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MULTI-DIMENSIONAL SQUARE WAVE EVALUATION AND SIMULATION EMPLOYING THE CUBIC INTERPOLATION PSEUDO-PARTICLE METHOD

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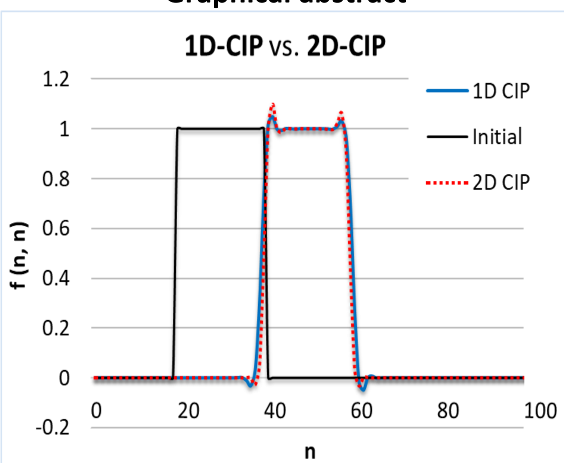
Graphical abstract



Abstract

The Cubic Interpolation Pseudo-Particle (CIP) method is used in various papers to simulate different phenomena. It solves hyperbolic-type equations and is more efficient than a first-order upwind scheme. This verification gives one the future to apply CIP to other or more complex geometries. In this paper, we simulate the numerically square wave propagation based on characteristic equations, using the CIP method using 1D-CIP and 2D-CIP. Based on the idea that the wave field and its spatial derivative propagate along the same characteristic curves obtained from a hyperbolic differential equation. In this research, we simulate the numerical propagation of the acoustic wave based on characteristic equations, using the CIP method with two dimensions 1D-CIP & and 2D-CIP. In addition, provides several numerical simulation behaviors in demonstrating how the CIP can accurately model the propagation of acoustic waves without much numerical dispersion. Moreover, the mean square error displayed the superiority of 2D-CIP with 0.5% over 1D-CIP. However, the characteristic-based CIP method is a particularly effective way to handle wave propagation to tackle the fluid's dynamic challenges and give accuracy, managing nonlinearities, and flexibility, making it a useful tool in numerical analysis.

Keywords: Cubic Interpolation pseudo-Particle (CIP), first order upwind, square wave, mean square error (MSE).

1.0 INTRODUCTION

Nowadays computational methods have become an important research approach for all industries. This approach promises good approximating results to the physical world. Due to technology enhancement, the researcher is still discovering the best computational method to solve some problem that has not been solved. In fluid flow problems, the Cubic Interpolated Pseudo-particle (CIP) method is a numerical tool used to predict fluid flow and heat transfer in industrial equipment containing nanofluids. It showed the ability to offer precise forecasts for intricate configurations and elevated nanoparticle concentrations. A numerical model that simulates a two-dimensional viscous flow with a distorted surface is used to improve the accepted approach [1, 2]. They also use the VOF/WLIC scheme [3] for interface capturing, which improves the results.

