Jurnal
Teknologi

Full Paper

A Single-Vendor Single-Buyer Integrated Production-Inventory Model with Quality Improvement and Controllable Production Rate

Article history
Received
7 February 2017
Received in revised form
12 July 2017
Accepted
1 November 2017

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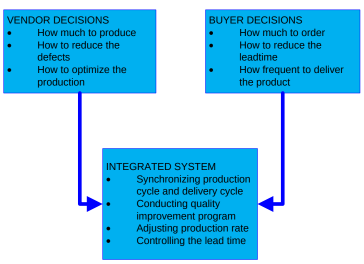
Graphical abstract



Abstract

In the classical vendor-buyer inventory models, the common unrealistic assumption is that all the items manufactured are in good quality. However, in reality, it can be observed that there may be some defective items produced and then delivered to the buyer. Thus, the existence of defective items would consequently give significant influence to system behavior. In addition, a manufacturing flexibility such as the capability to adjust production capacity becomes a key success factor for increasing system flexibility as well as reducing total cost. Here, we investigate how a quality improvement program and adjustable production rate can help the supply chain to reduce the total cost. This paper studies the effect of quality improvement and controllable production rate in joint economic lot size model consisting of single-vendor and single-buyer under stochastic demand. The model gives allowance to the vendor to adjust production rate and also to invest an amount of capital investment to reduce the defect rate. The lead time is comprised of production time and setup and transportation time. The model also considers a situation in which the shortages in buyer side are assumed to be partially backordered. To solve the model, an iterative algorithm is proposed to determine simultaneously safety factor, delivery lot size, delivery frequency, production rate and process quality for minimizing total cost is proposed. The result from this study shows that allowing the vendor to both adjust the production rate and reduce the defective product by adopting quality improvement policy can reduce the individual and total cost. In the example given, the proposed model gives significant total cost saving of 45.9% compared to the model without controllable production rate and quality improvement.

Keywords: Inventory, stochastic demand, production rate, quality improvement, supply chain

1.0 INTRODUCTION

Managing production-inventory system is a major supply chain management challenge. Recent studies in inventory management have proved that integrating production-inventory decisions through determining optimal production cycle and ordering cycle jointly can lead to reducing the total cost. In inventory literature, the problem dealing with making lot sizing decisions involving parties in supply chain is known as joint economic lot size (JELS). Goyal [1] was probably the first author who proposed JELS model. He developed vendor-buyer inventory model for a situation where the vendor produces a lot in an infinite rate and then delivers it to the buyer on a lot-for-lot basis. The results from this study showed that making inventory decisions jointly in vendor-buyer system can reduce total cost significantly. Subsequently, Banerjee [2] investigated a lot-for-lot policy whereby the vendor produces a batch at finite production rate and delivers it to the buyer equally. Goyal [3] developed Banerjee's [2] model by relaxing lot-for-lot assumption.

He demonstrated that producing a batch which is made of equal shipments can result in lower total cost, but all batches must be produced before the first lot is delivered to the buyer. Other researchers including Goyal [4], Hill [5, 6], and Jauhari *et al*. [7] concerned on investigating shipment policy in vendor-buyer model by allowing the shipment size is of an unequal size. Pujawan and Kingsman [8] argued that even the unequal sized shipment policy results in lower total cost, however it may produce some deficiencies which relate to higher idle-capacity cost and difficulties in operational planning and control. A comprehensive review on vendor-buyer inventory problem was presented by Ben-Daya *et al*. [9].

In many practical situations, it is proved that the product quality is not always perfect but would be influenced by the process reliability in the production system. In inventory model, there was a stream of research focused on investigating the effect of defective product in the production system. Rosenblatt and Lee [10] probably the first researchers who investigated the effect of imperfect production process on economic production quantity (EPQ) model. Porteous [11] also studied EPQ model considering quality improvement investment to reduce the defective products and setup cost. He used a logarithmic function to formulate the investment needed by the system to reduce the probability of out of control. Then the model was extended by other researchers into various models. Lee and Park [12] developed EPQ model considering imperfect production process, rework cost and warranty cost. He proposed a model that the production process may deteriorate and produce some defective products. The defective products are reworked at such cost before the shipment can be made. The warranty cost will incur when the defective products passed to the customer. Ouyang and Chang [13] investigated the effect of partial backorder and variable lead time on EPQ model with quality improvement. Ouyang *et al*. [14] extended the Ouyang and Chang [13]'s model by incorporating investment cost in quality improvement process, setup cost and lead time reduction. Chen *et al*. [15] proposed EPQ model considering imperfect production process and allowable shortage. They also investigated the effect of learning process on optimal production batch. The above mentioned papers considered a single stage model and neglecting the issue of integrated inventory model in supply chain system. However, it has been proved in many studies that the integrated inventory model can reduce the total cost significantly compared to the independent model.

