Full Paper

NUMERICAL INVESTIGATION OF AS-BUILT AND CARBON FIBER REINFORCED POLYMER RETROFITTED REINFORCED CONCRETE BEAM WITH WEB OPENINGS UNDER IMPACT LOADING
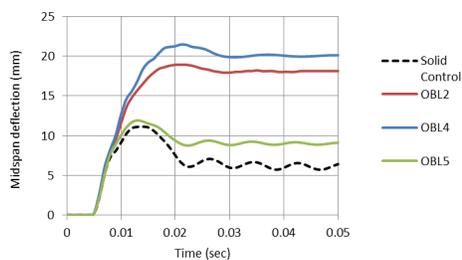
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Abstract

It is not uncommon to provide openings in beams due to utility needs such as mechanical, electrical and sewerage passages. However, no clear guidelines are available in code of practice to handle beam web openings and presence of openings in beams changes behavior of Reinforced Concrete beam into a more complicated one. This research work investigates response of RC beam with openings under impact loading and proposes application of carbon fiber reinforced polymer (CFRP) composites to restore lost beam performance due to the presence of web openings. 3D nonlinear finite element software LS-DYNA is used for model development, validation and parametric finite element analysis work. The accuracy of the nonlinear finite element models are verified using experimental results from literature. Further parametric studies are performed to optimize beam web openings on opening location, size and CFRP no. of layers and fiber orientation. Numerical results showed as compared to control, large opening cutouts in RC beam decreased impact resistance of a beam by 54%. Also, RC beam exhibited poor impact loading resistance close to loading point (mid span) and performed good near shear zone. As compared to control as-built RC beam, Carbon Fiber Reinforced Polymer (CFRP) composites strengthening reduced mid span deflection by 74% and improved beams brittle failure mode. Also, 90^0 fiber orientation complete wrapping resulted in reduction of shear cracking around opening however exhibited low overall impact resistance as compared with 0° fiber orientation.

Keywords: Carbon fiber reinforced polymer; Impact loading; Opening; Reinforced concrete beams, Finite element analysis

1.0 INTRODUCTION

In a building, there are mechanical, electrical and sewerage passage ways access that need to be considered during construction process. Often, a beam causes obstruction to these utilities and a need may arise to provide opening cutouts. However, it is important to know optimum size and location of proposed openings in order to minimize RC performance loss and also to prevent structural failure.

Previous studies showed opening cutouts change overall behavior of RC beam. Openings in RC beam decreases stiffness of a beam, lead excessive deflection and cracking and reduction in strength of a RC beam. Also, due to sudden changes in beam cross section dimensions high stress concentration occurs at opening corners [1,2].

Study on effect of opening shape indicated beams with circular opening and their diameter less than or equals to 44% of overall depth of the beam (without any special reinforcement provided in opening zone) showed similar characteristics to the beams without opening under monotonic loadings however circular opening beams with diameter more than 44% of the depth of the beam reduce the ultimate load carrying capacity of the RC rectangular beams. Comparison between the two shapes, circular opening showed more strength than its equivalent size of square opening with 9.58% difference in their ultimate load carrying capacity [3]. Similarly, an experimental comparative study of the maximum deflection location of the beams with openings and solid beam showed

that maximum deflection location of beams having openings shifts from the center to a point that is in between midspan of the beam and center of opening [4]. Furthermore, FRP wrapped beams exhibit more deflection at midspan than those at another two points. This can clearly show that because of FRP wrapping, shear resistance and the flexural stiffness of beams with openings improved. And the FRP sheet number of layers has significant effect on the stiffness and ultimate load of a beam. The failure modes and gain in the ultimate strength also depends on the FRP orientation, [5]. When flexural region is wrapped U-shape with FRP, is the most effective position of configuration with respect to load carrying capacity. Using FRP U-wrap in the flexural zone, both the flexural and shear capacity increase and this may also prevent failure in brittle manner.

Conventional methods of analysis and design of solid beams cannot be used for RC beams with openings. The effect of opening cutouts on response of a RC beam is unpredictable and codes of provisions have gaps in designing of beams with large opening sizes. These voids left ambiguity for practicing engineers when a need comes to cutout openings in structural members.

Most researches on application of CFRP materials were concentrated on the behavior of Solid Reinforced concrete beams under impact loads. In engineering practice, there are lots of situations in which RC beams with opening undergo dynamic or impact loading. In particular, the impact response of RC beams with web opening and strengthened using CFRP materials is main interest of this study.

Therefore, this research work aims to investigate influence of web openings on the behavior of RC beams under dynamic impact loading and proposes use of carbon fiber reinforced polymer (CFRP) to restore or increase lost performance due the presence of web openings. Also, optimization of web openings in terms of opening location and size are performed for minimum loss in RC beam performance.

2.0 METHODOLOGY

Experimental Specimen for Validation

Reinforced Concrete Beam under impact loading experimentally tested by [6] was adopted for verification of LS-DYNA finite element model. The specimens are solid RC beams, from this S1322 beam specimen is chosen. Herein after, this specimen is used as a control specimen for studying beams with opening such as opening size and location. The Experimental test specimen details and test set up are defined below for the analysed reinforced concrete beam. At the time of testing the concrete compressive strength was 42 MPa. On the compression side 2-D13 bars were used and 2-D22 bars on the tension side. For stirrups D 10 bars were applied. The yield strengths of D10, D13, and D22 were 295Mpa, 397 MPa and 418 MPa, respectively. The RC beam specimens as illustrated in Figure 1 have cross sectional dimensions of 250 mm in depth, 150 mm in width, and 1700 mm in length.

