ASEAN Engineering Journal

Full Paper

ON DIFFERENT FORMULATIONS FOR THE MULTI-PERIOD VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS PICKUP AND DELIVERY

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Article history
Received
27 October 2021
Received in revised form
17 May 2022
Accepted
05 July 2022
Published online
28 February 2023

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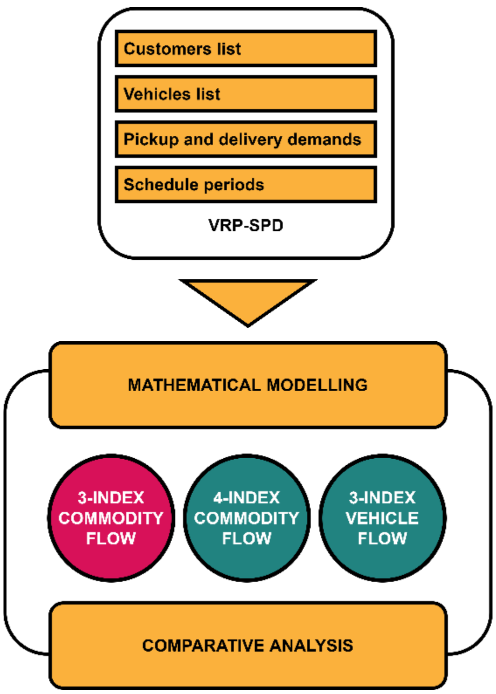

Abstract

In this paper, we extend the vehicle routing problem with simultaneous pickup and delivery (VRPSPD) with a consideration of multiple planning horizons. We propose three alternative mathematical formulations for Periodic-VRPSPD (P-VRPSPD) based on the available formulations for VRPSPD in the literatures, namely the three-index commodity flow formulation, four-index commodity flow formulation, and three-index vehicle flow formulation. We perform comparison analysis by conducting extensive numerical experiments on a set of instances with various complexities in order to evaluate the performance of these formulations. Overall, it is observed that the three-index commodity flow formulation returns the best results.

Keywords: Integer programming, Mathematical formulation, Periodic routing, Simultaneous pickup and delivery, Vehicle routing problem

1.0 INTRODUCTION

Over the last sixty years, the vehicle routing problem (VRP) and its variants have been vastly discussed. First introduced by Dantzig and Ramser [1], the VRP aims to establish a set of routing plans for a fleet of vehicles in order to satisfy the customer demands. The routing plan generally must begin and be complete at a designated depot, in which the fleet of vehicles must visit all the customers once under the compliance of several constraints. Until now, a considerable amount of effort has been dedicated by the research community to propose numerous variants of VRP in order to develop a model that can capture realistic situations. In this regard, one can consult the excellent works of Eksioglu et al. [2], Braekers et al. [3], and Vidal et al. [4], who provided a comprehensive review of the development of VRP variants.

Among the important variants of VRP is the VRP with simultaneous pickup and delivery (VRPSPD). In this VRPSPD, a fleet of identical vehicles must serve the delivery and/or pickup demands of a set of customers. In another words, certain customers have a delivery demand, others have a demand for pick up, and at least one customer has both delivery and pickup demands. According to Berbeglia et al. [5] and Battarra et al. [6], the class of pickup and delivery problems (PDP) can be classified into three main categories. In the first category, the commodity may consist of more than one start point and more than one finish point (many-to-many problems). Furthermore, any point of the route may be the start point and destination point of multiple commodities. For the second category, several commodities are delivered from a depot to several customers, while different commodities are picked at customers and sent to the depot (one-to-many-to-one problems). In the last category, each commodity has one start point and one endpoint (one-to-one problems). Within these categories, VRPSPD is the most studied problem in the second category since this situation occurs in many industries such as laundry service in hotels [7], long-distance road transport [8], supermarket chains [9], and fashion retailers [10].

On the other hand, the impact of considering multiple planning periods in VRP has also been actively studied. In this class of periodic VRP (PVRP), the aim is to determine the optimal routing plan to visit a set of customers in more than one planning horizon, in which the planning horizon may stand for working hours, working days, or even a certain more prolonged period. The need for a PVRP model particularly arises when a decision-maker must cope with a situation where the demand pattern of customers varies from one to another period of time, and to date, numerous studies have been dedicated to the development of PVRP-based models in a diverse array of applications, from the collection process of glass, metal, plastics or paper [11], animal waste [12], to routing of home healthcare nurses [13].

Nevertheless, there has been no study on the VRPSPD with multiple planning periods until now. This fact has been supported by the comprehensive review of Koç et al. [14]. Thus, looking back at the importance of the VRPSPD and PVRP, we intend to commence the study of the periodic vehicle routing problem with simultaneous pickup and delivery (P-VRPSPD). In this study, we develop three different formulations for the P-VRPSPD based on the available formulations of single-period VRPSPD from the works of Dell'Amico et al. [15], Montane and Galvao [16], and Rieck and Zimmerman [17]. Then, we conduct extensive numerical experiments to discuss the performance of these formulations and their applicability to be implemented in a real-life situation.

2.0 LITERATURE REVIEW

This section discusses the review of literature related to the P-VRPSPD. The PVRP is a classic variant of VRP with diverse applications. Generally, its application can be categorized into three distinct problems: routing for on-site service, pickup, and delivery [18]. The first PVRP application type is routing for on-site service. Evidently, periodic routing problems arise for arranging the route of the staff for service cases such as for the salesman and maintenance engineer. Hadjiconstantinou and Baldacci [19] examined the route for preventive maintenance staff from a utility company, while Blakeley et al. [20] optimized the periodic maintenance of escalators and elevators with variance in the interval of maintenance tasks.

In the pickup problem, PVRP has been widely applied for waste and garbage collection so far. Nuortio et al. [21] modeled the waste collection for the residential customers as a PVRP, while comparing the node routing with the arc routing paradigm. Angelelli and Speranza [22] compared various technologies for garbage collection using PVRP, such as evaluating the traditional pickup method and the use of more advanced trucks. Further, the waste collection problem was also extended for more specific case such as collection of recyclable materials [23], recycling paper [24], and waste vegetable oil [25], infectious waste at hospitals and clinics [26][27]. Besides the waste collection, various problems were also modeled as pickup PVRP, such as goat milk collection [28] and the collection of parts for use in auto parts manufacturing [29].

Then the third type of PVRP application is on the delivery problem. Here, the delivery PVRP has a vast array of cases, such as for store replenishment or delivery for retail stores [30][31], delivery of hospital amenities [32], and replenishment of vending machines [33]. Unfortunately, while the PVRP either for pickup or delivery problems has been extensively explored, the PVRP for the simultaneous pickup and delivery problem suffers from the negligence of previous authors. Meanwhile, the VRPSPD itself is an essential variant of VRP as it has a wide range of applications.

The VRPSPD is the extension of VRP in the PDP. In VRPSPD, a set of customers may have a delivery or pickup demand, and at least one of the customers has both the delivery and pickup demand [14]. Several variants of VRPSPD have been studied until now. Angelelli and Mansini [34] introduced the VRPSPD with time windows. Here, the customer can only be served within its time window. Hence the vehicle must wait if it arrives earlier than the time when the customer is available. Since the waiting time in VRPSPD with time windows is the point of concern, many studies have developed various methods to minimize this variable. For example, Fan [35] used maximizing customer satisfaction and minimizing the total cost as objectives since they are inversely proportional to the waiting time for the vehicle from the lower bound of the time window. Further studies in VRPSPD with time windows include the consideration of mixed pickups and deliveries [36], split loads [37], and hard time windows [38]. Moreover, other extensions of VRPSPD also exist, such as the problem with heterogeneous fleet [39][40], the multi-depot VRPSPD [41][42], and the stochastic VRPSPD [43][44].

