Jurnal Teknologi, bil. 26, Jun 1997 hlm. 41–56

CLOSED-FORM EXPRESSIONS OF WAVE INDUCED FORCES ON MEMBERS OF OFFSHORE STRUCTURES IN DEEP WATER

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Abstract. The paper deals with the estimation of equivalent nodal forces and moments which can be useful in the analysis of beam element based Finite Element (FE) models of offshore structures. Ocean wave forces do not follow any standard pattern, thus, if the global X-Y reference plane is horizontal, the X and Y co-ordinates will not vary along the length of a vertical beam element. Taking advantage of such a situation, a number of closed form expressions for equivalent nodal loads can be derived. The analytical diffraction force from the MacCamy and Fuchs' theory as well as the Morison equation based inertia and drag forces are considered here as wave loading. A step-by-step calculation procedure is also proposed which transfers complex member loads to the nodes of a FE model with beam elements, arbitrarily oriented in space.

1 INTRODUCTION

There are various books available, even from the fifties, which deal with framed structures, made up of many elements. In many cases attention is appropriately paid to formulate various matrices relating to framed structures, but when modelling external loads, discussions are often limited to concentrated joint loads and uniformly distributed loads. In the case of other types of member loads, the previous trend was to use engineering handbooks. Even today, besides concentrated and uniformly distributed loads, only a few standard types can be handled adequately by general purpose finite element software. In this study, it will be shown that a few sets of equations can be derived through algebraic manipulations from member loads of a very general nature acting on three-dimensional beams and the derived equations can be programmed easily to be used as a 'black box' later on.

This paper concentrates on water wave forces acting on offshore structures which can be adequately modelled as space frames for Finite Element calculations. The closed-form expressions derived are certainly capable of minimising computational time since equivalent nodal forces can be immediately calculated in the case of a large element instead of subdividing the element and then summing up individual contributions from each subdivision. In addition, the closed-form expressions can be very helpful to anyone who does not want to get into the complexities in understanding the interaction of offshore structures with the environmental loading.

2 CONVENTIONAL FINITE ELEMENT ANALYSIS

For an idealised space frame model representing an offshore structure for finite element (FE) analysis, the dynamic equation of motion of the system:

$$[M]\{\ddot{X}\} + [B]\{\dot{X}\} + [K]\{X\} = \{W(t)\} \tag{1}$$

Element in the load matrix, $\{W(t)\}$ may be the sum of two types of nodal load: (a) applied nodal load and (b) equivalent nodal load. Any load applied externally to a joint or a 'node' of a structure is classified as an applied nodal load. If the load is applied on a structural member, but not on its FE nodes, the equivalent nodal loads are to be calculated. The direct stiffness method of analysis assumes all nodes restrained and the reactions developed at the nodes are called fixed end moments and fixed end shears. The node actions which result from the loading of the members are equal in magnitude and opposite in direction to these fixed end moments and shears. In other words, a fixed end moment or shear can be transformed into an equivalent nodal load by simply reversing its sign. The fixed end actions for common loading conditions of a prismatic member and a few loading conditions of particular non-prismatic beam elements can be found in available engineering handbooks. In this paper, the Bernoulli-Euler theory is used to obtain closed-form expressions of equivalent nodal loads where the structural members are subjected to complicated wave loading.

3 SHAPE FUNCTIONS OF BERNOULLI-EULER BEAMS

The shape functions of a Bernoulli-Euler beam are reproduced here since they are used extensively in the next sections. In Fig. 1, the transverse deflections of a uniform beam element of length L, mass density ρ_s, elastic modulus E, cross-sectional area A and moment of inertia I, are shown. The displacement function is assumed as the product of nodal displacements and shape functions $\psi_i(x)$;

$$v(x,t) = \sum_{i=1}^{4} \psi_i(x) v_i(t) \tag{2}$$

The displacement function for a uniform beam is a cubic polynomial. The following shape functions are obtained from the boundary conditions;

$$\psi_1 = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3 \tag{3}$$

$$\psi_2 = x - 2L\left(\frac{x}{L}\right)^2 + L\left(\frac{x}{L}\right)^3 \tag{4}$$

$$\psi_3 = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3 \tag{5}$$

$$\psi_4 = -L\left(\frac{x}{L}\right)^2 + L\left(\frac{x}{L}\right)^3 \tag{6}$$

4 MACCAMY AND FUCHS' DIFFRACTION FORCE

A significant practical advantage of the MacCamy and Fuchs' solution [1] is that, while it accounts for the diffraction effects in an analytical form, it can be applied for any ratio of wave length to column diameter [2]. Although the solution is valid for a circular cylinder

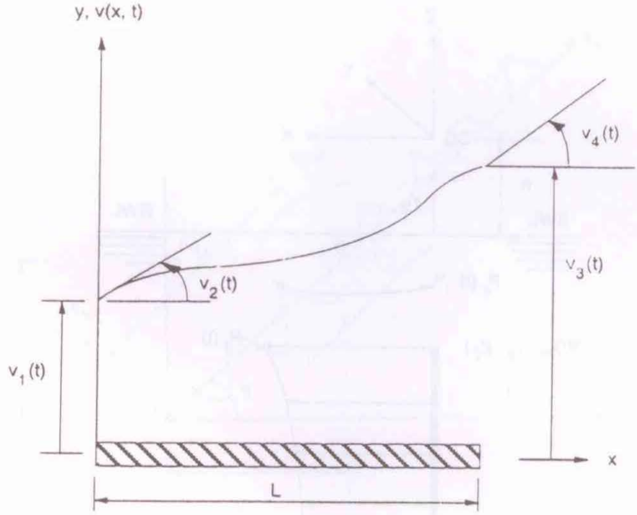

Fig. 1 The transverse deflections of Bernoulli-Euler Beam

resting on the sea bed, it can be applied with sufficient accuracy in practice for the calculation of wave inertia forces on floating columns of a platform [3]. This analytic solution seems to have a great practical value since 'panel and sources' methods are avoided, although the results are comparable. The equivalent diffraction forces on vertical beams are derived here and discussed in detail.

