Integrated Model Of Industrial Robot For Control Applications

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ABSTRACT

This paper deals with the development of an intergrated mathematical model of a robot manipulator. The model of the system comprises the mechanical part of the robot as well as the actuators and the gear trains. Two different approaches of deriving the integrated model are presented which results in two different forms of the integrated dynamic model of the robot manipulator in state space description. Both types of the integrated model are highly nonlinear, time varying, and represent a more realistic model of the robotic system. The integrated model and the approach are useful and suitbale for dynamic analysis and control synthesis purposes, and will provide a more efficient approach to the real situation.

INTRODUCTION

An Important initial step in the design of controllers for an industrial robot is to obtain a complete mathematical model of the industrial robot. Typical industrial robots can be modeled as an open kinematic chain of N-rigid bodies or links, connected in series by N joints. Normally, the joints are actuated by either electric or hydraulic actuators [12].

To improve the performance of robot manipulators, various control strategies have been proposed in the available literature [1-5, 8-12, 14-17]. However, in much of the literature, the dynamics of the actuators, which are part of the whole manipulator system, have generally been ignored and the drive torques of forces are modeled as pure torque or force [2, 4, 5, 8, 9]. This, in the majority of cases, is a simplification of a much more realistic model of the system [1].

Since the actuators are part of the whole system, it is necessary to form an integrated mathematical model comprising the mechanical part of the system and the actuators. Several authors have the complete model of the robotic system [1,3,10,11,14,16,17]. However, in some of the approaches [3,16], the method is too complicated.

The purpose of this paper is to give a unifying framework for the formulation of the complete mathematical dynamic model of an electrically driven robot manipulator in state variable form. Two approaches are presented. The formulation results in nonlinear time varying state equations which represent a more realistic model than the pure torque or force generator. In the first approach, the joint angles, velocities and the armature current of the actuating mechanisms are chosen as the state variables. The formulation of the overall intergrated model of the manipulator and the actuator dynamics is simple and straightforward. The resulting intergrated model can be decomposed into input decentralized form easily. However, the formulation also results in that the nonlinear, uncertain and coupling terms to lie outside the range space of the input matrix of the intergrated model state equation. In the second approach, a different set of state variables is chosen. The resulting structure of the integrated model is different from the prevoious one in that the nonlinear, uncertain and coupling terms lie in the range space of the input matrix of the derived state equation. However, the derivation is not as straightforward as the prevoius one as it is necessary to find the time derivative of the nonlinear, coupled dynamic equation of the mechanical part of the manipulator. The integrated dynamic of the robot manipulator derived are by no means represent a complete model of the robotic system since the drive system nonlinearities such as Coulomb friction, viscous friction, backlash, stiffening spring characteristic of the actuator are not included in integrated dynamic model derivation. The model and the approaches are useful and suitable for dynamic analysis and control synthesis purposes, and it will lead to a very efficient and convenient approach for applying a number of advanced control algorithms for controlling and industrial robot, such as multivariable control theory, model reference adaptive control techniques [2], decentralized control methods [10,11,14], and hierarchical control strategies [5,10].

MANIPULATOR DYNAMICS

A number of techniques for developing an efficient analytical model of a manipulator are available. Among these are the Lagrangian-Euler method [6], the Newton-Euler method [7], and the generalized d'Alembert formulation [15]. All of these method provide equations which describe the three-dimension system motion.

For an N degree-of freedom (dof) manipulator, the dynamic equations describing the motion of the manipulator can be written in the following matrix form as:

$$\mathrm{M}(\theta(t))\,\ddot{\theta}(t) + \mathrm{C}(\dot{\theta}(t),\,\dot{\theta}(t)) = \mathrm{T}(t) \qquad \text{Eqn. 1}$$

or

$$\mathrm{M}(\theta(t))\,\ddot{\theta}(t) + \mathrm{D}(\theta(t),\,\dot{\theta}(t)) + \mathrm{G}(\theta(t)) = \mathrm{T}(t). \qquad \text{Eqn. 2}$$

