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MAGNETO-CONVECTIVE INSTABILITY IN A HORIZONTAL VISCOELASTIC NANOFLUID SATURATED POROUS LAYER

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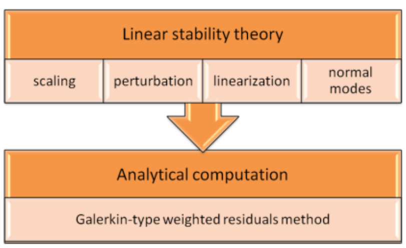
Graphical abstract



Abstract

Nanofluids have been shown experimentally to have high thermal conductivity. In this study, the convective instabilities in a horizontal viscoelastic nanofluid saturated by porous layer under the influences of gravity and magnetic field are investigated. The linear stability theory is used for the transformation of the partial differential equations to system of ordinary differential equations through infinitesimal perturbations, scaling, linearization and method of normal modes with two-dimensional periodic waves. The system is solved analytically for the closed form solution of the thermal Darcy-Rayleigh number by using the Galerkin-type weighted residuals method to investigate the onset of both stationary and oscillatory convection. The effects of the scaled stress relaxation parameter, scaled strain retardation parameter and Chandrasekhar number on the stability of the system are investigated. The scaled strain retardation parameter stabilizes while the scaled stress relaxation parameter destabilizes the nanofluid system. The system in the presence of magnetic field is more stable than the system in the absence of magnetic field.

Keywords: Nanofluid, viscoelastic, magnetic field, porous medium, instability, analytical

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1.0 INTRODUCTION

Nanofluids is a new class of fluids that exhibit superior properties [1]. The effect of nanoparticles on the heat transfer escalation is still an argument among researchers. Buongiorno [2] decided that the important slip mechanisms that produce the relative velocity between the nanoparticle and the base fluid are Brownian motion and thermophoresis. Controversial experimental findings and theories do not fully explain the mechanisms of the elevated thermal conductivity [2], [3], [4]. Kleinstreuer and Feng [4] reviewed various experimental and theoretical methods to predict the effective thermal conductivity in relation to the particle velocity. They emphasized on the necessity to consider several mechanisms to obtain predictive results and benchmark with new experimental data sets.

Nield and Kuznetsov [5] focused on the onset of instability in a porous medium in the solution of nanofluids. They used the models by Buongiorno [2] and applied the Darcy law for porous media to investigate both steady and oscillatory convection by using linear stability analysis and one-term Galerkin-type weighted residual method. The temperature and nanoparticle concentration at the upper and lower boundary are assumed of uniform temperature and nanoparticle concentration. They found the instability can be delayed or promoted depending on the region of concentration of nanoparticles. If the nanoparticles distribution is higher at the bottom fluid layer, the oscillatory convection is possible. Nield and Kuznetsov [6] considered the contribution of thermophoresis to the nanoparticle flux by an imposed temperature on the boundaries. They showed that the oscillatory convection cannot occur with the new imposed temperature boundary condition on the nanoparticle volume fraction. The effect of nanoparticles on steady convection is to destabilize. The Marangoni instability problems under the influence of a linear feedback control have been investigated by Kechil and Hashim [7]. Hamid *et al.* [8] investigated the instability subject to uniform temperature and uniform heat flux at the lower side of boundary in the presence of insoluble surfactant. Linear feedback control and insoluble surfactant delays the onset of convective instabilities.

The onset of convective instability in non-Newtonian nanofluids was studied by Nield [9], Yadav *et al.* [10], Kang *et al.* [11]. Yadav *et al*. [10] extended the work of Nield and Kuznetsov [12] for the non-Newtonian nanofluids of the Oldroyd type to investigate the stability in a horizontal rotating porous layer. Rotation and thermal conductivity variation parameter act as stabilizer. However, porosity and viscosity variation parameters act as destabilizer. A weakly nonlinear analysis based on the minimal representation of truncated Fourier series method is used to compute the concentration and thermal Nusselt numbers because the linear stability analysis does not predict the amplitude of convective motion. Shivakumara *et al.* [13] investigated the convective instability in a layer of porous medium saturated by the Oldroyd-B viscoelastic nanofluid. They found that the oscillatory convection is possible only if the strain retardation parameter is less than the stress relaxation parameter.