The real-life implementation of VRPSPD models has been documented by previous literature. Yin et al. [45] discussed the application of the split-load VRPSPD model to optimize the subsidiary system of China Railway Express. Drexl et al. [8] are concerned about the VRPSPD with time windows for the truckload business model in Germany. The data for the experiment includes 2,800 delivery and pickup demands that spread over 1,975 locations with 1,645 vehicles, 43 depots, and 157 additional stations. Belgin et al. [9] considered a two-echelon VRPSPD on a distribution system of 25 Turkish chain supermarkets. Then, another study from Wang et al. [46] presented a multi-objective heterogeneous VRPSPD with time windows, in which the objective functions were to minimize the makespan, the number of vehicles, total traveled distance, total waiting time due to early arrival, and total lateness due to late arrival. In a recent study, Zhang et al. [10] studied a multi-commodity many-to-many variant of the VRPSPD on a Singapore fashion retailer with 30 retail outlets supplied by a central warehouse.

Although the VRPSPD has been extensively studied and various extensions have been developed, the survey from Koç et al. [14] gave remarks on the lack of attention in several practical scenarios, one of these is the multi-period VRPSPD. The consideration of periodicity is important when the demand varies over the planning periods and this situation is not captured in the static model of VRPSPD, which has been vastly discussed by the likes of Dell'Amico et al. [15], Montane and Galvao [16], Subramanian et al. [47][48], and Rieck and Zimmerman [17]. Therefore, this study presents a novel extension of VRPSPD called as the P-VRPSPD and to the best of our knowledge, this is the first work to extend the VRPSPD with consideration of multiple planning horizons.

3.0 PROBLEM FORMULATION

In general, the P-VRPSPD can be defined in a graph $G = (V, A)$. Let us first denote n as the number of customers to be served and $\mathcal{D} = 0$ as the depot node. Using these notations, we can define $V = [0,1, \dots, n] = \mathcal{D} \cup N$ as the set of all nodes and A as the set of arcs between nodes (i, j) where $i \in V$, $j \in V$, and $N = V \backslash \{\mathcal{D}\}$ as the set of customers to be served. Let $T = [1, \dots, t]$ as the set of time periods, where t is the length of time periods. In every period t, each customer $i \in N$ will have non-negative delivery demand d_{it} and non-negative pickup demand p_{it} that must be satisfied in a single visit by a fleet of k homogenous vehicles with capacity Q. Then, the goal of the P-VRPSPD is to minimize the sum of travel cost c_{ij} which is incurred when the vehicle travels on arcs (i, j). This travel cost c_{ij} may represent the travel time, fuel cost, or even carbon emission incurred to travel from node i to j, but in this study, we assume that this cost represents the travel distance from node i to j.

The assumptions hold in P-VRPSPD are as follows. First, each vehicle performs at most one route per period. Second, the vehicle routes start and finish at the depot $\mathcal{D}$. Third, the commodity is assumed to be homogenous. Fourth, each customer is visited once and by one vehicle only for each period. Fifth, the sum of demand of a vehicle route cannot exceed Q. Last, within one period, at least one of the customers in N will have $d_{it} > 0$ and $p_{it} > 0$.

In this work, we derive several alternative formulations to model the P-VRPSPD based on the literature of VRPSPD. Three different alternatives are discussed, namely (1) the three-index commodity flow formulation (3-CF), (2) the four-index commodity flow formulation (4-CF), and (3) the three-index vehicle flow formulation (3-VF). These formulations are presented in the following sub-sections.

3.1 Three-index Commodity Flow Formulation (3-CF)

This 3-CF formulation extends the two-index commodity flow formulation for VRPSPD by Dell'Amico et al. [15]. There are three decision variables for 3-CF formulation. Let us denote x_{ijt} as a binary variable that takes the value of 1 if arc (i, j) is traversed in period t and 0 otherwise, let y_{ijt} be the integer amount of pickup commodity traversed on arc (i, j) in period t, and let z_{ijt} be the integer amount of delivery commodity traversed on arc (i, j) in period t. Then, the integer linear programming (ILP) for 3-CF formulation can be presented as follows.

$$\textbf{Min} \sum_{i \in V \backslash j} \sum_{j \in V \backslash i} \sum_{t \in T} c_{ij} x_{ijt} \tag{1.1}$$

subject to

$$\sum_{j \in V \backslash i} x_{ijt} = 1 \qquad \forall\, i \in N, t \in T \tag{1.2}$$

$$\sum_{j \in N} x_{0jt} \leq k \qquad \forall\, t \in T \tag{1.3}$$

$$\sum_{j \in V \backslash i} x_{ijt} = \sum_{j \in V \backslash i} x_{jit} \qquad \forall\, i \in V, t \in T \tag{1.4}$$

$$\sum_{j \in V \backslash i} y_{ijt} - \sum_{j \in V \backslash i} y_{jit} = p_{it} \qquad \forall\, i \in N, t \in T \tag{1.5}$$

$$\sum_{j \in V \backslash i} z_{jit} - \sum_{j \in V \backslash i} z_{ijt} = d_{it} \qquad \forall\, i \in N, t \in T \tag{1.6}$$

$$y_{ijt} + z_{ijt} \leq Q x_{ijt} \qquad \forall\, i, j \in A, t \in T \tag{1.7}$$

$$y_{ijt}, z_{ijt} \geq 0 \qquad \forall\, i, j \in A, t \in T \tag{1.8}$$

$$x_{ijt} \in \{0, 1\} \qquad \forall\, i, j \in A, t \in T \tag{1.9}$$

The objective function (1.1) defines the minimization of routing costs for all periods. This objective is subjected to a set of constraints. Constraint (1.2) guarantees that each customer is included in only one route within one period. Constraint (1.3) confirms that, for each period, the constructed vehicle routes cannot exceed the number of available vehicles. Constraints (1.4), (1.5), and (1.6) ensure the flow of pickup and delivery tasks. Constraint (1.7) limits the vehicle load for all routes and periods. Finally, Constraints (1.8) and (1.9) define the value restrictions of decision variables.

3.2. Four-index Commodity Flow Formulation (4-CF)

We now present the 4-CF formulation for the P-VRPSPD. The 4-CF formulation is adapted from the commodity flow formulation of VRPSPD proposed by Montane and Galvao [16]. In a way, this formulation is very similar to the 3-CF formulation in Subsection 3.1, with the main difference in the presence of additional index

k that define the vehicle deployment specifically. In this regard, let us introduce $K = [1, \dots, k]$ as a set of available vehicles.

The decision variables required for this 4-CF formulation are as follows. Let x_{ijtk} as a binary variable with a value of 1 if vehicle k travels on arc (i, j) in period t and 0 otherwise. Then, let y_{ijt} and z_{ijt} as integer variables to compute the pickup and delivery commodity traversed on arc (i, j) in period t, similar to the same decision variables in the 3-CF formulation. Using these variables, the ILP for 4-CF formulation can be presented as follows.

$$\mathbf{Min} \sum_{i\in V\backslash j} \sum_{j\in V\backslash i} \sum_{t\in T} \sum_{k\in K} c_{ij}x_{ijtk} \quad (2.1)$$

subject to

$$\sum_{j\in V\backslash i} \sum_{k\in K} x_{ijtk} = 1 \qquad \forall\, i \in N, t \in T \quad (2.2)$$

$$\sum_{j\in V} x_{0jtk} \le 1 \qquad \forall\, t \in T, k \in K \quad (2.3)$$

$$\sum_{j\in V\backslash i} x_{ijtk} = \sum_{j\in V\backslash i} x_{jitk} \qquad \forall\, i \in V, t \in T, k \in K \quad (2.4)$$

$$\sum_{i\in V\backslash j} y_{jit} - \sum_{i\in V\backslash j} y_{ijt} = p_{jt} \qquad \forall\, j \in N, t \in T \quad (2.5)$$

$$\sum_{i\in V\backslash j} z_{ijt} - \sum_{i\in V\backslash j} z_{jit} = d_{jt} \qquad \forall\, j \in N, t \in T \quad (2.6)$$

$$y_{ijt} + z_{ijt} \le Q \sum_{k\in K} x_{ijtk} \qquad \forall\, i,j \in A, t \in T \quad (2.7)$$

$$y_{ijt}, z_{ijt} \ge 0 \qquad \forall\, i,j \in A, t \in T \quad (2.8)$$

$$x_{ijtk} \in \{0,1\} \qquad \forall\, i,j \in A, t \in T, k \in K \quad (2.9)$$

The objective function (2.1) minimizes the total routing cost of all periods. Constraint (2.2) guarantees that each customer is visited once and only by one vehicle within one period. Constraint (2.3) ensures that each vehicle $k \in K$ can only perform one route in a single period. Similar to Equations (1.4)-(1.6), Constraints (2.4)-(2.6) are the flow conservation constraints. Constraint (2.7) guarantees that the vehicle capacity is not violated in all periods. Finally, Constraints (2.8) and (2.9) define the value restrictions of decision variables.