A number of researchers, including Huang [16], Yang and Pan [17], Ouyang *et al*. [18], Khan and Jaber [19], investigated imperfect production process in integrated vendor-buyer system. Huang [16] proposed a uniform distribution for percentage of defective product produced by vendor. The demand in buyer side was assumed to be deterministic and the vendor uses equal shipment size to deliver lot to the buyer. The vendor and buyer jointly determine the production cycle and ordering cycle to minimize total cost. The results from this study indicated that the proposed model performs better than the model without considering just in time (JIT) philosophy. Yang and Pan [17] developed vendor-buyer system considering quality improvement and stochastic demand. Facing a certain number of defective product in production system, the vendor conducted an investment to reduce the probability of the process can go out of control. Ouyang *et al*. [18] gave an extension to the previous work of Yang and Pan [17] by introducing controllable lead time and shortage cost. The lead time was assumed to be able to be reduced by a crashing cost. They proposed an iterative procedure to determine the optimal decision variables, including reorder point which was assumed as predetermined variable in previous study. Furthermore, Khan and Jaber [19] studied two-stage supply chain system consisting of the vendor and the buyer. Their study focused on investigating the effect of defective raw material parts received from suppliers in vendor-buyer system. Uthayakumar and Parvathi [20] investigated the effect of deterioration, credit period incentives and ordering cost reduction in vendor-buyer system under controllable lead time. The vendor and buyer decide upon a capital investment in reducing buyer ordering cost and coordinate both production and replenishment cycle. Further, Uthayakumar and Rameswari [21] investigated quality improvement, variable lead time and credit policy in deterministic vendor-buyer system. They adopted the formulation of Porteous [11] to formulate the quality improvement in vendor side. As the study discussed the credit policy, they also examined the behavior of the system in which the vendor gave allowance to the buyer to delay the payment.

Major research direction on inventory model is considering controllable production rate. This research was motivated by fact that the production rate in production system can be adjusted flexibly by inserting idle time over production run and setting the speed of production equipment (Buzacott and Ozkarahan [22]; Schweitzer and Seidmann [23]). Khouja and Mehrez [24] probably the first researchers to introduce EPQ model considering controllable production rate. They proposed two formulae of controllable production rate in EPQ model and proved that a model with adjusted production rate always performed better than the model with fixed production rate. They also proposed to increase the regular production rate up to the maximum production capacity. The retailer will be charged an additional cost to accomplish the increased production rate. Song *et al*. [26] extended Moon and Cha [25]'s model by proposing stochastic vendor-buyer system with variable production rate and lead time interactions. The vendor and buyer costs were modeled as the function of lead time, hence the effect of lead time reduction on both parties can be considered simultaneously.

Integrated vendor-buyer system dealing with stochastic demand and variable lead has also attracted considerable attention. Ben-Daya and Hariga [27] proposed integrated inventory model for single-vendor single-buyer system with a lead time formulated proportionally to the delivery lot. The extensions of the model were given by Hsiao [28] and Glock [29]. Hsiao [28] introduced a new lead time formulation in which the lead time of the first shipment was formulated by considering production time and setup and transportation time while the lead time for 2, 3,….n shipment was only transportation time considered in the model. Glock [29] modified the model by adopting unequal-sized shipment policy from Hill [5]. He proved that the unequal-sized shipment model resulted in lower total cost compared to equal-sized shipment model. Moreover, Glock [30] investigated different lead time reduction strategies in stochastic vendor-buyer model. The model considered a situation in which the production rate can be adjusted and the setup and transportation time can be reduced by a crashing cost. Jauhari *et al.* [31] studied imperfect production process and ajustable production rate in a periodic review integrated inventory model. Jauhari *et al*. [32] extended their previous model by allowing the inclusion of imperfect production process and inspection errors. Further, AlDurgam *et al.* [33] investigated variable production rate and full truckload constraint in single-vendor single manufacturer system. Although most of the above mentioned papers considered stochastic demand which may represent the real situation, the incorporation of quality improvement policy and controllable production rate were rarely found in the model.

Our review of inventory literature showed that the integrated vendor-buyer model has been discussed in the past, but none has studied the impact of incorporating quality improvement, controllable production rate and variable lead time, simultaneously. Thus, the interdependencies between these decision variables are not yet well understood. This paper develops integrated vendor-buyer model considering quality improvement, controllable production rate and variable lead time under stochastic demand. For better understanding, a comparison of the proposed model with some of the related works in the literature is given in Table 1. We propose a vendor-buyer model which determines shipment size, frequency of delivery, safety factor, process quality and production rate simultaneously. We propose an effective algorithm to solve the problem and present a numerical example to illustrate the application of the model.

2.0 METHODOLOGY

2.1 Model Development

To develop the integrated vendor-buyer model, we use the following notations:

D demand rate in units per unit time
σ standard deviation of demand per unit time
A order cost incurred by the buyer for each order size of nQ
F transportation cost per shipment
h_b holding cost per unit per unit time for buyer
h_v holding cost per unit per unit time for vendor
π backorder cost per unit backordered
π_0 marginal profit per unit for buyer
θ proportion of shortages that will be backordered
P production rate in units per unit time
a_1 per unit time cost for running the machine independent of production rate
a_2 the increase in unit machining cost due to one unit increase in production rate
K setup cost

Table 1. A Comparison of the proposed model with some related papers in the inventory literature

