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DOUBLE BOOTSTRAP CONTROL CHART FOR MONITORING SUKUK VOLATILITY AT BURSA MALAYSIA

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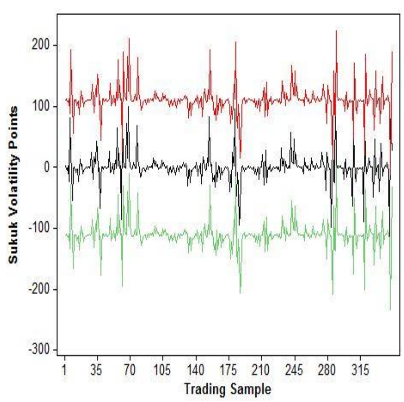
Graphical abstract



Abstract

The bootstrap approach on control limit has provided a solution in solving uncertainty estimation problem in control chart performance. However, the limitation of this standard chart has shown to be less efficient and invalidation at certain magnitude shift, especially the monitored sample data is assumed from skewed family distribution. Thus, in this study, a double bootstrap base-model and its control limit is developed in order to improve the efficiency and decrease the invalidation chart performance. In order to test the performance of proposed model, a simulation study using Average Run Length (ARL) and Type II Error rate were implemented. The result has shown that the proposed chart is sensitive and effective in detecting the shift process for small and medium size of skewed sample data. Also, it has found that the proposed chart shown to has better performance on large magnitude shift. The performance of the proposed model was investigated further using sukuk volatility data at Bursa Malaysia. The result revealed that the double bootstrap control chart is sensitive to small shifts process when it can detect changes in the volatility faster. In other words, it is efficient in monitoring the shifts process. Thus, the proposed model could help the traders in making a new decision, for example, either save/hold for a certain period, sell or buy the sukuk certificate.

Keywords: Double bootstrap, estimation, control chart, simulation, sukuk

Abstrak

Pendekatan bootstrap terhadap batas kawalan telah memberikan suatu penyelesaian masalah penganggaran ketidakpastian dalam pencapaian carta kawalan. Bagaiamanapun, batasan terhadap piawai carta menunjukkan ianya kurang berkesan dan ketidaksahihan pada anjakan magnitude tertentu, terutama pemantauan bagi sampel data yang diandaikan daripada keluarga taburan kepencongan. Oleh itu, dalam kajian ini, satu model berasaskan bootstrap berganda dan batas kawalannya dibangunkan bagi meningkatkan kecekapan dan mengurangkan ketidaksahihan pencapaian carta. Bagi menguji pencapaian terhadap model cadangan, satu kajian simulasi menggunakan Purata Panjang Larian (ARL) dan kadar Ralat Jenis II telah dijalankan. Keputusan menunjukkan bahawa model cadangan adalah sensitif dan berkesan dalam mengesan proses anjakan bagi kepencongan sampel data yang bersaiz kecil dan sederhana. Juga, didapati bahawa model cadangan menunjukkan pencapaian yang lebih baik pada anjakan magnitude besar. Pencapaian model cadangan disiasat dengan lanjut menggunakan data kemeruapan sukuk di Bursa Malaysia. Keputusan mendedahkan bahawa carta kawalan bootstrap berganda adalah sensitive kepada proses anjakan kecil apabila ianya dapat mengesan perubahan titik kemeruapan dengan lebih cepat. Dengan kata lain, ianya adalah cekap dalam memantau proses anjakan. Dengan itu, model cadangan boleh membantu peniaga dalam membuat keputusan baharu, contohnya, sama ada menyimpan/memegang untuk tempoh tertentu, menjual atau membeli sijil sukuk.

Kata kunci: Bootstrap berganda, penganggaran, *carta kawalan,* simulasi, sukuk

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1.0 INTRODUCTION

The moving centerline exponentially weighted moving average (MCEWMA) control chart was introduced by Mastrangelo in 1991 [1]. Since that, it has been widely used to monitor autocorrelated sample data due to its efficient performance in out-of-control process [2], [3]. The autocorrelated data sometimes collected in individual units or literally individual observation and the data could be distributed by non-normal such as skewed distribution [4]-[6]. According to previous literatures studies, [3] [6], [7], the direct monitoring on skewed distribution could increase the type II error rate and eventually produce inconsistent estimation of Average Run Length (ARL) in either small, medium or large magnitude shift of out-of-control process. Despite of these problem, some studies have designed a flexible chart for monitoring the skewed distribution, for example Gamma and Weibull distributions families, and multimodal sample data, [4], [7], [8]. Even though the flexible chart has given excellent performance in detecting the out-of-control process, the bootstrap approach could help improving the performance in terms of solving the incompatibility of approximating a non-normal distribution to Gaussian distribution and decrease the uncertainty estimation [9]-[11].