According to the MacCamy and Fuchs' theory, the net force in the direction of wave propagation per unit axial length of the cylinder is given by:

$$f_{MF}(t) = \frac{2\rho g H}{k} \frac{\cosh k(y_w + d)}{\cosh kd} \frac{1}{\sqrt{A(kr)}} \cos(kx_w - \omega t + \alpha_d) \tag{7}$$

where

$$A(kr) = J'^2_1(kr) + Y'^2_1(kr) \tag{8}$$

and

$$\alpha_d = \tan^{-1}\left(\frac{J'^2_1(kr)}{Y'^2_1(kr)}\right) \tag{9}$$

The wave axes xw and yw are chosen such that xw is positive in the direction of wave propagation and yw is positive upward, measured from the sea water line (SWL). The third wave axis, zw is the transverse axis in the horizontal plane. If we consider the centre of gravity of an offshore structure as the origin of the global XYZ axes system, and if we fix the origin of the wave axes system on the SWL, directly below or above (depending on the structure under consideration) the CG position (Fig. 2), a simple relation between the two axes systems is given by:

$$x_w = X\cos\theta + Y\sin\theta \tag{10}$$

$$y_w = Z \pm h \tag{11}$$

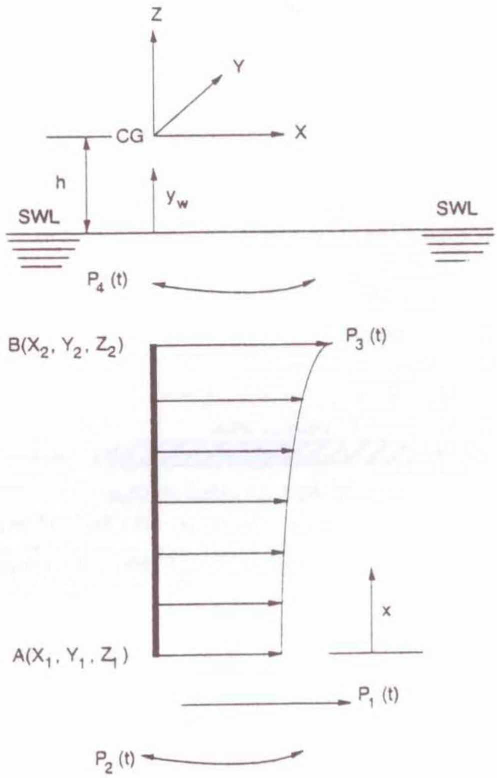

Fig. 2 The global and local co-ordinates of beam

In Fig. 2, it is shown that y_w is equal to $(Z + h)$. In the deep-water case we can replace the hyperbolic term in Eq. (7) by $\exp(kyw)$. Substituting Eqs. (10) and (11) into Eq. (7):

$$f_{MF}(t) = \frac{2\rho g H}{k} \exp \{k(Z \pm h)\} \frac{1}{\sqrt{A(kr)}} \cos\{k(X \cos\theta + Y \sin\theta) - \omega t + \alpha d\} \tag{12}$$

It is important to note that all the terms in Eq. (12) are shown in the global co-ordinates. If we define:

$$G_{MF}(t) = \frac{2\rho g H}{k} \exp (\pm kh) \frac{1}{\sqrt{A(kr)}} \cos\{k(X \cos\theta + Y \sin\theta) - \omega t + \alpha d\} \tag{13}$$

Note that $G_{MF}(t)$ does not depend on the global Z co-ordinates. Thus,

$$f_{MF}(t) = G_{MF}(t) \exp (kZ) \tag{14}$$

In Fig. 2, a vertical beam element in the FE model of an offshore structure is shown where $Z = Z_1 + x$ so far as $Z_1 \leq Z \leq Z_2$. With this in mind, the net horizontal force per unit length in Eq. (14) is rewritten as:

$$f_{MF}(t) = G_{MF}(t) \exp (kZ_1) \exp (kx) \tag{15}$$

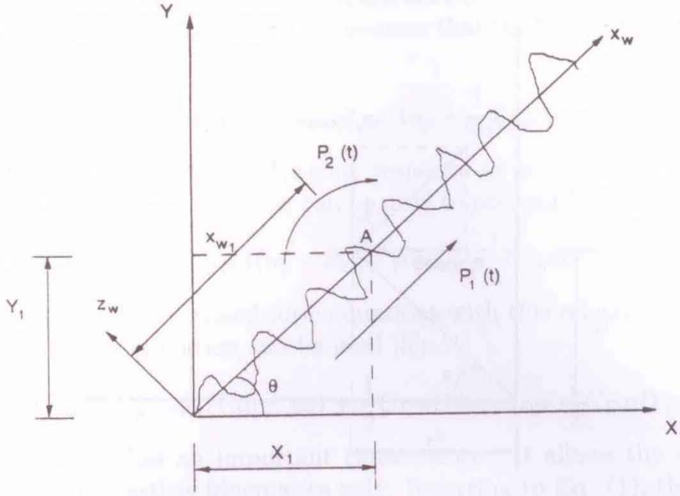

Fig. 3 The global and wave coordinates at node A

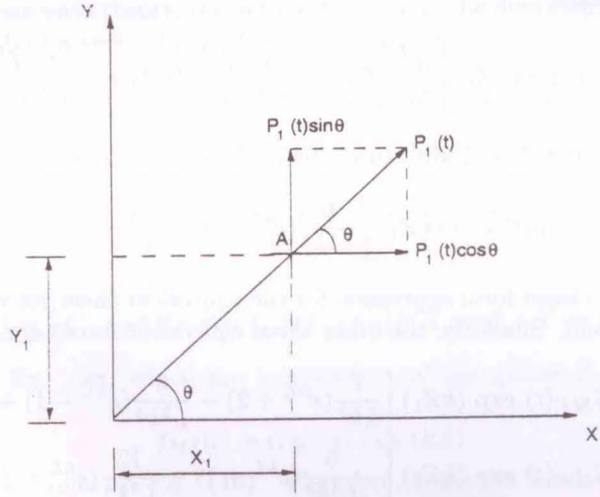

Fig. 4 The global components of equilavent nodal force at A

4.1 Equivalent Nodal Diffraction Forces and Moments

For Bernoulli-Euler beams, the equivalent forces and moments can be found by integrating the external member load with the shape functions:

$$P_i(t) = \int_0^L f(x,t)\psi_i(x)\,dx \tag{16}$$

Thus $P_1(t)$ is found by substituting Eqs. (15) and (3) into Eq. (16):

$$P_1(t) = \int_0^L G_{MF}(t)\,\exp\,(kZ_1)\,\exp\,(kx)\left[1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3\right]dx \tag{17}$$

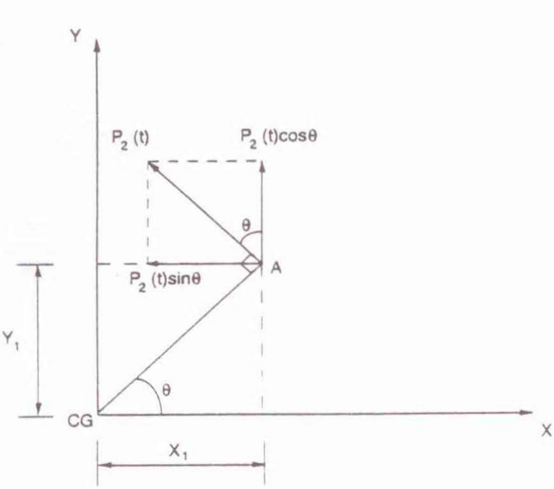

Fig. 5 The global components of equilavent nodal moment at A

After simplification,

$$P_1(T) = G_{MF}(t)\ \exp\ (kZ_1)\left[\int_0^L e^{kx}dx - \frac{3}{L^2}\int_0^L x^2e^{kx}dx - \frac{2}{L^3}\int_0^L x^3e^{kx}dx\right] \tag{18}$$

The final form of $P_1(t)$ is found after integrating Eq. (18) by parts:

$$P_1(t) = G_{MF}(t)\ \exp\ (kZ_1)\left[\frac{6}{L^2k^3}(e^{kL}+1) - \frac{12}{L^3k^4}(e^{kL}-1) - \frac{1}{k}\right] \tag{19}$$

Equation (19) is the closed-form expression for the equivalent shear force acting at the node A of the beam element. Similarly, the other three equivalent forces are found:

$$P_2(t) = G_{MF}(t)\ \exp\ (kZ_1)\left[\frac{2}{Lk^3}(e^{kL}+2) - \frac{6}{L^2k^4}(e^{kL}-1) + \frac{1}{k^2}\right] \tag{20}$$

$$P_3(t) = G_{MF}(t)\ \exp\ (kZ_1)\left[-\frac{6}{L^2k^3}(e^{kL}+1) - \frac{12}{L^3k^4}(e^{kL}-1) + \frac{e^{kL}}{k}\right] \tag{21}$$

$$P_4(t) = G_{MF}(t)\ \exp\ (kZ_1)\left[\frac{2}{Lk^3}(e^{kL}+1) - \frac{6}{L^2k^4}(e^{kL}-1) - \frac{e^{kL}}{k^2}\right] \tag{22}$$

The contributions in the equivalent global load vector, $\{W(t)\}$ are:

$$\{WA(t)\} = \begin{Bmatrix} P_1(t)\cos\theta \\ P_1(t)\sin\theta \\ 0 \\ -P_2(t)\sin\theta \\ P_1(t)\cos\theta \\ 0 \end{Bmatrix} \quad \{WB(t)\} = \begin{Bmatrix} P_3(t)\cos\theta \\ P_3(t)\sin\theta \\ 0 \\ -P_4(t)\sin\theta \\ P_4(t)\cos\theta \\ 0 \end{Bmatrix} \tag{23}$$

5 REVISED FORM OF MORISON EQUATION

The standard form of Morison equation [4] assumes that the structure, which is experiencing the forces, is rigid.

$$f_{MI}(t) + f_{MD}(t) = C_m \rho A \dot{u}_p + \frac{1}{2} C_D \rho D \left| u_p \right| u_p \tag{24}$$