Papers	Buyer policy	Demand	Lead time	Safety factor	Defect product	Production Rate	Quality investment	Backorder
Huang [16]	EOQ	Deterministic	Zero	No	Yes	Fixed	No	No
Yang and Pan [17]	Continuous review	Stochastic	Deterministic	No	Yes	Fixed	Yes	Full Backorder
Ouyang *et al*. [18]	Continuous review	Stochastic	Controllable	One type	Yes	Fixed	Yes	Full Backorder
Uthayakumar and Rameswari [21]	EOQ	Deterministic	Controllable	One type	Yes	Fixed	Yes	No
Song *et al*. [26]	Continuous review	Stochastic	Controllable	One type	No	Controllable	No	Full Backorder
Ben-Daya and Hariga [27]	Continuous review	Stochastic	Depend on lot size	Two types	No	Fixed	No	Full Backorder
Hsiao [28]	Continuous review	Stochastic	Depend on lot size	Two types	No	Fixed	No	Full Backorder
Glock [29]	Continuous review	Stochastic	Depend on lot size	One type	No	Fixed	No	Full Backorder
Jauhari *et al*. [31]	Periodic Review	Stochastic	Depend on lot size	One type	Yes	Controllable	No	Full Backorder
Jauhari *et al*. [32]	Periodic Review	Stochastic	Depend on lot size	One type	Yes	Controllable	No	Full Backorder
AlDurgham *et al*. [33]	Continuous review	Stochastic	Depend on lot size	One type	No	Controllable	No	Full Backorder
Proposed model	Continuous review	Stochastic	Depend on lot size	Two types	Yes	Controllable	Yes	Partial Backorder

w rework cost per unit defective product
β_0 original probability that production process can go out of control, $\beta_0 \geq \beta$
α capital investment per unit time
β the probability that production process can go out of control
n number of shipments, which is a positive integer
Q the size of equal shipments from the vendor to the buyer
k safety factor
T_w setup and transportation time
T_s transportation time

In this paper, we consider a supply chain model consisting of single-vendor and single-buyer for a single product. The buyer adopts continuous review policy to manage his/her inventory level. The buyer orders a lot of size nQ. The buyer incurs ordering cost A for each order size nQ and incurs transportation cost F for each shipment of size Q. The vendor produces nQ in each production batch with a finite production rate P (P>D) and has a responsibility to deliver a shipment size of Q to the buyer over n times. In each production run, the vendor incurs setup cost K. Shortages are permitted in the model and assumed to be partially backordered. The proportion of demand during stock-out period that will be backordered is denoted by θ. The demand during lead time of the first shipment is normally distributed with mean $DL(Q)$ and standard deviation σ. The demand during lead time of the j^{th} shipment is also normally distributed with mean DT_s and standard deviation σ for j= 2,3,....n. The lead time for the first shipment consists of production time and the sum of setup and transportation time and the lead time for 2,3,....n shipment is only transportation time considered in the model. The development of proposed model will be described below.

2.1.1 Vendor's Expected Cost Per Unit Time

The vendor's major cost consists of holding cost, setup cost, rework cost, investment cost for quality improvement and production cost. Vendor's holding cost is calculated from the average accumulation of the vendor's during production run. Figure 1 shows the inventory profile for the vendor. Vendor's inventory level is calculated by subtracting the accumulative delivery from vendor's accumulated inventory. The inventory level for vendor is determined as follows:

$$\frac{\left\{\left(nQ\left[\frac{Q}{P}+(n-1)\frac{Q}{D}\right]-\frac{n^2Q^2}{2P}\right)-\left(\frac{Q^2}{D}\left[1+2+.....+(n-1)\right]\right)\right\}}{nQ/D}$$
$$=\frac{Q}{2}\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\} \quad (1)$$

Since K is the setup cost incurred by the vendor for each production run D/nQ, the expected setup cost per unit time is formulated by KD/nQ. The expected number of defective product per production run is given by

$$\frac{n^2Q^2\beta/2}{nQ/D}=\frac{nQD\beta}{2} \quad (2)$$

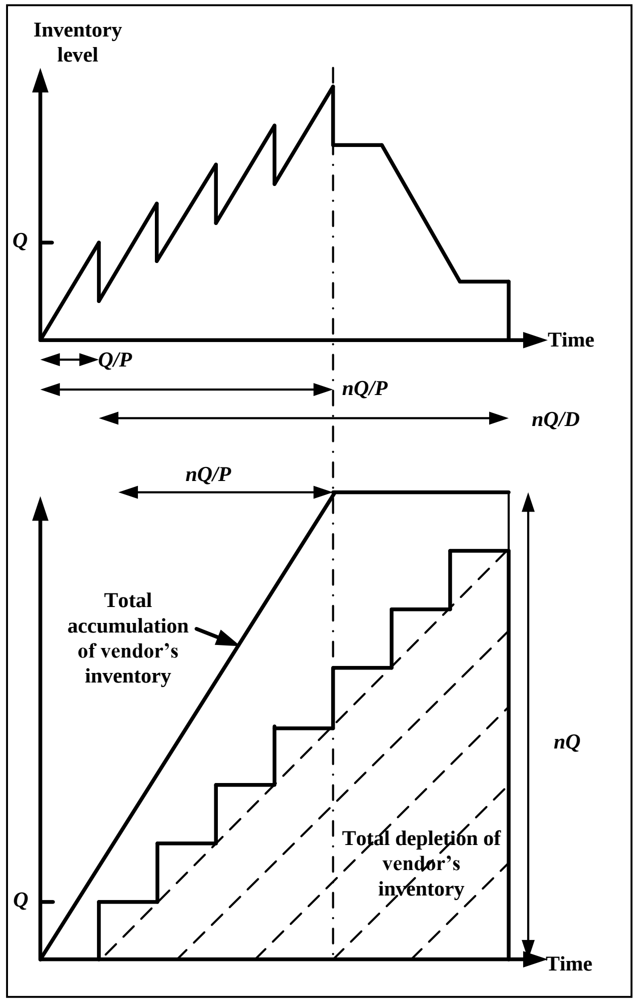


Figure 1 Vendor's inventory profile

Since w is the rework cost per unit item, the expected rework cost per unit time is as follows

