VALIDATION OF THE HULL GIRDER DEFLECTION OF A MULTIPURPOSE CARGO SHIP

Joynal Abedina, Francis Franklinb, S M Ikhtiar Mahmudb*

aSchool of Engineering, Newcastle University, NE1 7RU, United Kingdom
bNaval Architecture and Marine Engineering Department, Military Institute of Science and Technology, Mirpur Cantonment, Dhaka 1216, Bangladesh

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*Corresponding author
ikhtiar@name.mist.ac.bd

Graphical abstract

Abstract

When a ship encounters head-sea conditions, it can experience still water bending and wave-induced vertical moments. These moments can cause the hull girder to undergo longitudinal deflection, which can negatively impact the shaft alignment of the propulsion unit and the sealing of hatch covers and other onboard machinery. On the other hand, beam sea conditions can lead to transverse hull girder deflection, which is especially problematic for open-deck ships and can cause significant hatch-comeing deflection. A numerical strength check has been performed using FEA software FEMAP with NX Nastran and an analytical review based on Euler-Bernoulli’s beam theory to analyse the ship’s strength. The ship is represented as a simply supported beam, and both analytical and numerical methods are used to calculate deflection for different sea conditions (head and beam seas). A mesh sensitivity analysis has been done to ensure the accuracy of the numerical analysis, and the results from both techniques are compared to validate their accuracy. Finally, a numerical analysis is conducted to confirm the accuracy of the analytical study when considering the ship as a complex structure. Overall, this research underscores the greater significance of longitudinal deflection over transverse deflection in the hull girder of a multipurpose cargo ship. Combining both approaches, a comprehensive understanding of the ship’s hull girder strength and deflection behaviour is achieved, enhancing overall structural integrity and safety.

Keywords: Analytical, numerical, hull girder, deflection, simply supported beam

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1.0 INTRODUCTION

Ship structural deflection refers to the bending and deformation of a ship’s hull, which occurs due to different loads, such as loading and unloading cargo and waves [1]. The hull girder is a vital structural element of a ship that runs along its bottom and bears the weight of the machinery, cargo, and other structures. Excessive deflection can result in structural failure, cargo damage, and even capsizing in extreme circumstances [2]. Lightship weight distribution, load distribution, and wave-induced global loads all contribute to the vertical bending moment that results in ship hull girder deflection [3]. During severe weather conditions, dynamic loads can also contribute to hull deflection [4]. An important task that must be carried out beginning with the early design stages is the assessment of a ship’s hull deflection in calm and turbulent waters [1]. The hull girder’s bending moments caused by waves and shipload fluctuations can affect a ship’s performance [5]. The propulsion shafting of the ship may also be impacted by hull deflection [6]. Generally, a ship’s hull deflection can significantly impact its performance. A ship’s hull that has been deflected may have a positive displacement under hogging conditions and a negative displacement under sagging conditions [4]. Strength, deflection, and vibration are important considerations in designing a ship structure [7]. Studying previous research on a ship’s hull girder...
The deflection is necessary. Niebylski, J. (1970) introduced a mathematical model, and its analysis considers the actual deflections of hulls during construction. This model is currently used for manufacturing control and examining the impact of different factors on ship structures during sequential building stages. It is considered the most effective measurement method [8]. Antoniou, A.C. (1980) conducted a study on over 2000 observations of central deflection in shipyard plate panels. The study found that the plate slenderness ratio, stiffener thickness, plate aspect ratio, and weld throat thickness were all significant factors in determining deflection. Through regression analysis, the study determined the functional connection coefficients and the impact of each parameter on deflection. Additionally, the study proposed new formulas for predicting maximum deflection in specific scenarios [9]. Ziha, K. (2002) studied the impact of longitudinal deflections on bending moments and shear forces in merchant ship hulls. The study utilised approximations and concluded that these effects were conservative and not of significant concern. Generally, more precise calculations of these quantities are optional [4]. Lee, Y.J. and Kim, U.K. (2005) researched calculating hull deflection data in reverse using bending moments. They also examined ways to minimise bearing damage caused by hull deflection during the design phase. However, hull deflections from different loading scenarios significantly increased bearing offset [10]. Sverko, D. (2005) conducted a study on multiple ships of different sizes and types to measure hull girder deflection accurately. The data collected was then used to evaluate the shaft alignment design and determine its susceptibility to changes during ship operations through the ABS Shaft Alignment Optimisation program [11]. Naar, H. (2006) conducted a study on the prismatic hull girder of a post-Panamax passenger ship to analyse its maximum strength under hogging and sagging loading conditions. The coupled beam approach and the finite element method were utilised to evaluate the bending moment against the deflection of the hull girder. Both approaches yielded results that exhibited a significant correlation until the moment started to decrease [12]. Dardamakis, A. (2022) studied shaft alignment in a standard 10,000 containerships. Using automation in the process and minimal user pre-processing significantly reduced the time needed to calculate hull deflections. This approach is dependable and efficient in determining hull deflections and bearing offsets due to its low time and experience requirements. Additionally, the automation for calculating the sectional properties of each ship frame makes it an excellent tool [13].