Magnetic fluid consists of a carrier fluid and a suspension of magnetic particles [14]. The magnetic fluid reacts differently and has different applications based on the size of the magnetic particles. There are many applications of magneto-convection in viscoelastic fluid such as in chemical engineering, biomedical, industries and geophysics. The influences of thermal buoyancy and magnetic field in the classical Rayleigh-Bènard problem of magneto convection for a nanofluid layer produce a draglike force known as Lorentz force. Due to this Lorentz force on Rayleigh-Bènard convection, the nondimensional parameter, named as the Chandrasekhar number is introduced [15]. The influence of the viscoelasticity on convective thresholds in magnetic fluid was analyzed by Perez *et al.* [16]. Narayana *et al.* [17] and Gupta *et al.* [15] found that the effect of magnetic field is to stabilize the nanofluid layer for both stationary and oscillatory convection.

The purpose of this paper is consider the influence of the magnetic on the natural convection in a viscoelastic nanofluid saturated porous layer layer of Shivakumara *et al.* [13]. The critical thermal Darcy-Rayleigh numbers are determined by performing the linear stability analysis. The eigenvalue problem is solved analytically by using the Galerkin-type weighted residuals method.

The mathematical analysis of the problem is divided into two sections. Section 2.0 discusses the formulation of the model problem and the analysis of linear stability with the introduction of scaling variables, infinitesimal perturbations, linearization and superposition of the normal modes. The solutions of the Galerkin-type weighted residuals for the stationary and oscillatory convection are presented in Section 3.0 with the graphical illustrations and discussion of the results.

2.0 METHODOLOGY

Consider a horizontal layer thickness H of Darcy porous medium saturated by an incompressible and electrically conducting Oldroyd-B nanolfuid. The lower boundary of the layer is coincidence with the $x-$axis at $z=0$. The gravity, $\mathbf{g}=-g\hat{\mathbf{e}}_z$ is assumed to act vertically downwards in the presence of an applied transverse magnetic field $\mathbf{H}$. The induced magnetic field is assumed small and negligible. The set up of the problem is drawn in Figure 1.

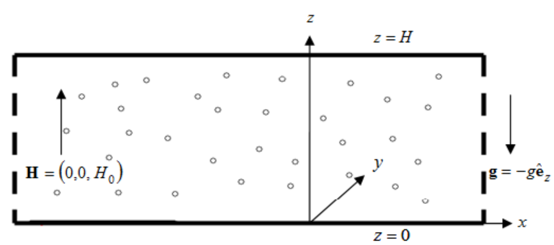


Figure 1 Schematic diagram of the physical problem

The conservation equations for the mass, momentum, energy and nanofluid concentration are ([2], [13], [6] and [18]):

$$\nabla\cdot\mathbf{v}=0, \tag{1}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\nabla p-\rho\mathbf{g}-\mathbf{J}\times\mathbf{B}\right)=-\frac{\mu}{K}\left(1+\lambda_2\frac{\partial}{\partial t}\right)\mathbf{v}, \tag{2}$$

$$(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f(\mathbf{v}\cdot\nabla)T=k\nabla^2T+\varepsilon(\rho c)_p\left[D_B\nabla\phi\cdot\nabla T+\frac{D_T}{T_c}\nabla T\cdot\nabla T\right], \tag{3}$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}(\mathbf{v}\cdot\nabla)\phi=D_B\nabla^2\phi+\frac{D_T}{T_c}\nabla^2T, \tag{4}$$

where $\mathbf{v}=(u,v,w)$ is the velocity, T is the temperature of the nanofluid, ϕ is the nanoparticle volume fraction, t is the time, $\mathbf{g}$ is the acceleration due to gravity, $\mathbf{J}$ is the current density, $\mathbf{B}$ is the magnetic induction vector, p is the pressure, $(\rho c)_m$ is the effective heat capacity of the porous medium, $(\rho c)_p$ is the effective heat capacity of the nanoparticles, $(\rho c)_f$ is the heat capacity of the base fluid, μ is the viscosity of the nanofluid, K is the permeability of the porous medium, ε is the porosity of the porous medium, λ_1 is the relaxation time, λ_2 is the retardation time, k is the thermal conductivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and T_c is the temperature at the upper wall.