Therefore, a bootstrap procedure is utilized in this study in order to improve the efficiency of control chart when the information data samples collected from individual observation of skewed distribution, is obtained. Instead of using the standard procedure [10], [11], a new angle of bootstrapping algorithm is considered using the modified steps of [12], [13].

Eventhough the bootstrap approach has improved the chart performance, Safiih *et al*. [14] has mentioned that single bootstrap chart shown poor monitoring function at greater shift magnitude. Due to this slight problem, a double bootstrap method is highlighted in this reserach. By taking the advantage of double bootstrap; particularly to decrease the biasness, shorten the interval length, reduce eror of model estimation[14]-[19],this model is also used to improve the efficiency of control chart in monitoring the greater shift magnitude.

In this study, a novel hybrid model of double bootstrap approach is proposed in order to yield more accurate results using moving centerline exponential weighted moving average (MCEWMA) model. The performance of the proposed model is compared with the existing MCEWMA model in terms of ARL estimation and type II error rate, using a selection of shift magnitud from out-of-control process. A Monte Carlo simulation from the Gamma distribution is included in this approach by generating three examples of individual observation from skewed family.

2.0 METHODOLOGY

2.1 Control Chart of Individual Observation

In this section, the moving centerline EWMA control chart is selected in order to monitor the individual observation, (*n*=1). The moving centerline EWMA has been recognized to construct standard exponentially weighted moving average control chart by giving great performance in detecting small shift in individual observation, as described by $(X_i = X_1,..., X_m)$ [20]-[22]. Thus, the base mathematical model of MCEWMA can be written as follows:

$$W_i = \lambda x_i + (1-\lambda)W_{i-1},\ i = 1,...,m \qquad (1)$$

where W_i denotes for base model of moving centerline EWMA for given sample data of i^{th}, x_i and the parameter is denoted as λ and $(1-\lambda)$ where it's value $\in (0,1]$ [23], [24]. As mentioned by Cox, Psarakis and Papleonida [25], [26], the parameter value was estimated by optimizing the λ. Due to the limited function of standard chart, where it only well functioned for independent and identically distributed (*iid*) sample data, Mastrangelo made an adjustment so that the serial correlation or autocorrelation data can be monitored without constructing any model of autoregressive (*p*,*d*) or AR (*p*,*d*) [1], [27], [28]. The *p* and *d* are the parameters of AR model. In fact, W_i is the one-step ahead center line of observed data, x_{i+1} is used to construct a set of control limit, while BK used the basic probability of the following equation:

$$P\left[W_i - L_e\sigma_e \le x_{i+1} - \le W_i + L_e\sigma_e\right] = 1-\alpha \quad (2)$$

where L_e and σ_e denote for sigma value of one-step ahead standard error and one-step ahead of standard deviation of equation (1) respectively, with $(1-\alpha)100$ level of significant. Considered an adjustment on equation (2) for construct upper limit and lower limit, thus it can be rewrited as following equation:

$$BK_{x_{i+1}} = W_i \pm L_e\sigma_e \quad (3)$$

where σ_e is consider to be calculated using the following equation:

$$\hat{\sigma}_e = \left[\sum_{i(c)=1}^{N}(\hat{e}_{i(c)})^2 \cdot \frac{1}{N}\right]^{\frac{1}{2}}, \quad i(c)=1,\ldots,n; \quad N \ne n \quad (4)$$

where N refer to sample size with individual observation. Based on previous study [12], [14], [29], the base-model of (1) is used as the center line in constructing the equation (3). This approach has proven to be well functioned in detecting the out-of-range points of serial correlation data of various assumption distribution. However, few studies [4], [7], [8] have pointed out that the equation (3) might lead to poor sensitiveness of detecting magnitude shift when dealing with skewed distribution, for example Gamma distribution family. Due to this, the control chart could eventually increase the average run length and also, the chart intend to increase the frequency of false detection point of out-of-process that will lead to greater type II error [4]-[6]. Motivated by this problem, in this research, a hybrid double bootstrap approach is proposed using the base-model of equation (1) and reconstructs the standard limit control of equation (3). Thus, the insensitiveness of chart monitoring could be decreased and eventually improved the efficiency of control chart.