Modern hulls of large oil carriers and cargo ships are designed to prevent bearing damage by avoiding hull deflection. This is an important measure to ensure the safety and efficiency of these ships. Hull girder deflections most significantly impact the bearing offset after the ship construction. Failure to consider hull deflection could lead to a poorly designed alignment with detrimental effects on bearing life. Yet the issue is that hull deflections are difficult to forecast and evaluate. One of the most crucial challenges today is considering hull girder deflections. The ability to estimate hull deflection with adequate accuracy is of utmost importance to ensure a robust alignment design and, thus, fewer alignment-related casualties [10].

Ships deform throughout their lifespans. Local buckling, heat effects, global bending moment, and welding during construction cause these deformations. Maintain the hull girder deflection within a range that allows machinery and equipment to operate correctly. Shafting and pipework may have issues, and eccentricity and inefficiency create more significant torsional moments in the primary shaft. Pipe deflection can cause blocked liquids and support issues. The deflection of the ship's hull as a beam is obtained by the second integration of the $M_o/EI$ curve. It is found that hull girder deflection depends upon moment of inertia (I) and elasticity (E). Excessive deflection reduces structural effectiveness. It may not cause structural failure, but excessive deflection cause misalignment of the ship's machinery and piping system, making these systems ineffective. The classification standards do not limit hull girder deflections. However, the L/D ratio is related to the factors that prevent excessive deflection [3].

Ship designers frequently consider the hull similar to a beam, and the distribution of weight-buoyancy discrepancy causes a longitudinal bending moment. When designing a ship's hull strength, it is common to consider the ship in two extreme conditions: floating on a wave the same length as the ship and with the crest at each end, called the "sagging condition," and floating on a wave with the crest amidship, called the "hogging condition." These two situations are the two most severe loading scenarios [14].

The following factors must be considered for the components of ships to limit a hull girder's deflection; however, there is no restriction from a strength perspective. For a ship with a larger L/D, a larger hull girder deflection will be anticipated, and attention will be paid to this [15].

1. The longitudinally installed pipes and rods on the top deck or bottom can expand and compress.
2. An increase in draft brought on by the hull girder's deflection.
3. The deflection of the hull girder causes secondary stress to be created.
4. Flexural vibration, or "whipping," of the hull girder.

Farias et al. (2023) conducted research, and they have proven that the deflection of a ship's hull significantly affects the alignment of its shaft. The study used the Stiffness Method, Finite Element Analysis, and hull girder approach to identify the optimal alignment configuration for different operating conditions. The study achieved alignment configurations that met the approval criteria in 91.1% of the scenarios studied by applying optimisation techniques. Additionally, a reliability study showed that alignment optimisation improves the suitability and safety of the ship's propulsion system. The article highlights the importance of optimisation in achieving satisfactory alignment configurations, which ultimately enhances the reliability of both the system and the ship. The study also shows the substantial impact of hull deflection on shaft alignment in medium-sized ships [16]. Zhou et al. (2023) investigated the effects of hull structural deformation on shaft alignment. The study divided hull deformation into global and local deformations and simplified them into two models: single-span and grillage beam. They then used the matrix displacement method to calculate the effect of hull deformation on shaft alignment. The study found that hull deformation is a significant factor in shaft alignment and that the matrix displacement method is an effective tool for calculating hull deformation. This research provides valuable insights into the importance of considering hull deformation when aligning shafts and highlights the usefulness of the matrix displacement method in this regard [17].
The study assesses the accuracy of an analytical technique in measuring a ship's hull girder deflection and compares it to the numerical approach. When designing a ship, designers need to consider the hull girder's deflection. This helps maintain the ship's structural integrity and prevent potential failure. Considering all relevant aspects and using advanced modelling and simulation technologies, a safe and effective ship can be constructed to withstand various loads and conditions throughout its service life.