This study simulated impact loading using a drop hammer impact loading machine, as shown in Figure 1. A drop hammer with a mass of 400 kg was dropped freely on the top surface of the RC beam at midspan; from a height of 0.6m for S1322 specimen was chosen. The striking head of the drop hammer

had a hemispherical tip with a radius of 90 mm. Specially designed devices were used to support the beam over a span of 1400 mm, it was allowed to freely rotate but preventing it from move out of displacement i.e simply supported.

The contact force occurred between the RC beam and the hammer was measured by a dynamic load cell that was rigidly tied to the drop hammer. And midspan deflection response was measured by using a laser displacement sensor, which has a thin rubber sheet placed on the bottom part of the RC beam as an aim to measure the response. The data was recorded by computer-based acquisition system of data at 100 kHz sampling rate.
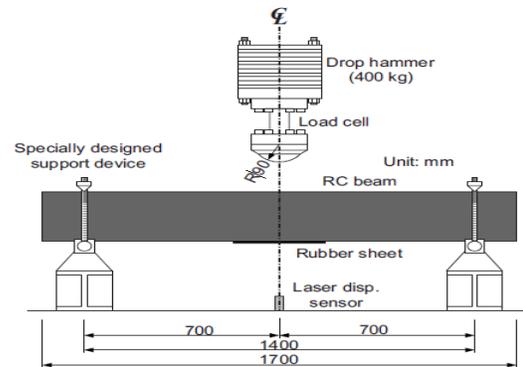
(a) Reinforcement arrangement
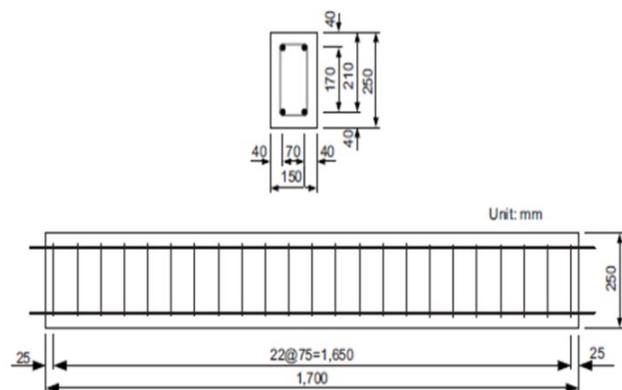
(b) Test set up

Figure 1. Detail of specimen and drop hammer impact test setup tested by [6]

Description of Specimens for Numerical Study

Specimens With Web Opening

A total of 13 specimens are studied to observe the opening effects. Geometry of specimens with varying opening sizes and opening locations parameters are explained below. A review of the literature [7] indicates that the following guidelines can be used to facilitate the selection of the size and location of web openings. Depth of openings should be limited around 50 % of overall beam depth. Openings on beams should not be placed closer than a half of the beam depth *D* near the supports in

order to avoid critical regions from shear failure and congestion of reinforcement. Following these size and location of openings are chosen and clearly seen in Figure 2 and Table 1.

Table 1. Configuration of Specimens with Opening

| Series	Beam Specimen	Location		Size of Opening		Area of Opening(mm²)
		lx(mm)	ly(mm)	l_o (mm)	d_o (mm)	
	S	-	-	-	-	-
	OBSA1	700	125	25	21.25	531.25
	OBSA2	700	125	50	42.5	2125
	OBSA3	700	125	75	63.75	4781.25
	OBSA4	700	125	100	63.75	6375
OBS	OBSA5	700	125	125	63.75	7968.75
	OBSA6	700	125	150	63.75	9562.5
	OBSB1	700	125	100	106.25	10625
	OBSB2	700	125	125	106.25	13281.25
	OBSC1	700	125	150	127.5	19125
	OBSC2	700	125	175	127.5	22312.5
	OBL2	250	125	100	106.25	10625
OBL	OBL4	450	125	100	106.25	10625
	OBL5	700	125	100	106.25	10625
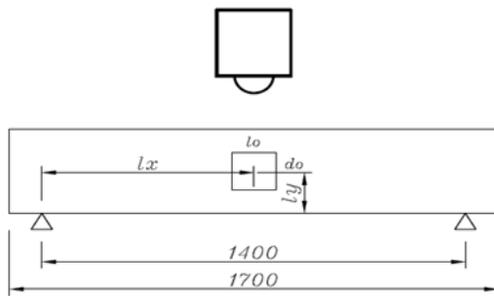
Figure 2. Orientation of openings on the beam

Where **lx** is distance of the opening from the center of the support to center of opening, **ly** is distance of the opening from the bottom of beam to center of opening, l_o length of opening and d_o depth of opening.

Specimens for CFRP Retrofitting

CFRP Number of layer and fiber orientation can influence performance of RC beams. Hence a total of 6 specimens were studied, which are taken from opening location and size specimens, are retrofitted. The specimens are shown in Table 2.

