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SPREADSHEET SOLUTIONS TO LAPLACE'S EQUATION: SEEPAGE AND FLOW NET

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Abstract. There has been an increasing interest in applying the electronic spreadsheets, traditionally used for the financial and accounting purposes, to solve complex engineering problems. This paper aims to describe the application of spreadsheets to solve the finite difference to Laplace's equation. The simple form of finite difference method also known as the relaxation method will be used for seepage analysis. The spreadsheet program used for this study was chosen based on the power, flexibility and the versatility of this package in various computing environments. Nevertheless any other spreadsheet programs can also perform the same task with varying levels of user friendliness.

Keywords: Spreadsheet modelling, flow net, seepage, finite difference method.

1.0 INTRODUCTION

There has been a tremendous growth in the development of powerful computer programs (written in various computer languages) which can be used to solve complex, tiresome and time consuming problems in civil engineering, water resources and environmental engineering. However, the use of these special programs, which are normally capable of solving specific tasks they were explicitly designed for, will be forced to the background, as a more versatile, flexible and powerful programs emerged.

The current trend is through the application of electronic spreadsheets such as Microsoft Excel[1] to solve these routine engineering calculations. The author will only discuss the application of Microsoft Excel to solve the finite difference method for seepage. Other spreadsheet programs such as Lotus 123[2] and Quattro Pro[3] may also be used to perform the same task. Olsthoorn [1] and Kleiner [2] demonstrated the application of an electronic spreadsheet to solve ground water and seepage problems. Hart [3] provided various applications of spreadsheet solutions of Laplace's equation to boundary value problems. He also compare the finite-difference solutions to Laplace's equation using the spreadsheet to the corresponding analytical solutions. They were in good agreement provided first, an appropriate number of iterations were used and second, the ratio of spreadsheet array size to the characteristic dimensions of the problem domain were adopted. Kelly and Bogardi [4] extended the use of spreadsheet to determine the flow direction. The flow direction was

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determined using the least-squares fitting of plane surfaces to water-level data utilizing the spreadsheet; the method can be easily applied to two- or three-dimensional flow. Dexter and Avery [5] utilized the spreadsheet program for water-balance technique to model surface hydrology. The selection of spreadsheet program in computer-based water-balance modelling is based on several advantages attributed to it's application. First, the compilation of code is much more simplified, as opposed to the conventional computer programs, thus offering a more streamlined programming. Second, the spreadsheets provide rapid iteration and calculation of all affected dependent variables as the independent variables are changed.

Interestingly Isakowitz et al., [6] show:

> *"Spreadsheet modelling represents one of the most pervasive and successful applications of personal computers. Since their introduction in 1970s, spreadsheet programs transformed the notion of end-user computing, creating a new computation paradigm that offers a unique combination of ease of use, on the one hand, and unprecedented modelling power, on the other".*

modelling with spreadsheets will remain as a unique approach which will continue to offer flexibility in computing environment. It's application will be further enhanced with the development of powerful spreadsheet programs or software with sophisticated graphing capability.

This paper will provide basic formulations followed by some procedures in adopting the spreadsheet modelling technique to solve seepage problem.

2.0 FINITE DIFFERENCE METHOD

The basic concept in the finite difference procedure is based on replacing continuous derivatives in equations governing the physical problem by the ratio of changes in the variables over a small but finite increment. As a result of these substitutions, the differential equation are transformed into the finite difference approximation, Strack [7].

Finite difference method is based on the solution of a continuous conceptual model with known properties at the boundary points. This continuous model will be replaced by a set of discrete points in a grid pattern with nodal points. The equation at each node will be solved to obtain the unknown value, such as potential or head, at that particular node. The basic grid either block-centered or mesh-centered will be regularly spaced with the different between nodes in the x direction, Δx, equal to that in the y direction, Δy.

2.1 Boundary Condition

The boundary condition has to be specified to solve the flow equations. The two basic types of boundaries are the Dirichlet boundary condition and the Neumann boundary condition. The condition for a Dirichlet boundary is when the head or potential is known along the entire boundary of the flow region, i.e. at all the boundary nodes. Meanwhile the Neumann condition refers to the condition where a flux across the boundary of flow region is known. These boundary conditions for a regional aquifer were clearly illustrated by Fetter [8]. Fetter suggested that the position of water table will be fixed for steady state of flow. This position will be the upper boundary and will be treated as the Dirichlet boundary. The boundary across which no flow takes place will be the Neumann boundary condition that gives $\delta h/\delta x$ or $\delta h/\delta y = 0$. Examples of these boundaries are the no flow boundary along the impermeable layer, vertical boundary of sheet pile wall or a cut-off wall and the no flow boundary of a ground water divide.

3.0 FORMULATION

The various steps in the formulation are summarized in Table 1.