2.0 INVESTIGATED SHIP IN THE RESEARCH

The investigated ship in this study is already an operational multipurpose cargo ship with a bulbous bow and transom, and the ship has a single-screw diesel engine. The investigated ship has a single cargo hold and is a double-skinned box. This ice-class multipurpose cargo ship can transport oversized freight, regular cargo, containers, and bulk grain. Table 1 lists the ship's most important characteristics.

Table 1 Principal particulars of the ship

<table>
<thead>
<tr>
<th>Items</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall</td>
<td>104.135 m</td>
</tr>
<tr>
<td>Length between perpendicular</td>
<td>98.535 m</td>
</tr>
<tr>
<td>Breadth moulded</td>
<td>15.25 m</td>
</tr>
<tr>
<td>Depth</td>
<td>7.45 m</td>
</tr>
<tr>
<td>Design Draught</td>
<td>4.90 m</td>
</tr>
<tr>
<td>Scantling Draught</td>
<td>5.60 m</td>
</tr>
<tr>
<td>Range of navigation</td>
<td>Unrestricted</td>
</tr>
<tr>
<td>Loading sequence</td>
<td>2R (2 Runs)</td>
</tr>
<tr>
<td>Propulsion</td>
<td>Self-propelled</td>
</tr>
</tbody>
</table>

For the construction of this ship, a longitudinal framing system was used. Figure 1 presents the midship section of the investigated ship.

2.1 Geometry and Scantlings Details

The cargo compartment of the multipurpose cargo ship has twin hull sides that are deep tanks. Figure 3 depicts the ship's double bottom, side shell, and transverse section. The stiffening at the bottom of the structure is made up of vertical plates, also known as floors, which will strengthen the bottom. Side stringers and beams of angles or channels reinforce the sides and decks. The transverse material provides transverse strength and prevents longitudinal buckling. The span thickness ratio is vital for resisting compressive stresses and preventing local deformation owing to water pressure.

3.0 HULL GIRDER DEFLECTION ANALYSIS

The hull girder deflection happens when a ship experiences a vertical bending moment, horizontal wave bending moment, and wave-induced torsional moment. The lightship weight distribution, the load distribution, and the wave-induced global loads can cause these moments. In addition, the amount of deflection due to shear is added to the amount of deflection due to bending, even though its amount is typically relatively small. The same factors that steadily raise nominal stress levels also gradually increase flexibility [3]. Material Properties of steel are used in this analysis are shown in Table 2. Generated FE model of the investigated ship in FEMAP is shown in figure 4.

3.1 Design Loads

A ship at sea is subjected to various loads, causing structural deformation and stress. The initial step is to assume accurate loads acting on the structure to construct a design. The load is gradually transferred from a local structural member to a more significant supporting element [7]. Global or primary loads act on the ship as a beam (hull girder), and primary response loads affect the ship's structural behaviour. On the other hand, local loads are defined to be applied to limited structural models (stiffened panels, single beams, plate panels). Individual structural components, such as plating panels, ordinary stiffeners, and significant supporting members, are subjected to local loads, which are pressures and stresses applied directly to them [18]. In this analysis, only hull girder loads have been used to investigate the hull girder structural deflection of this investigated ship.

3.1.1 Hull Girder Loads

There are static and dynamic components to ship hull girder loads, and still water bending moments and shear forces are the
most important. The ship’s hull girder can be considered a non-uniform beam subjected to variable loads along its length [19].

3.1.1.1 Still Water Bending Moments (Swbm)

Under one load condition, the still water bending moment at a given section of the ship is constant but varies from one load condition to the next. Each load condition’s duration is likewise a random variable. According to the above load cases, classification society rules expressly provide formulations for evaluating still water-bending moment values. The direct computation can also determine the bending moment of still water [20]. This investigation estimated the still water bending moment using the BV, NR 467 rules for the classification of steel ships [21].

