ASEAN Engineering Journal

Full Paper

THE EFFECT OF DIFFERENT DECODING TECHNIQUES WITH GAUSSIAN APPROXIMATION ON THE PERFORMANCE OF POLAR CODES

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Article history
Received
25 February 2022
Received in revised form
21 June 2022
Accepted
07 August 2022

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Graphical abstract

Abstract

In the last ten years, polar code research has piqued the interest of firms and researchers, particularly in the communication industry. Polar codes have been utilised as a coding method for the fifth-generation wireless standard (5G). However, the polar decoder does not adequately correct errors in successive cancellation (SC) decoding when dealing with short- to intermediate-length codes. However, SC decoding can correct errors more efficiently by using sequential cancellation list (SCL) decoding. The main drawback of SCL is its higher cost due to computational complexity and throughput. The present research investigates the effect of Gaussian approximation (GA) and different decoding approaches on the performance of polar codes. First, SC and SCL decoders are developed utilising amplitude shift keying modulation; a decoder using GA is then integrated. According to simulation data, the SCL, both with and without GA, exhibits a better block error rate (BLER) than SC. The maximum difference between the SCL decoder and SC decoder is 0.6 dB at BLER=0.1 for N=2048. Furthermore, at BLER=5.6 x 10^{-6}, the SCL decoder with GA performs better than the SC decoder for block lengths, N=1024, with a maximum difference of 2.72 dB. When the polar decoder with GA is utilised, enhancements are observed in polar code performance for various list sizes and block lengths, although time complexity is increased.

Keywords: Amplitude shift keying, fifth-generation wireless standard, Gaussian approximation, polar codes, successive cancellation list decoding

1.0 INTRODUCTION

Fifth generation (5G) cellular network technology represents the most current cellular network technology and has been widely used since early 2019. 5G technology allows customers to experience peak data speeds of several gigabits per second, ultra-low latency, more dependability, expanded network bandwidth, better connectivity, and more consistency. Over the past decade, polar codes have been introduced to build control channels to improve enhanced mobile broadband (eMBB) services. The first polar code algorithm was introduced by E. Arikan in 2009 [1]. These algorithms consist of code sequences that achieve symmetric capacity, a binary-input discrete memoryless channel [2], and linear block error-correcting code with symmetric channel characteristics. Low-density parity-check (LDPC) codes, which are used in a variety of current wireless applications, are well-known for their excellent performance with long block lengths and high code rates [3]. On the other hand, it performs poorly with short block lengths and low code rates. Therefore, polar codes are the best available candidates for controlling channels in 5G communication because of their vast application possibilities [4]. Generally, LDPC codes was dedicated for data channels while polar codes for control channels [5,6]. However, owing to their low computational complexity, polar codes with successive cancellation (SC) decoding cannot correct errors as well as other codes such as LDPC codes or turbo codes. Since the first polar code decoding method was introduced, this technique has been improved by the inception of a successive cancellation list (SCL) decoding scheme for 5G applications. The primary advantage of SCL decoding over SC decoding is that it can minimise the block error rate (BLER) more efficiently while maintaining a similar decoding speed.

SC decoding methods are the most common approach to decoding polar codes. It can be viewed as a greedy search algorithm in the code tree, as it only finds one decoding path by making step-by-step decisions with low complexity $O(N \log N)$ where N is block lengths [7]. Although these methods are not especially computationally complex, the decoding throughput is limited due to SC's serial nature. Furthermore, the length of polar codes exceeds SC decoding's capabilities and channel capacity. SC decoding is also inadequate regarding its ability to correct errors when used for short to intermediate code lengths [8]. While SCL enhances the error correction performance of SC for polar codes, it has a higher computational complexity and poorer output performance [9].