$$\frac{wnQD\beta}{2} \quad (3)$$

Therefore, the investment for quality improvement is formulated with logarithmic function

$$IC(\beta)=v\ln\left(\frac{\beta_0}{\beta}\right), \text{ for } 0<\beta\leq\beta_0 \quad (4)$$

with v=1/λ and λ denotes the percentage decrease in probability of out of control (β) per dollar investment in quality improvement. Thus, the opportunity cost of quality improvement investment can be determined as follows

$$\alpha v \ln\left(\frac{\beta_0}{\beta}\right) \tag{5}$$

In this paper, we adopt Khouja and Mehrez's [24] formulae to calculate the production cost in vendor side. The formula of production cost is given by

$$I(P)=\frac{a_1}{P}+a_2P \tag{6}$$

where a_1 and a_2 are non-negative real numbers that provide the best fit to estimate unit production cost function. Consequently, the expected cost per unit time of vendor is given by

$$ETC_V(n,Q,\beta,P)=\frac{Q}{2}h_V\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}$$
$$+\frac{DK}{nQ}+\frac{wnQD\beta}{2}+\alpha v\ln\left(\frac{\beta_0}{\beta}\right)+\left(\frac{a_1}{P}+a_2P\right)D \tag{7}$$

2.1.2 Buyer's Expected Cost Per Unit Time

The expected cost per unit time for buyer consists of ordering cost, transportation cost, holding cost and shortage cost. Since A is buyer's ordering cost and D/nQ is a frequency of order per unit time, the expected ordering cost can be formulated as DA/nQ. By considering frequency of delivery D/Q, the expected transportation cost for buyer per unit time is given by DF/Q.

In this paper, we use the safety stock's formulation from Hsiao [28]. He proposed a different formulation to estimate the safety stock of first shipment and jth shipment. The lead time of the first shipment is formulated as the sum of production time and setup and transportation time Q/P + T_w and lead time of j^{th} shipment is only T_s considered in the model. The demand during lead time of the first shipment is normally distributed with mean $DL(Q)$ and standard deviation σ. The safety stock of the first shipment can be represented as

$$S_1=k_1\sigma\sqrt{L(Q)}=k_1\sigma\sqrt{Q/P+T_w} \tag{8}$$

where k_1 is the safety factor of first shipment. Thus, the expected shortage of the first shipment is given by

$$b(s_1,L(Q))=\int_{s_1}^{\infty}(x_1-s_1)f(x_1)dx_1=\sigma\sqrt{Q/P+T_w}\,\psi(k_1) \tag{9}$$

where,

$$\psi(k_1)=f_S(k_1)-k_1\left[1-F_S(k_1)\right] \tag{10}$$

$fs(k)$ is a probability density function of standard normal distribution and $Fs(k)$ is cumulative distribution function of standard normal distribution.

The demand during lead time of the j^{th} shipment is also normally distributed with mean DT_s and standard deviation σ for j = 2,3,...n. The safety stock of j^{th} shipment is formulated as follows

$$S_2=k_2\sigma\sqrt{T_s} \tag{11}$$

where k_2 is the safety factor of j^{th} shipment. The expected shortage of j^{th} shipment is formulated by

$$b(s_2,L(T_s))=\int_{s_2}^{\infty}(x_2-s_2)f(x_2)dx_2=\sigma\sqrt{T_s}\,\psi(k_2) \tag{12}$$

where,

$$\psi(k_2)=f_S(k_2)-k_2\left[1-F_S(k_2)\right] \tag{13}$$

Considering (8) and (11), the formulation of k_2 can be reformulated as a function of k_2, that is

$$k_2=k_1\sqrt{\frac{Q/P+T_w}{T_s}} \tag{14}$$

Here, we modify the formulation of shortage by inserting the partial backorder formulation. By considering the partial backorder, some of shortages would be backordered and others would be lost sale. The expected number of backorder per cycle is given by

$$\sigma\theta\left[\sqrt{\frac{Q}{P}+T_w}\,\psi(k_1)+(n-1)\sqrt{T_s}\,\psi(k_2)\right] \tag{15}$$

The expected number of lost sales per cycle is given by

$$(1-\theta)\sigma\sqrt{Q/P+T_w}\,\psi(k_1) \tag{16}$$

Furthermore, the shortage cost per unit time is formulated as

$$\left(\frac{D\sigma}{nQ}(\pi+\pi_0(1-\theta))\right)\left[\sqrt{\frac{Q}{P}+T_w}\,\psi(k_1)+(n-1)\sqrt{T_s}\,\psi(k_2)\right] \tag{17}$$