The component $\mathrm{D}(\theta(t),\,\dot{\theta}(t))$ can be written in a different form as follows:

$$\mathrm{D}(\theta(t),\,\dot{\theta}(t)) = \mathrm{D}(\theta(t))\,\mathrm{H}(\dot{\theta}(t)) \qquad \text{Eqn. 3}$$

where

$$\mathrm{H}(\dot{\theta}(t)) = \begin{bmatrix} \dot{\theta}_1(t)\dot{\theta}_1(t) \\ \dot{\theta}_1(t)\dot{\theta}_2(t) \\ \vdots \\ \dot{\theta}_{N-1}(t)\dot{\theta}_N(t) \\ \dot{\theta}_N(t)\dot{\theta}_N(t) \end{bmatrix} \qquad \text{Eqn. 4}$$

and

$\mathrm{M}(\theta(t))$	:	$N \times N$ positive definite, bounded, and symmetrical inertia matrix
$\mathrm{D}(\theta(t), \dot{\theta}(t))$	:	$N \times 1$ vector incorporating the Coriolis, centrifugal, and gravitational forces
$\mathrm{D}(\theta(t), \dot{\theta}(t))$	:	$N \times 1$ vector of Coriolis and Centrifugal forces
$\mathrm{G}(\theta(t))$	:	$N \times 1$ vector of gravitational forces
T	:	$N \times 1$ vector of driving forces/torques
$\theta(t), \dot{\theta}(t), \ddot{\theta}(t)$	:	$N \times 1$ vectors of joint displacements, velocities, and accelerations respectively.

Equation (2) can be rewritten in state variable from [2,10]. Let the state variables for the N dof manipulator be:

$$\begin{aligned} x_{pi} &= \theta_i \\ x_{pi+N} &= \dot{\theta}_i, \quad i = 1, 2, \ldots, N, \end{aligned} \qquad \text{Eqn. 5}$$

and, hence, the 2N-dimensional state vector is:

$$\mathrm{X}_p(t) = [\theta^T, \dot{\theta}^T]^T = [x_{pi}, \dot{x}_{pi+N}]^T \qquad \text{Eqn. 6}$$

By using the notations:

$$G'(x_{pi}) : G'(x_{pi}) \begin{bmatrix} x_{p1} \\ \vdots \\ x_{pN} \end{bmatrix} = G(x_{pi}), \qquad \text{Eqn. 7}$$

$$E^T(\dot{x}_{pi}) = \left[\begin{array}{c|c|c} \dot{x}_{p1} I_N & \multicolumn{2}{c}{0^T} \\ \hline & \dot{x}_{p2} I_{N-1} & \begin{array}{c} 0^T \\ \hline \ddots \\ \hline \dot{x}_{pN} \end{array} \end{array}\right] \qquad \text{Eqn. 8}$$

where I_i, is the ixi identity matrix, in terms of the variables, equation (2) becomes:

$$\dot{X}_p(t) = A_p(X_p, t)X_p(t) + B_p(X_p, t)T(t) \qquad \text{Eqn. 9}$$

where

$$A_p(X_p, t) = \left[\begin{array}{c|c} 0 & I_N \\ \hline Ap1(X_p, t) & Ap2(X_p, t) \end{array}\right] \qquad \text{Eqn. 10}$$

$$Apl(X_p, t) = -M^{-1}(x_{pi})G'(x_{pi}) \qquad \text{Eqn. 11}$$

$$Ap2(X_p, t) = -M^{-1}(x_{pi})D(x_{pi})E(\dot{x}_{pi}) \qquad \text{Eqn. 12}$$

and

$$B_p(X_p, t) = \begin{bmatrix} 0 \\ \dots\dots \\ M^{-1}(x_{pi}) \end{bmatrix} \qquad \text{Eqn. 13}$$

Each element of the matrices (10) and (13) is a nonlinear function of the state variables, taking into account the contribution of the inertia matrix, Coriolis, centrifugal, and gravitational forces.

ACTUATOR DYNAMICS

For robot manipulators, permanent magnet D.C motors and electrohydraulic actuators are widely used as the actuating mechanism. Here, an electrically driven manipulator is considered where the actuator model can be represented by the following linear time invariant differential equations

$$J_{mi}\ddot{\theta}_{mi}(t) = -B_{vi}\dot{\theta}_{mi}(t) + K_{ti}\dot{I}_i(t) - T_{Li}(t)/N_i \qquad \text{Eqn. 14}$$

$$L_i\dot{I}_{mi}(t) = -K_{vi}\dot{\theta}_{mi}(t) - R_i\dot{I}_i(t) + V_i(t) \qquad \text{Eqn. 15}$$

where

J_{mi}	:	moment of inertia for ith motor (Kgm^2)
$\theta_{mi}(t)$	:	angular displacement for ith motor (rad)
B_{vi}	:	viscous friction coefficient for ith motor (Nm/rad/s)
K_{ti}	:	torque constant for ith motor (Nm/A)
K_{vi}	:	back emf constant for ith motor (V/rad/s)
L_i	:	armature inductance for ith motor (H)
R_i	:	armature resistance for ith motor (Ω)
$I_i(t)$	:	armature current for ith motor (A)
$T_{Li}(t)$	:	load due to ith joint of the manipulator on the motor
N_i	:	gear ratio at the ith joint.