An SCL decoder known as successive cancellation list flip (SCL-Flip) was recently introduced by Y. H. Pan et al. to improve the previous SCL scheme [10]. It is less complex than other decoders, and it saturates more quickly. A specific scheme, denoted as SCL-Flip-ω, was developed, and generalised at most ω times to flip the decision in the competition path during decoding. The results of the concatenated polar codes proposed by the researchers achieved a 0.13 dB performance gain when eight sizes of the SCL list were utilised compared to the original scheme for BLER reading, 10^{-3}. Differ from the 16 sizes of the SCL list, the results achieved 0.14 dB gain for BLER is 10^{-2}. Hence, the utilisation of the small list size indicated that the proposed algorithm yields better frame error rates than other algorithms while keeping implementation complexity low. The performance of SCL decoding to aid cyclic redundancy check (CRC) algorithm is comparable to LDPC and lowers complexity than turbo codes which has been proved by [11].

In 2017, Gaussian approximation (GA) was utilised to reduce density evolution despite its high computational complexity [12]. Due to its efficiency and complexity of construction, GA can improve the performance of polar codes, particularly the polar decoder. GA can also reduce density evolution without sacrificing the computational complexity of the polar codes' SC and SCL. Apart from that, researchers in [13] claimed that GA for polar codes algorithm had suffered the subchannel selection inaccuracy when the code length is quite long, resulting in catastrophic performance loss. However, this algorithm only involved the evaluation of decoder performance. Instead, this research only focused on their proposed metric, known as a cumulative-logarithmic error (CLE), using the polar codes approach for conventional approximate GA (AGA) evaluation.

The purpose of the current study is to explore how GA affects the decoding performance of SC and SCL in the additive white Gaussian noise (AWGN) channel with amplitude shift keying (ASK) modulation [14]. With this purpose in mind, this study was carried out to accomplish three goals. The first is to create SC and SCL decoders for the AWGN channel that use ASK modulation. The second is to integrate GA into polar decoders with the AWGN channel. The third goal is to examine how SC and SCL decoding processes perform when the proposed GA and ASK are applied.

The remainder of this paper is structured as follows. Section 2 overviews the relevant research on SC and SCL decoders. Section 3 discusses the algorithms utilised in this research, including polar code, SC, SCL, and GA algorithms. Section 4 explores the details of the decoding process, which is divided into three stages: developing SC and SCL with ASK modulation, integrating GA with a polar decoder, and analysing the BLER performances of the polar decoders. The performances of polar codes using GA, SCL, and SC decoders with different parameter values applied according to the simulation outcomes are analysed in Section 5. Finally, Section 6 offers some concluding remarks.

2.0 RELATED RESEARCH ON POLAR CODES

The belief propagation (BP) decoder is considered a reconfigurable decoder because it can decode both polar and low-density parity-check (LDPC) codes [15]. The proposed decoder can also improve hardware efficiency and lower the overall cost since only one decoder is implemented. Combined computational blocks were employed to replace a basic computational block (BCB) or a portion of the 0th check node computation with the reconfigurable BP decoder. The saving hardware resource denoted as σ has achieved a 73.06% reduction compared to the parallel LDPC and 15.53% compared to the LDPC with layered decoding. The complexity also reduced 73.02% of σ compared to pipelined BP polar decoder and 5.56% reduction compared to field-programmable gate array (FPGA) implementation of the polar decoder. It can be seen that the proposed decoder by this group has exhibited better hardware efficiency than a stand-alone LDPC or polar decoders with a comparable error performance.

Y. Shen *et al.* (2020) claimed that the previous SCL has long latency and that the SCL with BP decoding does not exhibit a worse error-correction performance than the previous SCL [16]. Recently, a bit-flipping method with BP decoding was proposed. However, this method falsely identifies erroneous bits in fixed flip sets. Therefore, the researchers proposed a generalised BP flip (BPF) decoding method in which multiple bits can be flipped during a single decoding process. This generalised BPF with merged sets performed very well in SCL-8, outperforming other decoding methods.