3.1 Darcy's Law

Based on Darcy's Law [9],

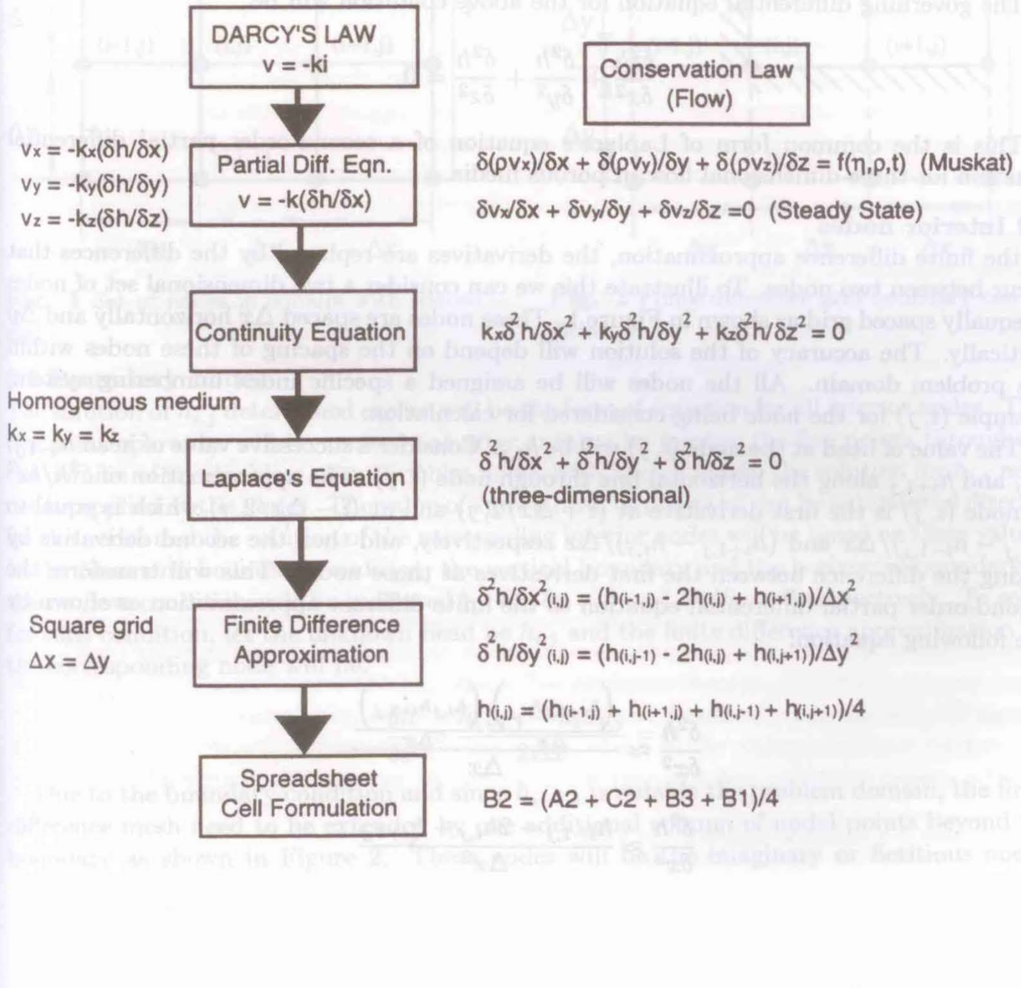
$$Q = -kiAT \qquad q = \frac{Q}{t} = -kiA$$

where Q and q are the total flow and the flow rate respectively; k is the permeability or hydraulic conductivity, i is the hydraulic gradient, $\delta h/\delta l$ (the head drop $h_2 - h_1$ occurs over the interval $l_2 - l_1$), and let the discharge velocity, $V = \frac{q}{A}$, then we will have Darcy's equation in a general differential form where:

$$v = -ki$$

$$v = -k\left(\frac{\delta h}{\delta l}\right)$$

Table 1 The flow of formulation for spreadsheet program


DARCY'S LAW
$v = -ki$
Conservation Law
(Flow)
$v_x = -k_x(\delta h/\delta x)$
$v_y = -k_y(\delta h/\delta y)$
$v_z = -k_z(\delta h/\delta z)$
Partial Diff. Eqn.
$v = -k(\delta h/\delta x)$
$\delta(\rho v_x)/\delta x + \delta(\rho v_y)/\delta y + \delta(\rho v_z)/\delta z = f(\eta,\rho,t)$ (Muskat)
$\delta v_x/\delta x + \delta v_y/\delta y + \delta v_z/\delta z = 0$ (Steady State)
Continuity Equation
$k_x\delta^2 h/\delta x^2 + k_y\delta^2 h/\delta y^2 + k_z\delta^2 h/\delta z^2 = 0$
Homogenous medium
$k_x = k_y = k_z$
Laplace's Equation
$\delta^2 h/\delta x^2 + \delta^2 h/\delta y^2 + \delta^2 h/\delta z^2 = 0$
(three-dimensional)
$\delta^2 h/\delta x^2_{(i,j)} = (h_{(i-1,j)} - 2h_{(i,j)} + h_{(i+1,j)})/\Delta x^2$
Square grid
$\Delta x = \Delta y$
Finite Difference
Approximation
$\delta^2 h/\delta y^2_{(i,j)} = (h_{(i,j-1)} - 2h_{(i,j)} + h_{(i,j+1)})/\Delta y^2$
$h_{(i,j)} = (h_{(i-1,j)} + h_{(i+1,j)} + h_{(i,j-1)} + h_{(i,j+1)})/4$
Spreadsheet
Cell Formulation
B2 = (A2 + C2 + B3 + B1)/4

The equation will also be true for x, y and z components in three-dimensional analysis.

$$v_x = -k_x\left(\frac{\delta h}{\delta x}\right) \qquad v_y = -k_y\left(\frac{\delta h}{\delta y}\right) \qquad v_z = -k_z\left(\frac{\delta h}{\delta z}\right)$$

where v_x, v_y and v_z are discharge velocities in the directions x, y and z respectively. (Cartesian coordinate).

Assuming that water is incompressible and no water is added to or removed from the system; the continuity equation for the steady state condition will then be equal to:

$$\delta\left(\frac{v_x}{\delta x}\right) + \delta\left(\frac{v_y}{\delta y}\right) + \delta\left(\frac{v_z}{\delta z}\right) = 0$$

By combining Darcy's 3-dimensional equation and the continuity equation we obtain:

$$\frac{\delta}{\delta_x}\left(-k_x\frac{\delta h}{\delta x}\right) + \frac{\delta}{\delta_y}\left(-k_y\frac{\delta h}{\delta y}\right) + \frac{\delta}{\delta_z}\left(-k_z\frac{\delta h}{\delta z}\right) = 0$$

This paper will only focus on the case where k is independent of x, y and z components; which is the case of flow through isotropic and homogeneous medium under steady-state condition.

The governing differential equation for the above condition will be:

$$\frac{\delta^2 h}{\delta x^2} + \frac{\delta^2 h}{\delta y^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

This is the common form of Laplace's equation of a second-order partial differential equation for three-dimensional flow in porous media.

3.2 Interior nodes

In the finite difference approximation, the derivatives are replaced by the differences that occur between two nodes. To illustrate this we can consider a two dimensional set of nodes at equally spaced grid as shown in Figure 1. These nodes are spaced Δx horizontally and Δy vertically. The accuracy of the solution will depend on the spacing of these nodes within the problem domain. All the nodes will be assigned a specific index numbering system, example (i, j) for the node being considered for calculation.