However, if the structure has a dynamic response or is a part of a floating body, the following form of the Morison equation can be used to account for the structural movement:

$$f_{MI}(t) + f_{MD}(t) = C_m \rho A (\dot{u}_p - \dot{u}_s) + \rho A \dot{u}_s + \frac{1}{2} C_D \rho D \left| u_p - u_s \right| (u_p - u_s) \tag{25}$$

It is not possible to derive closed-form equations with this relative velocity model. However, the following approximation can be used [5]:

$$\frac{1}{2} C_D \rho D \left| u_p - u_s \right| (up - us) + \frac{1}{2} C_D \rho D \left| u_s \right| u_s = \frac{1}{2} C_D \rho D \left| u_p \right| u_p \tag{26}$$

This approximation has an important consequence. It allows the calculation of wave forces based on water particle kinematics only. Referring to Eq. (1), the structural contributions can be handled separately.

5.1 Inertia Forces on a Vertical Cylinder

Based on Airy's linear wave theory, the net inertia force in the direction of wave propagation per unit axial length of a vertical cylinder can be written as:

$$f_{MI}(t) = \frac{1}{2} C_m \rho g A H k \ \exp\,(k y_w) \sin(k x_w - \omega t) \tag{27}$$

Equations (10) and (11) can replace the terms involving the wave axis system:

$$f_{MI}(t) = \frac{1}{2} C_m \rho g A H k \ \exp\,\{k(Z \pm h)\} \sin\{k(X \cos\theta + Y \sin\theta) - \omega t\} \tag{28}$$

If we again define:

$$G_{MI}(t) = \frac{1}{2} C_m \rho g A H k \ \exp\,\{(\pm k h)\} \sin\{k(X \cos\theta + Y \sin\theta) - \omega t\} \tag{29}$$

then the terms in Eq. (28) which are independent of the global Z co-ordinate, can be isolated. Thus,

$$f_{MI}(t) = G_{MI}(t) \exp\,(kZ) \tag{30}$$

Equation (30) is similar to Eq. (14). Therefore, from the similarity, the closed-form expressions of the equivalent nodal forces are written as:

$$P_1(t) = G_{MI}(t) \exp\,(kZ_1) \left[\frac{6}{L^2 k^3}(e^{kL} + 1) - \frac{12}{L^3 k^4}(e^{kL} - 1) - \frac{1}{k} \right] \tag{31}$$

$$P_2(t) = G_{MI}(t) \exp\,(kZ_1) \left[\frac{2}{L k^3}(e^{kL} + 2) - \frac{6}{L^2 k^4}(e^{kL} - 1) + \frac{1}{k^2} \right] \tag{32}$$

$$P_3(t) = G_{MI}(t) \exp\,(kZ_1) \left[-\frac{6}{L^2 k^3}(e^{kL} + 1) - \frac{12}{L^3 k^4}(e^{kL} - 1) + \frac{e^{kL}}{k} \right] \tag{33}$$

$$P_4(t) = G_{MI}(t) \exp\,(kZ_1) \left[\frac{2}{L k^3}(2e^{kL} + 1) - \frac{6}{L^2 k^4}(e^{kL} - 1) - \frac{e^{kL}}{k^2} \right] \tag{34}$$

The nodal load vectors $\{W_A(t)\}$ and $\{W_B(t)\}$ can be found straight from Eq. (23)

5.2 Drag Forces on a Vertical Cylinder

If the structural velocity terms are uncoupled from water particle velocities, as explained earlier, it is possible to find similar closed-form expressions for equivalent drag forces:

$$f_{MD}(t) = \pm C_D \rho g D H^2 k \text{ exp } (2ky_w) \cos^2(kx_w - \omega t) \tag{35}$$

The sign of Eq. (35) is the same as the sign of $\cos(kx_w - \omega t)$, i.e., if $\cos(kx_w - \omega t)$ is positive, $f_{MD}(t)$ will be positive. The equation (35) is now written in the global co-ordinate format, using Eqs. (10) and (11):

$$f_{MD}(t) = \pm\frac{1}{2} C_D \rho g D H^2 k \text{ exp } \{2k(Z \pm h)\} \cos^2\{k(X\cos\theta + Y\sin\theta) - \omega t\} \tag{36}$$

For a vertical cylinder, the global X and Y co-ordinates are constant throughout its length. So we again define:

$$G_{MD}(t) = \pm\frac{1}{8} C_D \rho g D H^2 k \text{ exp } \{(\pm 2kh)\} \cos^2 k(X\cos\theta + Y\sin\theta) - \omega t \tag{37}$$

Thus Eq. (36):

$$f_{MD}(t) = G_{MD}(t) \text{ exp } (2kZ) \tag{38}$$

Equation (38) is similar to Eq. (30) and replacing 'k' by '$2k$' in Eqs. (25) the closed-form expressions are obtained:

$$P_1(t) = G_{MD}(t) \text{ exp } (2kZ_1) \left[\frac{6}{4L^2k^3}(e^{2kL}+1) - \frac{3}{4L^3k^4}(e^{2kL}-1) - \frac{1}{2k}\right] \tag{39}$$

$$P_2(t) = G_{MD}(t) \text{ exp } (2kZ_1) \left[\frac{1}{4Lk^3}(e^{2kL}+2) - \frac{3}{8L^2k^4}(e^{2kL}-1) + \frac{1}{4k^2}\right] \tag{40}$$

$$P_3(t) = G_{MD}(t) \text{ exp } (2kZ_1) \left[-\frac{3}{4L^2k^3}(e^{2kL}+1) + \frac{3}{4L^3k^4}(e^{2kL}-1) + \frac{e^{2kL}}{2k}\right] \tag{41}$$

$$P_4(t) = G_{MD}(t) \text{ exp } (2kZ_1) \left[\frac{1}{4Lk^3}(2e^{2kL}+1) - \frac{3}{8L^2k^4}(e^{2kL}-1) - \frac{e^{2kL}}{4k^2}\right] \tag{42}$$