**Table 2.** CFRP Retrofitted Specimens Parameters

Series	Specimen	CFRP no. of layer	Wrap position	Fiber Orientation
	OBL5-L1	1	Complete wrap	0^0
OBL5	OBL5-L2	2	Complete wrap	0^0
	OBL5-L3	3	Complete wrap	0^0
	OBSC2-0	3	Complete wrap	0^0
OBSC2	OBSC2-90	3	Complete wrap	90^0
	OBSC2-0,90	3	Complete wrap	Top & bottom (0^0), Side (90^0)

### Finite Element Modelling In LS-DYNA

Material models, element types, mesh size etc. used for modelling process in LS-DYNA program are explained below. The experimental specimen experimentally tested by [6] is reproduced in LS DYNA. And further parametric studies of opening and CFRP retrofitting continue by modifying this control specimen.

Element Types

Solid element was utilized for modelling of concrete, support steel plate and drop hammer in LS-DYNA. A three-dimensional solid element i.e. 8-noded hexahedron is selected from solid element formulation. Beam element with Truss element formulation is utilized in this study for simulation of steel reinforcement bars. According to [8] this element has three degrees of freedom at each node and carries an axial force. Also, for modelling Carbon Fiber Reinforced Polymer laminates; the Belytschko-Lin-Tsay shell element was implemented as a computationally effective alternative to the HughesLiu shell element. All element formulations are shown in Figure 3.
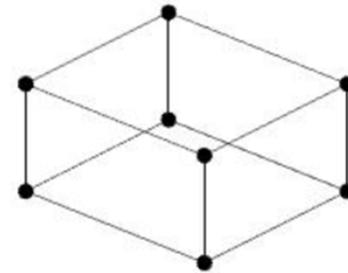
(a) solids
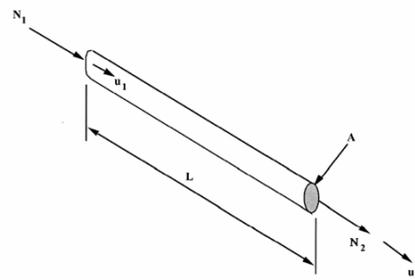
(b) rebar
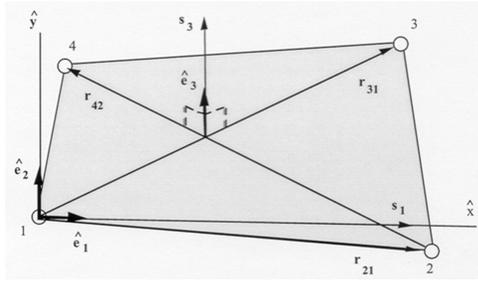
(c) CFRP

Figure 3. Element formulation for (a) solids (b) rebar and (c) CFRP [8]

Material Models

Concrete

Continuous Surface Cap Model, MAT_CSCM is widely used concrete material model available in the LS-DYNA program for RC structures under dynamic loads. This surface implements a multiplicative formulation in order to combine the cap (hardening compaction surface) with the failure (shear) surface smoothly and continuously, [9]. Numerical complexity of treating the compressive 'corner' region between the cap and failure surface is eliminated by the smooth intersection as shown in Figure 4.
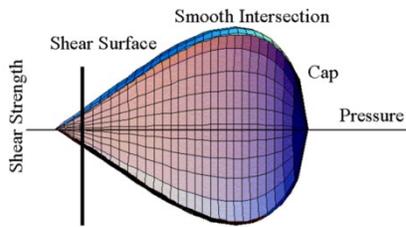
Figure 4. General shape of concrete model yield surface in two-dimensions [8].

Steel Reinforcement

For the steel bars (including longitudinal rebar and stirrups), a plastic kinematic material model (MAT_PLASTIC_KINEMATIC) was applied. This model is appropriate to model kinematic and isotropic hardening plasticity with the option of including rate effects. Formulation of this model is described in Figure 5.
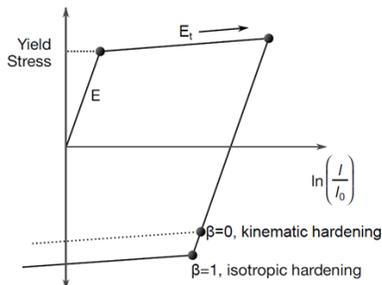

Figure 5: Elastic-plastic behaviour with kinematic and isotropic hardening. [8]

Steel Support Plate and Impact Cube

Material properties of steel support plate and Impact cube is modelled with Elastic material model 1(MAT_ELASTIC). This is an isotropic hypoelastic material and is applicable for shell, beam and solid elements in LS-DYNA.

Carbon Fiber Reinforced Polymer

Material model of ORTHOTROPIC ELASTIC (002) was used to model Carbon Fiber Reinforced Polymer. This material is suited for modelling the elastic-orthotropic behaviour of shells, solids and thick shells. This model will predict realistic behaviour for finite displacement and rotations as long as the strains are small. Unlike others, material Specifications are taken from Manufacturer specification of LINEA FRP system Technology (Italy). The Material properties are described in the Table 3.