3.1.1.2 Vertical Wave Bending Moment (Vwbm)

An additional vertical bending moment induced due to the waves must be considered to evaluate the total bending moment. This component depends upon the range of navigation of the ship. In this analysis, the investigated ship’s navigation range is unrestricted. Vertical wave bending moment is also calculated according to BV, NR 467 rules for the Classification of Steel Ships [21].

3.1.1.3 Horizontal Wave Bending Moment (Hwbm)

A horizontal wave bending moment occurs when the ship is in a beam and oblique sea [22]. According to the BV, NR 467 rules for the classification of steel ships, the horizontal wave bending moment at any hull transverse section has been calculated [21]. Bending moment of still water, horizontal and vertical are exhibit in Figure 2 for hogging and sagging condition.

![Figure 2 Comparison of the investigated ship's bending moments](image)

3.2 Hull Girder Strength Analysis

Hull girder strength analysis aims to determine the hull girder’s actual stress and stiffness for specified load cases resulting from loading conditions. Usually, the hull girder strength analysis aims to determine the local strength resulting from local loads rather than the longitudinal hull girder strength. FEMAP integrated with NX-NASTRAN has been used in this analysis to investigate the hull girder strength of the mentioned ship. This analysis was conducted using the "Net" thickness approach, which implies the strength analysis was done with corrosion deduction of the plate and stiffener thickness. The plating and stiffener corrosion deductions were calculated using the BV, NR 467 rules for steel ship classification. This method ensures the structural strength of the cargo ship in both “as-built” and “design life” conditions.

![Figure 1 Generated FE model of the investigated ship in FEMAP](image)

### Table 2 Material Properties of steel

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbols</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity modulus,</td>
<td>$E$</td>
<td>206 GPa</td>
</tr>
<tr>
<td>Density,</td>
<td>$p$</td>
<td>7850 kg/m$^3$</td>
</tr>
<tr>
<td>Poisson’s ratio,</td>
<td>$\nu$</td>
<td>0.30</td>
</tr>
<tr>
<td>Yield Stress,</td>
<td>$R_y$</td>
<td>235 (for Mild Steel)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>355 (Higher Tensile Steel)</td>
</tr>
</tbody>
</table>

3.2.1 Checking Criteria

Using Finite element software FEMAP, the strength check is carried out. The checking criteria are referred from BV, NR 467 rules for the Classification of Steel Ships [21].

The master allowable stress, $\sigma_{\text{Master}}$, in N/mm$^2$, is obtained from the following formula:

$$\sigma_{\text{Master}} = \frac{R_y}{\gamma_{RF}}$$  \hspace{1cm} (1)

where:

- $R_y$: yielding stress
- $\gamma_{RF}$: resistance partial safety factor
- $\gamma_{M}$: material partial safety factor

For mild steel (Grade A), the master allowable stress, $\sigma_{\text{Master}}$, is calculated as 219.42 MPa. $\sigma_{\text{Master}}$, the maximum allowed stress for high tensile steel (Grade AH-36), is estimated to be 331.77 MPa. It is necessary to confirm that the equivalent Von-
Mises stress $\sigma_{VM}$ is in accordance with the following formula for all sorts of analyses:

$$\sigma_{VM} \leq \sigma_{Master}$$  \hspace{1cm} (2)

### 3.2.2 Coordinate System

According to BV rules NR 467 (Part B, Chapter 1, Section 2) [21] the coordinate system of the ship is referred to as a right-hand coordinate system (see Figure 5):

- **Origin**: at the point where the ship's longitudinal plane of symmetry, the baseline, and the aft end of L meet.
- **X-axis**: longitudinal axis, positive forwards
- **Z axis**: transverse axis, positive towards portside
- **Y axis**: vertical axis, positive upwards

The coordinate system described in NR 467 is illustrated in Figure 5.