The value of head at the node (i, j) will be $h_{i,j}$. Consider a successive value of head $h_{i-1,j}$, $h_{i,j}$ and $h_{i+1,j}$ along the horizontal line through node (i, j). The approximation of $\delta^2 h/\delta x^2$ at node (i, j) is the first derivative at $(i + \Delta x/2, j)$ and at $(i - \Delta x/2, j)$ which is equal to $(h_{i,j} - h_{i-1,j})/\Delta x$ and $(h_{i+1,j} - h_{i,j})/\Delta x$ respectively, and then the second derivative by taking the difference between the first derivatives at those nodes. This will transform the second-order partial differential equation to the finite different approximation as shown by the following equation:

$$\frac{\delta^2 h}{\delta x^2} \approx \frac{\frac{\left(h_{i+1,j}h_{i,j}\right)}{\Delta x}\frac{\left(h_{i,j}h_{i-1,j}\right)}{\Delta x}}{\Delta x}$$

$$\frac{\delta^2 h}{\delta x^2} \approx \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{\Delta x^2}$$

and similarly

$$\frac{\delta^2 h}{\delta y^2} \approx \frac{h_{i,j-1} - 2_{i,j} + h_{i,j+1}}{\Delta y^2}$$

Based on the assumption that the elements in the grids are equally spaced, such that $\Delta x = \Delta y$, and for two dimensional analysis in x and y directions, the value of the head at node (i, j) at any point in the flow domain can be determined by taking the average value of head computed from the four surrounding nodes and can be expressed as:

$$h_{i,j} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4} \qquad (1.0)$$

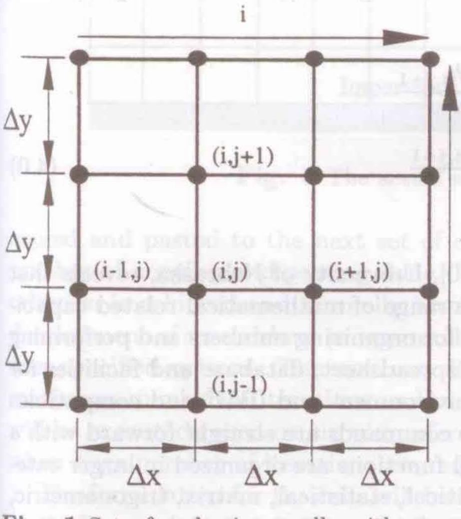


Fig. 1 Set of nodes in equally with spaced grid

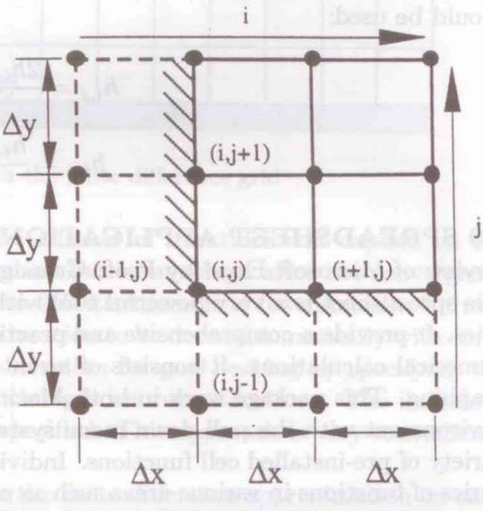


Fig. 2 Finite difference grid boundary condition

3.3 Boundary nodes

The solution of $h_{i,j}$ determined earlier will be the form of equation for all interior nodes. This five point operators will be created one after another by moving the five points throughout the domain of the problem. For Dirichlet nodes with known heads, the solution for $h_{i,j}$ need not be applied at the nodes. The value of known head or potential can be substituted directly for the nodes and the solution of the surrounding interior nodes will be based on these values. In the Neumann boundary condition, the vertical boundary and the horizontal boundary of the no flow condition will be indicated by $\delta h/\delta x = 0$ and $\delta h/\delta y = 0$ respectively. To solve for such condition, let the unknown head be $h_{i,j}$ and the finite difference approximation for the corresponding node will be:

$$\frac{\delta h}{\delta x} \approx \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta x} = 0$$

Due to the boundary condition and since $h_{i-1,j}$ is outside the problem domain, the finite difference mesh need to be extended by one additional column of nodal points beyond the boundary as shown in Figure 2. These nodes will be the imaginary or fictitious nodes.

In finite difference form, the condition of $\delta h/\delta x = 0$ denotes that $h_{i+1,j} = h_{i-1,j}$. From Equation 1.0, the general form for vertical no flow boundary node will be:

$$h_{i,j} = \frac{2h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4} \tag{2.0}$$

For a horizontal no flow boundary as in the impermeable boundary, $h_{i,j-i} = h_{i,j+1}$ for $\delta h/\delta y = 0$; and the equation will be:

$$h_{i,j} = \frac{h_{i-1,j} + h_{i+1,j} + 2h_{i,j+1}}{4} \tag{3.0}$$

For the combination of both a vertical and a horizontal boundaries such as the intersection of impervious boundaries (refer to node $(i.j)$ shown in Figure 2), the following equation should be used:

$$h_{i,j} = \frac{2h_{i+1,j} + 2h_{i,j+1}}{4}$$
$$h_{i,j} = \frac{h_{i+1,j} + h_{i,j+1}}{2} \tag{4.0}$$

4.0 SPREADSHEET APPLICATION

Review of Microsoft Excel by Prof. Grandgenett [10], University of Nebraska, reveals that this spreadsheet is a very powerful tool with a wide range of mathematical related capabilities. It provide a comprehensive and practical tool for organizing numbers and performing numerical calculations. It consists of an integrated spreadsheet, database and facilities for graphing. This package work in both Macintosh[4] environment and IBM[5] and compatibles environment with the pull down menu system. The commands are straight forward with a variety of pre-installed cell functions. Individual cell functions are organized in larger categories of functions in various areas such as mathematical, statistical, matrix, trigonometric, logical, financial and special function. The above is also true for most of the spreadsheet programs in the market today. Microsoft Excel for Macintosh is used in this paper, nevertheless any other spreadsheet programs can also be used to perform the same task with varying levels of user friendliness.