2.2 Double Bootstrap Approach on Base-model of Chart

The standard solution to improve the monitoring functional of a chart is to reconstruct the control limit using any method such as bootstrap [9], [11], [27]. However, the reconstruction sometimes doesn't seem to be the best solution to the family of skewed data, especially for Gamma distribution. Instead of reconstruction the limit, this research used a difference procedure where the part of double bootstrap is made to the base-model.

2.3 Replication Size of First and Second Sampling Data

According to Efron, Lola and Zainuddin [30], [31], the sampling procedure can be obtained by using a preferable size of replication, for example 1000 times, to obtain shorter interval and smaller standard error. However, a limitation size is preferred to precede standard iteration of sampling for double bootstrap method. As was used by Efron, Martin and Hall [17]-[19], the replication size considered to be smaller than 1000 times, for instance 500. However, despite of using a fix size, Akhmad *et al.* and Aparisi and García-Díaz [32], [33] has estimated the replication size by finding a converge value from bias estimation of bootstrap sampling size. Therefore, in this research, the replication size of single and double bootstrap follows the study of [34] so that the shorter interval and smaller error could be obtained in simulation study. Thus, the selected sizes are 1700 and 2200 for single and double bootstrap procedure respectively.

2.4 Algorithm of Double Bootstrap Approach

The algorithm for double bootstrapping the moving centerline EWMA control chart can be referred to the following steps:

Step1: Generate an individual observation of a sample set $(x_{n=1,i} = x_{1,1},\ldots,x_{1,m})$ randomly from residual of Gamma distribution, $e_i \sim \Gamma(\alpha,\beta)$ with the shape, $\alpha = 0.283$, skewness, $\beta = 0.049$ and θ=202.02. These parameter values are motivated by the study as reported by Lola and Zainuddin [31].

Step 2: Use the sample of step 1 to estimate the base-model of moving centerline EWMA, W_i and the parameter, λ=0.94. The residual base-mode can be estimated using the information of $\hat{W}_i$.

Step 3: The estimated residual base-model from step 2 is considered to be used in single bootstrap procedure where the replication size is *B*=1700. This replication would obtain a sequence of single bootstrap sample that can be shown in matrix forms as following recursion:

$$\hat{e}_i^{*(t)} = \begin{bmatrix} \hat{e}_1^{*(1)} & \cdots & \hat{e}_1^{*(1699)} & \hat{e}_1^{*(1700)} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{e}_{m-1}^{*(1)} & \cdots & \hat{e}_{m-1}^{*(1699)} & \hat{e}_{m-1}^{*(1700)} \\ \hat{e}_m^{*(1)} & \cdots & \hat{e}_m^{*(1699)} & \hat{e}_m^{*(1700)} \end{bmatrix} \quad (5)$$

where $\hat{e}_i^{*(t)}$ refer to sampling the residual base-model and (*t*) refer to sequence of single bootstrap sampling, $(t) = 1,\ldots,1700$. The notation of "*" refer to single bootstrap method.

Step 4: Obtain a sequence of independent data from this sampling single bootstrap set by making the adjustment calculation from residual base-model equation. A matrix form of independent sampling data as the following recursion:

$$x_i^{*(t)} = \begin{bmatrix} x_1^{*(1)} & \cdots & x_1^{*(1699)} & x_1^{*(1700)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m-1}^{*(1)} & \cdots & x_{m-1}^{*(1699)} & x_{m-1}^{*(1700)} \\ x_m^{*(1)} & \cdots & x_m^{*(1699)} & x_m^{*(1700)} \end{bmatrix} \quad (6)$$

Step 5: Calculate the mean of every row in equation (6) so that an independent sample of single bootstrap, $(x^* = x_1^*,...,x_i^*)$ can be obtained.

Step 6: Use the sample of single bootstrap from step 5 to estimate the base-model of single bootstrap moving centerline EWMA, W_i^* and it's residual, e_i^*. The estimation is considered the same parameter value as used in step 2.

Step 7: Repeat the step 3 until step 5 for double bootstrap procedure with the replication size is BB=2200. The independent sample of double bootstrap can be denoted as $(x^{**} = x_1^{**},...,x_i^{**})$.

Step 8: Estimate the base-model of double bootstrap moving centerline EWMA, W_i^{**} and it's residual, e_i^{**} using the sample in step 7. In order to form a control chart, use the estimation of W_i^{**} to be the centerline and estimate the control limit ($CL_{x_{i+1}^{**}}$) using the following recursion:

$$\mathrm{BK}_{x_{i+1}^{**}} = \hat{W}_i^{**} \pm 3\hat{\sigma}_{e^{**}} \quad (7a)$$

$$\mathrm{CL}_{x_{i+1}^{**}} = \hat{W}_i^{**} \quad (7b)$$

where L is considered to be $3\hat{\sigma}_{e^{**}}$ with $\hat{\sigma}_{e^{**}}$ refers to the standard deviation estimation of residual base-model of $\hat{W}_i^{**}$. The estimation can be referred to equation (4).