Table 3. CFRP Material Property

Material Property	Value
Fibre Density	1.8g/cm^3
Thickness	0.334mm
E-modulus	240 GPa
Tensile Strength	4700Mpa
Elongation	2%
Weight	600g/m^2

FE Geometry, Mesh, Loading and Boundary Condition

Reinforcement bars (longitudinal and stirrups), concrete and steel support plate in the numerical model is exactly the same geometry as experimental control beam specimen reported by [6]. But little geometry modification is made for drop hammer without changing the mass, drop height and material properties. In finite element analysis (FEA), the results accuracy and requested time of computing are determined by the mesh density (finite element size), [10]. According to theory of FEA, the FE models using fine mesh give highly accurate results but provide non-convergence solution. Appropriate mesh size could be selected by many trials until the solution converges to better solution. Shared node approach was implemented in this study, which means perfect bond and displacement continuity between concrete and steel elements. Similarly the interface between concrete and CFRP was modelled as a perfect bond.

The specimen has simply supported boundary condition on both sides, allowing it to freely rotate while preventing it from moving out of displacement. CONTACT AUTOMATIC GENERAL Contact type is chosen as single surface contact, that the contact is defined wholly by the slave side (the beam). All the mesh, loading and boundary condition are shown in Figure 6.
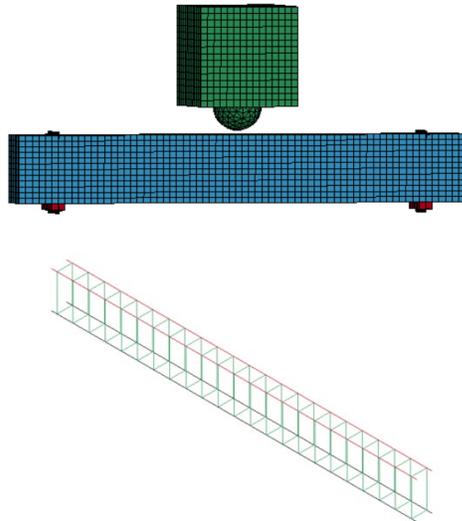
Figure 6. Geometry, mesh and boundary condition

Analysis Type

This study is about Reinforced concrete beams under impact loading, so explicit solver was used. Explicit dynamic solutions are well suited to physics that involve severe loads applied over a short period of time (on the order of micro or milliseconds). Explicit analysis can handle nonlinearities relatively easier as compared to the implicit analysis. This consists of treatment of contact, including large strains and material failure.

3.0 RESULTS AND DISCUSSION

Verification Study

The purpose of the verification process is to ensure that the material models (used for concrete and reinforcements), loading conditions and mesh size are adequate to model the response of the member and make sure that the simulation process is correct. To validate the finite element model, the experimental specimen S1322 Solid beam specimen of [6] is reproduced in LS DYNA, which is also used as control Beam specimen.

From Figure 7, the midspan deflection curve of the Finite element model has similar pattern with solid beam specimen from experiment and maximum deflection of FEM exhibit only 4.12% reduction. In addition, from Impact Force time history curves of both, the impact force increased rapidly to the peak due to the sudden interaction between the drop-weight hammer and the specimen then begins to reduce with sinusoidal vibration. Finally impact force diminished to zero due to the rebound of the drop weight. But the Peak Impact Force value of FEM shows only 5.68% increment. Generally, the curves showed clearly an acceptable agreement between the experimental results and finite element model throughout the entire range of behaviour.

As can be seen from Figure 7, crack patterns of Experimental specimen and Finite Element model show high failure near point of loading due to high stress concentration. Hence the FE model can successfully capture cracking process of the beam specimen. Therefore, these results further prove the ability of
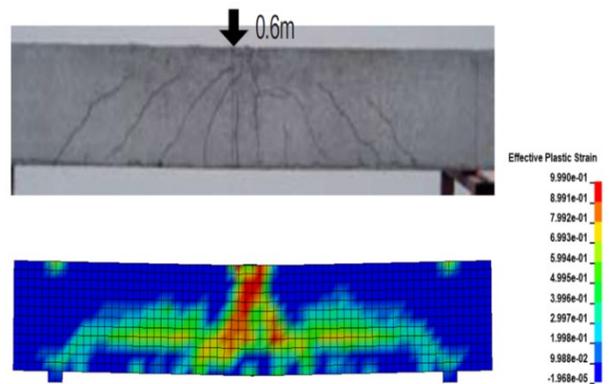
Finite Element model in predicting behaviour of the beam specimen.
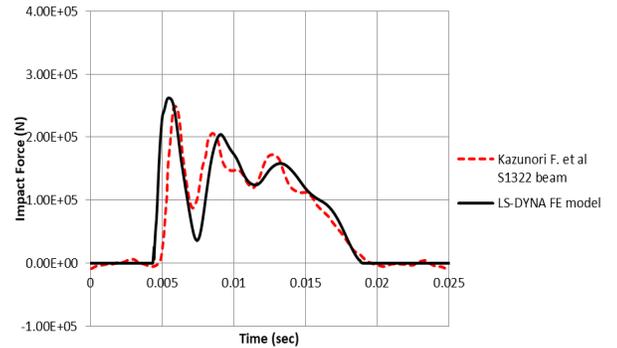
(a)   Mid span deflection
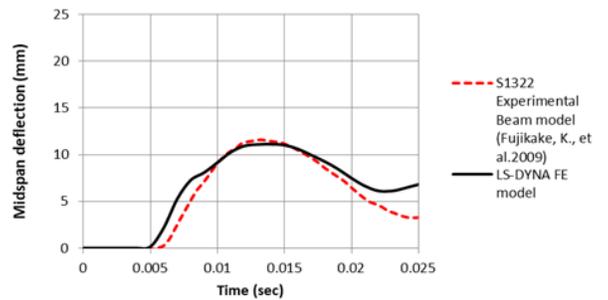
(b) Impact force time history