The nanofluid density ρ is

$$\rho \cong \phi\rho_p + (1-\phi)\{\rho_f[1-\beta(T-T_c)]\}, \quad (5)$$

where ρ_p is the density of nanoparticles, ρ_f is the density of the base fluid, β is the coefficient of thermal expansion and c is the specific heat at constant pressure.

The constitutive equations are

$$\mathbf{J} = \sigma(\mathbf{v}\times\mathbf{B}), \quad (6)$$

$$\mathbf{B} = \mu_m\mathbf{H}, \quad (7)$$

where σ is the electrical conductivity and μ_m e magnetic permeability. For a weakly electrically conducting fluid the constitutive equations (6) and (7) give the following Lorentz force

$$\mathbf{J}\times\mathbf{B} = -\sigma u\mu_m^2 H_0^2\hat{\mathbf{e}}_x - \sigma v\mu_m^2 H_0^2\hat{\mathbf{e}}_y. \quad (8)$$

The lower and upper boundaries are assumed to be rigid (no slip) with uniform temperature and no nanoparticles flux given by

$$w = 0,\ T = T_h,\ D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_C}\frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \quad (9)$$

$$w = 0,\ T = T_c,\ D_B\frac{\partial\phi}{\partial z} + \frac{D_T}{T_C}\frac{\partial T}{\partial z} = 0 \text{ at } z = H, \quad (10)$$

where T_h is the temperature at the lower wall. The introduction of the scaling quantities for the length, velocity, time, pressure, nanoparticles volume fraction and temperature [6] involves the nondimensional variables (denoted by asterisk $*$)

$$(x,y,z) = \frac{(x^*,y^*,z^*)}{H},\ (u,v,w) = \frac{H(u^*,v^*,w^*)}{\alpha_f},$$
$$t = \frac{\alpha_f}{\varepsilon H^2}t^*,\ p = \frac{K}{\mu\alpha_f}p^*, \quad (11)$$
$$\phi = \frac{\phi^* - \phi_0^*}{\phi_0^*},\ T = \frac{T^* - T_c^*}{T_h^* - T_c^*},$$

where $\alpha_f = \dfrac{k}{(\rho c)_f}$ is the thermal diffusivity of the porous medium, k is the effective thermal conductivity of the porous medium, T_h^* is the dimensionless temperature at the lower wall, T_c^* is the dimensionless temperature at the upper wall and ϕ_0^* is the reference value for the dimensionless nanoparticles volume fraction.

Using the scaling quantities (11), the conservation equations (1) - (4) are transformed to dimensionless equations,

$$\nabla^*\cdot\mathbf{v}^* = 0, \quad (12)$$

$$\left(1+\lambda_1\frac{\partial}{\partial t^*}\right)\{\nabla^* p^* - \phi^*\rho_p - (1-\phi^*)[\rho_f - \beta\rho_f(T^*-T_c^*)]\mathbf{g} + \sigma u\mu_m^2 H_0^2\hat{\mathbf{e}}_x + \sigma v\mu_m^2 H_0^2\hat{\mathbf{e}}_y\} = -\frac{\mu}{K}\left(1+\lambda_2\frac{\partial}{\partial t^*}\right)\mathbf{v}^* \quad (13)$$

$$(\rho c)_m\frac{\partial T^*}{\partial t^*} + (\rho c)_f(\mathbf{v}^*\cdot\nabla^*)T^* = k\nabla^{*2}T^* + \varepsilon(\rho c)_p\left[D_B\nabla^*\phi^*\cdot\nabla^*T^* + \frac{D_T}{T_c^*}\nabla^*T^*\cdot\nabla^*T^*\right], \quad (14)$$

$$\frac{\partial\phi^*}{\partial t^*} + \frac{1}{\varepsilon}(\mathbf{v}^*\cdot\nabla^*)\phi^* = D_B\nabla^{*2}\phi^* + \frac{D_T}{T_c^*}\nabla^{*2}T^*. \quad (15)$$