3.0 RESULTS AND DISCUSSION

3.1 Monte Carlo Simulation Study

The performance of proposed chart is estimated using ARL and type II error rate [34]-[36]. In order to examine the best performer, the estimation of proposed chart are considered to be compare to standard chart, standard chart with bootstrapping limit, and also single bootstrap chart. This comparison is conducted via Monte Carlo simulation study using R language by generating a single Gamma family distribution, $e_i \sim \Gamma(\alpha = 0.283, \beta = 0.049)$ and θ= 202.02 with three selected sample size of n = 30, 100, 300 for individual observation, n=1 [37] [38]. For the sake of simplicity, the sample size were classified to be small (n=30), medium (n=100) and large (n=300).The reason of selecting the parameters values was elaborated in section 2.3. The main purpose of selecting different sample size is to study the consistency of chart performance in terms of giving precise estimation begins from small sample until large sample data [12], [13], [39], [40]. The existing standard chart refers to moving centerline EWMA with standard limit,(W(BK)) and moving centerline EWMA with bootstrapping limit (W(BK*)), while for single bootstrap chart refers to bootstrap moving centerline EWMA with bootstrapping limit (W*). By running the algorithms in section 2.3 using R language, thus, the final result of control chart performance of ARL estimation and type II error rates can be referred to Table 1 and Figure 2, respectively. In order to complete the dependent sampling of $\hat{T}_n^{JJ}$, the replication size for W(BK) is considered to be 1000 times while for W(BK*) and W*, it is considered to follow the replication size of single bootstrap method, JJ= 1700.

3.2 Average Run Length Estimation

A comparison of ARL estimation of selection magnitude shift for all sample sizes as shown in Table 1. Based on Table 1, at n=30, the ARL estimation for all charts are found to be consistently decreased as small magnitude shift increase from δ=0.5 up to δ=1.0. However, at δ=2.0 and δ=3.0, both standard charts shows an increasing number of ARL, indicated that, both charts are slow detector of shift process. Even though the bootstrapping limit control reduced the ARL estimation, however, the efficiency of chart still unreliable due to the increasing estimation from 80.129 (δ=2.0) up to 217.052 (δ=3.0). Meanwhile, the single bootstrap chart shown positive performance where the ARL estimations are found to be smoothly decreased as the magnitude shift increase, starting from δ=0.5 up to δ=3.0. Instead of having greater value, the residual of base-model replication in single and double bootstrap procedures eventually reduce the standard deviation value, and thus, the ARL estimation has found to be smaller compared to standard chart with bootstrapping control limit.

At sample size of n=100, both standard charts given a smooth decreasing ARL estimation starting from the magnitude shift of δ=0.5 until δ=2.0. However, both charts are unreliable in detecting the shift process at large magnitude of δ=3.0. For standard chart, it need an average of 74 points to detect the out-of-control process, while the standard chart with bootstrapping limit need an average of 51 point to state the shift occurs. However, the double bootstrap need an average of 5 points to detect the shift process compare to the single bootstrap (8 points) and both standard charts. Means that, double bootstrap chart is statistically proven to be more reliable and efficient in monitoring the out-of-control process of individual observation. The efficiency of double bootstrap chart is indicated by its consistency in decreasing ARL estimation from the smallest until large magnitude shift. The decreasing results also illustrated graphically that can be refer to Figure 1.

Table 1 Comparison of ARL estimation using selection magnitude shift for n=30, 100, 300 sample size

n	δ	ARL			
		W(BK)[a]	W(BK*)[b]	W*[c]	W**[d]
30	0.5	486.113	317.426	192.910	61.948
	1.0	462.212	86.948	67.931	43.173
	1.5	83.355	55.607	47.868	28.497
	2.0	274.389	80.129	25.574	17.285
	3.0	399.250	217.052	20.420	7.362
100	0.5	432.529	147.668	54.472	45.929
	1.0	187.710	69.734	52.191	41.421
	1.5	165.641	66.554	46.042	26.186
	2.0	49.480	22.644	18.864	16.141
	3.0	73.968	50.951	8.033	5.455
300	0.5	125.205	105.466	41.192	25.637
	1.0	116.136	36.385	18.507	13.133
	1.5	165.574	44.346	12.987	10.352
	2.0	88.399	26.630	12.047	10.110
	3.0	74.615	45.346	17.869	2.888

[a,b]Refer to respective estimation of standard and single bootstrap control limit
[b,c]Refer to respective single bootstrap and double bootstrap of control chart.