A CIP-based Cartesian grid method proposes generating focused waves and validating it through experiments [4]. Using a CIP-based numerical coding, the study evaluates the water entrance of wedges in regular waves [1, 5]. The CIP method is also used for numerical simulation of nonlinear to demonstrate sound wave propagation in a time domain [4]. In [6] used the CIP method to study the flow past an oscillating square cylinder and analyze the effects of different oscillation amplitudes and frequencies. The Constraint Interpolated Profile (CIP) scheme was proposed as a stable and less dispersive scheme in CFD and applied to many difficult problems. Reducing memory requirements, a new directional-splitting CIP interpolation approach employs just the value and first-order derivatives of the physical field as computing variables. It may be readily expanded to solve advection equations of arbitrary dimensions while maintaining third-order accuracy even for non-uniform grid spacings [7]. The Constraint Interpolated Profile is a scheme or method where it more to explains interpolation between two particles using third-order accuracy to form the movement. It means that CIP will keep in existence the original forms without any change current times are changing. A universal solution for hyperbolic-type equations, the CIP approach was proposed [7], [8] and it thoroughly demonstrates as being effective. A simulation of square waves with two dimensions using the 1D-CIP method demonstrated in the literature [9]. With interactive spreadsheets and Visual Basic for Application (VBA) programming, the study looked for to demonstrate the propagation of Gaussian wave packets and wave deflecting in a square membrane. Gaussian beam waves traveling in the x-direction and 2D wave growth in a square membrane were solved analytically and used in the simulation. Equations of state and differential equations were used to calculate the two-dimensional wave vibration modes. In [1, 3], a simulation of square wave propagation is shown using a two-dimensional CIP-based numerical model. The outcomes showed a unique visual representation of the wave's vibration modes in the square membrane. According to [3], an improved numerical model that uses the Constrained Interpolation Profile -based approach to simulate two-dimensional incompressible viscous flow with a deformed free surface. In [10] the simulation of acoustic wave propagation is discussed by using the characteristic curves method and the CIP (Cubic Interpolated Profile) method, which is known for its low numerical dispersion and stable numerical calculations. It compares the CIP method with other finite difference schemes in solving the advection equation and highlights its advantages in accurately simulating waves with steep edges and high-frequency components, even with a simple orthogonal grid. The paper also derives the characteristic equations for describing acoustic wave propagation based on the equations of motion and continuity in acoustic media.

The stability and phase error analysis, along with numerical simulations, demonstrate the effectiveness and stability of the CIP method in acoustic wave simulations, even with reduced grid points. Furthermore, a numerical simulation approach employing the high-order difference method presented in [1, 11], and [12] to investigate the behavior of reciprocating sea waves on beaches with different topography toward those waves. Other numerical techniques such as the finite difference and finite element methods can be compared to CIP. While every technique has advantages, CIP stands out for its ease of use and capacity to manage nonlinearities [13]. In the applied sciences, the CIP technique is useful for a variety of issues, but it excels in modeling wave interactions and events. In addition to aiming for computing efficiency, nonlinear equation solution efficiency is also a goal [14]. In terms of flexibility, accuracy, and managing nonlinearities, the CIP approach compares favorably to other numerical techniques, making it an invaluable tool in numerical analysis. Particularly in one-dimensional settings, the CIP technique effectively simulates nonlinear acoustic wave propagation while minimizing numerical dispersion problems related to abrupt pressure changes [15]. Additionally, it studies fluid dynamics in shear-driven cavities, demonstrating its capacity to mimic fluid motion patterns in two dimensions at various Reynolds numbers [16]. The flow generator in the framework is a constrained interpolation profile (CIP) approach, and the multiphase flow model is based on the Navier-Stokes equations. By comparing the results of computation with the available experimental data, the model's validity is verified, and the wave profile and velocity-pressure domain exhibit good agreements. This idea is used in various other industries. It is applied to study the impact of waves on offshore structures and marine crafts to ascertain their integrity under adverse sea conditions as well as it is applied in branches such as oil and gas [17], where understanding the behavior of oil in water emulsions is helpful in improving recovery and refining processes. Other researchers [18] used the CIP method in solving shallow water equations to imitate natural disasters like river flooding, tsunamis, and tidal actions for disaster preparedness and for managing water resources. The proposed model can reproduce nonlinear flow phenomena and accurately capturing complex free surface flow. However, [1] focused on the effects of wave parameters and the position of the wedge impacting the water surface on the velocity and pressure field of the fluid and the impact force on the wedges. A discretization technique is typically unavoidable when creating a numerical simulation.