(c) Crack pattern

Figure 7. Comparison of experimental and finite element analysis results

Effect of Different Opening Sizes and Locations

This portion presents analysis results of specimens with web opening. Impact load and midspan deflection responses are considered, which helps as an important performance index to assess the level of damage on RC beams subjected to impact loadings, [6]. Three and ten specimens are considered for

location and size comparison respectively. Results are also shown in Table 4.
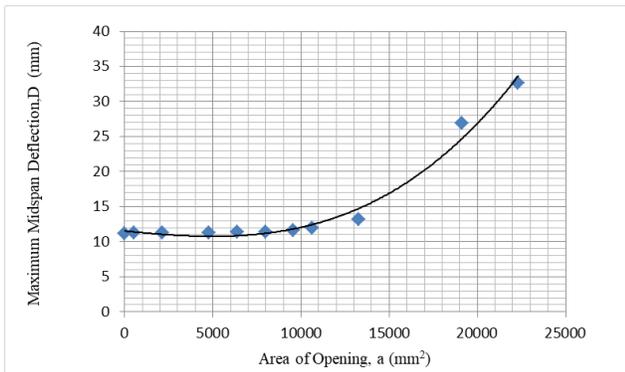
Midspan Deflection Response

The effect of location of opening on deflection response is illustrated in Figure 8 (a). Midspan deflection becomes larger when opening located around shear zone (OBL4). Furthermore, twelve specimens with different opening size were analysed and results are shown in Figure 8 (b). Significant change in midspan deflection is observed when depth of opening is higher than 42.5% of overall depths of the beam. And maximum midspan deflection versus opening size relation is expressed in simple regression equation shown below with correlation coefficient of 0.9825.

$$D = 2E\text{-}12a^3 + 1E\text{-}08a^2 - 0.0003a + 11.5 \tag{1}$$
$$R^2 = 0.9825$$
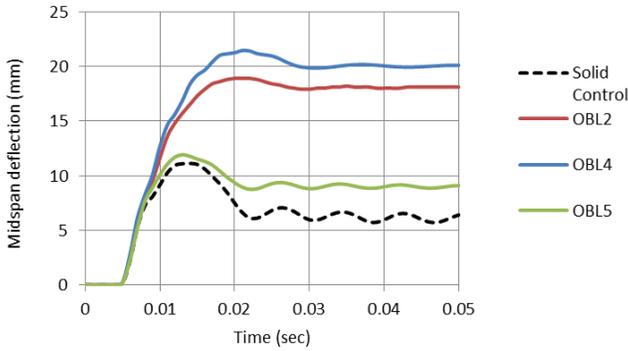
(a) Opening location effect

(b) Opening size effect

Figure 8. Midspan deflection response of specimens with opening

Impact Force Time History

Impact force response due to opening size and location are shown in Figure 9. The period of vibration and also vibration frequency increased as the opening location moves towards center, indicating softening of the beam. The Impact force resistance of specimens decrease as Location of opening approaches from shear to flexure zone. Furthermore, the impact resistance of beams significantly reduced when depth of

opening is higher than 42.5% of overall depth of the beam. That implies a large opening. For small openings there is small change in Peak impact force behaviour. But for large openings, the peak impact force increases rapidly, refer Table 4. And peak impact force versus opening size relation is expressed in simple regression equation shown below with correlation coefficient of 0.9901.

$$F = 1E\text{-}11a3 - 6E\text{-}07a2 + 0.0002a + 257.89 \tag{2}$$
$$R^2 = 0.9901$$
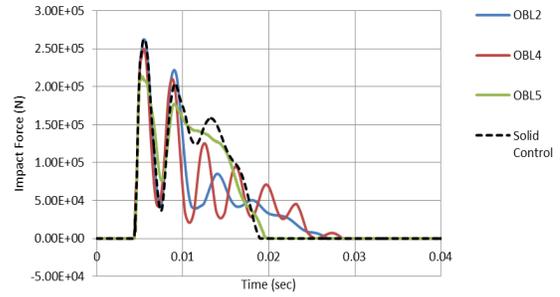
(a) Opening location
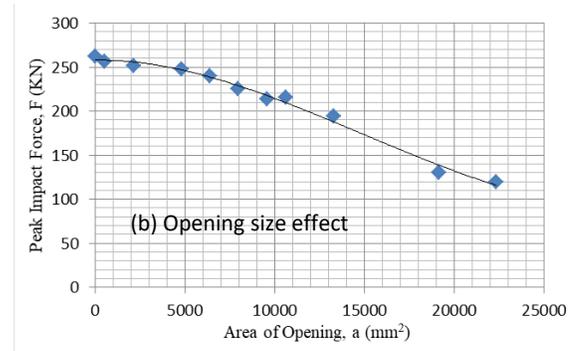