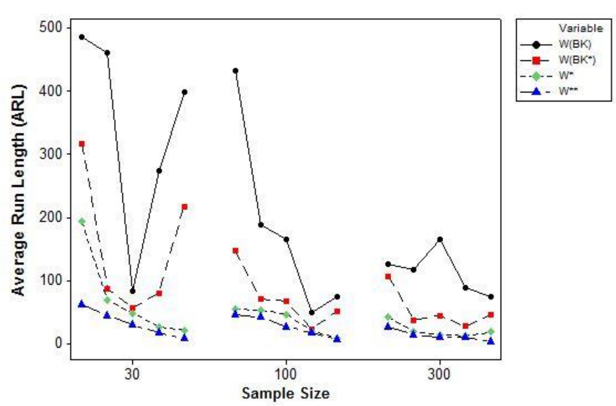


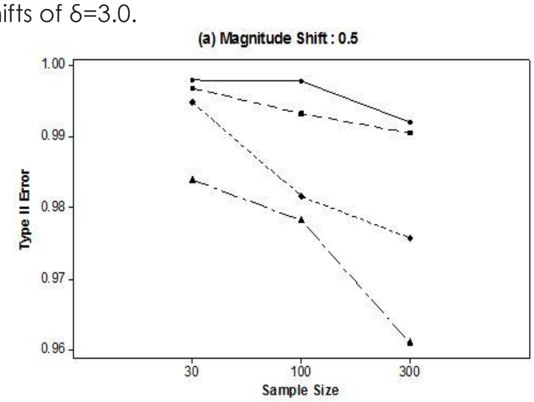
Figure 1 Comparison of average run length estimation for five selection magnitude; first point is δ=0.5, second point is δ=1.0, third point is δ=1.5, fourth point is δ=2.0, fifth point is δ=3.0

Interestingly, at *n*=300 in Figure 1, both standard charts shown its consistency of decreasing the ARL estimation only at small magnitude shift of δ=0.5 and of δ=1.0. At the magnitude shift of δ=1.5, both charts detected greater point to state the out-of-control process. However, the standard chart shown decreasing result starting from δ=2.0 until δ=3.0. Instead of producing the same result with standard chart, the bootstrapping limit chart continues to produce inconsistent ARL estimation starting from of δ=1.5 up to δ=3.0.

Moreover, unexpected result shown by single bootstrap chart that detected at magnitude shift of δ=3.0. The ARL estimation inconspicuously increases from 12 points to 17 points of detecting the out-of-control process. Based on this scenario, it means that single bootstrap not well perform at large magnitude shift of large sample size. However, the proposed chart, on the other hand, is still reliable to continue decreasing ARL estimation smoothly starting from magnitude shift of δ=0.5 up to δ=3.0. At large magnitude, for example δ=3.0, the double bootstrap chart only need 3 points to declare the out-of-control process, means that the chart is fast detector of shift occurrence. For further observation of chart performance, type II-error estimation has calculated to find the probability of false detection of in-control points using all charts.

3.3 Type II Error Rate Estimation

The estimation for type II error using standard chart, standart chart with bootstrapping limit, single bootstrap chart and double bootstrap chart are shown graphically on Figure 2. Based on the figure, at *n*=30, the rate of type II error of all charts are above 0.8 for all magnitude shifts. At smallest sample size, the double bootstrap chart has shown smallest rate than other charts. Even though, the single bootstrap chart found to produce lower rate, but the estimation of magnitude shift of δ=0.5 until δ=1.5, are close to the rate of standard bootstrapping limit chart. At δ=2.0 and δ=3.0, the rate become stable at above 0.97 compare to standard charts that have greater rate of type II-error. Meanwhile, for the standard charts, the rates are found to be above 0.98 means that the false detection of in-control points is greater compare to single and double bootstrap chart. Even though the bootstrapping limit chart can improve the performance of standard chart, the chart shown inconspicuous differ estimation of standard chart at large magnitude shifts of δ=3.0.