![Figure 5](image)

**Figure 5** Modelling coordinate system

### 3.2.3 Boundary Condition

The cantilever beam concept is implemented within the finite element (FE) model to comprehensively examine the hull girder's longitudinal strength attributes, aiming to obtain results of heightened conservatism. The simple supported beam concept is used in deflection analysis to obtain more accurate deflection values. Boundary conditions of cantilever beam are shown in Table 3. Both sides of a simply supported beam have been subjected to bending moment and boundary conditions. On fwd. and aft side of the FE model, bending moments are applied and are constrained by supported boundary conditions (Tables 4 and 5). Under the main deck, rigid elements are constructed, transferring the load to numerous nodes. A rigid element connects free edge nodes to other nodes in the same plane, enabling them to work as one unit. To construct two boundary conditions, two rigid components must be used [23]:

1. **Constraint**: a rigid element at the model's aft and fwd. with zero degrees of freedom to clamp.
2. **Moment**: To establish a hogging/sagging condition, a bending moment is applied to a rigid element in the fore and aft parts of the model.

<table>
<thead>
<tr>
<th>Boundary conditions (Cantilever)</th>
<th>Translations in directions</th>
<th>Rotation around axes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node at the aft end</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Node at the fore end</td>
<td>Fixed Free Free Free Free</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4 Boundary conditions (For Head Sea simple supported)

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Translations in directions</th>
<th>Rotation around axes</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Node at the aft end</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Node at the fore end</td>
<td>Fixed Free Free Free Free</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5 Boundary conditions (For Beam Sea simple supported)

<table>
<thead>
<tr>
<th>Boundary conditions</th>
<th>Translations in directions</th>
<th>Rotation around axes</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Node at the aft end</td>
<td>Fixed Fixed Fixed Fixed</td>
<td></td>
</tr>
<tr>
<td>Node at the fore end</td>
<td>Fixed Free Free Free Free</td>
<td></td>
</tr>
</tbody>
</table>

### 3.2.4 Longitudinal Hull Girder Strength Analysis

Sagging is the worst-case scenario due to the higher stress value. As a result, the sagging condition is considered for longitudinal hull girder strength analysis. The stress values in the midship areas are investigated because the applied maximum bending moment corresponds to the value in the midship section.

Validation is done by comparing typical stress values obtained from beam theory and direct calculations with finite element analysis (Figure 6).

![Figure 6](image)

**Figure 6** Comparison of hull girder stress between beam theory and direct calculation at midship (Sagging condition)

It can be shown in Figure 6 that the stress discrepancy between beam theory and direct calculation is about 5%. This divergence is acceptable in the beam theory idea context [24].
To meet the strength checking criterion, normal stress in critical areas should be less than the master permitted stress indicated in section 3.2.1. Figure 7 shows that the hatch coaming top plate (higher tensile plate) has maximum normal stress value 242 MPa lower than the master allowable stress 331.77 MPa. Also, Figure 8 exhibits maximum stress value of 239 MPa in hogging condition.

3.3 Hull Girder Deflection Calculation

By equating the resistive moment to the bending moment, the elastic curve equation for a beam is obtained [3].

$$EI \frac{d^2y}{dx^2} = M(x) \quad (3)$$

In this equation, y is the deflection, E is the material's modulus of elasticity, and I is the moment of inertia of the beam's cross-section about a horizontal axis passing through its centroid.

Calculating a ship's bending-related deflection is similar to doing so for a beam. An intermediate $$\frac{M_B}{I}$$ curve's second integration is used to calculate the deflection of a free-free supported ship with a variable moment of inertia.

$$\frac{M_B}{I} = \frac{dy}{dx} \quad (4)$$

$$\frac{dy}{dx} = \frac{1}{E} \left[ \int \frac{M_B}{I} \, dx \right] + a \quad (5)$$

$$y = \frac{1}{E} \left[ \int \frac{M_B}{I} \, dx \, dx \right] + ax + b \quad (6)$$

Where:

- y is the deflection,
- a is the first constant of integration of the $$\frac{M_B}{I}$$ curve,
- b is the second constant of integration of the $$\frac{M_B}{I}$$ curve.

The change in slope is determined by the first integration of the $$\frac{M_B}{I}$$ curve, and the ordinates of the curve are equal to the areas under the $$\frac{M_B}{I}$$ curve represented by:

$$\frac{dy}{dx} = \frac{1}{E} \left[ \int \frac{M_B}{I} \, dx \right] + a \quad (7)$$

The end slope is the integration constant, a, and it is not zero since the ends of the hull girder are free. The total slope is equal to the sum of the end ordinates, and the axis of the slope curve is a line parallel to the baseline. The point of maximum deflection is typically close to the maximum ordinate of the $$\frac{M_B}{I}$$ curve, at which the slope curve crosses the axis [3].