Identifying the information that lost inside the grid cell between these separated points will be the fundamental objective of a numerical methodology. However, most previous numerical approaches did not address the actual solution within the grid cell, and the resolution was constrained by the grid size. Whereas the potential limitations and challenges associated with CIP approaches have a lot to offer in terms of accuracy and efficiency for some situations, there are drawbacks as well, including issues with stability, computing cost, complexity, and problem-specific applicability. It is essential to continue researching and developing solutions to these problems in order to increase the resilience and adaptability of CIP approaches.

2.0 NUMERICAL METHOD

One numerical technique for resolving advection equations with minimal numerical diffusion is cubic interpolated pseudo particle (CIP). The CIP scheme may be considered a kind of semi-Lagrangian technique as it employs a Lagrangian invariant solution [7]. It well demonstrated that the CIP approach is a universal solution for equations of the hyperbolic model. The CIP, which originally meant for cubic interpolated pseudo-particle, evolved to represent for cubic interpolated propagation [2]. The principal points of the CIP scheme in both one and two-dimensional cases are briefly covered in this section.

2.1 Mathematical One-Dimensional CIP Method

CIP tracks back the advection characteristic utilizing gradient, the partial derivative f. The Courant number is also a factor in this procedure.

$$\Delta x < | (i + 1) - i |, CFL = C \Delta t/\Delta x < 1 \quad (1)$$

Characteristic of advection determined the profile after time Δt. Advection equation.

$$\partial f / \partial t + u . \partial f / \partial x = 0 \quad (2)$$

The advection equation provides a straightforward translation of function f with velocity u when velocity u is constant.

$$f (xi, t+\Delta t) = f (xi+ u \Delta t, t) \quad (3)$$

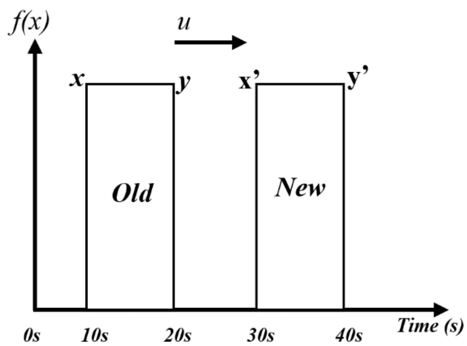


Figure 1 Advection's characteristic

In Figure 1, the starting at 10s, the profile moves with a velocity of u to the positions x, y at time Δt. Both the new and old profiles were identical. The quadratic polynomial f(x)= ax3+bx2+cx+d could possibly be used to interpolate the profile between two points in a grid if two values of *f(x)* are provided. The cubic polynomial required four unknowns to be solved.

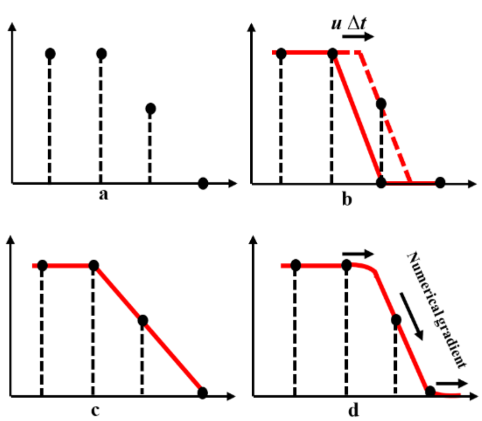


Figure 2 The concept of the CIP method

Figure 2 illustrates the CIP method's basic concept. The solution to the advection equation provides an understandable translational motion of a wave with a velocity when the velocity is constant. In Figure 2(b), the solid line's initial profile advances in a continuous image that's comparable to that of a dashed red line. So, in this moment, exact details and the solution at grid spots are shown by circles. Nevertheless, if the dashed line in Figure 2a is removed, the profile information inside the grid cell is lost, making it difficult to visualize the original profile. Instead, it is normal to visualize a profile like the one depicted by the solid line in Figure 2c. Therefore, numerical circulation is obtained even with the exact solution as seen in Figure 2c when we produce the profile by linear interpolation [8]. In contrast to traditional high-order approaches, the CIP scheme—which is seen in Figure 2d—illustrates an alternative method for reconstructing the profile inside a grid cell.