The dimensionless corresponding boundary conditions from (9) and (10) are

$$w^* = 0, T^* = T_h^*, D_B\frac{\partial\phi^*}{\partial z^*} + \frac{D_T}{T_c^*}\frac{\partial T^*}{\partial z^*} = 0 \text{ at } z^* = 0, \quad (16)$$

$$w^* = 0, T^* = T_c^*, D_B\frac{\partial\phi^*}{\partial z^*} + \frac{D_T}{T_c^*}\frac{\partial T^*}{\partial z^*} = 0 \text{ at } z^* = H. \quad (17)$$

The non dimensionalized system of (12) - (17) after dropping the asterisk are,

$$\nabla\cdot\mathbf{v} = 0, \quad (18)$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)[\nabla p + Rm\hat{\mathbf{e}}_z + Rn\phi\hat{\mathbf{e}}_z - RaT\hat{\mathbf{e}}_z + Q(u\hat{\mathbf{e}}_x + v\hat{\mathbf{e}}_y)] = -\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\mathbf{v}, \quad (19)$$

$$M\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)T = \nabla^2 T + \frac{N_B}{Le}\nabla\phi\cdot\nabla T + \frac{N_A N_B}{Le}\nabla T\cdot\nabla T, \quad (20)$$

$$\frac{\partial\phi}{\partial t} + (\mathbf{v}\cdot\nabla)\phi = \frac{1}{Le}\nabla^2\phi + \frac{N_A}{Le}\nabla^2 T, \quad (21)$$

subject to the dimensionless boundary conditions

$$w = 0,\ T = 1,\ \frac{\partial\phi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \quad (22)$$

$$w = 0,\ T = 0,\ \frac{\partial\phi}{\partial z} + N_A\frac{\partial T}{\partial z} = 0 \text{ at } z = 1, \quad (23)$$

with the following nondimensional parameters [13],

$$\Lambda_1 = \frac{\alpha_f}{\varepsilon H^2}\lambda_1, \Lambda_2 = \frac{\alpha_f}{\varepsilon H^2}\lambda_2, Le = \frac{\alpha_f}{\varepsilon D_B}, M = \frac{(\rho c)_m}{(\rho c)_f \varepsilon},$$
$$Q = \frac{\sigma \mu_m^2 H_0^2 K}{\mu}, \; Rm = \frac{\left[\phi_0^* \rho_p + \rho_f\left(1-\phi_0^*\right)\right] gKH}{\mu \alpha_f},$$
$$Ra = \frac{\left[\rho_f \beta\left(T_h^* - T_c^*\right)\right] gKH}{\mu \alpha_f}, Rn = \frac{\left[\left(\rho_p - \rho_f\right)\phi_0^*\right] gKH}{\mu \alpha_f}, \tag{24}$$
$$N_A = \frac{D_T\left(T_h^* - T_c^*\right)}{D_B T_c^* \phi_0^*}, \; N_B = \frac{(\rho c)_p}{(\rho c)_f}\phi_0^*,$$

where Λ_1 is the scaled stress relaxation parameter, Λ_2 is the scaled strain retardation parameter, Le is the Lewis number, M is the heat capacity ratio, Q is the Chandrasekhar number, Rm is the basic density Darcy-Rayleigh number, Ra is the thermal Darcy-Rayleigh number, Rn is the concentration Darcy-Rayleigh number, N_A is the modified diffusivity ratio and N_B is the modified particle density increment.

2.1 Perturbation Solution

The fluid is at rest in the reference steady basic state with the velocity, hydrodynamic pressure, temperature and nanoparticle volume fraction varying in the $z-$direction [6] are

$$\mathbf{v}_b = 0, \; p = p_b(z), \; T = T_b(z), \; \phi = \phi_b(z). \tag{25}$$