6 A GENERAL APPROACH FOR INCLINED MEMBERS

Not all members in an offshore structure are vertical. In this section a general way of handling complicated loads on an inclined member, arbitrarily oriented in space, is discussed. It is not possible to find the closed-form expressions of equivalent nodal vector in the most general case. But the following approach may offer a better understanding to the problem.

The beam element AB in Fig. 6 is divided into N equal parts. The equivalent load vector is found by superimposing the contributions of external loads acting on each division of the beam.

The local co-ordinates at the midpoint of the jth. division:

$$x_j = (j-1)\frac{L}{N} + \frac{1}{2}\frac{L}{N} = (j - \frac{1}{2}), \quad y_j = 0, \quad z_j = 0 \tag{43}$$

The corresponding global co-ordinates are:

$$X_j = X_1 + \frac{(X_2 - X_1)(j - \frac{1}{2})}{N} \tag{44}$$

$$Y_j = Y_1 + \frac{(Y_2 - Y_1)(j - \frac{1}{2})}{N} \tag{45}$$

$$Z_j = Z_1 + \frac{(Z_2 - Z_1)(j - \frac{1}{2})}{N} \tag{46}$$

The next step is to calculate the external load per unit length in the global X, Y and Z directions. In most cases it may be straight-forward to specify the member load per unit length in the global directions but not when the ocean-wave forces are concerned. In the following section, an extension of the Morison equation is used for demonstration.

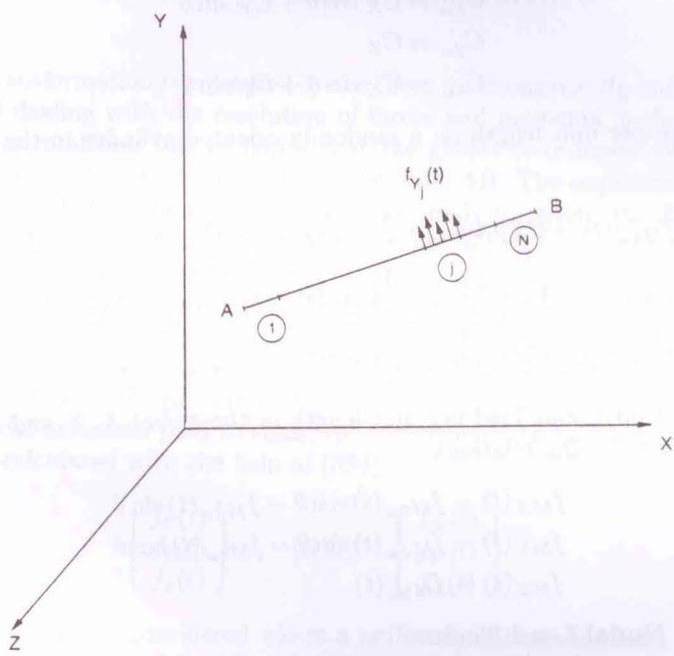

Fig. 6 The global member load $f_Y j(t)$ on jth division of AB

6.1 Morison Wave Forces on an Inclined Member

The formulation for an inclined cylinder [6] is based on so-called independence principles. It states that the forces on the inclined cylinder can be decomposed into their normal and tangential components and the tangential component can be neglected. However, noting that the water particle motion in waves is orbital, the original Morison equation also neglects the tangential component of force on the vertical cylinder.

Using vector algebra, the components of horizontal and vertical water particle velocities are found which are normal to the axis of the inclined member. These normal velocity components along the wave x_w, y_w and z_w directions are:

$$u_{nx_w} = u_p - C_{x_w}(C_{x_2}u_p + C_{y_w}v_p) \tag{47}$$

$$u_{ny_w} = v_p - C_{y_w}(C_{x_2}u_p + C_{y_w}v_p) \tag{48}$$

$$u_{nz_w} = -C_{z_w}(C_{x_2}u_p + C_{y_w}v_p) \tag{49}$$

The acceleration components along x_w, y_x and z_w can be obtained by differentiating Eq. (47), (48) and (49) with respect to time. The direction cosines in wave coordinates are to be calculated. It is better to find their relationship with the direction cosines in the global co-ordinates. From Fig. 6,

$$C_X = \frac{X_2 - X_1}{L} \quad C_Y = \frac{Y_2 - Y_1}{L} \quad C_Z = \frac{Z_2 - Z_1}{L} \tag{50}$$