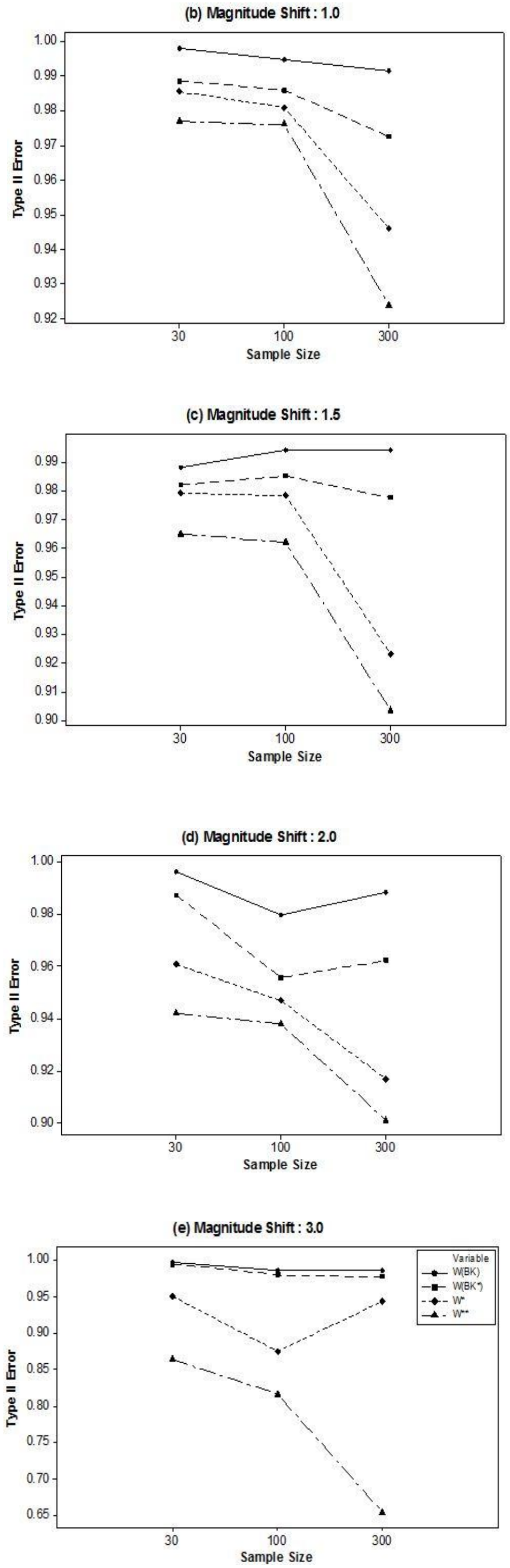


Figure 2 Type II error rate for selection magnitudes; (a) magnitude shift = 0.5, (b) magnitude shift =1.0, (c) magnitude shift =1.5, (d) magnitude shift =2.0, (e) magnitude shift =3.0

At *n*=100, the rate of double bootstrap chart is shown to be closer to single bootstrap chart at magnitude shift of δ=1.0. This might due to the greater difference of ARL estimation between the both charts. Also, the rate estimated by the chart turned out to has inconspicuous decreasing value at δ=0.5 to δ=1.5 with the rate of 0.9782 to 0.9759, due to the same range value of ARL estimation. Based on Table 1, the estimation values are 45.929 and 41.421 for δ=0.5 and δ=1.5 respectively. Moreover, at this sample size, bootstrapping limit chart shown greater rate at all magnitude shifts.

Furthermore, at *n*=300, the rate of double bootstrap chart shown to be consistently decrease as the magnitude shift increase. A small number of estimation decrease from magnitude shift of δ=1.5 to δ=2.0 with the rate from 0.9034 to 0.9011, due to the small difference of points in out-of-control process detected by double bootstrap chart. Meanwhile, rather than having consistently decreasing rate, single bootstrap chart shown a slight increasing rate from magnitude shift of δ=2.0 to δ=3.0 with the value from 0.9170 to 0.9440. Means that, the single bootstrap has poor sensitivity in detecting the out-of-control point at large magnitude shifts of large sample of individual observation. In terms of producing consistency estimation at all magnitude shifts, double bootstrap chart produce a consistent decreasing rate as sample size increase. For single bootstrap chart, it is clear that the chart has produce decreasing value at small magnitude shift and δ=2.0. However, at δ=3.0, the chart shown its weakness where a dramatic change of rate is detected from the increase sample size of *n*=100 to *n*=300. On the other hand, bootstrap limit chart is consistently, decreased of estimation only for magnitude shift of δ=0.5 up to δ=1.5, and starts to show its inconsistency at large magnitude. Meanwhile, as expected, the standard chart only showed the decreasing rate at small magnitude shifts.