3.3 Three-Index Vehicle Flow Formulation (3-VF)

The third formulation is adapted from the work of Rieck and Zimmerman [17], who proposed a two-index vehicle flow model for VRPSPD. Different from the previous commodity flow formulations which satisfy the amount of pickup and delivery commodity traversed on each arc, this vehicle flow formulation only defines the routes of vehicles [14].

Our 3-VF formulation requires three decision variables. First, let us denote x_{ijt} as a binary variable to specify if arc (i, j) is used in period t and 0 otherwise. Then, let integer variables D_{it} as the amount of delivery goods that must be delivered to node i in period t and L_{it} as the load of the vehicle right after visiting customer i in period t. The ILP formulation for the 3-VF model can be written in the following way.

$$\mathbf{Min} \sum_{i\in V\backslash j} \sum_{j\in V\backslash i} \sum_{t\in T} c_{ij}x_{ijt} \quad (3.1)$$

subject to

$$\sum_{i\in V\backslash j} x_{ijt} = 1 \qquad \forall\, j \in N, t \in T \quad (3.2)$$

$$\sum_{i\in V\backslash j} x_{jit} = 1 \qquad \forall\, j \in V, t \in T \quad (3.3)$$

$$\sum_{j\in N} x_{0jt} \le k \qquad \forall\, t \in T \quad (3.4)$$

$$d_{it} \le D_{it} \le Q \qquad \forall\, i \in V, t \in T \quad (3.5)$$

$$p_{it} \le L_{it} \le Q \qquad \forall\, i \in N, t \in T \quad (3.6)$$

$$D_{it} \ge D_{jt} + d_{it} - M(1 - x_{ijt}) \qquad \forall\, i \in V\backslash j, j \in N\backslash i, t \in T \quad (3.7)$$

$$L_{jt} \ge D_{jt} - d_{jt} + p_{jt} \qquad \forall\, j \in N, t \in T \quad (3.8)$$

$$L_{jt} \ge L_{it} + d_{jt} + p_{jt} - M(1 - x_{ijt}) \qquad \forall\, i \in N\backslash j, j \in N\backslash i, t \in T \quad (3.9)$$

$$D_{it} \ge 0 \qquad \forall\, i \in V, t \in T \quad (3.10)$$

$$L_{it} \ge 0 \qquad \forall\, i \in N, t \in T \quad (3.11)$$

$$x_{ijt} \in \{0,1\} \qquad \forall\, i,j \in A, t \in T \quad (3.12)$$

Objective (3.1) aims to minimize the total routing cost. Constraints (3.2) and (3.3) guarantee that each customer is served once, and the entering vehicle will depart from the customer node. Constraint (3.4) limits the number of vehicles that can be deployed in one period. Constraints (3.5) and (3.6) are the vehicle capacity constraints. Constraint (3.7) confirms the consistency of the delivery load along the route and Constraints (3.8)-(3.9) define a load of vehicles after serving the customers. Lastly, Constraints (3.10), (3.11), and (3.12) restrict the value of decision variables.

3.4 Comparison Between Formulations

Here, several observations on the presented formulations are discussed to emphasize the difference between them. First, it can be observed that the 3-CF formulation is basically a simplified version of the 4-CF formulation, where the index k for deciding the usage of vehicle k is omitted from the decision variable x_{ijt}. This omission of index k leads to a more compact formulation in the 3-CF formulation. In this regard, the information of which vehicle k should traverse which arc (i,j) can still be derived from the amount of pickup and delivery commodities traversed on arc (i,j) in period t (y_{ijt} and z_{ijt}), since each arc $(i,j) \in A$ is basically only traversed by one vehicle at a single period t. Secondly, it can be observed too that the 3-CF and 4-CF formulations belong to the family of multi-commodity flow problems, wherein the context of VRP, was first discussed by Baldacci et al. [49]. This type of problem combines the assignment constraints to model the route of vehicles, while the movement of pickup and delivery goods is modeled with multicommodity flow constraints, without the requirement to use sub tours elimination constraints. On the other hand, the 3-VF formulation is a vehicle flow-based formulation, which is based on the *standard* classical formulation of VRP presented in Laporte et al. [50]. This type of formulation clearly defines the vehicle routes along with the graph and generally requires a set of constraints to eliminate the sub tours, such as the classical Dantzig-Fulkerson-Johnson [51], Miller-Tucker-Zemlin [52], arrival time constraints [53], or transit load constraints which are used in this presented formulation.

3.5 Case illustration

In order to give a better understanding of the P-VRPSPD, we dedicate this subsection to presenting an illustration of the P-VRPSPD case and the optimal solution produced from such a case. Let us consider the following example of a small P-VRPSPD instance with $n = 8, k = 3, t = 3$, and $Q = 80$. Table 1 presents a list of delivery and pickup demands from all n customers for each period t. In this regard, for clarity purpose we retain the value of d_{0t} and p_{0t} as zeros for all $t \in T$. Then, consider a symmetric adjacency matrix in Table 2 that presents the travel costs to travel on arcs (i,j).

Using the information in Table 1 and Table 2, we can obtain the optimal solution for this case illustration as presented in Figure 1. It is observed from the optimal solution that the solutions for $t = 1$ and $t = 2$ are generally similar, due to the usage of a symmetric adjacency matrix. However, it is important to note that the solution for $t = 3$ is different from the other time periods, in which only a single vehicle is deployed in this period. This stems from the fact that the total demands in $t = 3$ are lower than the total demands in $t = 1$ and $t = 2$ (see Table 1), so that the Q (capacity) of a single vehicle is not violated to serve all demands and, it turns out, can return a lower sum of travel cost. Additionally, we can also observe that although the total delivery and pickup demands for each period t are largely exceeding the value of Q, a single vehicle might be sufficient to satisfy all the demands. Thus, compared to the classical VRP that only considers delivery demand, this observation illustrates the complexity of the simultaneous presence of delivery and pickup demands,

Table 1 List of Demands

i	$t=1$		$t=2$		$t=3$	
	d_{i1}	p_{i1}	d_{i2}	p_{i2}	d_{i3}	p_{i3}
0	0	0	0	0	0	0
1	2	15	11	6	3	4
2	10	19	14	13	5	18
3	14	1	9	13	16	11
4	18	8	12	9	7	5
5	3	17	14	11	15	5
6	10	4	1	4	9	10
7	9	7	9	14	4	14
8	17	7	12	13	2	6
Total	83	78	82	83	61	73

Table 2 Adjacency Matrix

i	j								
	0	1	2	3	4	5	6	7	8
0	0	45	14	28	55	79	83	47	73
1		0	50	93	79	61	48	62	24
2			0	74	86	49	9	31	10
3				0	42	66	91	76	57
4					0	63	61	17	52
5						0	7	85	21
6							0	67	88
7								0	89
8									0

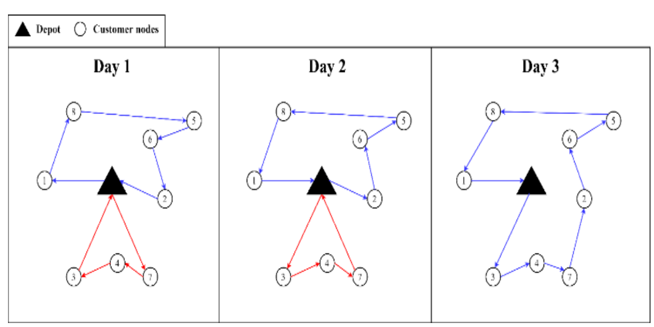

Figure 1 Optimal solution for the case illustration

4.0 COMPUTATIONAL EXPERIMENT

In this section, we provide a discussion on the design of computational testing and the results obtained from the experiments. All mathematical formulations are coded in Python 3.7 and executed with GUROBI 9.0.2 in the same hardware to guarantee a fair comparison.