Now, the expected total cost per unit time for buyer is given by

$$ETC_B(n,k_1,k_2)=\frac{D}{nQ}(A+nF)$$
$$+\left(\frac{D\sigma}{nQ}(\pi+\pi_0(1-\theta))\right)\left[\sqrt{\frac{Q}{P}+T_w}\,\psi(k_1)+(n-1)\sqrt{T_s}\,\psi(k_2)\right]$$
$$+h_b\left(\frac{Q}{2}+k_1\sigma\sqrt{Q/P+T_w}+(1-\theta)\sigma\sqrt{Q/P+T_w}\,\psi(k_1)\right) \tag{18}$$

2.1.3 Expected Total Cost Per Unit Time

The expected total cost per unit time can be formulated by adding the expected total cost incurred by the vendor into the expected total cost incurred by the buyer, that is

$$
\begin{aligned}
ETC_T(n, Q, k_1, \beta, P) &= ETC_V(n, Q, \beta, P) + ETC_B(n, Q, k_1, k_2) \\
&= \frac{Q}{2}h_v\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}+\frac{DK}{nQ}+\left(\frac{a_1}{P}+a_2P\right)D \\
&\quad+\frac{D}{nQ}(A+nF)+\frac{wnQD\beta}{2}+\alpha v\ln\left(\frac{\beta_0}{\beta}\right) \\
&\quad+h_b\left(\frac{Q}{2}+k_1\sigma\sqrt{Q/P+T_w}+(1-\theta)\sigma\sqrt{Q/P+T_w}\,\psi(k_1)\right) \\
&\quad+\left(\frac{D\sigma}{nQ}(\pi+\pi_0(1-\theta))\right)\left[\sqrt{\frac{Q}{P}+T_w}\,\psi(k_1)+(n-1)\sqrt{T_s}\,\psi(k_2)\right]
\end{aligned}
\tag{19}
$$

Subject to $0 < \beta \le \beta_0$.

2.2 Solution

To solve the above model, we temporarily ignore the constraint $0 < \beta \le \beta_0$ and relax the integer requirement on n. For fixed n, we take the first derivatives of $ETC_T(n, Q, k_1, \beta, P)$ with respect to k_1, Q, P and β respectively, and find the following equations:

$$
\begin{aligned}
\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial k_1} &= \sigma h_b\sqrt{Q/P+T_w}\,\big(1-(1-\theta)[1-F_s(k_1)]\big) \\
&\quad-\frac{D\sigma(\pi+\pi_0(1-\theta))}{nQ}\left([1-F_s(k_1)]+(n-1)\left[1-F_s\left(k_1\sqrt{\frac{Q/P+T_w}{T_s}}\right)\right]\right)\sqrt{Q/P+T_w}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial Q} &= \frac{\left(h_v\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}+h_b\right)}{2}-\frac{D\left(\frac{A+K}{n}+F\right)}{Q^2} \\
&\quad+\frac{wnD\beta}{2}+\frac{h_bk_1\sigma}{2P\sqrt{Q/P+T_w}}+\frac{h_b(1-\theta)\sigma\psi(k_1)}{2P\sqrt{Q/P+T_w}}+\frac{D(\pi+\pi_0(1-\theta))\sigma\psi(k_1)}{2nPQ\sqrt{Q/P+T_w}} \\
&\quad-\frac{D(\pi+\pi_0(1-\theta))\sigma\left(\sqrt{Q/P+T_w}\,\psi(k_1)+(n-1)\sqrt{T_s}\,\psi(k_2)\right)}{nQ^2}
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial P} &= \frac{Q}{2}h_v\left(\frac{nD-2D}{P^2}\right)+\left(a_2-\frac{a_1}{P^2}\right)D \\
&\quad-\frac{h_bk_1\sigma Q}{2P^2\sqrt{Q/P+T_w}}-\frac{h_b(1-\theta)\sigma\psi(k_1)Q}{2P^2\sqrt{Q/P+T_w}}-\frac{\frac{D\sigma}{n}(\pi+\pi_0(1-\theta))\psi(k_1)}{2P^2\sqrt{Q/P+T_w}}
\end{aligned}
\tag{22}
$$

$$
\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial \beta} = \frac{wnQD}{2}-\frac{\alpha v}{\beta}
\tag{23}
$$

The minimum value of $ETC_T(n, Q, k_1, \beta, P)$ occurs at the point (Q, k_1, β, P) which satisfies $\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial Q}=0$, $\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial k_1}=0$, $\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial \beta}=0$ and $\frac{\partial ETC_T(n,Q,k_1,\beta,P)}{\partial P}=0$ simultaneously. It can be seen that (19) is convex in k_1 and β, but might not be convex in Q and P. By setting (20)-(23) equal to zero, we will find:

$$
\frac{[1-F_s(k_1)]+(n-1)\left[1-F_s\left(k_1\sqrt{\frac{Q/P+T_w}{T_s}}\right)\right]}{F_s(k_1)+\theta[1-F_s(k_1)]}=\frac{h_bnQ}{D(\pi+\pi_0(1-\theta))}
\tag{24}
$$