Depending on the loading, the bending moment may cross its baseline at one or more points. According to the size of the regions on the other side of the baseline, the slope curve would have matching points of a maximum or minimum slope, and the $$\frac{M_B}{I}$$ curve would have corresponding points of zero value in this case [3].

The second integral of the $$\frac{M_B}{I}$$ curve, which is the deflection curve, is represented by:

$$y = \frac{1}{E} \left[ \int \frac{M_B}{I} \, dx \, dx \right] + ax + b \quad (8)$$

The deflection curve's constant of integration, b, is equal to zero because the ends of the hull girder are free. The deflection curve will close at the ends of the baseline if the slope curve is integrated about the curve's axis [3].

The slope curve's constant of integration, a, is derived from the deflection by the fact that when x = length L, y = 0, and:

$$a = -\frac{1}{E} \left[ \frac{\int M_B \, dx \, dx}{L} \right] \quad (9)$$

3.3.1 Analysing The Effects Of Hull Girder Deflection

The deflection line of the ship's hull is frequently presented as a second-order symmetric parabola:

$$w(x) = w_m \frac{x^2}{L_{WL}^2} \quad (10)$$

Where w(x) is a general hull deflection at section x, w_m is a specific ship hull deflection at amidships, and L_{WL} is the length of the waterplane.

Evidently, the deflection line of a ship's hull is neither symmetrical nor parabolic, yet the variations from the parabolic form are often of modest significance. It has been demonstrated experimentally and numerically that a parabola can satisfactorily fit the hull deflection data. On the other hand, it is impractical and, in most situations, impossible to establish the hull deflection shape on board with greater accuracy. Since the precise location is practically unknown, another assumption about the location of the maximal deflection at the center of flotation, LCF, may simplify the draft survey process without significantly affecting the accuracy of the displacement calculation [4].
3.3.2 Analytical Determination Of Ship’s Hull Girder Deflection As A Simply Supported Beam

The maximum bending moment applying rule calculations is derived from analytically calculating the highest deflection value. Section 3.1 displays bending moments used in the hull girder deflection analysis. The moment of inertia (I) must be measured in multiple transverse sections of the ship along its whole length in order to ascertain its distribution. The midship area will have the greatest moment of inertia. Due to the hull form, the cross-sectional areas drop as the sections come closer to the ends of the ship, and as a result, the moments of inertia also decrease. The distribution of the moment of inertia over the length of the ship is shown in Figures 9 and 10.

Figure 9 Distribution of the moment of inertia (Y axis) along ship length m^4

Figure 10 Distribution of the moment of inertia (Z axis) along ship length m^4

In order to obtain a mathematical approach to the distribution of the moment of inertia near the ends of the ship, a parabolic equation has been obtained. The deflection is obtained by the second integration of the M_b/I curve. This mathematical operation is done by integrating twice the function obtained in the software Excel when adding a trend line to the MB/I curve. Figures 11 and 12 show the MB/I curve along the ship’s length for Sagging and hogging loading conditions for head sea.

The distribution of the moment of inertia close to the ship’s ends has been mathematically modelled using a parabolic equation. The second integration of the MB/I curve is used to obtain the deflection. This mathematical process is carried out by twice integrating the function that Excel’s program produced when a trend line was added to the MB/I curve. Figures 11, 12 and 13 display the MB/I curve for head sea loading under sagging, hogging and beam sea situations respectively along the ship’s length.

The MB/I function the approximation of the original curve with R-squared values for the sagging-head sea, hogging-head sea, and beam sea of 0.9318, 0.9322, and 0.8846, respectively. The R-squared, or the coefficient of determination, measures how well the data fit the curve.
Hull girder deflections can be found using equations 4 and 5. Figures 14, 15, and 16 show the hull girder deflection for the investigated ship’s loading at sagging-head sea, hogging head sea, and beam sea conditions.