4.1 Generation of Test Instances

With respect to the available benchmark in VRPSPD (e.g., Dethloff [54] and Salhi and Nagy [55]), we find no sufficient instance for the P-VRPSPD since there is no study available yet on this topic. Henceforth, in this study, we generate a set of test instances with various complexities to evaluate the performance of all proposed formulations. This dataset is generated randomly using the random seed function of Python 3.7. It consists of 80 instances with different combinations of n, k, and t. The locations of customer nodes are generated randomly with uniform distribution in a $(5000x5000)$ cartesian plane, and we assume that the location of $\mathcal{D}$ is in the middle of the plane. Then, we calculate the Manhattan distance between nodes to create the travel costs c_{ij} for all $(i,j) \in A$. For clarity, the usage of Manhattan distance to simulate the road network for ground vehicles is particularly common in drone-routing research [56][57]. Finally, the complete configurations for generating this dataset are presented in Table 3.

Table 3 Dataset Configurations

Variable	Description	Value
$\|N\|$	Number of customers	$\{10,20,30,40,50\}$
$\|K\|$	Number of available vehicles	$\{5,10,15,20\}$
$\|T\|$	Number of time periods	$\{5,10,15,20\}$
Q	Vehicle capacity	200
(D_x, D_y)	Coordinates (x, y) of depot	$(2500, 2500)$
(n_x, n_y)	Coordinates (x, y) of customer nodes	$U(0, 5000)$
d_{it}	Delivery demand of customers	$U(0, 20)$
p_{it}	Pickup demand of customers	$U(0, 20)$

4.2 Results and Discussions

The computational tests are performed on a personal computer with Intel® Core™ i7-10700 CPU 2.90 GHz with 32 GB of DD4 RAM and a Windows 10 operating system. The complete results are presented in Tables 4-9, which is structured as follows. The first part of the table (columns 1st to 4th) describes the instance details, which are varied by the number of customers ($|N|$), number of available vehicles ($|K|$), and the number of time periods ($|T|$). For each instance, we run each formulation in two hours running time (7200 seconds). Then, the second part of the table (column 5th to 10th) respectively presents the number of constraints and variables required by the corresponding formulation, the obtained solution value along with the obtained lower bound (LB), the optimality gap (which presents the percentage gap between the obtained solution and the LB), and the running time required. In the cases when a formulation does not obtain any solution for a given instance after 7200 seconds, we write the corresponding cell as 'inf'.

The experiment results provide two important observations. First, it can be easily observed that the 3-CF formulation is superior to the other formulations. The 3-CF formulation can provide optimal solutions for all instances with up to 40 customers, 20 available vehicles, and 20 periods of time. Even for the instances with 50 customers, the 3-CF formulation is able to find optimal solutions for some instances and still provides high quality solutions with less than 5% optimality gap for most of them. On the other hand, the 4-CF and 3-VF formulations are only able to provide optimal solutions for instances with up to 20 customers. The 4-CF formulation is even unable to find any solution for instances with more than 40 customers. Second, the 3-CF formulation is also proven to be more computationally efficient. The comparison of running time between these three formulations is visually presented in Figure 2. It is easily observed that the 4-CF and 3-VF formulations require higher running time in general, while the 3-CF formulation is even still able to solve some instances with 50 customers with less than two hours of running time.

We offer two explanations for the results presented above. First, compared to the 4-CF formulation, the 3-CF formulation involves only three-index decision variables (x_{ijt}, y_{ijt} and z_{ijt}) instead of the four-index one (x_{ijt}). Omitting the requirement to declare which vehicle k to traverse through a certain arc (i,j) at period t leads to the amount reduction of decision variables and constraints required to build the model [58], which in turn, reduces the size of the search space as well. Secondly, similar to the 3-CF formulation, the 3-VF formulation also employs three-index decision variables as well. However, it can be observed in Equations (3.7) and (3.9) that the 3-VF formulation makes use of big-M coefficients [59]. The presence of such big-M constraints, which are known to have a weak continuous relaxation [60], leads to an inefficient enumeration of large-size instances and deteriorates the performance of the solver as the size of the problem grows.

All in all, these observations lead us to endorse the usage of the 3-CF formulation. It has been proven that the 3-CF formulation is able to provide (near-)optimal solutions for P-VRPSPD cases with a realistic size (50 customers) in a reasonable time. It must be noted that the main challenge of periodic routing problems is the development of a single model that can cope with the different situations in multiple time periods. In a way, this challenge can be viewed as an effort to efface the requirement to execute the model to aid the decision-making process in every single time period. By looking at the periodic routing problems this way, spending one-two hour to obtaining a high-quality decision support to plan the routing of vehicles for the whole next month (in the cases of $|T| = 20$) becomes a reasonable choice. Nevertheless, the P-VRPSPD is a generalization of the VRPSPD and the classical VRP, which belong to the class of NP-hard problems, and the NP-hardness of P-VRPSPD can be tracked in Figure 2 from the exponentially increasing running times of all formulations. Thus, it would still be in the interest of operational researchers to develop an efficient heuristic solution for this problem, especially for large-size instances with more than 50 customers.

