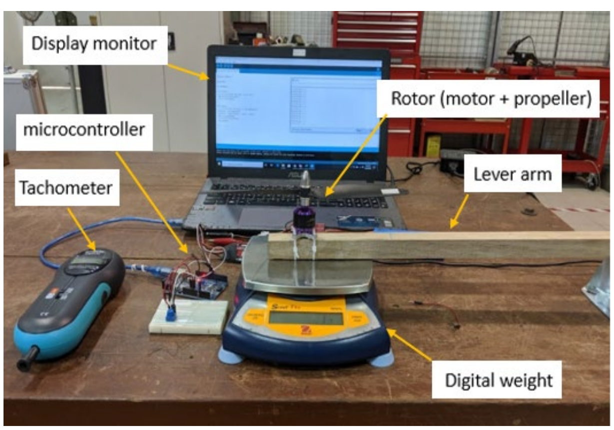

Figure 3 Static Thrust Test Setup

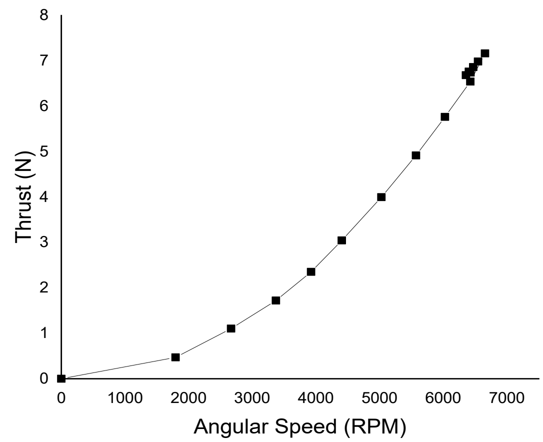

Figure 4 Static Thrust Test

Figure 5 shows the graph of force lift test for all motors. The force-lift generated by the propeller on certain rotational speed is calculated based on Newton's second law, $F = mg$, where m is the mass of the rotor while g is the gravity of the earth. Then, it is linearized to obtain the equation of thrust force F_i generated by motor i, where i = 1, 2, 3, 4, 5 and 6.

$$F_i = a_i + b_i \times (s_t)_i \tag{23}$$

Note that; s_t is the triggering signal from program speed while a_i and b_i are the thrust factors. The thrust factor a_i is assumed as zero, for ideal system while b_i is calculated from the slope of the graph and the linearized force line is shown in Figure 6.

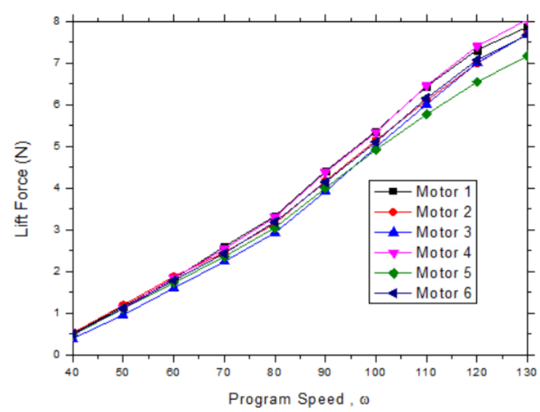

Figure 5 Force Lift Test for Six Motors

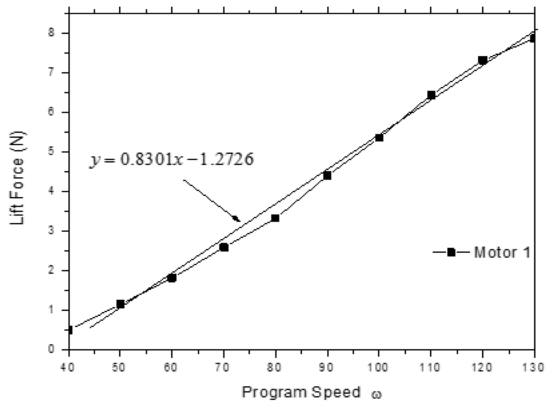

Figure 6 Linear Intersection Line for Motor 1

Based on the momentum theory, the thrust force of propeller is proportional to square of rotational speed, therefore the thrust factor, *b* of the maximum speed of each motor is calculated with the formula in Equation (21).

$$F_i = b\omega^2 \tag{24}$$

where F is the force and ω is the angular velocity. From Figure 6, as the speed is maximum, triggering signal is 130, $\omega = 746\ rad/s, F = 7.671N$. Thus, from Equation (24),

$$b = F/\omega^2 = 7.671/746^2 = 1.378\ x\ 10^{-5} \tag{25}$$

Based on linear equation in Figure 6,

$$y = 0.8301x - 1.2726 \tag{26}$$

and comparing with $y = mx + c$.

$$m = 0.8301;\ and\ a_1 + b = 0.8301 \tag{27}$$

After substituting equation (25) into equation (27), we obtained the thrust factor for rotor 1 as,

$$a_1 = 0.830 \tag{28}$$

This procedure is repeated for motors 2 until 6. After the force lift test for all motors were done and calculated, the average thrust factor or thrust constant, C_T in unit *Ns* is

$$C_T = 8.683 \times 10^{-5} Ns^2 \tag{29}$$

The drag factor or torque constant, C_Q can be determined by $C_Q = C_T L$; where L is the arm length of hexarotor.