$$
Q=\sqrt{\frac{2D\left\{\left(\frac{A+K}{n}+F\right)+\frac{(\pi+\pi_0(1-\theta))\sigma\left(\sqrt{Q/P+T_w}\,\psi(k_1)+(n-1)\sqrt{T_s}\,\psi(k_2)\right)}{n}\right\}}{\left(h_v\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}+h_b\right)+wnD\beta+\frac{h_bk_1\sigma}{P\sqrt{Q/P+T_w}}+\frac{h_b(1-\theta)\sigma\psi(k_1)}{P\sqrt{Q/P+T_w}}+\frac{D(\pi+\pi_0(1-\theta))\sigma\psi(k_1)}{nPQ\sqrt{Q/P+T_w}}}}
\tag{25}
$$

$$
\begin{aligned}
P &= \sqrt{\frac{a_1D+Qh_vD-\frac{Qh_vnD}{2}+\left(\frac{h_bk_1\sigma Q}{2\sqrt{Q/P+T_w}}+\frac{h_b(1-\theta)\sigma\psi(k_1)Q}{2\sqrt{Q/P+T_w}}+\frac{\frac{D\sigma}{n}(\pi+\pi_0(1-\theta))\psi(k_1)}{2\sqrt{Q/P+T_w}}\right)}{a_2D}} \\
&= \sqrt{\gamma}
\end{aligned}
\tag{26}
$$

$$
P = \text{Max}\,\{P_{min}, \text{Min}\,\{P_{max}, \sqrt{\gamma}\,\}\}
\tag{27}
$$

$$
\beta=\frac{2v\alpha}{wnQD}
\tag{28}
$$

As can be seen in (24)-(28), there is relationship between one another. The above formulations (P,Q,k) are relatively different with other related models since there are considerations on partial backorder and two types of safety stock. Therefore, a new procedure is needed to find the optimal values of decision variables. Here, a procedure is proposed to find the convergence values of Q, k_1, β and P. The procedure to solve the above problem is as follows.

Step 1 : Set n=1 and $ETC_T(Q^{n-1}, k_1^{n-1}, \beta^{n-1}, P^{n-1}) = \infty$

Step 2 : Set $\beta=\beta_0$ and compute

$$
P=\left[\sqrt{\frac{a_1}{a_2}}\right]
$$

$$
Q=\left[\sqrt{\frac{2D\left(\frac{A+K}{n}+F\right)}{\left(h_v\left\{n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right\}+h_b\right)+wnD\beta}}\right]
$$

where $[x]$ is the nearest integer to x

Step 3 : Compute k_1 from (24)

Step 4 : Compute β from (28)

Step 5 : Compute P from (27). If $\gamma < 0$ set $P = P_{min}$

Step 6 : For a given previous value of k_1, β and Q, compute new values of Q from (25), of k_1 from (24), of β from (28) and of P from (27).

Step 7 : Repeat steps 3 to 6 until no change occurs in the values of Q, k_1, β and P.

Step 8 : Compute ETC_T $(Q^n, k_1^n, \beta^n, P^n)$ from (19). If $ETC_T(Q^n, k_1^n, \beta^n, P^n) \leq ETC_T(Q^{n-1}, k_1^{n-1}, \beta^{n-1}, P^{n-1})$ repeat steps 2-7 with $n=n+1$, otherwise go to step 9.

Step 9 : Compute $ETC_T(Q^*, k_1^*, \beta^*, P^*) = ETC_T(Q^{n-1}, k_1^{n-1}, \beta^{n-1}, P^{n-1})$. Then Q^*, k_1^*, β^*, P^* and n^* are the solution for the proposed problem.

3.0 RESULTS AND DISCUSSION

In order to illustrate the above procedure, let us consider the proposed method to solve a numerical example. This example is a modified version from the one given in Jauhari [31] to fit the present supply chain configuration. The complete values of the parameters used in this numerical example are shown in Table 2.

Table 2 The values of the parameters used in the numerical example

Parameters	Values	Parameters	Values
D	1,000 units/year	w	15/unit
σ	5 units/year	β_0	0.002
A	\$50/order	λ	0.0025
F	\$50/shipment	K	\$400/setup
h_b	\$5/unit/year	P_{min}	1,500 units/year
h_v	\$4/unit/year	P_{max}	5,000 units/year
π	\$50/unit	a_1	\$2,500
π_0	\$150/unit	a_2	\$0.0004/unit
θ	0.5	T_w	0.1 year
α	\$0.1/year	T_s	0.05 year

Employing the proposed algorithm, the solution of the above problem is presented in Table 3. The optimal number of deliveries is $n = 4$ and the optimal shipment size is $Q^* = 153$ units. Since $n^* = 4$, the optimal vendor's production batch is 612 units and the delivery interval is 0.153 years or 53.55 days (1 years = 350 days). The optimal production rate is 2,178.816 units and the length of production time is 0.28 years or 98.3 days. The optimal total annual cost for all parties involved in the supply chain is \$4,382.344. The probability of the production process can go out of control is $\beta = 8.71 \times 10^{-6}$ which is smaller than β_0. The illustration of the impact of standard deviation of demand changes on model's behavior is provided in Table 4.

It can be seen that as the standard deviation of demand continually increases, the buyer and vendor cost increases as well. Facing more uncertain demand, the buyer will keep more safety stock to increase the inventory level and reduce the probability of stock out. The shipment quantity is reduced gradually which leads to the increase in delivery frequency. Interestingly, the vendor can maintain the production rate in such level to reduce the lead time. For the vendor, the increase in demand uncertainty will give slight impact to his cost. However, the increase in cost is mainly absorbed by the buyer. Nevertheless, in view of supply chain perspective, the risk of demand uncertainty should be maintained jointly in some equitable manner to increase the efficiency of an integrated decision.