If u is a constant velocity, the solution in (1) gives a straightforward translation movement of a wave. Like the dashed line of a continuous representation, the initial profile as a solid line in Figure 2a, advances. At grid locations, the answer—which is the same as the exact solution—is now represented by spot circles. It is easy to predicts a profile like that given by the solid line in Figure 2c, but if we remove the dashed line as shown in Figure 2b, the information from the profile inside the grid cell is lost and it becomes harder to image the original profile. This approach is commonly used in literature to establish equations involving hyperbolic type equations [2]. As a result, this section will employ this approach to illustrate two dimensions. Whenever we use the location variable to (2) for differentiation, we obtain:

$$\partial fx/\partial t + u . \partial fx/\partial x = 0 \quad (4)$$

Thus, the translation of fx with velocity u is represented by fx =∂f/∂x in (4), which corresponds inside (2). The unique idea within the CIP technique is that, after one step, we provide the profile at each node in accordance with (4) and we use (2) and (4) to track the time course of both f and fx . When we build the profile, we can significantly reduce the numerical diffusion using this limitation.

Using f and fx at nearby grid points, a bounding polynomial is used in the CIP approach to estimate spatial values in the grid interval as follows:

$$F_i(x)= a_i X^3+ b_i X^2+f_{x,\, i} X + f_i \quad (5)$$

In this case, X= x - xi. The interpolated functions and its initial derivatives are continuous at each end, which is achieved by determining the coefficients of (a) as well as (b) in (4). Therefore, we have

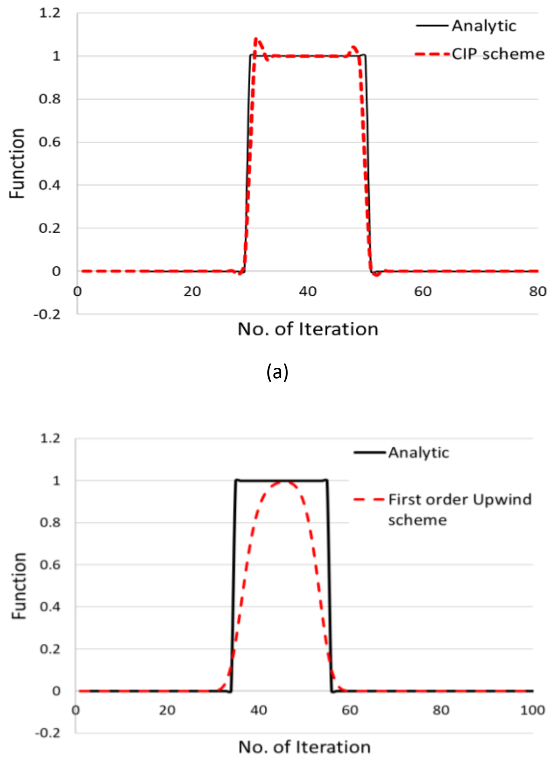
$$a_i= [(f_{x,i} +f_{x,i-1}) /(\Delta x^2)]-[2(f_i-f_{i-1})/(\Delta x^3)], \quad (6)$$

$$b_i= [(2f_{x,i} +f_{x,i-1}) /(\Delta x)]-[3(f_i-f_{i-1})/(\Delta x^2)] \quad (7)$$

Where Δx= xi – xi-1. Once Fi(x) are determined for all grid intervals, the spatial derivative is calculated as

$$F_{x,i} (x)=(3a_i X+2b_i)X+ f_{i,j} \quad (8)$$

The advection equation is compared between the First Order Upwind Scheme [2], and the CIP Scheme with an analytical solution, as shown in Figure 3.