Soft bit decisions, upon which decoding is based, are made at the decoder output using the log-likelihood-ratio (LLR) values of the transmitted message bits [17]. List decoders are among the recently proposed decoder architectures. The SC decoder is widely used because it can reduce complexity, as the complexity of the linear mapping of all decoder components is $2N$ when this decoder is employed. Meanwhile, the complexity of a SC decoder is $O(N \log N)$. SC makes decisions bit by bit and, thus, can make only one decision at a time. The researchers constructed a polar encoder and decoder in MATLAB, carried out an extensive decoder analysis, and implemented several SC components in Verilog. Channel polarisation is an appealing notion owing to its dependability and simplicity. The proposed polar codes with SC reduced implementation and have become the most promising error correction codes developed so far.

Other researchers proposed an improved method for rapid polar code decoding that significantly reduced the time required for SC-based decoding [18]. Most of the subcodes used to quickly decode polar codes were made up of three multi-nodes polar code subcodes. An evaluation of the influence on SC and SCL decoding latency showed that the proposed technique is significantly faster than other contemporary decoding systems. The SC-tree search was avoided and, thus, performance was significantly improved by decoding the subcodes recognised by these patterns, referred to as nodes. From an algebraic perspective, each frozen bit limits the codeword's potential values. Because polar codes are recursive, the components (or nodes) of the SC tree can be decoded in the same manner. Furthermore, path metrics were correctly computed using the proposed quick decoding method.

A novel early termination (ET) scheme that considers additional checkpoints has been proposed to reduce the long decoding and response times associated with polar codes [19]. If the incoming signal decoding process fails, the decoding process should be terminated as soon as possible. Shorter decoding intervals allow for more complex post-processing if a connection is temporarily interrupted, resulting in a faster reaction time. Many ET techniques have been proposed to forecast decoding failure for polar codes. Including parity check elements increased the likelihood that ET would be triggered, thus speeding up the ET scheme. Also, the time and energy needed for ET processes can be reduced by implementing a strategy based on distributed parity bits as long as the time required for the decoding stage can be minimised. Additional checkpoints can be selected in an ET strategy without reducing the message bit coding rate.

The LLR has been employed to choose a single bit during SC decoding. All sub-canals were almost fully polarised as the code length approached infinity, and the data were successfully decoded. While this seems to indicate that the polarisation code had reached the channel's symmetric capacity, the BLER of the SC algorithm is insufficient when dealing with codes of a short or medium length. A previously presented GA-optimised SC Flip (G-SCF) decoding method minimised the likelihood that each subchannel would be decoded as a threshold value obtained by the approximation. Thus, the number of LLR formula recursions was also minimised [20].

The SC Flip (SCF) method can improve BLER efficiency, but it can correct only one mistake, and its complexity is substantial at a low signal-to-noise ratio. Meanwhile, the segment SCF algorithm can decode many error bits by segmentation while reducing complexity at low signal-to-noise ratios. The G-SCF method builds on this foundation by employing an approximate Gaussian construction technique to further optimise the process. According to [21], polar codes perform well but face enormous challenges, while deep learning models have limited sizes. A model comprising an SC decoder with a convolutional neural network (CNN), along with additive correlated Gaussian noise channels, that can overcome this limitation was proposed. The model showed improved decoding performance and reduced the limitations imposed on deep learning by code length. The novel foundation was presented as a CNN-SC decoder capable of estimating coloured noise and improving performance in correlated noise channels with a high correlation coefficient.

Research contribution regarding GA implementation architecture for SCL and SC decoding for polar codes has good references. However, most research did not focus on GA implementation architecture and ASK modulation with either SC or SCL decoder for their polar codes algorithm [21–24]. Besides, several research papers were proposed over a decade ago [25], [26]. For instance, an analysis of polar codes over the AWGN channel based on Gaussian approximation was proposed by D. Wu et al. [27]. In this paper, SC decoding was employed to decode the message and estimate the exact BLER with the help of Gaussian approximation. The BLER performances of this method have almost the same performance as the previous Monte Carlo method. Even though the method comes with lower complexity and maintains good performance, this method was proposed with BPSK modulation. Similar to research in [28], BPSK modulation was also utilised to modulate the encoded message along with the segmented SC-flip (SSCF) decoding algorithm. This approach achieved 46.99% average complexity reduction with extra minimised BLER. Nevertheless, the error rate performance for SCL decoding with ASK modulation using the LM-rate demapper Gaussian approximation (LM-DGA) method was also not disappointing [29]. The polar codes approach with high-order modulating outperformed state-of-art-LDPC codes. [30] also indicated that their design of multilevel coding (MLC) with SCL decoding for CRC-concatenated polar codes, which uses ASK modulation, improved the error rate performance. Based on the review, most previous research rarely proposes the GA approach and ASK modulation in their works. Therefore, a further investigation of the effects of GA on the performance of the SC/SCL method with ASK modulation is made in this research paper.