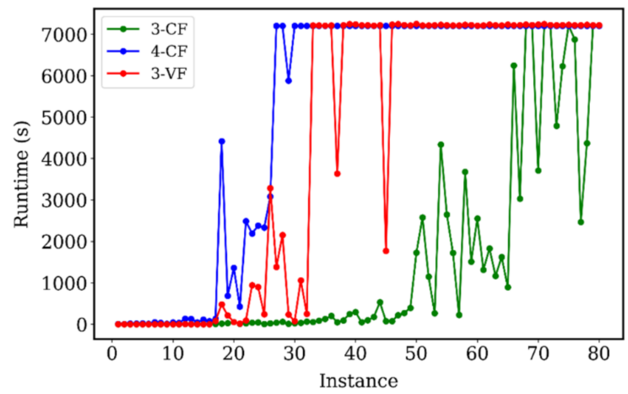

Figure 2 Comparison of running time between formulations

Table 4 Computational Results for 3-CF Formulation

No.	Instance			3-CF Formulation					
	$\|N\|$	$\|K\|$	$\|T\|$	Constraints	Variables	Solution	LB	Gap (%)	Time (s)
1	10	5	5	1860	1650	89170	89170	0	<1
2	10	5	10	3720	3300	171960	171960	0	<1
3	10	5	15	5580	4950	280170	280170	0	<1
4	10	5	20	7440	6600	464760	464760	0	<1
5	10	10	5	1860	1650	87630	87630	0	<1
6	10	10	10	3720	3300	207820	207820	0	1
7	10	10	15	5580	4950	273180	273180	0	<1
8	10	10	20	7440	6600	425040	425040	0	1
9	10	15	5	1860	1650	84680	84680	0	<1
10	10	15	10	3720	3300	215480	215480	0	1
11	10	15	15	5580	4950	246420	246420	0	1
12	10	15	20	7440	6600	314880	314880	0	3
13	10	20	5	1860	1650	99860	99860	0	<1
14	10	20	10	3720	3300	209880	209880	0	1
15	10	20	15	5580	4950	277080	277080	0	<1
16	10	20	20	7440	6600	422920	422920	0	<1
17	20	5	5	6710	6300	120726	120726	0	3
18	20	5	10	13420	12600	238050	238050	0	15
19	20	5	15	20130	18900	360026	360026	0	26
20	20	5	20	26840	25200	500660	500660	0	53
21	20	10	5	6710	6300	140274	140274	0	9
22	20	10	10	13420	12600	265654	265654	0	22
23	20	10	15	20130	18900	426444	426444	0	36
24	20	10	20	26840	25200	579982	579982	0	42
25	20	15	5	6710	6300	125210	125210	0	5
26	20	15	10	13420	12600	220292	220292	0	21
27	20	15	15	20130	18900	403570	403570	0	36
28	20	15	20	26840	25200	512072	512072	0	59
29	20	20	5	6710	6300	142828	142828	0	12
30	20	20	10	13420	12600	258162	258162	0	20
31	20	20	15	20130	18900	330902	330902	0	34
32	20	20	20	26840	25200	403568	403568	0	64
33	30	5	5	14560	13950	159780	159780	0	55
34	30	5	10	29120	27900	322300	322300	0	85
35	30	5	15	43680	41850	451370	451370	0	128
36	30	5	20	58240	55800	638688	638688	0	201
37	30	10	5	14560	13950	158868	158868	0	47
38	30	10	10	29120	27900	296208	296208	0	91
39	30	10	15	43680	41850	503350	503350	0	246
40	30	10	20	58240	55800	600804	600804	0	292
41	30	15	5	14560	13950	139640	139640	0	47

42	30	15	10	29120	27900	312904	312904	0	94
43	30	15	15	43680	41850	459392	459392	0	175
44	30	15	20	58240	55800	612088	612088	0	529
45	30	20	5	14560	13950	157456	157456	0	73
46	30	20	10	29120	27900	306720	306720	0	76
47	30	20	15	43680	41850	469738	469738	0	218
48	30	20	20	58240	55800	678160	678160	0	274
49	40	5	5	25410	24600	184966	184966	0	396
50	40	5	10	50820	49200	342898	342898	0	1727
51	40	5	15	76230	73800	517428	517428	0	2579
52	40	5	20	101640	98400	629672	629672	0	1145
53	40	10	5	25410	24600	172216	172216	0	266
54	40	10	10	50820	49200	334892	334892	0	4337
55	40	10	15	76230	73800	513366	513366	0	2647
56	40	10	20	101640	98400	709172	709172	0	1722
57	40	15	5	25410	24600	165708	165708	0	226
58	40	15	10	50820	49200	345330	345330	0	3678
59	40	15	15	76230	73800	510762	510762	0	1507
60	40	15	20	101640	98400	697012	697012	0	2552
61	40	20	5	25410	24600	180792	180792	0	1313
62	40	20	10	50820	49200	387618	387618	0	1826
63	40	20	15	76230	73800	547454	547454	0	1166
64	40	20	20	101640	98400	735258	735258	0	1622
65	50	5	5	39260	38250	188168	188168	0	895
66	50	5	10	78520	76500	396360	396360	0	6245
67	50	5	15	117780	114750	586672	586672	0	3030
68	50	5	20	157040	153000	817638	806605	1,35	7200
69	50	10	5	39260	38250	195354	194223	0,58	7200
70	50	10	10	78520	76500	411572	411572	0	3709
71	50	10	15	117780	114750	559952	554521	0,97	7200
72	50	10	20	157040	153000	759898	747034	1,69	7200
73	50	15	5	39260	38250	203040	203040	0	4790
74	50	15	10	78520	76500	410588	410588	0	6227
75	50	15	15	117780	114750	611570	591089	3,35	7200
76	50	15	20	157040	153000	792462	792462	0	6873
77	50	20	5	39260	38250	208296	208296	0	2468
78	50	20	10	78520	76500	379038	379038	0	4368
79	50	20	15	117780	114750	593132	591466	0,28	7200
80	50	20	20	157040	153000	763416	756841	0,86	7200

Table 5 Computational Results for 4-CF Formulation

No.	Instance			4-CF Formulation					
	$\|N\|$	$\|K\|$	$\|T\|$	Constraints	Variables	Solution	LB	Gap (%)	Time (s)
1	10	5	5	2100	3850	89170	89170	0	4
2	10	5	10	4200	7700	171960	171960	0	6
3	10	5	15	6300	11550	280170	280170	0	15
4	10	5	20	8400	15400	464760	464760	0	18
5	10	10	5	2400	6600	87630	87630	0	21
6	10	10	10	4800	13200	207820	207820	0	12
7	10	10	15	7200	19800	273180	273180	0	49
8	10	10	20	9600	26400	425040	425040	0	31
9	10	15	5	2700	9350	84680	84680	0	8
10	10	15	10	5400	18700	215480	215480	0	40
11	10	15	15	8100	28050	246420	246420	0	31
12	10	15	20	10800	37400	314880	314880	0	133
13	10	20	5	3000	12100	99860	99860	0	127
14	10	20	10	6000	24200	209880	209880	0	43
15	10	20	15	9000	36300	277080	277080	0	110
16	10	20	20	12000	48400	422920	422920	0	73

17	20	5	5	7150	14700	120726	120726	0	131
18	20	5	10	14300	29400	238050	238050	0	4417
19	20	5	15	21450	44100	360026	360026	0	688
20	20	5	20	28600	58800	500660	500660	0	1360
21	20	10	5	7700	25200	140274	140274	0	424
22	20	10	10	15400	50400	265654	265654	0	2487
23	20	10	15	23100	75600	426444	426444	0	2190
24	20	10	20	30800	100800	579982	579982	0	2378
25	20	15	5	8250	35700	125210	125210	0	2334
26	20	15	10	16500	71400	220292	220292	0	3081
27	20	15	15	24750	107100	404152	380982	5,73	7200
28	20	15	20	33000	142800	512850	478802	6,64	7200
29	20	20	5	8800	46200	142828	142828	0	5875
30	20	20	10	17600	92400	258394	250041	3,23	7200
31	20	20	15	26400	138600	330902	324161	2,04	7200
32	20	20	20	35200	184800	403854	384451	4,80	7200
33	30	5	5	15200	32550	159928	152558	4,61	7200
34	30	5	10	30400	65100	323306	305117	5,63	7200
35	30	5	15	45600	97650	451370	437085	3,16	7200
36	30	5	20	60800	130200	639020	593299	7,15	7200
37	30	10	5	16000	55800	158988	151291	4,84	7200
38	30	10	10	32000	111600	296208	287813	2,83	7200
39	30	10	15	48000	167400	504168	456604	9,43	7200
40	30	10	20	64000	223200	616192	501959	18,54	7200
41	30	15	5	16800	79050	139640	131970	5,49	7200
42	30	15	10	33600	158100	313396	306064	2,34	7200
43	30	15	15	50400	237150	461172	438380	4,94	7200
44	30	15	20	67200	316200	620874	567872	8,54	7200
45	30	20	5	17600	102300	157456	151593	3,72	7200
46	30	20	10	35200	204600	306720	295065	3,80	7200
47	30	20	15	52800	306900	470370	440457	6,36	7200
48	30	20	20	70400	409200	696656	639220	8,24	7200
49	40	5	5	26250	57400	188328	171049	9,17	7200
50	40	5	10	52500	114800	inf	311859	inf	7200
51	40	5	15	78750	172200	inf	469856	inf	7200
52	40	5	20	105000	229600	inf	576733	inf	7200
53	40	10	5	27300	98400	173338	160685	7,30	7200
54	40	10	10	54600	196800	inf	305369	inf	7200
55	40	10	15	81900	295200	inf	455272	inf	7200
56	40	10	20	109200	393600	inf	637198	inf	7200
57	40	15	5	28350	139400	inf	154524	inf	7200
58	40	15	10	56700	278800	inf	312392	inf	7200
59	40	15	15	85050	418200	inf	451197	inf	7200
60	40	15	20	113400	557600	inf	622522	inf	7200
61	40	20	5	29400	180400	inf	159194	inf	7200
62	40	20	10	58800	360800	inf	341459	inf	7200
63	40	20	15	88200	541200	inf	476036	inf	7200
64	40	20	20	117600	721600	inf	647044	inf	7200
65	50	5	5	40300	89250	inf	169168	inf	7200
66	50	5	10	80600	178500	inf	365212	inf	7200
67	50	5	15	120900	267750	inf	525464	inf	7200
68	50	5	20	161200	357000	inf	738704	inf	7200
69	50	10	5	41600	153000	inf	170426	inf	7200
70	50	10	10	83200	306000	inf	363921	inf	7200
71	50	10	15	124800	459000	inf	483523	inf	7200
72	50	10	20	166400	612000	inf	641890	inf	7200
73	50	15	5	42900	216750	inf	163298	inf	7200
74	50	15	10	85800	433500	inf	344568	inf	7200
75	50	15	15	128700	650250	inf	530795	inf	7200
76	50	15	20	171600	867000	inf	674342	inf	7200