(a)

(b)

Figure 3 Comparison solution of the advection equation using (a) First Order Upwind Scheme and (b) CIP Scheme with analytical solution

After all, the advected profile is given by:

$$f_i^{\,n+1}=F_i\,(x_i+\xi)=a_i\,\xi^3+b_i\,\xi^2+f_{x,i}\,\xi+f_i\,, \quad (9)$$

$$f_{x,i}^{\,n+1}=F_{x,i}\,(x_i+\xi)=3a_i\,\xi^2+2b_i\,\xi+f_{x,i}. \quad (10)$$

where $\xi=-c_i\,\Delta t$ and the superscript n indicates the time. Start by implementing the CIP approach to a square wave's evolution. The results of comparing the predictions made by CIP and first order upwind approaches as the wave advances from its original position are displayed in Figure 2. In these computations, we know that the CIP method gives better accuracy compared to the first order upwind method to advection equation.

2.2 Mathematical Two-Dimensional CIP Method

In this case, the equation of the CIP method is not similar to the one-dimensional. Therefore, the calculation must be done to obtain the equation. In this paper, we are considering the two dimensional as the research. The equations are showed step by step as are below, as shown in Figure 4.: -

The governing equation shows as.

$$\partial f/\partial t + u_x\,.\,\partial f/\partial x + u_y.\partial f/\partial y = 0. \quad (11)$$

From the third order accuracy.

$$F_{i,j}(x,\,y)=A1_{i,j}\,x^3+A2_{i,j}\,x^2y+A3_{i,j}\,x^2+A4_{i,j}\,xy+Gx+A5_{i,j}\,y^3+A6_{i,j}\,xy^2+A7_{i,j}\,y^2+Hy+J \quad (12)$$

Where *G, H*, and *J* can be obtained once origin is (0, 0) as:

$$F_{i,j}\,(0,0)=J=f_{i,j}, \quad (13)$$

$$(\partial F_{i,j}\,(0,0))/\partial x=G=\partial x f_{i,j}, \quad (14)$$

$$(\partial F_{i,j}\,(0,0))/\partial y=H=\partial y f_{i,j}. \quad (15)$$

The equation becomes;

$$F_{i,j}(x,\,y)=A1_{i,j}\,x^3+A2_{i,j}\,x^2y+A3_{i,j}\,x^2+A4_{i,j}\,xy+\partial x\,f_{i,j}\,x+A5_{i,j}\,y^3+A6_{i,j}\,xy^2+A7_{i,j}\,y^2+\partial y\,f_{i,j}\,y+f_{i,j}. \quad (16)$$

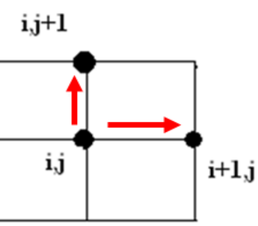


Figure 4 Particle move in x and y-direction.

The boundary conditions to implement CIP codes, as shown in Table 1 below.

Table 1 Boundary Conditions

Parameters	1D-CIP	2D-CIP
ΔX	0.1	0.1
ΔY	------------------	0.1
Δt	0.1 and 0.2	0.1 and 0.2
No. of iteration	100 and 200	100 and 200
Ux	0.1 and 0.2	0.1 and 0.2
Uy	--------------------	0.1 and 0.2
i	20 to 40	20 to 40
j	--------------------	20 to 40

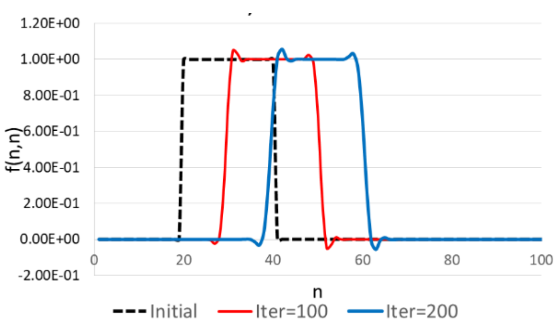
3.0 RESULTS AND DISCUSSION

The following calculation is performed in order to show the difference in calculation results depending on the code programming. Two methods (1D-CIP, 2D-CIP) are compared. One has the movement in one direction, and another has two dimensional movements as shown in the following analytical solutions.