3.0 POLAR CODES ALGORITHMS

A. Polar encoder

Polar codes are linear block codes with a length of *N=2^n* and a rate of *R=K/N*, where *K* is the number of coded bits. The transformation matrix, $G^{\otimes n}$, described in Equation (1), can be used to create polar codes.

$$x = uG^{\otimes n} \tag{1}$$

Vector $u = \{u_0, u_1, \dots, u_{N-1}\}$ is encoded into vector $x = \{x_0, x_1, \dots, x_{N-1}\}$. The matrix $G^{\otimes n}$ is obtained as the $n - th$ Kronecker product of the polarising kernel as shown in Equation (2).

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

Of the available bit channels (*N*), the polar encoding method chooses K information bits and assigns them to reliable bit channels. The unchosen bit channels represent the frozen set of F in u and are given the known value. A flag s_i is added to each bit channel to differentiate between frozen and information bits, as shown in Equation (3).

$$s_i = \begin{cases} 0 \; if \; u_i \in F \\ 1 \; otherwise \end{cases} \tag{3}$$

B. SC decoder

Polar codes are decoded by SC decoding based on [1, 12]. The estimated bits are denoted by $\hat{u}_1^N$, and the received codeword is denoted by y_1^N. The decoding rule of the SC decoder is as follows:

u_i is assigned to the fixed value when the source bit u_i is frozen; otherwise, it is calculated using Equation (4).

$$\hat{u}_i = \begin{cases} 0 \; h(y_1^N, \hat{u}_1^i) > 0 \\ \;\; 1 \; otherwise \end{cases} \tag{4}$$

The metric value $h(y_1^N, \hat{u}_1^i)$ is calculated as shown in Equation (5).

$$h(y_1^N, \hat{u}_1^{i-1}) = M_N^{(i)}(u_i = 0, \hat{u}_1^{i-1} \middle| y_1^N) - M_N^{(i)}(u_i = 1, \hat{u}_1^{i-1} | y_1^N) \quad (5)$$

In the Equation (5), $M_N^{(i)}$ is the logarithmic posteriori probability, which can be determined recursively from the logarithmic posteriori probabilities of the channel denoted as $W. M_1^{(1)}(x|y)$ is calculated using Equation (6).

$$M_1^{(1)}(x|y) = log \frac{W(y|x)}{\sum_y W(y|y)} \quad (6)$$

Here, $(y|x)$ represents the channel transition probabilities, $x \in \chi, y \in \gamma$.

C. SCL decoder

Assuming that the transmitted codeword is W_0^{N-1} and the obtained code word is y_0^{N-1} the log-likelihood ratio of the approximate $\hat{u}_i$ of the information bits u_i can be expressed as in Equation (7).

$$L_N^i = (y_0^{N-1}, u_0^{i-1}) = ln \frac{W_N^{(i)}(y_0^{N-1}, u_0^{i-1}|0)}{W_N^{(i)}(y_0^{N-1}, u_0^{i-1}|1)} \quad (7)$$