Equations (14) and (15) can be combined to form a single third-order differential equation as follows :

$$\dddot{\theta}_{mi}(t) + \frac{(B_{vi}L_i + J_{mi}R_i)}{J_{mi}L_i}\ddot{\theta}_{mi}(t) + \frac{(K_{vi}K_{ti} + B_{vi}R_i)}{J_{mi}L_i}\dot{\theta}_{mi}(t) + \frac{R_i T_{Li}(t)}{N_i J_{mi} L_i} + \frac{\dot{T}_{Li}(t)}{N_i J_{mi}} = \frac{K_{ti}V_i(t)}{J_{mi}L_i} \qquad \text{Eqn. 16}$$

where $\dot{T}_{Li}(t)$ is the time derivative of the load due to the ith joint of the manipulator on the ith motor, i.e., $T_{Li}(t)$.

To reflect the actuator dynamics to the manipulator side of the gearing mechanism, we use the following identity

$$\theta_{\mathrm{mi}}(t) = N_i\theta_i(t), \quad N_i >> 1. \qquad \text{Eqn. 17}$$

In the following , two different forms of actuator dynamics in state space description are given. The first form is based on equations (14) and (15) where the joint position, joint velocity and armature current are chosen as state variables.

By defining a 3×1 state vector of the ith actuator to be :

$$X_{Ai}(t) = [\theta_i(t), \dot{\theta}_i(t), I_i(t)]^T, \qquad \text{Eqn. 18}$$

equations (14) and (15) can be rewritten in state variable form as follows :

$$\dot{X}_{Ai}(t) = A_{Ai}X_{Ai}(t) + B_{Ai}U_i(t) + F_{Ai}T_{Li}(t), \qquad \text{Eqn. 19}$$

where

$$A_{Ai} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \dfrac{-B_{vi}}{J_{mi}} & \dfrac{K_{ti}}{N_i J_{mi}} \\ 0 & \dfrac{-F_{vi}}{L_i} & \dfrac{-R_i}{L_i} \end{bmatrix}, \quad F_{Ai} = \begin{bmatrix} 0 \\ -\dfrac{1}{N_i^2 J_{mi}} \\ 0 \end{bmatrix} \qquad \text{Eqn. 20}$$

$$B_{Ai} = \begin{bmatrix} 0 \\ 0 \\ 1/L_i \end{bmatrix}, \quad U_i(t) = V_i(t).$$

$X_{Ai}(t)$: $n_i$xl state vector of the ith actuator, where n_i is the order of the ith actuator
$U_i(t)$: scalar input to the ith actuator
$T_{Li}(t)$: the load acting on the ith actuator due to the manipulator itself (from equation 1 or 2).

and $n_i = 3$. A_{Ai}, B_{Ai} and F_{Ai} are the system, input and load distribution matrices for the ith actuator respectively.

For N actuators (N dof robot), the augmented dynamic equation of the actuators can be written in the compact form which is as follows:

$$\dot{X}_A(t) = A_A X_A(t) + B_A U(t) + F_A T_L(t) \qquad \text{Eqn. 21}$$

$$X_A(t_o) = X_{Ao}, \qquad \text{Eqn. 22}$$

where

$$\mathrm{X}_A(t) = [\mathrm{X}_{A1}^{\mathrm{T}}(t), \mathrm{X}_{A2}^{\mathrm{T}}(t), \ldots, \mathrm{X}_{AN}^{\mathrm{T}}(t)]^{\mathrm{T}}$$
$$\mathrm{U}(t) = [\mathrm{U}_1(t), \mathrm{U}_2(t), \ldots, \mathrm{U}_N(t)]^{\mathrm{T}}$$
$$\mathrm{T}_L(t) = [\mathrm{T}_{L1}(t), T_{L2}(t), \ldots, T_{LN}(t)]^T \qquad \text{Eqn. 23}$$
$$\mathrm{A}_A = \operatorname{diag}[\mathrm{A}_{A1}, \mathrm{A}_{A2}, \ldots, \mathrm{A}_{AN}]$$
$$\mathrm{B}_A = \operatorname{diag}[\mathrm{B}_{A1}, \mathrm{B}_{A2}, \ldots, \mathrm{B}_{AN}]$$
$$\mathrm{F}_A = \operatorname{diag}[\mathrm{F}_{A1}, \mathrm{F}_{A2}, \ldots, \mathrm{F}_{AN}]$$

and $\mathrm{X}_A(t)$ is an $N \times 1$ vector, where $\mathrm{N} = \sum_{i=1}^{N} \mathrm{n}_i$.