**Figure 14** Longitudinal hull girder deflection along ship length (Sagging-head Sea)

![Figure 14](image)

**Figure 15** Longitudinal hull girder deflection along ship length (Hogging-head Sea)

![Figure 15](image)

Figures 14 and 15 show that the longitudinal hull girder deflection is almost the same for hogging and sagging loading scenarios, with the highest deflection value obtained at midship, around 190 mm.

**Figure 16** Transverse hull girder deflection along ship length (Beam Sea)

![Figure 16](image)

The maximum transverse hull girder deflection value at midship is approximately 15 mm, as shown in Figure 16.

### 3.3.3 Numerical Determination Of Ship’s Hull Girder Deflection As A Simply Supported Beam

To verify the accuracy of the analytical deflection calculation, a FE model was employed to determine the ship’s hull girder deflection as a simply supported beam based on section 3.2 and the load specified in section 3.1. Femap software was used with the NX Nastran solver to conduct a static analysis. The longitudinal deflection results for both sagging and hogging in Head Sea can be found in Figures 17 and 18, while Figure 19 displays the transverse deflection of the hull girder for Beam Sea.

**Figure 17** Longitudinal hull girder deflection along ship length (Sagging Head Sea)

![Figure 17](image)

**Figure 18** Longitudinal hull girder deflection along ship length (Hogging Head Sea)

![Figure 18](image)

**Figure 19** Transverse hull girder deflection along ship length (Beam Sea)

![Figure 19](image)

### 3.3.4 Mesh Sensitivity Analysis

Mesh sensitivity analysis is a technique used in numerical simulations to determine the optimal mesh size and quality for accurate results. It involves varying the mesh size and comparing the results to determine the optimal mesh size for the simulation. The technique is used in ship modelling to investigate the effects of mesh size and quality on the accuracy.
of the simulation results. The analysis can help improve the accuracy of the simulation and provide insights into the behavior of the ship model under different conditions.

To address discrepancies between numerical and analytical determinations of ship hull girder deflection, a mesh sensitivity analysis was conducted to ensure accurate comparisons. The convergence curves are as follows:

![Graph 1](image1.png)

**Figure 20** Comparison of Analytical vs Numerical deflection (Hogging Head Sea)

![Graph 2](image2.png)

**Figure 21** Comparison of Analytical vs Numerical deflection (Sagging Head Sea)

![Graph 3](image3.png)

**Figure 22** Comparison of Analytical vs Numerical deflection (Beam Sea)

![Graph 4](image4.png)

**Figure 23** Comparison of Analytical vs Numerical deflection (Beam Sea)

The data presented in Figures 20, 21, 22 and 23 indicate that Mesh Density 100 is the point at which the results stabilise. These results are similar to Figures 17, 18, and 19. However, there is a slight deviation of approximately 8% from the analytical values. According to Euler and Bernoulli’s beam theory, it is important to note that this deviation still falls within the acceptable range.

### 3.3.5 Numerical Determination Of Ship’s Hull Girder Deflection As A Complex Structure

The FE model used in section 3.2 is applied to calculate hull girder deflection, and the load mentioned in section 3.1 calculates the hull girder deflection. The next step is to analyse the ship after establishing the constraints listed in section 3.2.3 and applying all the loads. This can be done using the solver NX Nastran to create a new Static Analysis in the Femap software. The computed hull girder longitudinal deflection for head sea (sagging and hogging) is shown in Figures 24 and 25.

![Graph 5](image5.png)

**Figure 24** Longitudinal hull girder deflection along ship length (Sagging Head Sea)

![Graph 6](image6.png)

**Figure 25** Longitudinal hull girder deflection along ship length (Hogging Head Sea)

Figures 24 and 25 show that the midship region experiences a maximum longitudinal hull girder deflection of about 130 mm during head sea. Figure 26 represents the transverse hull girder deflection at Beam Sea.

![Graph 7](image7.png)

**Figure 26** Transverse hull girder deflection along ship length (Beam Sea)
The maximum transverse hull girder deflection value at midship is shown in Figure 26 to be approximately 6 mm.

### 3.3.6 Difference Between The Analytical And Numerical Determination Of Hull Girder Deflection

The section properties from the midship scantling calculation have been taken to calculate the girder deflection and assume that the ship is a simply supported beam at the ends. In the FEA environment (FEMAP), we simulate the identical transverse section.

The distinction between analytical and numerical hull girder deflection determination is shown in Table 6.