Thus, the torque constant, C_Q becomes;

$$C_Q = 2.388 \times 10^{-5}\ Nms^2 \tag{30}$$

Motor Speed Test

Theoretically, timing-pulses are applied to the Electronic Speed Controller (ESC) to determine the speed of the brushless motor. The length of the pulse will decide how fast the motor turns. Shorter duration pulse turns the motor slower while longer duration pulse turns the motor faster. The ESC uses a 50Hz Pulse-Width Modulated (PWM) signal from the controller and with a constant duty cycle, the speed of the motor can be adjusted by changing the frequency value, which can be accomplished by varying the timing-pulse from 1ms to 2ms.

For this speed testing, a dc-brushless motor is used, and different speed sets were programmed in the microcontroller linked to the ESC. The timing-pulse for each ESC produced was varied by increment of 10 and the motor speed was measured by using the tachometer or speed sensor as shown in Figure 7. Tachometer is placed in vertical position above the running rotor and speed of rotor is captured. The results in Figure 8 shows that the motor speed start to increase when the program speed is assigned to 40 rpm and keep increasing until reach the maximum at program speed 130 rpm, and then the motor turn slower after it. The desired speed of each motor will be used to control the throttle input of the hexarotor. Figure 7 and Figure 8 show the experimental set-up for the speed testing as well as the result of speed test for all motors respectively.

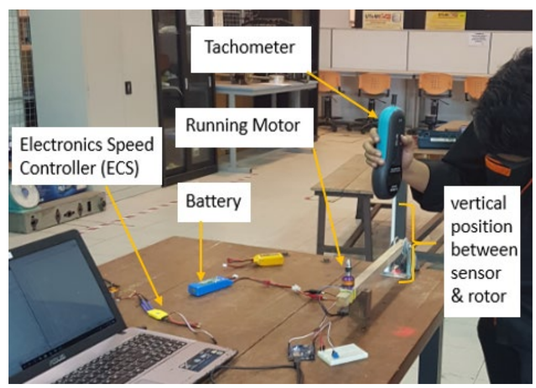

Figure 7 Motor Speed Test Setup

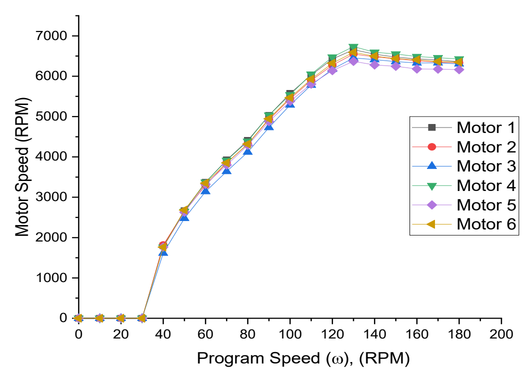

Figure 8 Speed Test for All Motors

The calibration result for speed test, static thrust test or force lift test and physical measurement for the parameter identifications of hexarotor are shown in Table 3. The mathematical modeling of hexarotor is derived and finalized at equations (18) and (19).

In general, modeling is the construction of physical or mathematical equations of the real system. Therefore, modeling is important to reflect the behavior of real systems through a set of mathematical equations. It served many purposes such as solving a problem in a short period or for economic reasons, to ease the manipulation of variables of systems. In this research, modeling of hexarotor is done to acquire testbed model so that parameter identification can be applied, and designed controllers can be validated and tested.

Table 3. Parameters Identification of Hexarotor

Parameter Names	Symbol	Value	Unit
Thrust constant (lift)	c_T	8.683×10^{-5}	Ns^2
Torque constant (drag)	c_Q	2.388×10^{-5}	Nms^2
Thrust factor rotor 1	a_1	0.830	N
Thrust factor rotor 2	a_2	0.821	N
Thrust factor rotor 3	a_3	0.845	N
Thrust factor rotor 4	a_4	0.877	N
Thrust factor rotor 5	a_5	0.771	N
Thrust factor rotor 6	a_6	0.859	N
Moment of Inertia	I_{xx}	0.02197	kgm^2
Moment of Inertia	I_{yy}	0.02162	kgm^2
Moment of Inertia	I_{zz}	0.04904	kgm^2

4.0 CONCLUSION

In this paper, a mathematical modeling for hexarotor using Hamiltonian approach has been proposed. This Hamiltonian modeling is more compact and easier to be used as compared to the model by Newtonian and Lagrangian approaches. Knowing that, the mathematical modeling of the of flight dynamics with the accurate parameters values are the fundamental and important task of developing an UAV control system. Thus, the parameters identification of hexarotor using both experimental and formula computation also have been presented. The outcome from this research works demonstrates an undemanding, yet effective method of modeling an UAV, and is useful for designing nonlinear controller to perform the important UAV tasks such as taking off, hovering, and landing.

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