From Figs. 2 and 3 and Eqs. (10) and (11):

$$C_{x_w} = C_X \cos\theta + C_Y \sin\theta \tag{51}$$

$$C_{y_w} = C_Z \tag{52}$$

$$C_{z_w} = C_X \cos\theta + C_Y \sin\theta \tag{53}$$

Now the forces per unit length on a randomly oriented cylinder in the wave coordinate system:

$$f_{Mx_w}(t) = C_m \rho A \dot{u}_{x_w} + \frac{1}{2} C_D \rho D u_{x_w} \sqrt{u_{nx_w}^2 + u_{ny_w}^2 + u_{nz_w}^2} \tag{54}$$

$$f_{My_w}(t) = C_m \rho A \dot{u}_{y_w} + \frac{1}{2} C_D \rho D u_{y_w} \sqrt{u_{nx_w}^2 + u_{ny_w}^2 + u_{nz_w}^2} \tag{55}$$

$$f_{Mz_w}(t) = C_m \rho A \dot{u}_{z_w} + \frac{1}{2} C_D \rho D u_{z_w} \sqrt{u_{nx_w}^2 + u_{ny_w}^2 + u_{nz_w}^2} \tag{56}$$

Finally the hydrodynamic load per unit length in the global X, Y and Z directions can be found from the following relations:

$$f_{MX}(t) = f_{Mx_w}(t)\cos\theta - f_{Mz_w}(t)\sin\theta \tag{57}$$

$$f_{MY}(t) = f_{Mx_w}(t)\sin\theta - f_{Mz_w}(t)\cos\theta \tag{58}$$

$$f_{MZ}(t) = f_{My_w}(t) \tag{59}$$

6.2 Equivalent Nodal Load Vector

In Section 6.1, $f_{MX}(t)$, $f_{MY}(t)$ and $f_{MZ}(t)$ are established as an example of member load components in the global axes system. It may be possible to specify member load directly or in a relatively easier way in other cases. But in the next step, it will be necessary to calculate the components of these forces in the local axes system. Harrison [7] has given the relations between two sets of concurrent orthogonal forces in equilibrium in matrix format. According to this Eulerian method, the force transformation matrix can be written as:

$$[RM] =$$

$$\begin{bmatrix} -\cos\alpha_E\cos\beta_E & \sin\alpha_E\cos\gamma_E - \cos\alpha_E\sin\beta_E\sin\gamma_E & -\sin\alpha_E\sin\gamma_E - \cos\alpha_E\sin\beta_E\cos\gamma_E \\ -\sin\alpha_E\cos\beta_E & -\cos\alpha_E\cos\gamma_E - \sin\alpha_E\sin\beta_E\sin\gamma_E & \cos\alpha_E\sin\gamma_E - \sin\alpha_E\sin\beta_E\cos\gamma_E \\ \sin\beta_E & -\cos\beta_E\sin\gamma_E & -\cos\beta_E\cos\gamma_E \end{bmatrix} \tag{60}$$

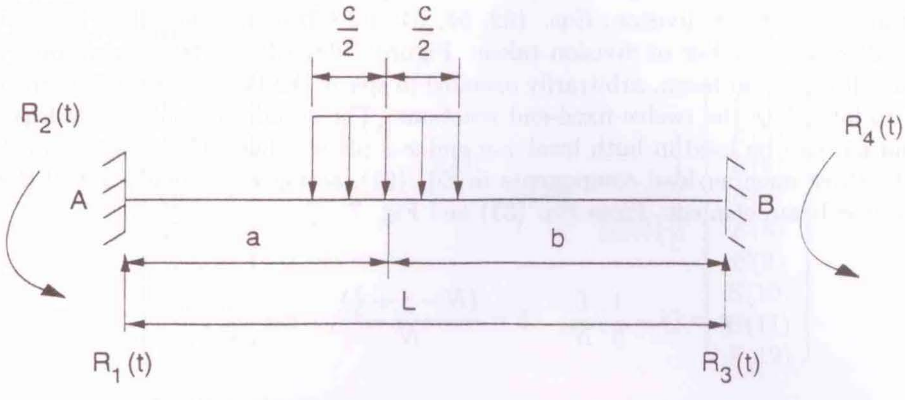

Fig. 7 Fixed-end forces from uniformly distributed load of length c

The Eulerian transformation represented by the three rotations α_E, β_E and γ_E is the most convenient way of dealing with the resolution of forces and moments in three dimensions. In visualising these rotations, one imagines that the global co-ordinate system has been moved to coincide with the end A of the beam element, AB. The sequence of rotations of α_E about the Z axis, then β_E about the Y axis and finally γ_E about the X axis are what is done to make the global co-ordinate system coincide with the local system. The final rotation γ_E is relevant only to members which otherwise would not be bent about principal axes by the end moments. The angle γ_E is, in fact, the only one of the three angles needs to be given as an input. the other angles are evaluated from the element projections. It is important to note that for circular members γ_E is zero. The coefficients in the matrix $[RM]$ are discussed in more detail in Ref. [8].

With some careful attention paid to signs, the components of $f_X(t)$, $f_Y(t)$ and $f_Z(t)$ in the local axes are calculated with the help of [RM]:

$$\left\{\begin{array}{c} f_x(t) \\ f_y(t) \\ f_z(t) \end{array}\right\} = -[RM]^{-1} \left\{\begin{array}{c} f_X(t) \\ f_Y(t) \\ f_Z(t) \end{array}\right\} \tag{61}$$