#### Table 6 Difference between the analytical and numerical determination of hull girder deflection.

<table>
<thead>
<tr>
<th>SI No</th>
<th>Load cases</th>
<th>Hull girder deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>The analytical values for</td>
</tr>
<tr>
<td></td>
<td></td>
<td>analyseship</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The analytical values for</td>
</tr>
<tr>
<td></td>
<td></td>
<td>the cargo hold</td>
</tr>
<tr>
<td>1</td>
<td>Hoggng (Head Sea)</td>
<td>190.5</td>
</tr>
<tr>
<td>2</td>
<td>Sagging (Head Sea)</td>
<td>190.9</td>
</tr>
<tr>
<td>3</td>
<td>Beam sea</td>
<td>15.6</td>
</tr>
</tbody>
</table>

### 4.0 DISCUSSION

The Euler-Bernoulli’s beam theory is used in this study to evaluate the stress and deflection resulting from vertical or lateral hull bending moments. The method assumes a consistent cross-section along the hull’s length and relies on Euler-Bernoulli’s beam theory. In this study, there is a variation between the analytical and numerical deflection values when a ship is considered a simply supported beam. This difference is allowable according to Euler-Bernoulli’s beam theory concept. However, due to the complex geometries of ship constructions, numerical determination of deflection produces more precise results. The numerical deflection calculation is utilised to achieve more precise outcomes for intricate geometries. The deflection of the hull girder is limited to 1 mm per metre of ship length as per International Standards [3]. Although the classification rules do not explicitly mention any restrictions on hull girder deflections, the standard that protects excessive deflection is linked to the L/D (Length to Depth) ratio. Based on the analysis, the ship’s numerical deflection exceeds the international standard, and the prescribed deflection value for the study vessel, according to international standards, is approximately 105 mm. The numerical deflection is greater due to two reasons:
1. The application of net scantlings
2. The utilisation of solely the Cargo hold model, not the entire ship’s drawing.

Applying the gross scantlings and utilising the entire ship model will unquestionably decrease numerical deflection. Table 6 confirms that the hull girder deflection metric about the cargo hold model length has been confirmed to match the numerical value obtained from the complex model in head sea conditions. However, a disparity arises under beam sea conditions. This divergence is because the numerical model considers the bulkhead, which decreases deflection. However, incorporating these bulkhead effects in analytical calculations is still a challenge. Nonetheless, this precision confirms the hull girder deflection’s authenticity.

To ensure safe and secure transport, it is important to evenly distribute cargo and use robust materials like steel to mitigate hull stresses such as hogging, sagging, and shearing, and to reduce hull girder deflection.

This research has developed a model to optimise ship design parameters, with a focus on achieving multiple objectives, such as weight reduction, production costs efficiency and identifying critical ship structural components that significantly impact the overall strength of the ship structure. The main aim is to reduce manufacturing costs by minimising the steel used in the ship’s construction while ensuring compliance with all essential safety standards.

### 5.0 CONCLUSIONS

This investigation aimed to assess the longitudinal strength and deflection of a ship’s hull girder. A 3D finite element model was used to examine the cargo hold and calculate the ship’s linear longitudinal strength and deflection to achieve this.

To validate the findings, both numerical and analytical methods are used to evaluate the strength of the hull girder. The hull girder’s longitudinal deflection is estimated for both upward bending (Hogging) and downward bending (Sagging) scenarios. The ship is represented as a beam in the analytical technique, and the deflection is calculated based on the bending of the hull girder. In contrast, the numerical approach uses Finite Element Analysis to determine the hull girders’ deflection directly. The deflection of the transverse hull girder is significant for the deflection of the hatch during open-deck ship operations. The ship undergoes transverse hull girder deflection in beam sea conditions, while longitudinal hull girder deflection occurs in head sea conditions. After analysing analytical and numerical estimations, the study confirms that longitudinal deflection is more significant than transverse deflection in the hull girder. Calculating hull girder deflection can be time-consuming, so evaluating it analytically early in the ship design process is more effective. Ensuring precision and reliability requires validating analytical results with numerical results. Combining both approaches offers a thorough comprehension of the ship’s hull girder strength and deflection behaviour, enhancing its overall structural integrity and safety.

### Acknowledgement

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References