For the finite difference application a template will has to be created to include the problem domain and all applicable boundaries. The layout of the boundaries such as the sheet pile, the base of the dam and the impervious boundary with limitation to only straight horizontal and vertical lines, can be included in the template. The finite difference grid with the value of potential or head at each node can also be incorporated in the template based on the actual setup of the problem domain as shown in Figure 3.

The first template is usually in the form of unformulated cells with the appropriate known heads values at the Dirichlet boundary nodes. All unknown values at the other nodes will be set to zero. Only the second template will consist of the formulation of the cell. The formula for the cell will be based on the finite difference approximation discussed earlier with respect to the location of the node in the problem domain. The iteration will start with the second template and the next iteration will be executed as the template is being

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[5] IBM is a registered trademark of International Business Machines Corporation

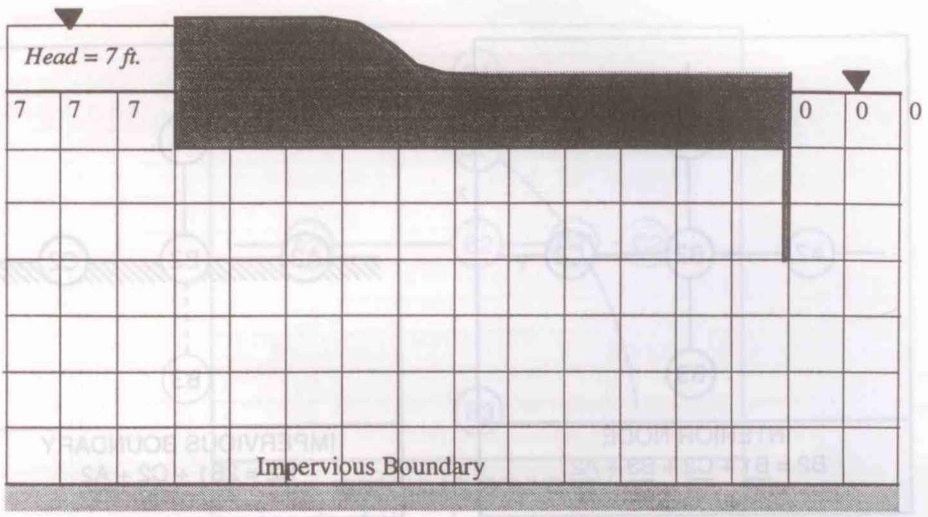


Fig. 3 The actual setup with the finite difference grid

copied and pasted to the next set of cells. The number of iterations will depend on the number of copying and pasting done. The equation has been solved after the convergence criterion is achieved and normally involved 50 to 250 iterations in a few minutes. Current development of spreadsheet programs allow the iteration to be done automatically with some user specified options. The user may limit the iteration for goal seeking by specifying the maximum number of iterations and maximum changes between iterations. The iterations will be executed based on circular reference as specified. This approach is very convenience when huge number of iterations are required.

The formulation for the cells will be based on the finite difference approximation on each node. Equations 1.0, 2.0, 3.0 and 4.0 will be transformed into the cell formulations as shown below:

If B2 represent the required potential or head value, then:

i) Equation 1.0 will be; $B2 = (A2 + C2 + B3 + B1)/4$
for all intermediate nodes.

ii) Equation 2.0 will be; $B2 = (2 * C2 + B3 + B1)/4$
for vertical boundaries.

iii) Equation 3.0 will be; $B2 = A2 + C2 + 2 * B1)/4$
for horizontal no flow boundaries such as the impermeable bed.

iv) Equation 4.0 will be; $B2 = (C2 + B1)/2$
for the intersection between two Neumann boundaries.

Figure 4 summarize the cell formulation for various nodal points in the flow domain, which include the interior nodes, nodes on impervious boundary, nodes on intersection of two impervious boundaries and nodes on interface of deferring permeability. Figure 5 shows the cell formulation for irregular boundaries such as the phreatic surface, recharge or discharge face, Kleiner [2].