The basic state is perturbed by infinitesimal disturbances

$$\mathbf{v} = \mathbf{v}', \; p = p_b + p', \; T = T_b + T', \; \phi = \phi_b + \phi', \tag{26}$$

where p_b is the constant reference pressure. Performing basic calculus and algebraic manipulation, the linearised perturbed system of (18) - (23) are,

$$\nabla \cdot \mathbf{v}' = 0, \tag{27}$$

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[\nabla_H^2 Rn\phi' - \nabla_H^2 RaT' + Q\nabla_z^2 w'\right] = -\left(1 + \Lambda_2 \frac{\partial}{\partial t}\right)\nabla^2 w', \tag{28}$$

$$M\frac{\partial T'}{\partial t} - w' = \nabla^2 T' + \frac{N_B}{Le}\left(N_A \frac{\partial T'}{\partial z} - \frac{\partial \phi'}{\partial z}\right) - 2\frac{N_A N_B}{Le}\frac{\partial T'}{\partial z}, \tag{29}$$

$$\frac{\partial \phi'}{\partial t} + N_A w' = \frac{N_A}{Le}\nabla^2 T' + \frac{1}{Le}\nabla^2 \phi', \tag{30}$$

subject to boundary conditions at $z = 0$ and $z = 1$ are

$$w' = 0, \; T' = 0, \; \frac{\partial \phi'}{\partial z} + N_A \frac{\partial T'}{\partial z} = 0, \tag{31}$$

where ∇_H^2 is the two-dimensional Laplacian operator, ∇_z^2 is the one-dimensional Laplacian operator with respect to $z-$plane and ∇^2 is the three-dimensional Laplacian operator.

2.2 Normal Modes

The system (27) - (31) constitutes a linear boundary-value problem that can be solved using the method of normal-modes. The solutions are sought in the form of

$$(w', T', \phi') = [\Gamma(z), \Theta(z), \Phi(z)]e^{(st + i\alpha_x + i\alpha_y)} \tag{32}$$

where $\Gamma(z), \Theta(z)$ and $\Phi(z)$ are the vertical velocity, temperature and nanoparticles volume fraction amplitudes, respectively. α_x and α_y are the wave number in the x and y direction, respectively. s is a dimensionless complex growth rate given by $\Re(s)$ is the growth rate and $\Im(s)$ is the frequency. For neutral stability, $\Re(s) = 0$ and when $\Im(s) = 0$, stationary convection sets in and when $\Im(s) \neq 0$, the convection is oscillatory. The substitution of (32) in (27) - (31) yields,

$$(1 + \Lambda_1 s)\left(QD^2\Gamma + \alpha^2 Ra\Theta - \alpha^2 Rn\Phi\right) = -(1 + \Lambda_2 s)\left(D^2 - \alpha^2\right)\Gamma, \tag{33}$$

$$\Gamma + \left(D^2 - \frac{N_A N_B}{Le}D - \alpha^2 - Ms\right)\Theta - \frac{N_B}{Le}D\Phi = 0, \tag{34}$$

$$N_A\Gamma - \frac{N_A}{Le}\left(D^2 - \alpha^2\right)\Theta - \left[\frac{1}{Le}\left(D^2 - \alpha^2\right) - s\right]\Phi = 0, \tag{35}$$

and the boundary conditions at $z = 0$ and $z = 1$ become

$$\Gamma = 0, \; \Theta = 0, \; D\Phi + N_A D\Theta = 0, \tag{36}$$

where $D \equiv \frac{d}{dz}$ and $\alpha = \left(\alpha_x^2 + \alpha_y^2\right)^{1/2}$. When $Q = 0$, the system (33) - (36) is reduced to the problem of Shivakumara *et al*. [13].

The Galerkin-type weighted residuals method with Γ, Θ and Φ in the form of series solution is used to obtain an approximate solution to the system (33) - (36)

$$\Gamma(z) = \sum_{i=1}^{N} I_i \Gamma_i(z), \; \Theta(z) = \sum_{i=1}^{N} J_i \Theta_i(z), \; \Phi(z) = \sum_{i=1}^{N} K_i \Phi_i(z), \tag{37}$$

where I_i, J_i and K_i are unknown coefficients and $i = 1,2,3,\ldots,N$, where N is the natural number.