Table 5 gives a description about the impact of production cost on model's behavior. It can be seen that facing an increase in production cost, the cost incurs by the vendor increases significantly while the cost of buyer is considerably constant. This is undertandable because the production activity occurs at vendor side, hence the buyer will not be directly affected by the increase in production cost. Looking at the results from the Table 5, we also find that as production cost increases, the vendor tends to produce with lower production rate to prevent a suffer from a high production cost.

Other interesting point to note here is that, when we look carefully to the behaviors of P and β on Tables 4 to 7, we will find that both parameters have similar trends. If the production rate increases, the probability of the production process can increase out of control as well. This point suggests that the production planner should carefully adjust the production rate because this decision also gives an impact to the process capability. Having larger production rate, the system may decrease the lead time which leads to the decrease in inventory cost, but it may influence the increase in quality improvement cost.

The impact of the change in λ on proposed model is presented in Table 6. As λ gets larger, the process quality (β) decreases. Having a constant value of capital investment, the increase in λ will give the vendor a higher capability to reduce the defect rate which can lead to reducing the vendor cost. Producing larger batch in each production run, the vendor will have more inventory but it will result in less setup frequency. As the increase in holding cost is less than the decrease in setup cost, the vendor receives the benefit from having a large production batch.

Moreover, having larger λ, the buyer cost is decreased due to the sharp decrease in ordering frequency and safety factor. It can be noted that here vendor has a chance to adjust production rate and to decrease the probability of defect rate which will give positive impact to the both parties in supply chain. Although the buyer is not directly affected by vendor's policy, he can get an advantage from shorter lead time and lower ordering frequency.

Table 7 gives an explanation on how the increase in capital investment can affect the model's behavior. As can be seen in Table 6, the vendor cost increases significantly due to the increase in capital investment. flexibly, it will give the system an opportunity to maintain the cost. However, the decrease of production and holding cost on vendor side cannot meet the increase in capital investment, thus the vendor cost increases. It can also been seen that, as the capital investment increases, β will increase as well. It can be understood because λ is set in a constant value, hence the becomes very expensive for the system. For the case of low capital investment ($\alpha \leq \$0.5$), the cost incurred by the buyer is relatively constant. However, when the capital investment increases to higher values ($\alpha > \$0.5$) the buyer cost will increase significantly due to the increase in safety stock.

Table 3 The optimal solution for the proposed problem

n	Q	P	k_1	β	Total cost
1	389	2,873.674	4,68295	1.37×10^{-5}	4,817.823
2	243	2,508.32	4,106961	1.09×10^{-5}	4,500.424
3	185	2,314.633	3,826584	9.6×10^{-6}	4,408.507
4	153	2,178.816	3,654795	8.71×10^{-6}	4,382.344
5	133	2,068.522	3,538604	8.02×10^{-6}	4,383.655

Table 4 The effect of the change in σ on proposed model

σ	n	Q	k_1	P	β	Vendor cost	Buyer cost	Total cost
10	4	153	1.981	2,185	$10^{-6} \times 8.71$	3,567.10	839.50	4,406.60
50	5	132	1.956	2,123	$10^{-6} \times 8.08$	3,576.24	1,018.80	4,595.05
100	5	131	1.960	2,182	$10^{-6} \times 8.14$	3,577.16	1,251.59	4,828.74
150	5	130	1.965	2,239	$10^{-6} \times 8.20$	3,579.04	1,482.28	5,061.32
200	6	116	1.946	2,187	$10^{-6} \times 7.66$	3,590.90	1,697.20	5,288.10
250	6	115	1.951	2,239	$10^{-6} \times 7.72$	3,593.26	1,920.78	5,514.03
300	7	105	1.936	2,189	$10^{-6} \times 7.25$	3,607.03	2,131.88	5,738.91

Table 5 The effect of the change in a_1 and a_2 on proposed model

a_1	a_2	n	Q	k_1	P	β	Vendor cost	Buyer cost	Total cost
1,000	0.00025	6	129	1.884	1,500	$10^{-6} \times 6.89$	2,427.29	799.12	3,226.41
1,500	0.0003333	6	129	1.884	1,500	$10^{-6} \times 6.89$	2,885.63	799.12	3,684.74
2,500	0.0004	4	153	1.981	2,179	$10^{-6} \times 8.71$	3,567.08	815.26	4,382.34
3,500	0.001	6	127	1.892	1,579	$10^{-6} \times 6.99$	5,213.29	800.94	6,014.22
5,000	0.002	6	129	1.884	1,500	$10^{-6} \times 6.89$	7,718.96	799.12	8,518.07
7,000	0.004	6	129	1.884	1,500	$10^{-6} \times 6.89$	12,052.29	799.12	12,851.41