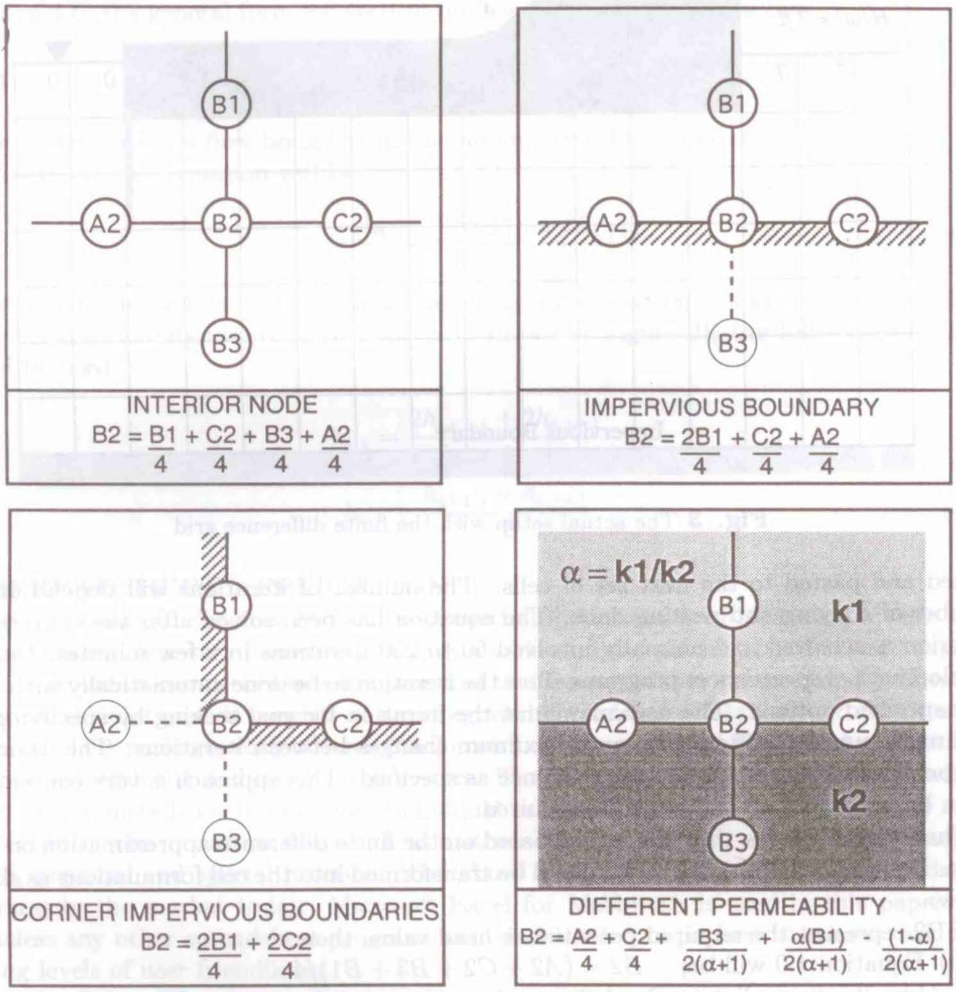


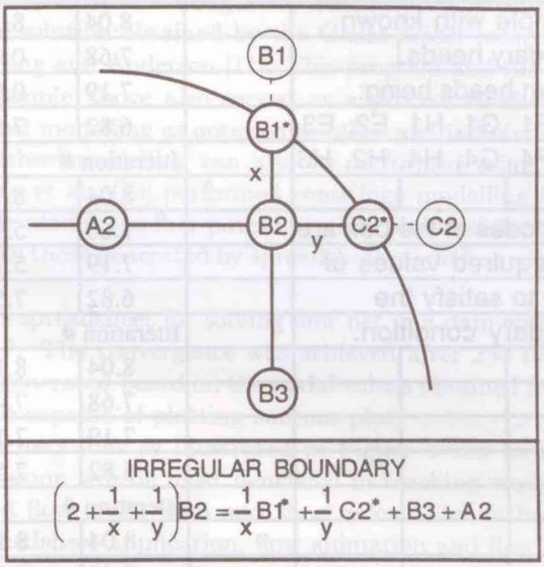
Fig. 4 Summary of cell formulation for various type of boundary

4.1 Over-relaxation

Sometime a large number of iterations are necessary to obtain the convergence, thus requires vast amount of time. The number of iterations may be reduced considerably by using over-relaxation, (Olsthoorn [1], Hart [3]). Over-relaxation relates the value for the potential at a node after an iteration to the potentials of its neighbouring nodes and its value from the preceding iteration. Let $h^*_{i,j}$ be the new value after one iteration and the corresponding change in the values be Δh.

Therefore, for all interior nodes, (from Equation 1):

$$h^*_{i,j} = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4}$$


B1
B1*
x
A2
B2
y
C2*
C2
B3
IRREGULAR BOUNDARY
$\left(2+\frac{1}{x}+\frac{1}{y}\right)B2=\frac{1}{x}B1^{*}+\frac{1}{y}C2^{*}+B3+A2$

Cell formulation for irregular boundary, phreatic/free surface, entrance face or discharge face.

Fig. 5 The formulation for irregular boundary

$$\Delta h = h^*_{i,j} - h_{i,j}$$

$$h = \frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4} - h_{i,j}$$

For the preceding iteration $h^*_{i,j} = h_{i,j} + \Delta h$, but for over-relaxation the increament $\alpha\Delta h$ will be used instead of Δh, therefore $h^*_{i,j} = h_{i,j} + \alpha\Delta h$; where α is the over-relaxation coefficient, $(1 < \alpha < 2)$, and a typical value between 1.5 – 1.7 is capable of decreasing the number of iterations required for convergence by an order of magnitude.

The new value for the node will be:

$$h^*_{i,j} = (1-\alpha)h_{i,j} + \frac{\alpha}{4}\Big(h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}\Big)$$

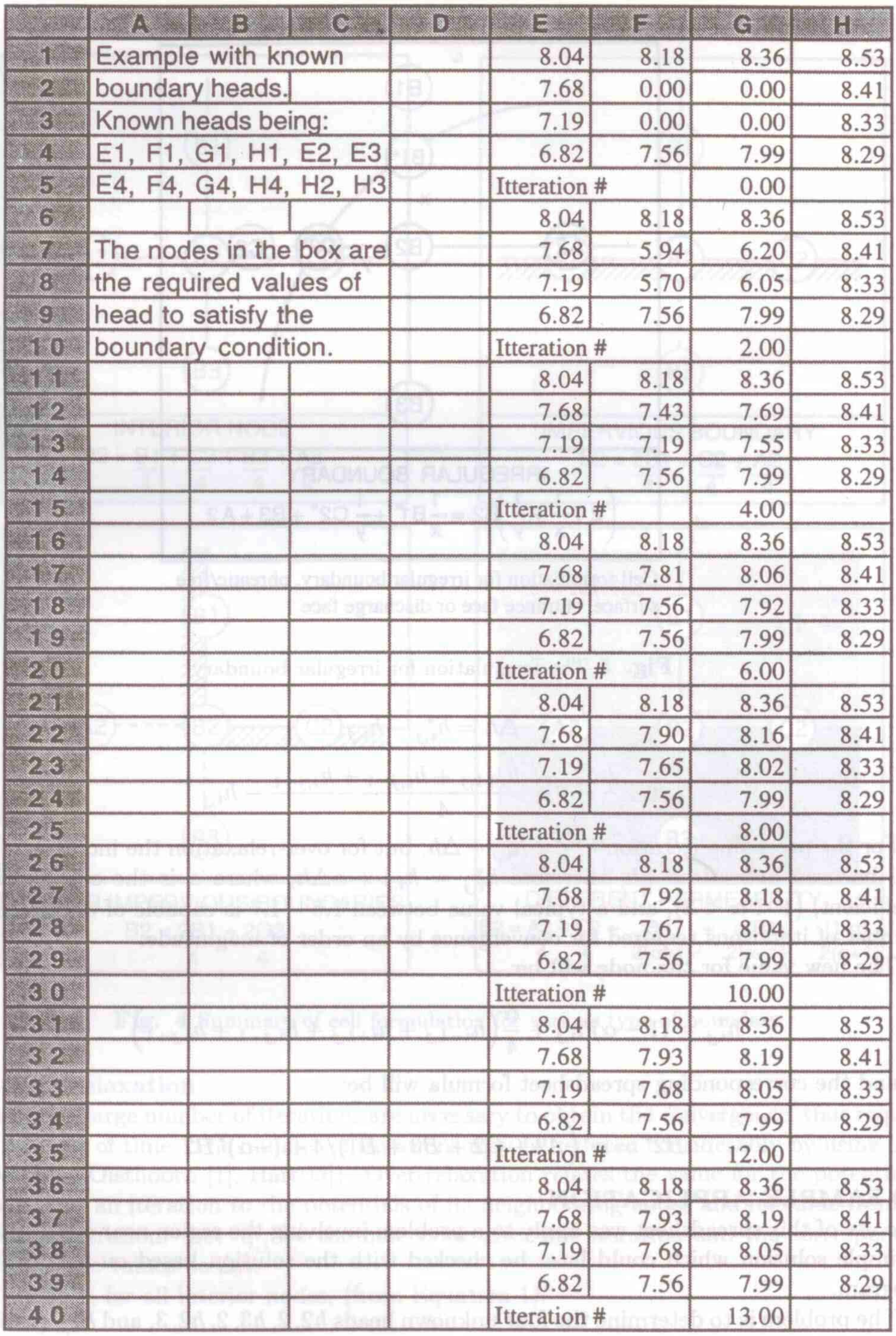
and the corresponding spreadsheet formula will be:

$$B2^* = \alpha^*\Big(A2 + C2 + B3 + B1)/4 + (-\alpha)^* B2$$

5.0 SAMPLE APPLICATION

The use of this spreadsheet was apply to a problem involving the region near a well to show a simple solution which could later be checked with the solution based on a FORTRAN program.

The problem is to determine the four unknown heads $h2, 2, h3, 2, h2, 3$, and $h_{3,3}$ (as shown in Figure A1), knowing the boundary head around them. A series of observation wells were located around the problem domain and the water level measured in all the wells will be the



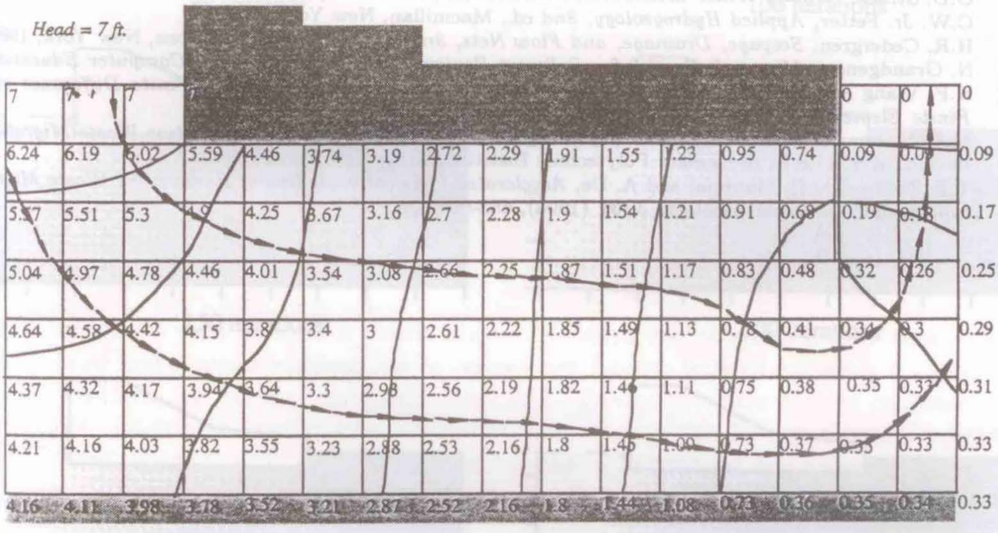
	A	B	C	D	E	F	G	H
1	Example with known				8.04	8.18	8.36	8.53
2	boundary heads.				7.68	0.00	0.00	8.41
3	Known heads being:				7.19	0.00	0.00	8.33
4	E1, F1, G1, H1, E2, E3				6.82	7.56	7.99	8.29
5	E4, F4, G4, H4, H2, H3				Itteration #		0.00	
6					8.04	8.18	8.36	8.53
7	The nodes in the box are				7.68	5.94	6.20	8.41
8	the required values of				7.19	5.70	6.05	8.33
9	head to satisfy the				6.82	7.56	7.99	8.29
10	boundary condition.				Itteration #		2.00	
11					8.04	8.18	8.36	8.53
12					7.68	7.43	7.69	8.41
13					7.19	7.19	7.55	8.33
14					6.82	7.56	7.99	8.29
15					Itteration #		4.00	
16					8.04	8.18	8.36	8.53
17					7.68	7.81	8.06	8.41
18					7.19	7.56	7.92	8.33
19					6.82	7.56	7.99	8.29
20					Itteration #		6.00	
21					8.04	8.18	8.36	8.53
22					7.68	7.90	8.16	8.41
23					7.19	7.65	8.02	8.33
24					6.82	7.56	7.99	8.29
25					Itteration #		8.00	
26					8.04	8.18	8.36	8.53
27					7.68	7.92	8.18	8.41
28					7.19	7.67	8.04	8.33
29					6.82	7.56	7.99	8.29
30					Itteration #		10.00	
31					8.04	8.18	8.36	8.53
32					7.68	7.93	8.19	8.41
33					7.19	7.68	8.05	8.33
34					6.82	7.56	7.99	8.29
35					Itteration #		12.00	
36					8.04	8.18	8.36	8.53
37					7.68	7.93	8.19	8.41
38					7.19	7.68	8.05	8.33
39					6.82	7.56	7.99	8.29
40					Itteration #		13.00	

Fig. 6 Spreadsheet solution for the region-near-a-well example

known boundary heads. The solution in Figure 6, which show the convergence in the 12th iteration checks with the solution obtained by the Gauss-Seidel computer program in the literature provided by Wang and Anderson [11]. This program and the solution is included in Appendix A. The example above also served as a process to check the validity and applicability of spreadsheet modelling as opposed to other method of numerical computing. The validation of spreadsheet modelling can also be performed using physical modelling. Mahmud [12] and Zimmie et al. [13] performed centrifuge modelling for two-dimensional seepage flow in small scale model; the flow patent visually observed in the centrifuge models are in good agreement with those generated by spreadsheet modelling performed on the same flow domain.