3.4 The Application to Volatility of Sukuk Investment at Bursa Malaysia

The Islamic certificate, known as sukuk, has become alternative investment in Bursa Malaysia trading market. In order to apply the double bootstrap control chart using real situation sample data, thus, a time series sukuk data under Islamic Medium-Term Notes program is considered in this research. Motivated by the study of [31], this research has considered monitoring the volatility points of sukuk under stock code of VN100268. The size of selected sample is 348 with the trading price information starting from 30 August 2010 until 28 October 2015. All the information used for this research can be search on the web of http://www.bursamalaysia.com. Instead of directly applied the sample price data, a specified calculation needed to be converted from the trading price into a set of sukuk sample return [41] [42]. In order to find an approximation of the distribution, the sukuk return has to be considered to undergo a fit distribution test using Kolmogorov-Smirnov and Anderson-Darling methods. Thus, the result can be referred in Table 2. The results shown that sukuk return can be approximated to few skewed family distribution for example, Gamma, Generalized Gamma and Weibull. According to the small estimation of Kolmogorov-Smirnov and Anderson-Darling, the preferred distribution assumption for sukuk return is Gamma.

Table 2 Goodness of fit summary for sukuk returns

Distribution Approximation	K-S[a]	A-D[b]
Dagum	0.3189	134.38
Gamma	0.2126	71.161
Generalized Gamma	0.2911	111.79
Inverse Gaussian	0.3215	114.84
Pearson VI	0.3259	117.92
Pearson V	0.3243	118.85
Weibull	0.2851	110.16

[a,b]Refer to respective estimation of Kolmogorov-Smirnov and Anderson-Darling.

Next, the volatility points of sukuk are estimated using the base-model (1) and proceeded to apply the proposed algorithm of double bootstrap control chart in section 2.4. Thus, the complete chart can be graphically referred to Figure 3. Based on the figure, VN100268 sukuk has shown high volatile at its beginning and ending of trading. Means that, sukuk has experienced dramatic change of its trading price and has gained high profit in that period. Moreover, at the trading activity of 81th point until 130th point and 188th point until 283th point, the sukuk has low volatility, meaning that, the sukuk price was relatively stable. The upper and lower limit of double bootstrap chart represents the maximum and minimum volatility points of sukuk respectively over the time. Based on the figure, there is no evidence of shifting process occur even though the drastic fluctuation recorded at the end of trading period. However, in order to observe the performance of

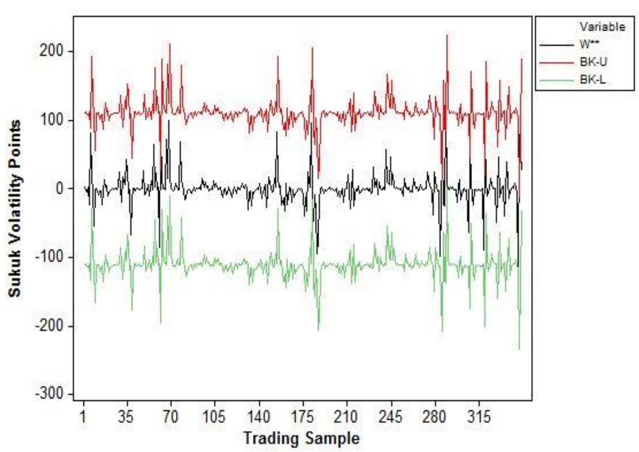


Figure 3 Volatility points of sukuk sample starting from 30 August 2010 until 28 October 2015

double bootstrap chart, this research considered to estimate the ARL and type II error rate for small magnitude shift.

Recall from the simulation study, where most of the charts shows a great performance at small magnitude shift of δ=0.5 up to δ=1.5, of large generated sample size, *n*=300. By taking this advantage, the volatility points of sukuk also considered to be monitored using the proposed design chart. Since the main purpose of using double bootstrap method is to reduce the residual in base-model (1) so that the efficiency of control limit will be increased. In order to show how well the double bootstrap base-model could perform, standard error of its control limit has estimated. For comparison, the estimation standard error of all tested charts in simulation study also considered and the result can be referred to Table 3. From this table, having standard base-model in chart could lead to less accuracy statistical estimation due to its greater standard error in all magnitude shifts.

Table 3 Standard error of tested control limit charts for small magnitude shift using *m*=348 sample size of sukuk

δ	W(BK)[a]	W(BK*)[b]	W*[c]	W**[d]
0.5	2.55E-06	1.47E-06	1.45E-06	1.03E-06
1.0	2.62E-06	1.45E-06	1.37E-06	9.60E-07
1.5	2.59E-06	1.72E-06	1.25E-06	9.02E-07

The abbreviation of [a,b,c,d] can be referred to Table 1. W**=double bootstrap chart, BK-U=upper control limit and BK-L=lower control limit.