Table 6 The impact of the change in λ on proposed model

λ	n	Q	k_1	P	β	Vendor cost	Buyer cost	Total cost
0.000025	1	115	2.605	2,618	$10^{-3} \times 4.63$	6,203.97	1,184.75	7,388.72
0.000125	2	174	2.203	2,507	$10^{-4} \times 3.06$	5,797.95	892.31	6,690.26
0.000625	4	144	2.007	2,199	$10^{-5} \times 3.70$	4,111.23	818.18	4,929.41
0.003125	4	154	1.978	2,177	$10^{-6} \times 6.92$	3,522.86	815.12	4,337.98
0.015625	5	135	1.945	2,061	$10^{-6} \times 1.26$	3,370.53	805.52	4,176.05

Further, the vendor tends to decrease the production lot which leads to having lower inventory level and tries to adjust production rate to higher value to reduce the production cost and lead time. Facing this situation, the vendor seems to decrease the holding cost and production cost to decrease the impact of the increase in capital investment. It is clear that by allowing the vendor to control production rate

The performance of the proposed model compared to three models, i.e. a model with fixed production rate, a model with fixed quality, a model with fixed production rate and quality, is presented in Table 8. When comparing the proposed model with the fixed production rate model, it is obvious that there is a total cost improvement for about 1.98%. It also can be proved that if the proposed model is compared to fixed process quality model, it gives significant reduction in total cost (45.7%). Furthermore, if the proposed model is compared with the model without controllable production rate and quality improvement, it will give more significant saving (45.9%). This implies that by allowing the vendor to control the production rate and to invest an amount of money to reduce the defect rate, both parties in supply chain system can reduce the cost.

Table 7 The impact of the change in α on proposed model

α	n	Q	k_1	P	β	Vendor cost	Buyer cost	Total cost
0.1	4	153	1.981	2,179	$10^{-6} \times 8.71$	3,567.08	815.26	4,382.34
0.3	4	147	1.998	2,192	$10^{-5} \times 2.72$	3,947.01	816.86	4,763.87
0.5	4	142	2.012	2,204	$10^{-5} \times 4.69$	4,264.78	819.27	5,084.06
0.7	4	136	2.031	2,217	$10^{-5} \times 6.86$	4,544.59	823.62	5,368.21
0.9	3	159	2.080	2,342	$10^{-4} \times 1.006$	4,779.26	841.81	5,621.07

Table 8 The comparison of proposed model and fixed quality and production rate models

Model	n	Q	k_1	P	β	Vendor cost	Buyer cost	Total cost
Fixed production rate	3	182	2.027	3,000	$10^{-6} \times 9.768$	3,625.47	845.29	4,470.76
Fixed process quality	1	165	2.479	2,667	$10^{-3} \times 2$	7,027.16	1,046.77	8,073.92
Fixed production rate and process quality	1	166	2.477	3,000	$10^{-3} \times 2$	7,064.64	1,044.71	8,109.35
Proposed model	4	153	1.981	2,179	$10^{-6} \times 8.714$	3,567.08	815.26	4,382.34

4.0 CONCLUSIONS

In this paper, we examined the effect of incorporating quality improvement and controllable production rate in JELS model. We intended to minimize total cost by simultaneously determine safety factor, delivery lot size, delivery frequency, process quality and production rate. To determine the solution of the model, we proposed an algorithm.

The proposed model is more applicable since it considers some factors such as stochastic demand, imperfect production, process quality and adjustable production rate. The results show that considering all factors simultaneously in the model rather than just single factor, such as fixed production rate or fixed quality lead to better economic choice for a supply chain system like one in this paper. The proposed model can help the managers to keep the inventories in the supply chain more efficiently. It can assist the managers to decide the optimum ordering quantity, quality level and level of production rate. The managers can determine the appropriate level of inventory to deal with demand uncertainty. As demand uncertainty increases, the optimum ordering quantity and safety factor can be determined precisely by the managers. The results indicate that the ordering quantity and the safety factor should be raised to minimize the impact of increased demand uncertainty.

The existence of the defective items and the quality investment made by the vendor significantly impact the system's performance. As we assumed that the production system is imperfect, the investment in quality improvement becomes important activity that should be done by the vendor to reduce the probability of defect. The model gives a guidance to managers to decide the optimum quality level that should be achieved by the production system. Clearly, the optimum process quality is significantly influenced by the percentage decrease in probability of out of control per dollar investment (λ). Therefore, the managers must be able to make a good plan so that the investment can produce maximum benefits.

Furthermore, the level of production rate can also be determined precisely by adopting the proposed model. It is proved that speeding or slowing the production rate significantly affect the supply chain's performance which cautions managers not to only focus on minimizing traditional costs (ordering, holding, setup, transportation) as the impact of production rate will go unnoticed.

The model developed here contains some limitations. We ignore the inspection process which is usually done by the buyer to screen out the defective items from the received lots. However, if it is considered in the model there would be different calculation of holding cost. In that situation, the holding cost can be divided into two types, which are holding cost for good items and holding cost for defective items. In addition, the rework process is assumed to be 100% perfect. In real situations, the rework may fail so that the resulting items cannot be sold to the primary market.

Future studies can look into incorporating inspection cost in vendor-buyer system. Here, the vendor might produce defective products due to deterioration in production process. The buyer might also conduct inspection activity to assure that the products delivered from vendor are always in good quality. Hence, incorporating inspection policy such as 100% inspection or sampling inspection into the model will give different insights. The study can also be extended by incorporating raw material procurement decision. Future study may also investigate the impact of defective item on both raw material and finished product on vendor side.

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