The application of this spreadsheet for solving flow net in a dam with a sheet pile cutoff wall is shown in Figure 7. The convergence was achieved after 250 iterations in about 4 minutes. The flow net is generated based on the nodal values obtained from the spreadsheet iteration, using a software capable of plotting contour plot.

Snap shots at various iterations as illustrated in Figure 8 can be utilized to animate the flow. Such flow animation will be most beneficial in teaching students about flow net construction, seepage and flow under dam and sheet piles. The author have developed a package dealing with spreadsheet application, flow animation and flow net construction as a 3 hours session for a graduate course, Seepage and Drainage, offered at Rensselaer.



7	7	7	7											0	0	0
6.24	6.19	6.02	5.59	4.46	3.74	3.19	2.72	2.29	1.91	1.55	1.23	0.95	0.74	0.09	0.09	0.09
5.57	5.51	5.3	4.9	4.25	3.67	3.16	2.7	2.28	1.9	1.54	1.21	0.91	0.68	0.19	0.18	0.17
5.04	4.97	4.78	4.46	4.01	3.54	3.08	2.66	2.25	1.87	1.51	1.17	0.83	0.48	0.32	0.26	0.25
4.64	4.58	4.42	4.15	3.8	3.4	3	2.61	2.22	1.85	1.49	1.13	0.78	0.41	0.34	0.3	0.29
4.37	4.32	4.17	3.94	3.64	3.3	2.98	2.56	2.19	1.82	1.46	1.11	0.75	0.38	0.35	0.33	0.31
4.21	4.16	4.03	3.82	3.55	3.23	2.88	2.53	2.16	1.8	1.45	1.09	0.73	0.37	0.35	0.33	0.33
4.16	4.11	3.98	3.78	3.52	3.21	2.87	2.52	2.16	1.8	1.44	1.08	0.73	0.36	0.35	0.34	0.33

ITTERATION No. 250

Fig. 7 Application of spreadsheet programming for a dam with cutoff wall

6.0 CONCLUSION

In conclusion, the electronic spreadsheet can be a powerful tool to solve engineering problems such as the application for finite difference analysis presented herein. The ease, versatility and the user friendliness of this package will further encourage its application in a wider and unlimited engineering applications. modelling with spreadsheet will remain as a unique approach which will continue to offer flexibility in computing environment. It's application will be further enhanced with the development of powerful spreadsheet programs or software with sophisticated graphing capability. Spreadsheet application also provide an opportunity to be utilized as a great teaching tool, such as the visualization of fluid flow in porous media as presented in this paper. There is great potential that the application of spreadsheets will be able to be extended to other engineering applications; solving rather complex engineering problems. This paper maybe able to generate more interest in using spreadsheet for various engineering applications apart from it's traditional use for financial and accounting purposes.

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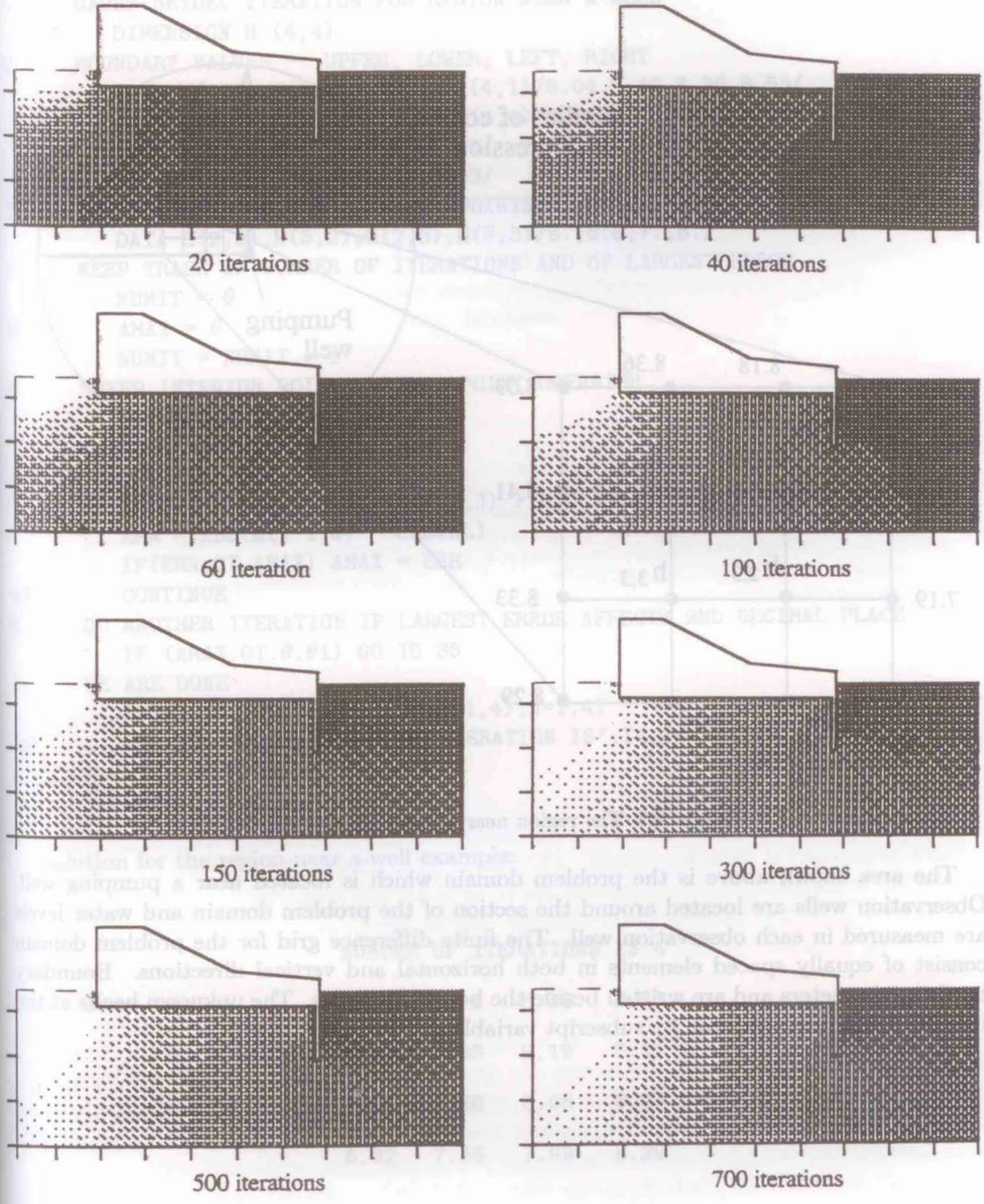