In this equation, the transfer likelihood of the ith subchannel is expressed by $W_N^{(i)}(y_0^{N-1}, u_0^{i-1}|u_i)$. Thus, $\hat{u} = \delta(L_N^{(i)})$ in the formula $\delta(x) = \frac{1}{2}(1 - sign(x))$. In the SCL decoding algorithms [12], L represents the width of the path search, which is a crucial parameter. If the path search width is less than L, the SCL technique must decode the path as much as possible. When the two branches keep extending down, copying the data from the parent node to the two branches is required. Meanwhile, when the decoder expands the path beyond L in the decoding process, these paths must be eliminated to ensure that the reserved decoding path does not exceed the L bar. Therefore, the path metrics (PM) is needed to determine the final decoded output (when the path exceeds L) and which part needs to be deleted.

$$PM_l^i = -\ln\left(P\left[U_0^i[l] | Y = y_0^{N-1}\right]\right) \quad (8)$$

$$= PM_l^{(i-1)} + ln\left(1 + e^{-(1-2\hat{u}_i[l] \cdot L}\right) \quad (9)$$

$$= PM_l^{(i-1)} + ln\left(1 + e^{-(1-2\hat{u}_i[l] \cdot L_N^j[l]}\right) \quad (10)$$

According to [12, 31], PM is well-approximated as

$$PM_l^i \approx \begin{cases} PM_l^{i-1} \; if \; \hat{u}_i[1] = \delta(L_N^{(i)}) \\ PM_l^{i-1} + \left|L_N^{(j)}[l]\right| \; otherwise \end{cases} \quad (11)$$

D. GA algorithm

Refer to [10], L_N^i is the LLR of the subchannel $W_N^{(i)}$. The LLRs, L_N^i, follow a Gaussian n distribution in an AWGN channel. Additionally, each received symbol, y_i, of the LLRs follow the rule presented as follows:

$L_N^i(y_i) \sim N\left(\frac{2}{\sigma^2}, \frac{4}{\sigma^2}\right)$ is the transmitted all-zero codeword. Thus, the recursive function of the mean can be expressed as Equations (12) and (13).

$$E\left[L_N^{(2i-1)}\right] = \emptyset^{-1}\left(1 - \left(1 - \emptyset\left(E\left[L(_{\frac{N}{2}}^{(2i-1)}\right]\right)\right)\right) \quad (12)$$

$$E\left[L_N^{(2i)}\right] = 2E\left[L_{N/2}^{(i)}\right] \quad (13)$$

In these equations, $E\left[L_N^{(2i)}\right] = \frac{2}{\sigma^2}$ and $E[\cdot]$ represents the mean of a random variable. Complicated integration computations are simplified by applying a two-segment approximation function of $\emptyset(x)$, as shown in Equation (14).

$$\emptyset(x) = \begin{cases} 1 - \frac{1}{\sqrt{4\pi x}} \int_{-\infty}^{\infty} tan \frac{u}{2} e^{-\frac{(u-x)^2}{4x}} du & x > 0 \\ 1 & x = 0 \end{cases} \quad (14)$$

In Equation (14), there is a point of discontinuity $\emptyset_{chung}(x)$, as described as in Equation (15):

$$\lim_{x \to 0} \emptyset_{chung}(x) = e^{0.0218} > \emptyset(0) = 1 \quad (15)$$

4.0 METHODOLOGY

The research objectives were achieved using MATLAB software to model and simulate the polar codes. First, the SC and SCL decoders were constructed, and ASK modulation was included in the AWGN channel. Then, GA was integrated into the channel's polar decoders in the AWGN channel. The final step was to analyse the BLER and complexity performances of the SC and SCL decoders with ASK modulation and GA. Overall, the methodology was carried out in two stages. In Stage 1, the system integrated a polar encoder and decoder with ASK modulation in the AWGN channel. In this stage, GA was not integrated yet into the system as it is to evaluate BLER performance without GA. The block diagram of the proposed communication system is shown in Figure 1.