(b) Opening size

**Figure 9.** Impact force response of specimens with opening

Crack Pattern

Damage and Failure patterns can be expressed in terms of Effective Plastic strain of concrete in LS-DYNA. It is clear that damage extent increases as opening sizes increase. So, we are going to see effect of opening location on the beam. OBL2 specimen, opening located near support exhibits shear failure and brittle damage as shown in Figure 10. But specimen with center opening show relatively lower damage.
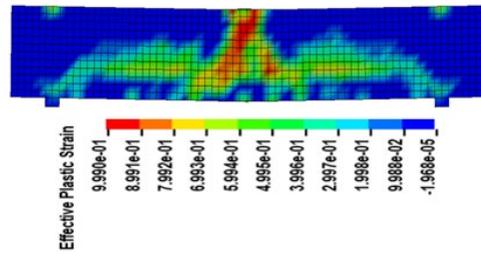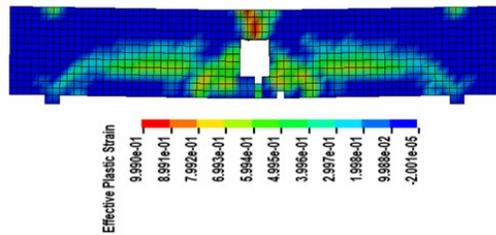
(a) Solid control
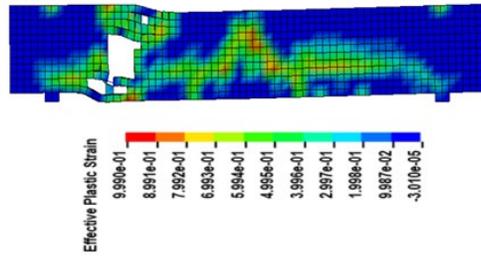
(b) OBL2
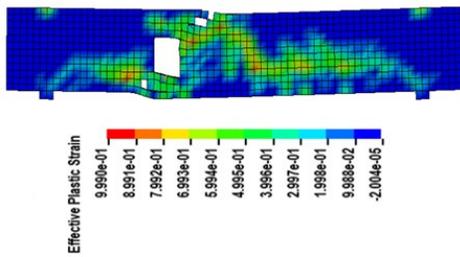
(c) OBL4

(d) OBL5

Figure 10. Failure modes of opening location specimens

**Table 4.** Peak Impact Force and Maximum Midspan Deflection of Varying Openings

Beam Specimen	Peak Impact Force P_{max} (KN)	Decrease in Impact force(%) difference	Maximum Midspan deflection δ_{max}(mm)	Increase in Midspan Deflection (%)
Solid	262.111	-	11.122	-
OBSA1	256.463	-2.155	11.282	+1.439
OBSA2	251.407	-4.084	11.303	+1.627
OBSA3	247.988	-5.388	11.33	+1.870
OBSA4	239.415	-8.659	11.422	+2.697
OBSA5	225.587	-13.935	11.433	+2.796
OBSA6	213.239	-18.646	11.636	+4.621
OBSB1	215.516	-17.777	11.914	+7.121
OBSB2	193.824	-26.053	13.167	+18.387
OBSC1	130.174	-50.336	26.928	+142.114
OBSC2	119.899	-54.256	32.607	+193.176
OBL2	261.095	-0.38	18.937	70.266
OBL4	249.985	-4.626	21.443	92.798
OBL5	215.516	-17.776	11.914	7.121

Effect of Number and Orientation of CFRP Layer

This portion presents analysis results of specimen's retrofitted using CFRP laminate, which were studied earlier with opening. To find out the effect of number of layer and fiber orientation three specimens are considered for each of them. Results are also shown in Table 5.

Midspan Deflection Response

FE results showed that effective usage of CFRP laminates in concrete is better solution to the reduction of midspan deflection and deformation of the structure. Figure 11 shows that the vibration frequency of retrofitted specimens increased as number of layer increased and higher than that of unstrengthen (control). Deflection decreased significantly even more than solid specimen. Considering fiber orientation, the results confirmed that as fibers wrapped longitudinally are better solution to the reduction of midspan deflection and deformation of the structure than along cross-section. When fibers placed longitudinally, it can reduce flexural failure.
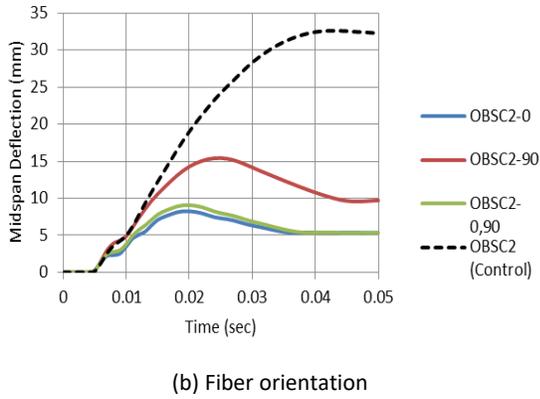