In Fig. 7, a classic case is considered where a uniformly distributed load, $f(t)$ of length c is acting on a beam element, AB of length L, at a distance $(= a)$ from the node A. The four fixed end reactions can be found from a standard engineering handbook:

$$R_1(t) = \frac{f(t)cb}{L} + \frac{\{R_2(t) + R_4(t)\}}{L} \tag{62}$$

$$R_2(t) = \frac{f(t)c}{12L^2}[c^2(L-3b) + 12ab^2] \tag{63}$$

$$R_3(t) = \frac{f(t)ca}{L} - \frac{\{R_2(t) + R_4(t)\}}{L} \tag{64}$$

$$R_4(t) = -\frac{f(t)c}{12L^2}[c^2(L-3b) + 12ab^2] \tag{65}$$

If member load per unit length acting at the midpoint of any division is approximated as uniform load over the division, Eqs. (62, 63, 64 and 65) can be useful. The accuracy will depend on the number of division taken. Figure 7 describes a two-dimensional case. For a three-dimensional beam, arbitrarily oriented in space, the law of superposition will be assumed to formulate the twelve fixed-end reactions. The standard results from Eqs. (62, 63, 64 and 65) can be used in both local x-y and x-z planes. Let us use a subscript 'j' to specify the three member load components in Eq. (61), acting at the midpoint of the jth. division of the beam element. From Eq. (31) and Fig. 7:

$$a=(j-\frac{1}{2})\frac{L}{N}, \quad b=\frac{(N-j+\frac{1}{2})}{N}, \quad c=\frac{L}{N} \tag{66}$$

The reactions corresponding to the member load component, $f_{y_j}(t)$ in the local y direction are found by substituting Eq. (66) into Eqs. (62 - 65).

$$R_{y1}(t)=\frac{F_{y_j}(N-j+\frac{1}{2})L}{N^2}+\frac{F_{y_j}(t)L}{4N^4}\left\{1-4(j-\frac{1}{2})(N-j+\frac{1}{2})\right\}(2j-N-1) \tag{67}$$

$$R_{y2}(t)=\frac{F_{y_j}(t)L^2}{12N^4}\left\{N-3(N-j+\frac{1}{2})+12(N-j+\frac{1}{2})^2(j-\frac{1}{2})\right\} \tag{68}$$

$$R_{y3}(t)=\frac{F_{y_j}(j-\frac{1}{2})L}{N^2}+\frac{F_{y_j}(t)L}{4N^4}\left\{1-4(j-\frac{1}{2})(N-j+\frac{1}{2})\right\}(2j-N-1) \tag{69}$$

$$R_{y4}(t)=-\frac{F_{y_j}(t)L^2}{12N^4}\left\{N-3(j-\frac{1}{2})+12(N-j+\frac{1}{2})(j-\frac{1}{2})^2\right\} \tag{70}$$

Similarly the reactions in the local x-z plane can be found after replacing $f_{y_j}(t)$ by $f_{z_j}(t)$ in Eqs. (67). With some careful attention paid to signs, the twelve fixed-end reactions can be written as:

$$E(1)=-f_{y_j}(t), \quad E(2)=-R_{y_1}(t), \quad E(3)=-R_{z_1}(t) \tag{71}$$

$$E(4)=0, \quad E(5)=-R_{z_2}(t), \quad E(6)=-R_{y_2}(t) \tag{72}$$

$$E(7)=0, \quad E(8)=-R_{y_3}(t), \quad E(9)=-R_{z_3}(t) \tag{73}$$

$$E(10)=0, \quad E(11)=-R_{z_4}(t), \quad E(12)=-R_{y_4}(t) \tag{74}$$

However, in most cases the member load in the local x direction, $f_{x_j}(t)$ (tangential component of forces) is neglected.

Now the force transformation matrix [RM] can be used again to relate the twelve fixed-end reactions, E to the equivalent joint load vectors $\{W_A(t)\}$ and $\{W_B(t)\}$ in the global directions:

$$\left\{\begin{matrix} W_A(1) \\ W_A(2) \\ W_A(3) \\ W_A(4) \\ W_A(5) \\ W_A(6) \\ W_B(1) \\ W_B(2) \\ W_B(3) \\ W_B(4) \\ W_B(5) \\ W_B(6) \end{matrix}\right\} = \left\{\begin{matrix} [RM] & & & \\ & [RM] & & \\ & & [RM] & \\ & & & [RM] \end{matrix}\right\} \left\{\begin{matrix} E(1) \\ E(2) \\ E(3) \\ E(4) \\ E(5) \\ E(6) \\ E(7) \\ E(8) \\ E(9) \\ E(10) \\ E(11) \\ E(12) \end{matrix}\right\} \tag{75}$$

6.3 Numerical Verifications

Several FORTRAN subroutines are developed to check the important equations presented in the previous section. The external forces and the fixed-end reactions should maintain equilibrium at the nodes. So arbitrary values are assigned to the member load components and the equilibrium condition is verified.

The general step-by-step procedure described earlier is further verified by considering a few classic cases where the reaction forces can be found from a standard engineering handbook. The hydrostatic loading on a vertical panel is a typical example of triangularly distributed load. The 'exact' reaction forces and moments are shown in Fig. 8. The beam is divided into 10 segments and the member load per unit length acting at the midpoint of each division is approximated as uniform load over the division. Figure 9 shows the approximated load case. The fixed-end reactions are found by superimposing the contributions of external loads acting on each division of the beam. The values are shown in Table 1 where $L = 30$ units and $f(t) = 20$ units. The approximation is found to be reasonable.