77	50	20	5	44200	280500	inf	183383	inf	7200
78	50	20	10	88400	561000	inf	320484	inf	7200
79	50	20	15	132600	841500	inf	512714	inf	7200
80	50	20	20	176800	1122000	inf	656172	inf	7200

Table 6 Computational Results for 3-VF Formulation

No.	Instance			4-CF Formulation					
	$\|N\|$	$\|K\|$	$\|T\|$	$\|N\|$	$\|K\|$	$\|T\|$	LB	$\|N\|$	$\|K\|$
1	10	5	5	1420	655	89170	89170	0	<1
2	10	5	10	2840	1310	171960	171960	0	1
3	10	5	15	4260	1965	280170	280170	0	1
4	10	5	20	5680	2620	464760	464760	0	2
5	10	10	5	1420	655	87630	87630	0	<1
6	10	10	10	2840	1310	207820	207820	0	<1
7	10	10	15	4260	1965	273180	273180	0	1
8	10	10	20	5680	2620	425040	425040	0	1
9	10	15	5	1420	655	84680	84680	0	<1
10	10	15	10	2840	1310	215480	215480	0	1
11	10	15	15	4260	1965	246420	246420	0	2
12	10	15	20	5680	2620	314880	314880	0	3
13	10	20	5	1420	655	99860	99860	0	<1
14	10	20	10	2840	1310	209880	209880	0	<1
15	10	20	15	4260	1965	277080	277080	0	1
16	10	20	20	5680	2620	422920	422920	0	2
17	20	5	5	4820	2305	120726	120726	0	79
18	20	5	10	9640	4610	238050	238050	0	478
19	20	5	15	14460	6915	360026	360026	0	211
20	20	5	20	19280	9220	500660	500660	0	55
21	20	10	5	4820	2305	140274	140274	0	18
22	20	10	10	9640	4610	265654	265654	0	92
23	20	10	15	14460	6915	426444	426444	0	942
24	20	10	20	19280	9220	579982	579982	0	899
25	20	15	5	4820	2305	125210	125210	0	240
26	20	15	10	9640	4610	220292	220292	0	3287
27	20	15	15	14460	6915	403570	403570	0	1380
28	20	15	20	19280	9220	512072	512072	0	2154
29	20	20	5	4820	2305	142828	142828	0	235
30	20	20	10	9640	4610	258162	258162	0	75
31	20	20	15	14460	6915	330902	330902	0	1060
32	20	20	20	19280	9220	403568	403568	0	250
33	30	5	5	10220	4955	159928	152607	4,58	7200
34	30	5	10	20440	9910	322300	314205	2,51	7200
35	30	5	15	30660	14865	451798	436957	3,28	7200
36	30	5	20	40880	19820	638992	613563	3,98	7200
37	30	10	5	10220	4955	158868	158868	0	3634
38	30	10	10	20440	9910	296208	284729	3,88	7200
39	30	10	15	30660	14865	506974	467862	7,71	7200
40	30	10	20	40880	19820	600922	569236	5,27	7200
41	30	15	5	10220	4955	139640	137287	1,69	7200
42	30	15	10	20440	9910	313412	298045	4,90	7200
43	30	15	15	30660	14865	459392	458538	0,19	7200
44	30	15	20	40880	19820	612758	587814	4,07	7200
45	30	20	5	10220	4955	157456	157456	0	1772
46	30	20	10	20440	9910	306720	292553	4,62	7200
47	30	20	15	30660	14865	472916	427451	9,61	7200
48	30	20	20	40880	19820	678160	654546	3,48	7200
49	40	5	5	17620	8605	188154	168615	10,38	7200
50	40	5	10	35240	17210	349808	313479	10,39	7200
51	40	5	15	52860	25815	548868	440183	19,80	7200
52	40	5	20	70480	34420	643618	567367	11,85	7200

53	40	10	5	17620	8605	172728	162728	5,79	7200
54	40	10	10	35240	17210	341100	308038	9,69	7200
55	40	10	15	52860	25815	525182	468695	10,76	7200
56	40	10	20	70480	34420	729526	647633	11,23	7200
57	40	15	5	17620	8605	167102	154877	7,32	7200
58	40	15	10	35240	17210	348416	313786	9,94	7200
59	40	15	15	52860	25815	535880	455645	14,97	7200
60	40	15	20	70480	34420	726248	642515	11,53	7200
61	40	20	5	17620	8605	183586	163093	11,16	7200
62	40	20	10	35240	17210	392488	365731	6,82	7200
63	40	20	15	52860	25815	572884	497207	13,21	7200
64	40	20	20	70480	34420	762764	646854	15,20	7200
65	50	5	5	27020	13255	201522	167066	17,10	7200
66	50	5	10	54040	26510	434358	353519	18,61	7200
67	50	5	15	81060	39765	630916	533326	15,47	7200
68	50	5	20	108080	53020	914254	727202	20,46	7200
69	50	10	5	27020	13255	216880	163348	24,68	7200
70	50	10	10	54040	26510	444684	358422	19,40	7200
71	50	10	15	81060	39765	640300	501309	21,71	7200
72	50	10	20	108080	53020	803968	686244	14,64	7200
73	50	15	5	27020	13255	212148	179062	15,60	7200
74	50	15	10	54040	26510	443406	371609	16,19	7200
75	50	15	15	81060	39765	675374	530172	21,50	7200
76	50	15	20	108080	53020	876912	686009	21,77	7200
77	50	20	5	27020	13255	222624	189737	14,77	7200
78	50	20	10	54040	26510	402052	335062	16,66	7200
79	50	20	15	81060	39765	679630	523265	23,01	7200
80	50	20	20	108080	53020	821484	683790	16,76	7200

5.0 SUMMARY AND FUTURE RESEARCH DIRECTIONS

In this study, we propose and discuss three alternative formulations for P-VRPSPD. These formulations are developed based on the available formulations for VRPSPD in literature [15]-[17]. We a perform comparison analysis on a set of generated instances for P-VRPSPD since there is no available literature on this topic until now. Overall, it is observed that the 3-VF formulation is superior to the other formulations based on the solution quality and computational efficiency.

Finally, there are numerous exciting directions for future works in P-VRPSPD. This study shows that the P-VRPSPD, being a generalization of a NP-hard problem, turns out to be a difficult problem to solve. Thus, future works may attempt to develop a tighter formulation for the P-VRPSPD, in which the works of Subramanian et al. [47][48] seem to be good starting points. Another direction in this regard is to develop an effective and efficient heuristic solution for this problem. Researchers can also explore incorporating other constraints to make the P-VRPSPD becomes more realistic, since some assumptions hold in the problem seems to be unrealistic. Some initial ideas are to relax the requirement to satisfy all demands and to allow a customer to be visited more than once, which seems to be more relevant in the case of pickup and delivery.

Acknowledgement

We thank the anonymous referees for their insightful suggestions.