(a) CFRP number layer

(b) Fiber orientation
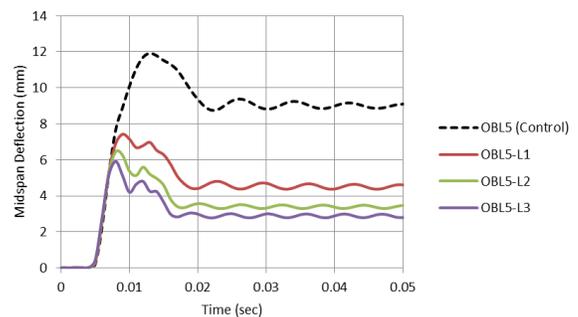
Figure 11. Midspan deflection response of CFRP retrofitted specimens

Table 5. Peak Impact Force, Maximum Midspan Deflection & Energy Dissipation of CFRP Number of Layer and Fiber Orientation.

Beam Specimen	Peak Impact Force P_{max}(KN)	Increase in Impact force(%)	Midspan deflection δ_{max} (mm)	Decrease in Midspan deflection (%)	Absorbed energy (P – D) E_P (J)
OBL5	215.516	-	11.914	-	1771
OBL5-L1	236.741	9.84	7.4227	-37.69	1304
OBL5-L2	244.154	13.28	6.4496	-45.86	1240
OBL5-L3	247.937	15.04	5.9229	-50.28	1199
OBSC2	119.899	-	32.607	-	2056
OBSC2-0	131.097	9.33	8.2571	-74.67	785
OBSC2-90	125.990	5.08	15.43	-52.67	1383
OBSC2-0,90	131.008	9.26	9.087	-72.13	859

Impact Force Time History

As number of layers increased, large impact force develops as shown in Figure 12. The impact force depends on the structural stiffness; the larger the stiffness, the higher the inertia and impact forces, [11]. Therefore, effective usage of CFRP laminates in concrete is a better solution for supporting the opening region and improving the maximum load-carrying capacity. In order to improve flexural capacity of beams, the FRP fibres should be joined parallel to the direction of the longitudinal reinforcement i.e., 0^0 orientation.
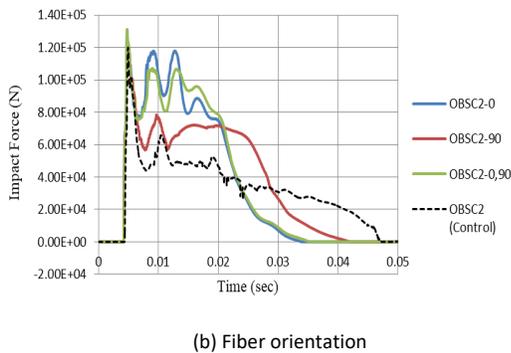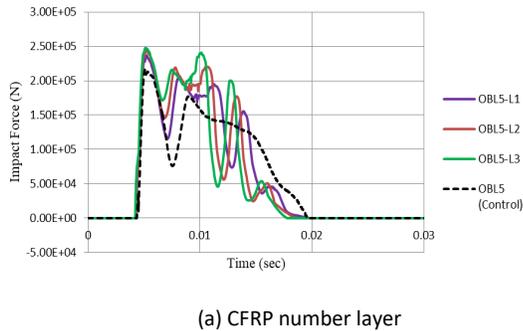

(a) CFRP number layer

(b) Fiber orientation

Figure 12. Effect CFRP on impact force time history

Crack Pattern

In retrofitted specimens even though the cracks around opening were developed, the beam could still resist further impact loading. This quality came from the composite action, which indirectly increased the shear capacity by preventing crack formation as shown in Figure 13 for different number of layers. FE results show that flexural strength is improved as fiber wrapping include longitudinal orientation.

(a) OBL5 control

(b) OBL5-L1

(c) OBL5-L2
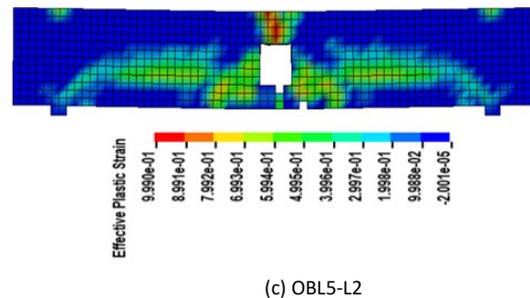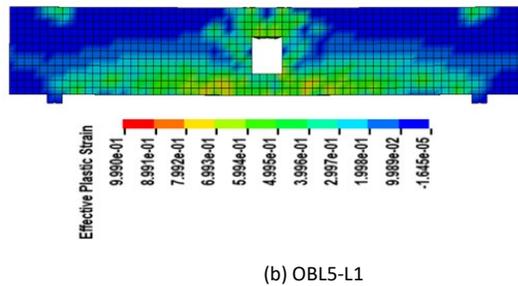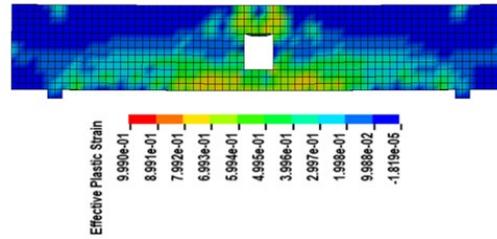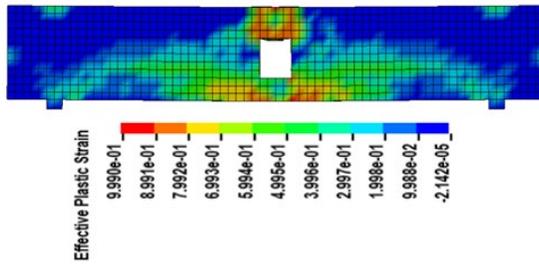
(d) OBL5-L3