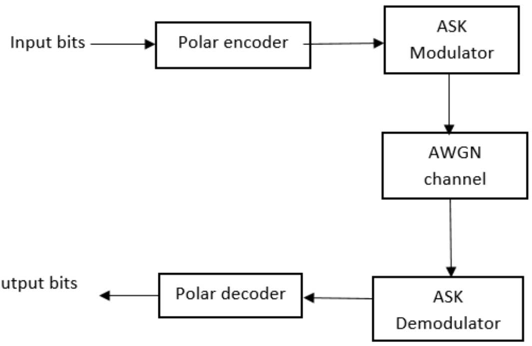

Figure 1 Block diagram of the communication system for Stage 1

The polar encoder (See Figure 1) was integrated into the system as stated in Section 3A. The input bits (10^4-10^9 bits) for the polar encoder were converted into frozen bits before modulation with ASK took place, and then all modulated bits were sent through the AWGN channel using the transmitter. After the receiver received the modulated message bits, all bits were demodulated and sent to the polar decoder. In this work, two types of decoding were used in the polar decoder: SC and SCL decoder, as elaborated in Sections 3B and 3C. The polar decoder decoded the bits after ASK modulation and produced them as decoded output bits. MATLAB software was used to implement the proposed algorithm to integrate all encoder, modulator and channel schemes as a simulation system. The simulation included one distinct ASK modulation (4-ASK) on the polar decoder. This simulation's parameters are 1024 and 2048 block lengths (*N*), and a rate (*R*) of 1/2.

The SC and SCL decoders were simulated with varying parameters (details of which are provided in Table 1). The list sizes (*L*) used in Stage 1 were 4, 8, 16, and 32. *L* is utilised only in SCL decoding because its complexity depends on the list size, which is *O(LNlogN).* This list improved the performance of successive cancellations for finite code lengths [18]. It works to improve the error-correction performance of SC during the decoding process. Path metrics were used (as in Equation (8)-(11) in Section 3C) to determine which path needs to be deleted and which path for the final decoded output. This parameter was needed to ensure that the reserved decoding path does not exceed the *L* bar.

Table 1 Parameters for Stage 1 Simulation

Parameters	SC	SCL
Block length (*N*)	1024, 2048	
List size (*L*)	-	4, 8. 16, 32
Rate	1/2	
Modulation	4-ASK	

In Stage 2, the GA was integrated with the polar decoder (Figure 2). The GA algorithm (based on Section 3D) estimated channel conditions based on demodulated signals decoded by the polar decoder. Block-wise maximum-likelihood was used to approximate the distributions of intermediate values arising in the decoder. This GA approach substantially simplifies the analysis of error probability. The output values, known as Gaussian random variables with mean and variance, were produced. This allows only the expected value of log-likelihood ratios to be computed, significantly decreasing the complexity. The parameters used for the simulations are the same as those for Stage 1 (see Table 1).

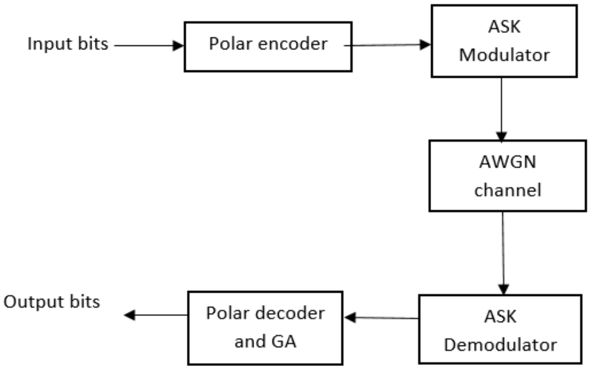

Figure 2 Block diagram for the polar decoder with GA.

5.0 RESULTS AND DISCUSSION

The effects of the GA and decoding techniques on polar codes performances are analysed in this section.

5.1 Decoding Performance of SC and SCL with ASK Modulation

Figure 3 shows the BLER performances of the SC and SCL with ASK modulation. *N*=1024 was used for both decoding algorithms, and *L*=4 was applied to SCL decoding. The SCL decoder with ASK modulation exhibited better BLER performance than the SC decoder, with a maximum difference of 1.2 dB at BLER=0.4. Hence, SCL achieves better BLER performance than SC. Figure 4 illustrates the BLER performance of the polar decoder for *N*=2048 and different values of *L*. The graph depicts that for all values of *L*, SCL outperforms SC. For example, SCL with *L*=32 and *N*=2048 performs SC at BLER=0.01, with a maximum difference of 2 dB. Meanwhile, for *L*=4, SCL has a maximum difference of 0.5 dB at BLER=0.01.