References

[1] Dantzig, G. B. and Ramser, J. H. 1959. The truck dispatching problem. *Management Science,* 6: 80–91, 1959. DOI: https://doi.org/10.1287/mnsc.6.1.80

[2] Eksioglu, B., Vural, A. V. and Reisman, A. 2009. The vehicle routing problem: A taxonomic review. *Computers & Industrial Engineering,* 57: 1472-1483. DOI: https://doi.org/10.1016/j.cie.2009.05.009

[3] Braekers, K., Ramaekers, K., and Van Nieuwenhuyse, I. 2016. The vehicle routing problem: State of the art classification and review. *Computers & Industrial Engineering,* 99: 300–313. DOI: https://doi.org/10.1016/j.cie.2015.12.007.

[4] Vidal, T., Laporte, G. and Matl, P. 2020. A concise guide to existing and emerging vehicle routing problem variants. *European Journal of Operational Research,* 286: 401-416. DOI: https://doi.org/10.1016/j.ejor.2019.10.010.

[5] Berbeglia, G., Cordeau, J. F., Gribkovskaia, I. and Laporte, G. 2007. Static pickup and delivery problems: a classification scheme and survey. *TOP,* 15: 1-31. DOI: https://doi.org/10.1007/s11750-007-0009-0.

[6] Battarra, M., Cordeau, J. F. and Iori, M. 2015. Pickup-and-delivery problems for goods transportation in: P. Toth, D. Vigo (Eds.). *Vehicle Routing: Problems, Methods, and Applications*. *MOS-SIAM Series on Optimization*. Chapter 6: 161–191.

[7] Çatay, B. 2010. A new saving-based ant algorithm for the Vehicle Routing Problem with simultaneous Pickup and Delivery. *Expert Systems With Applications,* 37: 6809-6817. DOI: https://doi.org/10.1016/j.eswa.2010.03.045.

[8] Drexl, M., Rieck, J., Sigl, T. and Press, B. 2013. Simultaneous Vehicle and Crew Routing and Scheduling for Partial- and Full-Load Long-Distance Road Transport. *Business Research,* 6: 242-264. DOI: https://doi.org/10.1007/BF03342751.

[9] Belgin, O., Karaoglan, I. and Altiparmak, F. 2018. Two-echelon vehicle routing problem with simultaneous pickup and delivery: Mathematical model and heuristic approach. *Computers & Industrial Engineering,* 115: 1-16. DOI: https://doi.org/10.1016/j.cie.2017.10.032.

[10] Zhang, Z., Cheang, B., Li, C. and Lim, A. 2019. Multi-commodity demand fulfillment via simultaneous pickup and delivery for a fast fashion retailer. *Computers & Operations Research,* 103: 81-96. DOI: https://doi.org/10.1016/j.cor.2018.10.020.

[11] Hemmelmayr, V., Doerner, K. F., Hartl, R. F. and Rath, S. A heuristic solution method for node routing based solid waste collection problems. *Journal of Heuristics,* 19: 129-156. DOI: https://doi.org/10.1007/s10732-011-9188-9.

[12] Coene, S., Arnout, A. and Spieksma, F. C. R. 2010. On a periodic vehicle routing problem. *Journal of the Operational Research Society,* 61: 1719–1728. DOI: https://doi.org/10.1057/jors.2009.154.

[13] An, Y. J., Kim, Y. D., Jeong, B. J. and Kim, S. D. 2012. Scheduling healthcare services in a home healthcare system. *Journal of the Operational Research Society,* 63: 1589-1599. DOI: https://doi.org/10.1057/jors.2011.153.

[14] Koç, Ç., Laporte, G. and Tükenmez, İ. 2020. A review of vehicle routing with simultaneous pickup and delivery. *Computers & Operations Research,* 122: 104987. DOI: https://doi.org/10.1016/j.cor.2020.104987.

[15] Dell'Amico, M., Righini, G. and Salani, M. 2006. A branch-and-price approach to the vehicle routing problem with simultaneous distribution and collection. *Transportation Science,* 40: 235–247. DOI: https://doi.org/10.1287/trsc.1050.0118.

[16] Montané, F. A. T. and Galvão, R. D. 2006. A tabu search algorithm for the vehicle routing problem with simultaneous pick-up and delivery service. *Computers & Operations Research,* 33: 595–619. DOI: https://doi.org/10.1016/j.cor.2004.07.009.

[17] Rieck, J. and Zimmermann, J. 2013. Exact Solutions to the Symmetric and Asymmetric Vehicle Routing Problem with Simultaneous Delivery and Pick-Up. *Business Research,* 6: 77–92. DOI: https://doi.org/10.1007/BF03342743.

[18] Campbell, A. M. and Wilson, J. H. 2012. Forty Years of Periodic Vehicle Routing. *Networks,* 60: 45–58. DOI: https://doi.org/10.1002/net.21527.

[19] Hadjiconstantinou, E. and Baldacci, R. 1998. A multi-depot period vehicle routing problem arising in the utilities sector. *Journal of the Operational Research Society,* 49: 1239–1248. DOI: https://doi.org/10.1057/palgrave.jors.2600641.

[20] Blakeley, F., Bozkaya, B., Cao, B., Hall, W. and Knolmajer, J. 2003. Optimizing periodic maintenance operations for schindler elevator corporation. *Interfaces,* 33: 67–79. DOI: https://doi.org/10.1287/inte.33.1.67.12722.

[21] Nuortio, T., Kytöjoki, J., Niska, H. and Bräysy, O. 2006. Improved route planning and scheduling of waste collection and transport. *Expert Systems With Applications,* 30: 223–232. DOI: https://doi.org/10.1016/j.eswa.2005.07.009.

[22] Angelelli, E. and Speranza, M. G. 2002. The periodic vehicle routing problem with intermediate facilities. *European Journal of Operational Research,* 137: 233–247. DOI: https://doi.org/10.1016/S0377-2217(01)00206-5.

[23] Bommisetty, D., Dessouky, M. and Jacobs, L. 1998. Scheduling collection of recyclable material at northern Illinois university campus using a two-phase algorithm. *Computers & Industrial Engineering,* 35: 435-438. DOI: https://doi.org/10.1016/s0360-8352(98)00127-2.

[24] Baptista, S., Oliveira, R. C. and Zúquete, E. 2002. A period vehicle routing case study. *European Journal of Operational Research,* 139: 220-229. DOI: https://doi.org/10.1016/S0377-2217(01)00363-0.

[25] Aksen, D., Kaya, O., Salman, F. S. and Akça, Y. 2012. Selective and periodic inventory routing problem for waste vegetable oil collection. *Optimization Letters,* 6: 1063–1080. DOI: https://doi.org/10.1007/s11590-012-0444-1.

[26] Shih, L. H. and Chang, H. C. 2001. A routing and scheduling system for infectious waste collection. *Environmental Modeling & Assessment,* 6: 261–269. DOI: https://doi.org/10.1023/A:1013342102025.

[27] Shih, L. H. and Lin, Y. T. 2019. Optimal routing for infectious waste collection. *Journal of Environmental Engineering,* 125: 479–484. DOI: https://doi.org/10.1061/(ASCE)0733-9372(1999)125:5(479).

[28] Claassen, G. D. H. and Hendriks, T. H. B. 2007. An application of Special Ordered Sets to a periodic milk collection problem. *European Journal of Operational Research*, 180: 754–769. DOI: https://doi.org/10.1016/j.ejor.2006.03.042.

[29] Alegre, J., Laguna, M. and Pacheco, J. 2007. Optimizing the periodic pick-up of raw materials for a manufacturer of auto parts. *European Journal of Operational Research*, 179: 736–746. DOI: https://doi.org/10.1016/j.ejor.2005.03.063.

[30] Rademeyer, A. L. and Benetto, R. 2012. A rich multi-period delivery vehicle master routing problem case study. in: *Computers and Industrial Engineering 42 Proceedings*, Cape Town, South Africa.

[31] Ronen, D. and Goodhart, C. A. 2008. Tactical store delivery planning. *Journal of the Operational Research Society*, 59: 1047–1054. DOI: https://doi.org/10.1057/palgrave.jors.2602449.