The load acting on the ith actuator $\mathrm{T}_{Li}(t)$ is given by the ith element of the vector $\mathrm{T}(t)$, of equation (1). Thus,

$$\mathrm{T}_L(t) = T(t) \qquad \text{Eqn. 24}$$

Remark – If the motor armature inductance is negligible, the third-order actuator model can be reduced to a second-order model ($\mathrm{n}_i = 2$), and the 2×1 state vector becomes $\mathrm{X}_{Ai}(t) = [\theta_i(t), \dot{\theta}_i(t)]^{\mathrm{T}}$, and elements of matrices A_{Ai}, B_{Ai}, F_{Ai} are as follows:

$$\mathrm{A}_{Ai} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{\mathrm{B}_{vi}\mathrm{R}_i + \mathrm{K}_{ti}\mathrm{K}_{vi}}{\mathrm{J}_{mi}\mathrm{R}_i} \end{bmatrix}, \mathrm{B}_{Ai} = \begin{bmatrix} 0 \\ \dfrac{\mathrm{K}_{ti}}{\mathrm{N}_i\mathrm{J}_{mi}\mathrm{R}_i} \end{bmatrix}, \mathrm{F}_{Ai} = \begin{bmatrix} 0 \\ -\dfrac{1}{\mathrm{N}_i^2\mathrm{J}_{mi}} \end{bmatrix}$$

From equation (16), we can obtain the second form of the actuator dynamic model in state variable form. Here the amature current is replaced by joint acceleration as state variable. Let the state vector for the ith actuator is define as follows:

$$\mathrm{X}_{Bi}(t) = [\theta_i(t), \dot{\theta}_i(t), \ddot{\theta}_i(t)]^{\mathrm{T}} \qquad \text{Eqn. 25}$$

Then, from equation (16), using (17) and (24), the actuator dynamic model can be rewritten in the following form:

$$\dot{\mathrm{X}}_{Bi}(t) = \mathrm{A}_{Bi}\mathrm{X}_{Bi}(t) + \mathrm{B}_{Bi}\mathrm{U}_i(t) + \mathrm{F}_{Bi}\mathrm{T}_i(t) + \mathrm{W}_B\dot{\mathrm{T}}_i(t), \qquad \text{Eqn. 26}$$

where

$$\mathrm{A}_{Bi} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\dfrac{\mathrm{K}_{vi}\mathrm{K}_{ti} = \mathrm{B}_{vi}\mathrm{R}_i}{\mathrm{J}_{mi}\mathrm{L}_i} & -\dfrac{\mathrm{B}_{vi}\mathrm{L}_i + \mathrm{J}_{mi}\mathrm{R}_i}{\mathrm{J}_{mi}\mathrm{L}_i} \end{bmatrix}, \mathrm{B}_{Bi} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{\mathrm{K}_{ti}}{\mathrm{J}_{mi}\mathrm{L}_i\mathrm{N}_i} \end{bmatrix} \qquad \text{Eqn. 27}$$

$$\mathrm{F}_{Bi} = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{\mathrm{R}_i}{\mathrm{N}_i^2\mathrm{J}_{mi}\mathrm{L}_i} \end{bmatrix}, \mathrm{W}_{Bi} = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{1}{\mathrm{N}_i^2\mathrm{J}_{mi}} \end{bmatrix}, \mathrm{U}_i(t) = \mathrm{V}_i(t).$$

$\mathrm{X}_{Bi}(t)$: 3×1 state vector of the ith actuator
$\mathrm{U}_i(t)$: scalar input to the ith actuator
$\mathrm{T}_i(t)$: the load acting on the ith actuator due to the manipulator itself (from equation 1 or 2),

and A_{Bi}, B_{Bi}, F_{Bi}, and W_{Bi}, are the system, input, load distribution and rate of load distribution matrices respectively, for the ith actuator.