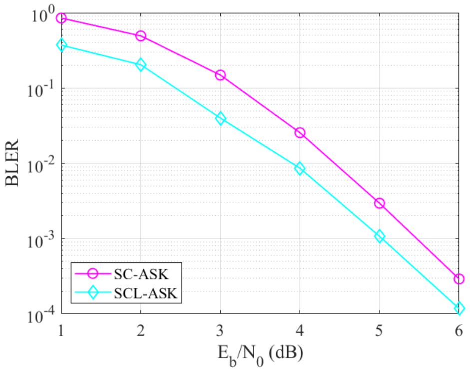

Figure 3 BLER performances of SC and SCL with ASK modulation (N=1024)

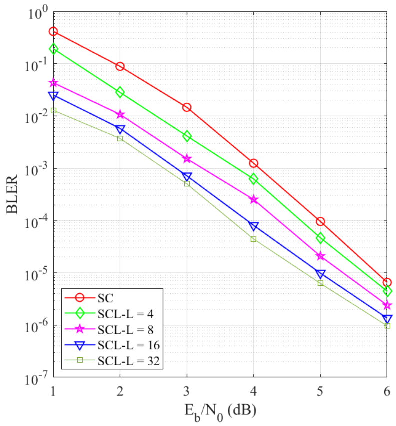

Figure 4 BLER performances of polar decoders for different values of L (N=2048)

5.2 Decoding Performance of SC and SCL with Gaussian Approximation

Figure 5 presents the BLER performances of SC and SCL with GA for *N*=1024 and *N*=2048, *R*=1/2, and (for SCL only) *L*=32. As shown in Figure 5, the SCL decoding algorithms outperformed the SC algorithms. Moreover, both algorithms perform better when

N=2048 than when *N*=1048. A maximum difference of 0.6 dB was observed for SCL at BLER=1.4 x10^{-9}. For SC, the maximum difference was 1 dB at BLER=6.2 x 10^{-6}. The SC and SCLs' decoding performances for different list sizes (*L*) are presented in Figure 6. In all cases, *N*=1024 and *R*=0.5. When *L*=32, SCL outperforms SC by a maximum difference of 2.77 dB at BLER=3 x 10^{-6}. SCL's BLER improvement is less when *L*=4 than when *L*=32, with a 1.6 dB maximum difference compared to SC. It is concluded that SCL achieved better results as the list size grows, as larger list sizes allow the model to find the correct path for decoding polar codes without sacrificing precision and reduce density evolution using GA.

Meanwhile, Figure 7 shows the BLER performances of the SC and SCL models with and without GA. In this simulation, *L* was set to 32, with *N*=1024. SCL with GA outperformed SCL without GA, with a maximum difference of 2.72 dB at BLER=5.6 x 10^{-6}. Similarly, the SC decoder with GA outperformed the SC decoder without GA, with a maximum difference of 2.7 dB at BLER=0.1.

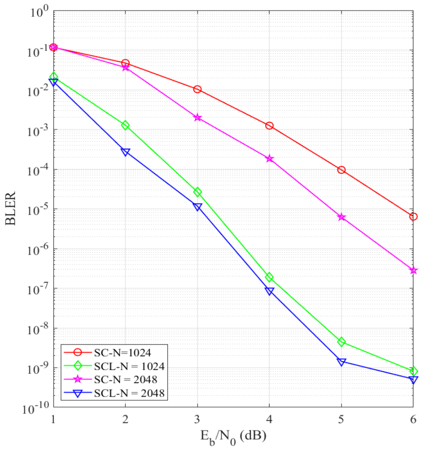

Figure 5 BLER performances of SC and SCL with GA for varying values of *N*.