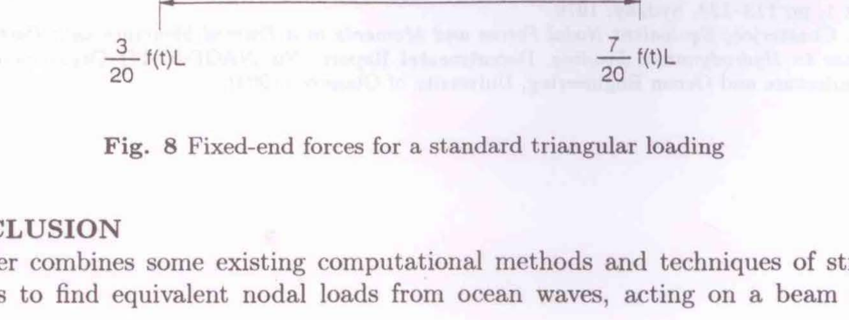

Fig. 8 Fixed-end forces for a standard triangular loading

7 CONCLUSION

This paper combines some existing computational methods and techniques of structural mechanics to find equivalent nodal loads from ocean waves, acting on a beam element

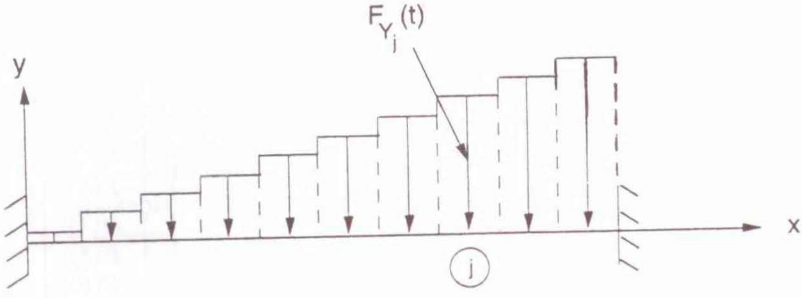

Fig. 9 The approximated load case

based FE model of an offshore structure. In the process, some closed form expressions are formulated which can be very helpful to anyone who does not want to get into the complexity of the interaction of waves with the structure. The added advantage is that the closed-form expressions are analytically integrated and simplified and they avoid computationally expensive numerical integration. For a large structure, that will considerably decrease the computer run time and storage requirements.

Besides closed-form expressions, a step-by-step equivalent load calculation procedure is also presented for beam elements, arbitrarily oriented in space. The procedure proposed here, can achieve a reasonable solution depending on the number of divisions of an element. It is worth noting that it can also deal with any other form of member loads in addition to forces from ocean waves. For programming purposes, in fact, it can be stored as a separate module which can be utilised later in the actual FE analysis.

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Table 1 The contributions from each division of the beam

Division No.	F_{Y_i}	E(2)	E(6)	E(8)	E(12)
1	-1	2.9715	3.9225	0.0285	-0.2775
2	-3	8.4375	28.9125	0.5625	-5.2875
3	-5	12.6375	62.8125	2.3625	-21.1875
4	-7	15.0675	92.6625	5.9325	-50.1375
5	-9	15.5115	109.8220	11.4885	-89.9775
6	-11	14.0415	109.9730	18.9585	-134.2770
7	-13	11.0175	93.1125	27.9825	-172.0870
8	-15	7.0875	63.5625	37.9125	-188.4380
9	-17	3.1875	29.9625	47.8125	-163.8370
10	-19	0.5415	5.2725	56.4585	-74.5275
Total	-	90.5010	600.0150	209.4990	-899.9850
Theory	-	90.0000	600.0000	210.0000	-900.0000

LIST OF IMPORTANT SYMBOLS

α_d	=	Phase lag of diffraction force
$\alpha_E, \beta_E, \gamma_E$	=	Eulerian transformation angles
C_X, C_Y, C_Z	=	Direction cosines in the global co-ordinates
$\{E\}$	=	Twelve member-end reactions
$F_{MF}(t)$	=	Diffraction force per unit length of a vertical cylinder according to MacCamy and Fuchs theory
$F_{MI}(t)$	=	Inertia force per unit length of a vertical cylinder according to the Morison equation
$F_{MD}(t)$	=	Drag force per unit length of a vertical cylinder according to the Morison equation
$F_{Mx_w}(t)$	=	Component of the normal force per unit length along X_w according to Borgman
$J_1'(kr)$		Derivative with respect to 'kr' of Bessel function of the first kind
k	=	Wave number
$P_i(t)$	=	Equivalent force or moment at the ends of a 2D beam
θ	=	Wave direction with respect to global X axis
$[RM]$	=	Eulerian transformation matrix
$R_i(t)$	=	Fixed-end force or moment at the ends of a 2D beam
$R_{yi}(t)$	=	Fixed-end force or moment at the ends of a 3D beam under the action of external load in the local y direction
u_p	=	Horizontal instantaneous water particle velocity
u_s	=	Velocity of the incremental length, dl, of the element
$u_{nx_w}, u_{ny_w}, u_{nz_w}$	=	Components along wave axes of the normal (to the element axis) velocity
$v(x,t)$	=	Displacement function
v_ρ	=	Vertical instantaneous water particle velocity
$\{W_A(t)\}$	=	Equivalent joint load vector at the node A
x, y, z	=	Local Cartesian co-ordinates
x_j, y_j, z_j	=	Midpoint local co-ordinates of the jth division of the element
X, Y, Z	=	Global Cartesian co-ordinates
x_w, y_w, z_w	=	Wave Cartesian co-ordinates
$f_i(x)$	=	Shape function
$Y_1'(kr)$	=	Derivative with respect to 'kr' of Bessel function of the second kind