3.0 RESULT AND DISCUSSIONS

The appropriate trial functions satisfying the boundary conditions (36) are chosen as

$$\Gamma_1 = z - z^2,\ \Theta_1 = z - z^2,\ \Phi_1 = N_A\left(z^2 - z\right). \tag{38}$$

The important critical parameter is the thermal Darcy-Rayleigh number Ra which determines the onset of instability. The Galerkin-type weighted residuals method yields the nontrivial solution for thermal Darcy-Rayleigh number given by

$$Ra = \frac{1}{\alpha^2}\left[\left(\frac{1+\Lambda_2 i\omega}{1+\Lambda_1 i\omega}\right)\gamma + 10Q\right]\left(\gamma + Mi\omega\right) - \left[\frac{\left(\gamma + Mi\omega\right)N_A Le + \gamma N_A}{\gamma + i\omega Le}\right]Rn, \tag{39}$$

where $\gamma = 10 + \alpha^2$ and $\omega = \Im(s)$.

Separating the real and imaginary parts, the expression for thermal Darcy-Rayleigh number, Ra in the form of $Ra = Ra_r + i\omega Ra_i$, where

$$Ra_r = \frac{\gamma}{\alpha^2}[\gamma\left(\frac{1+\Lambda_1\Lambda_2\omega^2}{1+\Lambda_1^2\omega^2}\right) - \left(\frac{\Lambda_2 - \Lambda_1}{1+\Lambda_1^2\omega^2}\right)M\omega^2 + 10Q\alpha^2] - \left[\frac{\gamma^2\left(N_A Le + N_A\right) + MN_A Le^2\omega^2}{\gamma^2 + \omega^2 Le^2}\right]Rn, \tag{40}$$

and

$$Ra_i = \frac{\gamma}{\alpha^2}\left(\frac{1+\Lambda_1\Lambda_2\omega^2}{1+\Lambda_1^2\omega^2}\right) + \frac{\gamma^2}{\alpha^2}\left(\frac{\Lambda_2 - \Lambda_1}{1+\Lambda_1^2\omega^2}\right) + 10QM - \left[\frac{\left(M - Le - 1\right)\gamma N_A Le}{\gamma^2 + \omega^2 Le^2}\right]Rn. \tag{41}$$

3.1 Stationary Convection

Setting $\omega = 0$ for the case of stationary instability produces,

$$Ra^{stat} = \frac{\left(10+\alpha^2\right)^2}{\alpha^2} + 10Q\left(10+\alpha^2\right) - \left(Le+1\right)N_A Rn. \tag{42}$$

3.2 Oscillatory Convection

For the onset of oscillatory convection $Ra_i = 0$ and $\omega \neq 0$, gives the following expression for $Ra = Ra^{osc}$,

$$Ra^{osc} = \frac{\left(10+\alpha^2\right)}{\alpha^2}[\left(10+\alpha^2\right)\left(\frac{1+\Lambda_1\Lambda_2\omega^2}{1+\Lambda_1^2\omega^2}\right) - \left(\frac{\Lambda_2-\Lambda_1}{1+\Lambda_1^2\omega^2}\right)M\omega^2 + 10Q\alpha^2] - \left[\frac{\left(10+\alpha^2\right)^2\left(N_A Le + N_A\right) + MN_A Le^2\omega^2}{\left(10+\alpha^2\right)^2 + \omega^2 Le^2}\right]Rn. \tag{43}$$

In this section, the results are focused on the effects of magnetic field on the stationary and oscillatory instabilities. The neutral stability curves of the thermal Darcy-Rayleigh number is plotted in the plane (α, Ra). Figure 2 shows the neutral stability curves with and without the effect of magnetic field for the variations of thermal Darcy-Rayleigh number, Ra as a function of the wave number α for various values of the scaled stress relaxation parameter, Λ_1.