[32] Banerjea-Brodeur, M., Cordeau, J. F. Laporte, G. and Lasry, A. 1998. Scheduling linen deliveries in a large hospital. *Journal of the Operational Research Society*, 49: 777–780. DOI: https://doi.org/10.1057/palgrave.jors.2600581.

[33] Rusdiansyah, A. and Tsao, D. B. 2005. An integrated model of the periodic delivery problems for vending-machine supply chains. *Journal of Food Engineering*, 70: 421–434. DOI: https://doi.org/10.1016/j.jfoodeng.2004.05.073.

[34] Angelelli, E. and Mansini, R. 2002. The vehicle routing problem with time windows and simultaneous pick-up and delivery. in: Klose, A. Speranza, M.G. and Van Wassenhove, L. N. *Quantitative Approaches to Distribution Logistics and Supply Chain Management, Lecture Notes in Economics and Mathematical Systems*. Springer-Verlag, Berlin: 249–267.

[35] Fan, J. 2011. The vehicle routing problem with simultaneous pickup and delivery based on customer satisfaction. in: *Procedia Engineering*, 15: 5284-5289. DOI: https://doi.org/10.1016/j.proeng.2011.08.979.

[36] Wang, H. F. and Chen, Y. Y. 2013. A coevolutionary algorithm for the flexible delivery and pickup problem with time windows. *International Journal of Production Economics*, 141: 4–13. DOI: https://doi.org/10.1016/j.ijpe.2012.04.011.

[37] Wang, Y., Ma, X., Lao, Y., Wang, Y. and Mao, H. 2013. Vehicle Routing Problem Simultaneous Deliveries and Pickups with Split Loads and Time Windows. *Transportation Research Record*, 2378: 120-128.

[38] Wang, C., Mu, D., Zhao, F. and Sutherland, J. W. 2015. A parallel simulated annealing method for the vehicle routing problem with simultaneous pickup-delivery and time windows. *Computers & Industrial Engineering*, 83: 111–122.DOI: https://doi.org/10.1016/j.cie.2015.02.005.

[39] Qu, Y. and Bard, J. F. 2013. The heterogeneous pickup and delivery problem with configurable vehicle capacity. *Transportation Research Part C: Emerging Technologies*, 32: 1–20.DOI: https://doi.org/10.1016/j.trc.2013.03.007.

[40] Avci, M. and Topaloglu, S. 2016. A hybrid metaheuristic algorithm for heterogeneous vehicle routing problem with simultaneous pickup and delivery. *Expert Systems With Applications*, 53: 160–171.DOI: https://doi.org/10.1016/j.eswa.2016.01.038.

[41] Nagy, G. and Salhi, S. 2005. Heuristic algorithms for single and multiple depot vehicle routing problems with pickups and deliveries. *European Journal of Operational Research*, 162: 126-141. DOI: https://doi.org/10.1016/j.ejor.2002.11.003.

[42] Li, J., Pardalos, P. M., Sun, H., Pei, J. and Zhang, Y. 2015. Iterated local search embedded adaptive neighborhood selection approach for the multi-depot vehicle routing problem with simultaneous deliveries and pickups. *Expert Systems With Applications*, 42: 3551–3561. DOI: https://doi.org/10.1016/j.eswa.2014.12.004.

[43] Wang, C. and Qiu, Y. 2011. Vehicle routing problem with stochastic demands and simultaneous delivery and pickup based on the cross-entropy method. in: *Lecture Notes in Electrical Engineering*. DOI: https://doi.org/10.1007/978-3-642-25646-2_7.

[44] Zhang, T., Chaovalitwongse, W. A. and Zhang, Y. 2012. Scatter search for the stochastic travel-time vehicle routing problem with simultaneous pick-ups and deliveries. *Computers & Operations Research*, 39: 2277–2290. DOI: https://doi.org/10.1016/j.cor.2011.11.021.

[45] Yin, C., Bu, L. and Gong, H. 2013. Mathematical model and algorithm of split load vehicle routing problem with simultaneous delivery and

pickup. *International Journal of Innovative Computing, Information and Control*, 9: 4497–4508.

[46] Wang, J., Zhou, Y., Wang, Y. Zhang, J., Chen, C. L. P. and Zheng, Z. 2016. Multiobjective Vehicle Routing Problems with Simultaneous Delivery and Pickup and Time Windows: Formulation, Instances, and Algorithms. *IEEE Transactions on Cybernetics*, 46: 582–594. DOI: https://doi.org/10.1109/TCYB.2015.2409837.

[47] Subramanian, A., Uchoa, E., Pessoa, A. A. and Ochi, L. S. 2011. Branch-and-cut with lazy separation for the vehicle routing problem with simultaneous pickup and delivery. *Operations Research Letters*, 39: 338–341. DOI: https://doi.org/10.1016/j.orl.2011.06.012.

[48] Subramanian, A., Uchoa, E., Pessoa, A. A. and Ochi, L. S. 2013. Branch-cut-and-price for the vehicle routing problem with simultaneous pickup and delivery. *Optimization Letters*, 7: 1569–1581. DOI: https://doi.org/10.1007/s11590-012-0570-9.

[49] Baldacci, R., Hadjiconstantinou, E. and Mingozzi, A. 2004. An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation., *Operations research*, 52(5): 723-738. DOI: https://doi.org/10.1287/opre.1040.0111

[50] Laporte, G., Nobert, Y. and Desrochers, M. 1985. Optimal routing under capacity and distance restrictions., *Operations research*, 33(5): 1050-1073. DOI: https://doi.org/10.1287/opre.33.5.1050

[51] Dantzig, G., Fulkerson, R. and Johnson, S. 1954. Solution of a large-scale traveling-salesman problem. *Journal of the operations research society of America*, 2(4): 393-410DOI: https://doi.org/10.1287/opre.2.4.393

[52] Miller, C. E., Tucker, A. W. and Zemlin, R. A. 1960. Integer programming formulation of traveling salesman problems., *Journal of the ACM (JACM)*, 7(4): 326-329. DOI: https://doi.org/10.1145/321043.321046

[53] Solomon, M. M. and Desrosiers, J. 1988. Survey paper—time window constrained routing and scheduling problems., *Transportation science*, 22(1): 1-13. DOI: https://doi.org/10.1287/trsc.22.1.1

[54] Dethloff, J. 2001. Vehicle routing and reverse logistics: The vehicle routing problem with simultaneous delivery and pick-up. *OR Spektrum*, 23: 79–96. DOI: https://doi.org/10.1007/PL00013346.

[55] Salhi, S. and Nagy, G. 1999. A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. *Journal of the Operational Research Society*, 50: 1034-1042. DOI: https://doi.org/10.1057/palgrave.jors.2600808.

[56] de Freitas, J. C. and Penna, P. H. V. 2020. A variable neighborhood search for flying sidekick traveling salesman problem. *International Transactions in Operational Research*, 27: 267–290. DOI: https://doi.org/10.1111/itor.12671.

[57] Murray, C. C. and Chu, A. G. 2015. The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery. *Transportation Research Part C: Emerging Technologies*, 54: 86–109. DOI: https://doi.org/10.1016/j.trc.2015.03.005.

[58] Furtado, M. G. S., Munari, P. and Morabito, R. 2017. Pickup and delivery problem with time windows: a new compact two-index formulation. *Operations Research Letters*, 45(4): 334-341. DOI: https://doi.org/10.1016/j.orl.2017.04.013

[59] Camm, J. D., Raturi, A. S. and Tsubakitani, S. 1990. Cutting big M down to size. *Interfaces*, 20(5): 61-66. DOI: https://doi.org/10.1287/inte.20.5.61

[60] Belotti, P., Bonami, P., Fischetti, M., Lodi, A., Monaci, M., Nogales-Gómez, A. and Salvagnin, D. 2016. On handling indicator constraints in mixed integer programming. *Computational Optimization and Applications*, 65(3): 545-566. DOI: https://doi.org/10.1007/s10589-016-9847-8