20 iterations

40 iterations

60 iteration

100 iterations

150 iterations

300 iterations

500 iterations

700 iterations

Fig. 8 Snap shots at various stages of iteration

APPENDIX A

Example with Dirichlet Boundary Conditions: Region-near-a-well [11]

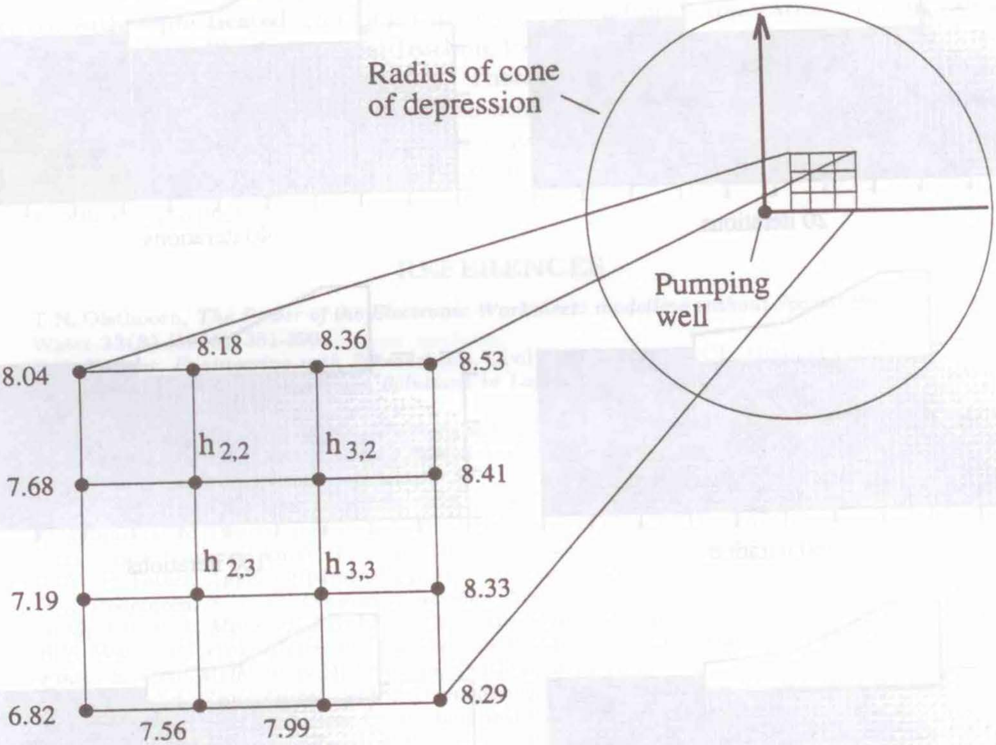

Fig. A1 The region near a pumping well [11]

The area shown above is the problem domain which is located near a pumping well. Observation wells are located around the section of the problem domain and water levels are measured in each observation well. The finite difference grid for the problem domain consist of equally spaced elements in both horizontal and vertical directions. Boundary heads are in meters and are written beside the boundary nodes. The unknown heads at the interior points are indicated by subscript variables.

Computer program using Gauss-Seidel iteration for the region-near-a-well [11]

```
C     GAUSS-SEIDEL ITERATION FOR REGION NEAR A WELL
         DIMENSION H (4,4)
C     BOUNDARY VALUES -- UPPER, LOWER, LEFT, RIGHT
         DATA H(1,1),H(2,1),H(3,1),H(4,1)/8.04,8.18,8.36,8.53/
         DATA H(1,4),H(2,4),H(3,4),H(4,4)/6.82,7.56,7.99,8.29/
         DATA H(1,2),H(1,3)/7.68,7.19/
         DATA H(4,2),H(4,3)/8.41,8.33/
C     INITIAL GUESSES FOR INTERIOR POINTS
         DATA H(2,2),H(3,2),H(2,3),H(3,3)/8.,8.5,7.,8./
C     KEEP TRACK OF NUMBER OF ITERATIONS AND OF LARGEST ERROR
         NUMIT = θ
35       AMAX = θ
         NUMIT = NUMIT + 1
C     SWEEP INTERIOR POINTS WITH 5-POINT OPERATOR
         DO 4θ J=2,3
         DO 4θ I=2,3
         OLDVAL = H(I,J)
         H(I,J) = (H(I-1,J) + H(I+1,J) + H(I,J-1) + H(I,J+1))/4
         ERR = ABS(H(I-1,J) - OLDVAL)
         IF(ERR.GT.AMAX) AMAX = ERR
4θ       CONTINUE
C     DO ANOTHER ITERATION IF LARGEST ERROR AFFECTS 2ND DECIMAL PLACE
         IF (AMAX GT.θ.θ1) GO TO 35
C     WE ARE DONE
         PRINT 5θ,NUMIT,((H(I,J),I=1,4),J=1,4)
50       FORMAT (///1X,NUMBER OF ITERATION IS',I4,///4(4F10.2//))
         STOP
         END
```

Solution for the region-near-a-well example:

```
        NUMBER OF ITERATIONS IS 4

          8.04    8.18    8.36    8.53

          7.68    7.93    8.19    8.41

          7.19    7.68    8.05    8.33

          6.82    7.56    7.99    8.29
```