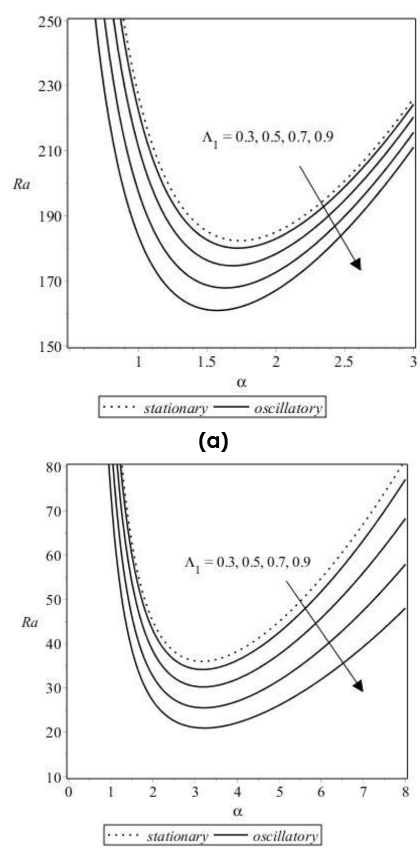
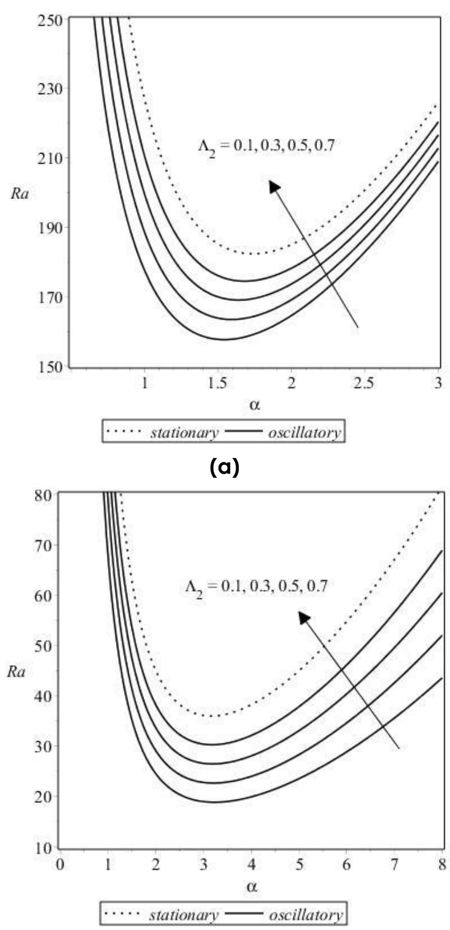


Figure 2 Neutral stability curves of the oscillatory and stationary convection (a) with magnetic field $(Q=1)$ and (b) without magnetic field $(Q=0)$ for various values of scaled stress relaxation parameter, Λ_1 on the thermal Darcy-Rayleigh number, Ra as function of α when $N_A = 2, Rn = 1, \Lambda_2 = 0.1, Le = 1$ and $M = 1$

It is observed that, an increase in the value of the scaled stress relaxation parameter decreases the thermal Darcy-Rayleigh number. Therefore, the effect of the relaxation parameter is to advance or promote the onset of convection in a viscoelastic nanofluid porous layer and the system is more unstable. The critical wave number for both system (with and without magnetic field) decreases as Λ_1 decreases. The minimum value of α known as the critical wave number denoted by α_c is 1.51 for the system with magnetic field and 3.10 for the system without magnetic field when $\Lambda_1 = 0.9$. The critical thermal Darcy-Rayleigh number is at 19.20 for the convection without magnetic field and 156.26 for the convection with magnetic field. Therefore, the system in the presence of magnetic field is more stable than the system in the absence of magnetic field.



(b)

Figure 3 Neutral stability curves of the oscillatory and stationary convection (a) with magnetic field $(Q = 1)$ and (b) without magnetic field $(Q = 0)$ for various values of scaled strain retardation parameter, Λ_2 on the thermal Darcy-Rayleigh number, Ra as function of α when $N_A = 2, Rn = 1, \Lambda_1 = 1, Le = 1$ and $M = 1$

The effect of the scaled strain retardation parameter is shown in Figure 3. It was shown that as the scaled strain retardation parameter increases the oscillatory thermal Darcy-Rayleigh number increases. The scaled strain retardation parameter delays the onset of convection in a viscoelastic nanofluid porous layer thus stabilizes the system. The critical wave number for both system (with or without magnetic field) decreases as Λ_2 increases. $\alpha_c = 1.479$ for the system with magnetic field and $\alpha_c = 3.10$ for the system without magnetic field when $\Lambda_2 = 0.1$. The critical thermal Darcy-Rayleigh number is at 17.10 for the convection without magnetic field and 152.73 for the convection with magnetic field. This shows that the system in the presence of magnetic field is more stable than the system in the absence magnetic field.