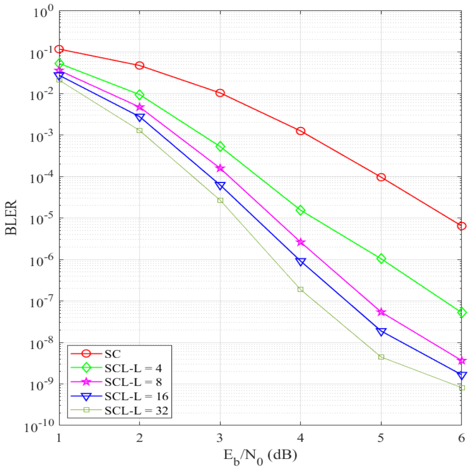

Figure 6 BLER performances of SC and SCLs for different values of *L* (*N*=1024).

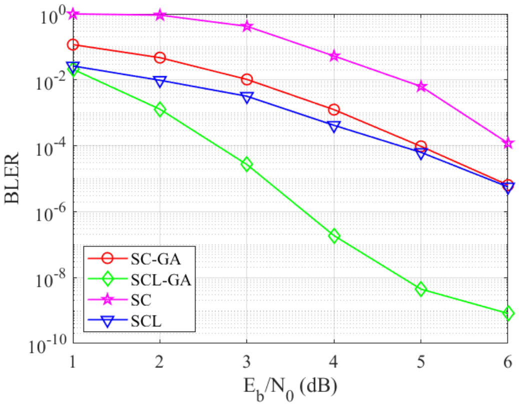

Figure 7 BLER performances of SC and SCL with and without GA.

This research evaluated both SC and SCL decoders regarding BLER with ASK modulation and GA. The results evaluated the BLER performance of both decoder and modulation scheme with the presence of GA. From the outcomes, it can be evaluated that GA presence has improved the BLER performance for both decoders, and SCL outperforms SC for both situations where ASK modulation was used with and without GA. It also improved the SC decoder's performance when GA was used in the system.

5.3 Computational complexity performances of SC and SCL with Gaussian approximation

The time complexity of the polar decoder was measured according to the total number of LR. The SC decoder has a time complexity of $O\ (NlogN)=(N+N\log(N))$, while the SCL decoder has a time complexity of $O\ (LNlogN)=L(N+N\log(N))$. Table 2 lists the time complexities for the SC and SCL decoder for *N*=1024 and different levels of *L* for SCL. SC has a lower time complexity (0.11x10^5) than all versions of SCL. The SCL with *L*=32 has the highest computational complexity of 3.6x10^5. The time complexity of SCL increases as the list size increases, as list size contributes to the complexity of polar decoders.

Table 2 Time complexities for SC and SCL with N=1024.

Polar decoders	Time complexity (x10^5)
SC	0.11
SCL (L=4)	0.45
SCL (L=8)	0.90
SCL (L=16)	1.80
SCL (L=32)	3.60

6.0 CONCLUSION

Compared to the SC decoder, the SCL decoder with ASK modulation exhibited improved BLER performance, with a maximum difference of 1.2 dB at BLER=0.4. Furthermore, the SCL decoder with GA outperformed the same decoder without GA by a maximum of 2.72 dB at BLER=5.6 x 10^{-6}. A similar finding was produced regarding the SC decoder, as the SC decoder with GA presented a more efficient BLER performance than that without GA, with a maximum difference of 2.7 dB at BLER=0.1. Regarding the SCL decoder, list size significantly impacted BLER performance. However, as the list size increases, so does the time complexity of the SCL model. Future research could improve upon the present study, particularly by considering polar coding with cyclic redundancy check (CRC) or different types of modulation and channels. For example, ASK modulation could be replaced with QAM or PSK to compare polar code performances in BiAWGN channels.

Acknowledgement

This research is fully supported by Fundamental Research Grant Scheme (FRGS) grant from Ministry of Higher Education (MOHE), FRGS/1/2021/ICT09/UITM/02/1. The authors fully acknowledged MOHE and Universiti Teknologi MARA for the approved fund which makes this important research viable and effective.

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