Figure 13. Failure modes of specimens with different number of CFRP layer

Figure 14. Effect of number of layers on Hysteretic loops of impact force vs. displacement (P-δ).

Hysteretic loops of Impact Force-Displacement and Energy Absorption

It can be clearly seen in Figure 14, the irregularity of the loop becomes more complex when number of layer increases. The configuration of (P – D) loops are very irregular but without loss of generality it could be approximated roughly to triangular for absorbed energy calculation, [12]. The Absorbed energy value is computed by integrating the impact force vs. displacement (P-δ) loop-area in LS-DYNA software for each beam, denoted by E_P.
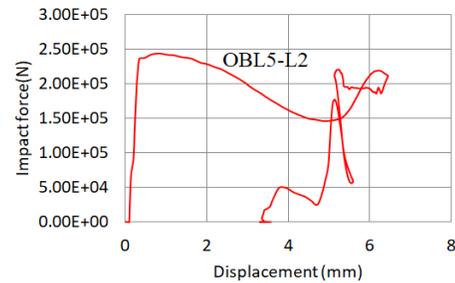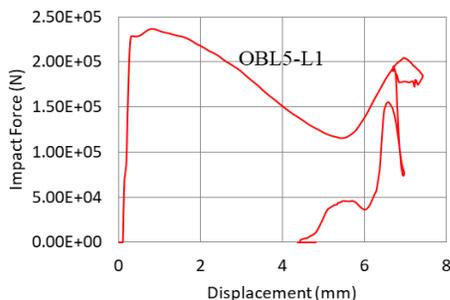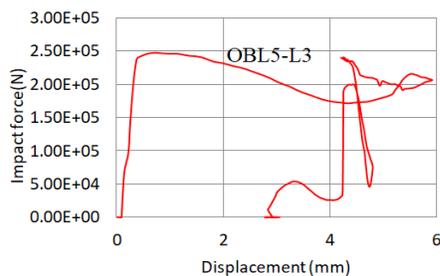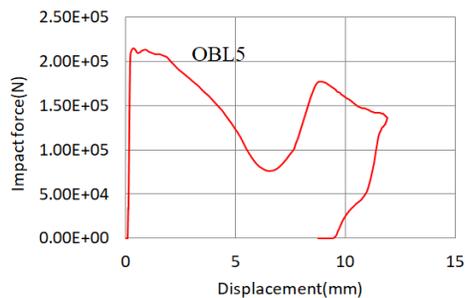

From Table 5, it can be noticed that the absorbed energy E_P decreases with the increasing CFRP number of layers, indicating the more energy is dissipated or released by means of other mechanisms, such as concrete cracking, kinetic energy due to the vibrations, material hysteresis and local damage instead of overall specimen deformation. Therefore, CFRP material has a good ductility and energy dissipation capacity.

4.0   CONCLUSION

The nonlinear finite element analysis results of Reinforced concrete beams under impact loading performed in this study involved two groups of specimens. First effect of web opening on 13 reinforced concrete beams investigated followed by analysis of RC beams retrofitted with Carbon fiber reinforced polymer (CFRP) composites. Next, conclusions inferred from results of this study are presented.

- Over all finite element analysis results showed good agreement with experimental results. This showed employed finite element procedures including modelling of geometry, material models, meshing and loading conditions were accurate enough to predict response of as-built and CFRP strengthened reinforced concrete beam under impact loading.
- Numerical investigation results indicated opening location across depth of a RC beam influences deflection response of a RC beam under impact loading. When opening location depth is below 42.5% of overall depths of a beam, no change of deflection value (maximum of 5%) is observed.
- In RC beams with large opening at flexure zone, excessive flexural cracks and failure are found at tension zone around openings and over all flexure failure mode is exhibited. Also, as compared to solid control beam, large opening cut outs in a RC beam decreased impact force resistance by 54% max whereas deflection is increased by 193%.
- RC beam exhibited poor impact loading resistance close to loading point (mid span) and performed good near shear zone.
- Carbon Fiber Reinforced Polymer fabrics significantly increased the capacity of concrete beams with web opening to resist impact loading and improve serviceability by reducing the maximum deflection.

- For all specimens as number of layer increase load carrying capacity improved.
- 0º CFRP fiber orientation along the beam with three-layer application improved midspan deflection up to 74% reduction than control specimen. And it is the most impact resistant. Furthermore 90^0 orientation with complete wrapping resulted in reduction of shear cracking around the opening but exhibits low overall impact resistance.
- Use of CFRP indirectly increased shear capacity by preventing formation cracks near opening.

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