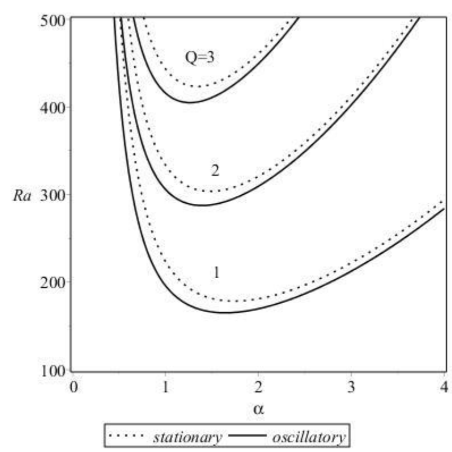


Figure 4 Neutral stability curves of the oscillatory and stationary convection for different values of Chandrasekhar number, Q on the thermal Darcy-Rayleigh number, Ra as function of α when $Le = 1, Rn = 2, N_A = 2, \Lambda_1 = 1, \Lambda_2 = 0.5$ and $M = 1$

The neutral stability curve is displayed for various values of Chandrasekhar number, Q in Figure 4. The critical thermal Darcy-Rayleigh number increases for both oscillatory and stationary convection with the increases of Chandrasekhar number.

Figure 5 illustrates the value of the critical thermal Darcy-Rayleigh number for the oscillatory convection as functions of (a) Λ_1, (b) Λ_2 for several values of Chandrasekhar number. As the value of Λ_1 increases, the critical thermal Darcy-Rayleigh number decreases while as the value of Λ_2 increases, the critical thermal Darcy-Rayleigh number increases. Thus, stress relaxation parameter promotes the onset of oscillatory convection while strain retardation parameter delays the onset of convection.

4.0 CONCLUSION

The influences of the magnetic field on the neutral convection in viscoelastic nanofluids were studied analytically. The system of conservation equations was solved using the linear stability theory and method of weighted residuals. The influences of physical parameters, particularly, the scaled stress relaxation parameter, scaled strain retardation parameter and the Chandrasekhar number were examined. The effect of the scaled stress relaxation parameter advances the onset of oscillatory convection while the effect of scaled strain retardation parameter delays the onset of oscillatory convection. The stationary instability is independent of the scaled stress relaxation and strain retardation parameters. The magnetic field delays both oscillatory and stationary convection. The system in the presence of magnetic field is more stable.

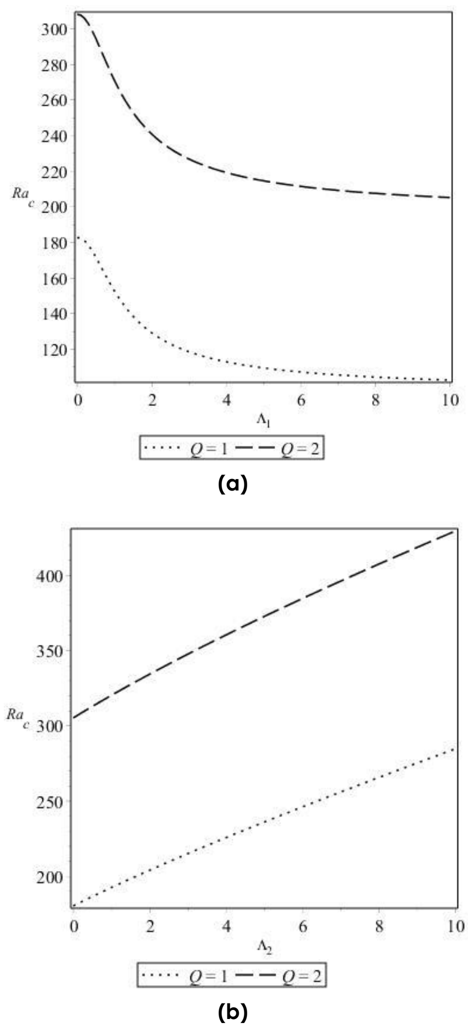


Figure 5 The critical thermal Darcy-Rayleigh number as functions of (a) Λ_1 and (b) Λ_2 on the oscillatory convection for several values of Q

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