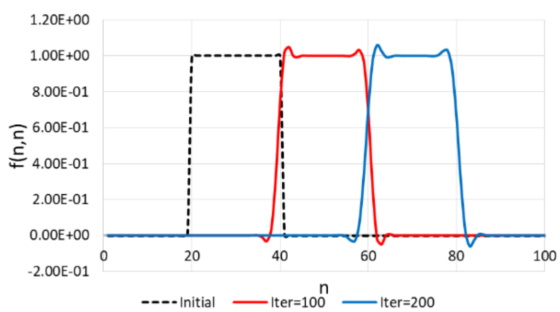
For 1D-CIP method we considered the velocity u= 1.0 m/s which is in one direction for all solution of this method (1D-CIP) and delta t= 0.1, after substituting it in,

$$t' = dt * No.\ of\ iteration. \tag{17}$$

To get the Numerical (t') =10s, and (t') =20s, these are the time moving of profile, for 100 and 200 iterations with velocity u =0.1 m/s, as shown in Figure 5a. As well as the analysis procedure to get the Numerical t' =20 s, in this time moving of profile, for 100 and 200 iterations as shown in Figure 5b.



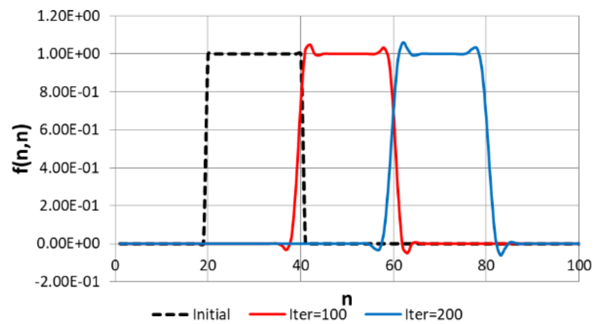
(a)



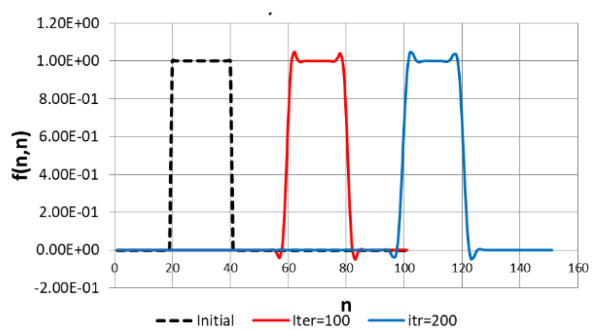
(b)

Figure 5 1D-CIP for velocity u= 0.1 m/s, and time t=10 s for 100 and 200 iteration

And as seen in Figure 6, the same numerical analysis method was used with a velocity of u = 0.2 m/s. The wave is clearly moving in time Δt, and the flow is depending on time displacement and iteration count as predicted.



(a)



(b)

Figure 6 1D-CIP for velocity u= 0.2 m/s, AND time t=10 s and t=20 s for 100 and 200 iteration

In the case of the 2D-CIP approach, the outcomes were identical to those of the preceding operation. This indicates that, when evaluated in a single direction, the behavior of the 1D-CIP and 2D-CIP in solution is the same. However, as Figures 7 and 8 demonstrate, the wave's two-directional characteristic has improved the movement significantly under borderline conditions.

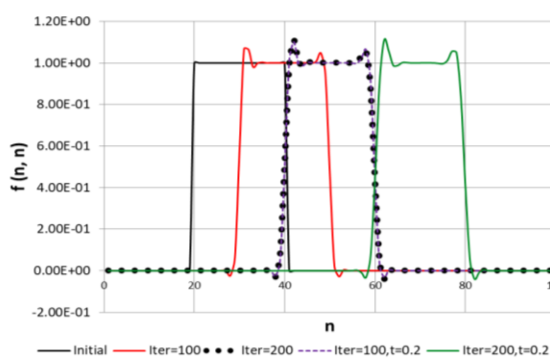


Figure 7 The 2D-CIP for velocity u= 0.1 m/s, and time t=10 and 20 s for 100 and 200 iteration.