Meanwhile, by using bootstrapping control limit, the chart could be well performed starting from magnitude shift of δ=0.5 to δ=1.0, however, as due to the greater standard error at magnitude shift of δ=1.5, the bootstrapping limit chart might be less effective in detecting the shift process. In the other hand, bootstrapping the base-model could provide better performance due to the small standard error resulted from both single and double bootstrap control limit. The consistent decreasing in estimation and smallest standard error given by double bootstrap indicated that the residual of base-model (1) has been reduced and by using this efficient limit, the chart will be more sensitive to detect the shifting process.

Table 4 ARL estimations charts for small magnitude shift using *m*=348 sample size of sukuk

δ	W(BK)[c]	W(BK*)[d]	W*[e]	W**[f]
(a) ARL				
0.5	131.804	62.723	10.051	5.054
1.0	135.851	64.899	9.029	2.147
1.5	69.193	64.512	5.455	1.559
(b)Rate of error[a]				
0.5	0.992413	0.984057	0.900505	0.802131
1.0	0.992639	0.984591	0.889243	0.534316

δ	W(BK)[c]	W(BK*)[d]	W*[e]	W**[f]
(a) ARL				
0.5	131.804	62.723	10.051	5.054
1.5	0.985548	0.984499	0.816676	0.358608
(c) False Alarm[b]				
	0.002300	0.002299	0.002050	0.002030

[a,b]Refer to the respective type II error and Type I Error estimation rate for all charts using the tested magnitude shift. [c,d,e,f]Refer to the abbreviation in Table 1.

From Table 4, as expected, standard chart shown an increasing ARL estimation and type II error rate at magnitude shift at δ=1.0 and decreasing at δ=1.5. However, even though the bootstrap procedure has helped to decrease the residual of base-model of bootstrapping limit chart until magnitude shift of δ=1.0, a greater ARL and type II error rate are found at this magnitude. In the other hand, the single and double bootstrap chart are found to have a smoothly decreasing ARL and type II error rate consistently from δ=0.5 until δ=1.5. In terms of application, the double bootstrap chart has detected 2 volatility points of fluctuation occur over the length period. The fast detection could lead to positive returns information for traders.

Additionally, a Type I Error or false alarm estimation also considered in this part of application study. The purposed is to examine how reliable the tested charts when the process is not shifted. Using nominal ARL_0=500, and the in-control mean, $\mu_0 = 0.0$ and standard deviation, $\sigma_0 = 1.0$, the result for the false alarm rate can be referred to Table 4. Based on the result, it shown that standard chart has the highest rate than bootstrapping chart. Suppose that, the estimate rate should be approaching the nominal rate of 0.002 (1/ARL0) so that the chart could be proven statistically efficient due to its sensitivity of detecting the false out-of-control points. In that case, the double bootstrap chart has shown a closest value to the nominal rate compare to single bootstrap chart. This indicates that, double bootstrap chart has shown an efficient chart in giving the less detection for false points.

4.0 CONCLUSION

The aim of this research is to reduce the residual of base-model control chart in order to increase the chart efficiency, so that, the chart could be well applied to skewed distribution family, for example Gamma. Using a residual bootstrap procedure onto base-model of moving centerline exponentially weighted moving average chart, a double bootstrap chart has designed in this research and its performance tested by estimating the ARL, type II error rate in simulation study and, also type I error rate considered in application study of sukuk investment sample data. In both studies, the sample of individual observation assumed to follow Gamma distribution and for performance comparison, standard and other bootstrapping chart also considered. Based on simulation result, the double bootstrap chart has found to improve the bootstrapping limit chart and standard chart at large magnitude shift of δ=2.0 and δ=3.0 in both small and medium sample size of individual observation. At large sample size, double bootstrap has helped to improve the efficiency of standard chart and bootstrapping limit chart at magnitude shift of δ=1.5 and δ=3.0 respectively. Moreover, the proposed chart found to well perform at large magnitude shift and improved the single bootstrap chart due to its insensitiveness of detecting the shift process. In case of type II error rate, double bootstrap has showed to be a reliable chart due to its high sensitivity in detecting the false out-of-control points at all magnitude shift compared to other tested charts. Meanwhile, in application study, the double bootstrap has shown to be the best detector of shifts process due to its smallest estimation of ARL, type II error rate, and type I error rate. Even though the objective of this research seems to be fulfilled, however, the biasness of double bootstrap control chart remains questionable and it will be tested in future.

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