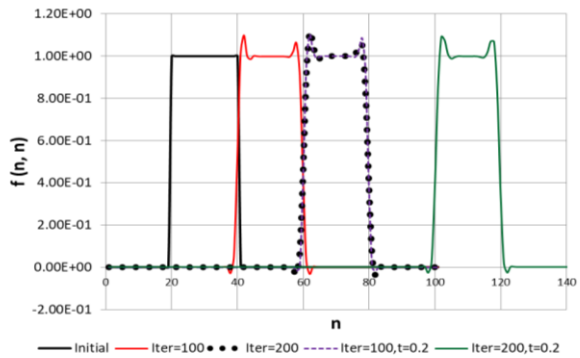


Figure 8 The 2D-CIP for velocity u= 0.2 m/s, and time t=10 and 20 s for 100 and 200 iteration.

In Figure 9, two different velocities u= (0.1 and 0.2) m/s are compared with t=10 s for 100 and 200 iterations. The mean squared error (MSE) considered to measures the amount of error between 1D and 2D-CIP. It assessed the average squared difference between the observed and predicted values which led the superiority of 2D-CIP over 1D-CIP by 0.5%, as illustrate in Figure 10.

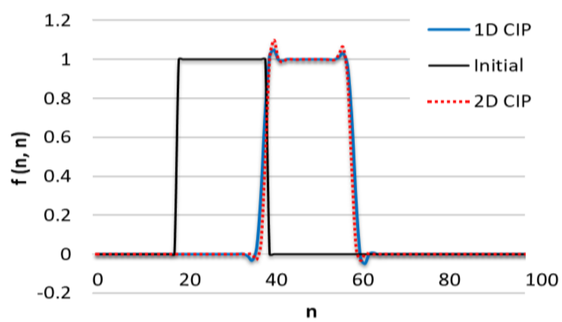


Figure 9 The 1D-CIP and 2D-CIP schemes comparison square wave propagation

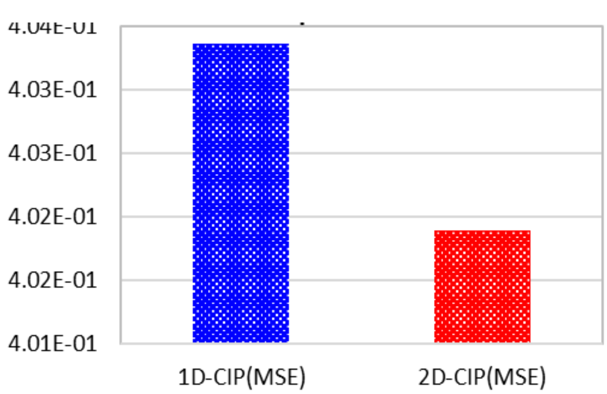


Figure 10 Mean square error between 1D-CIP and 2D-CIP schemes

4.0 CONCLUSION

The paper demonstrates how the steady flow may be simulated using an approach called the CIP is utilized to tackle different fluid flow problems. Applying the CIP approach helps to improve spatial accuracy up to the third order. The cubic polynomial is utilized in the CIP method to interpolate the profile between grid points. Analytical solution and 1D- and 2D-CIP findings are compared. Moreover, the mean square error considered to demonstrate the superiority of 2D-CIP over 1D-CIP. There are two crucial factors that must be considered while solving the fluid flow for certain boundary conditions. The first concern is the shift in time delta t. The flow must be dependent on the passage of time. The potential future research directions in CIP methods are to extend and enhance it for three-dimensional simulations, which often need more processing power and demand careful interface and boundary condition management in order to maximize accuracy while maintaining computational economy.

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Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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