For N dof of robot manipulator, the augmented dynamic equation of the actuators can be written in the compact form as follows:

$$\dot{X}_B(t) = A_B X_B(t) + B_B U(t) + F_B T(t) + W_B \dot{T}(t), \qquad \text{Eqn. 28}$$

$$X_B(t_o) = X_{Bo}, \qquad \text{Eqn. 29}$$

where

$$\begin{aligned}
X_B(t) &= [X_{B1}^T(t), X_{B2}^T(t), \ldots, X_{BN}^T(t)]^T \\
U(t) &= [U_1(t), U_2(t), \ldots, U_N(t)]^T \\
T(t) &= [T_1(t), T_2(t), \ldots, T_N(t)]^T \\
A_B &= \text{diag}\,[A_{B1}, A_{B2}, \ldots, A_{BN}] \\
B_B &= \text{diag}\,[B_{B1}, B_{B2}, \ldots, B_{BN}] \\
F_B &= \text{diag}\,[F_{B1}, F_{B2}, \ldots, F_{BN}] \\
W_B &= \text{diag}\,[W_{B1}, W_{B2}, \ldots, W_{BN}]
\end{aligned} \qquad \text{Eqn. 30}$$

and $X_B(t)$ is an $N \times 1$ vector, where $N = \sum_{1}^{N} 3$.

MANIPULATOR AND ACTUATOR DYNAMIC MODEL INTEGRATION

In this section, two different state space representations of the manipulator and actuator dynamic model integration are presented. The formulation of the first form of the integrated model is based on the actuator dynamic model described by equation (21), while the derivation of the second form of the integrated model of the manipulator and its actuating mechanism is based on the actuator dynamic model in term of equation (28). Let us refer to these two forms of the integrated robot manipulator model as Form A and B respectively.

Form A

This method is based on the dynamic equation of the manipulator in state variable form of equaiton (9) and the actuators dynamic described by equation (21). Let the transformation between the manipulator state vector $X_p(t)$ and the actuator state vector $X_A(t)$ be Z_A, such that,

$$X_P(t) = Z_A X_A(t), \qquad \text{Eqn. 31}$$

where the $2N \times 3N$ transformation matrix Z_A has the following form:

$$Z_A = \left[\begin{array}{cccc}
100 & & & \mathbf{0} \\
 & 100 & & \\
 & & \ddots & \\
\mathbf{0} & & & 100 \\
\hdashline
010 & & & \mathbf{0} \\
 & 010 & & \\
 & & \ddots & \\
\mathbf{0} & & & 010
\end{array}\right] \qquad \text{Eqn. 32}$$

Substitution of equation (31) into (9), gives

$$Z_A \dot{X}_A(t) = A_p(X_A(t), t) Z_A X_A(t) + B_p(X_A(t), t) T(t). \quad \text{Eqn. 33}$$

Form equation (34), the driving forces/torques $T(t)$ can be obtained as

$$T(t) = B_p^{\dagger}(X_A(t), t) Z_A \dot{X}_A(t) - B_p^{\dagger}(X_A(t), t) A_p(X_A(t), t) Z_A X_A(t), \quad \text{Eqn. 34}$$

where $B_p^{\dagger}(X_A(t), t)$ is the Penrose-Pseudoinverse of $B_p(X_A(t), t)$:

$$B_p^{\dagger}(X_A(t), t) = [B_p^{T}(X_A(t), t) B_p(X_A(t), t)]^{-1} B_p^{T}(X_A(t), t). \quad \text{Eqn. 35}$$

Using (24), substituting (37) into the actuators state equation (21), gives the state equation of the integrated system model as:

$$\dot{X}_A(t) = A(X_A, t) X_A(t) + B(X_A, t) U(t), \quad \text{Eqn. 36}$$

where

$$A(X_A, t) = [I_N - F_A B_p^{\dagger}(X_A(t), t) Z_A]^{-1} [A_A - F_A B_p^{\dagger}(X_A(t), t) A_p(X_A(t), t) Z_A] \quad \text{Eqn. 37}$$

$$B(X_A, t) = [I_N - F_A B_p^{\dagger}(X_A(t), t) Z_A]^{-1} B_A. \quad \text{Eqn. 38}$$

In this method, it is required to find the pseudoinverse of the matrix $B_p(X_A(t), t)$. In the following, the existence and the uniqueness* of the matrix $B_p^{\dagger}(X_A(t), t)$ will be shown.

Since the manipulator inertia matrix $M(X_A(t), t)$ is always symmetric and nonsingular, the following properties of the inertia matrix hold for any value of $X_A(t)$:

$$M(X_A(t), t) = M^{T}(X_A(t), t) \quad \text{Eqn. 39}$$
$$[M^{-1}(X_A(t), t)]^{T} = M^{-1}(X_A(t), t).$$

Hence from (13),

$$B_p^{T}(X_A(t), t) = [0 \vdots M^{-1}(X_A(t), t)]. \quad \text{Eqn. 40}$$

This gives

$$B_p^{T}(X_A(t), t) B_p(X_A(t), t) = [M^{-1}(X_A(t), t)]^2, \quad \text{Eqn. 41}$$

and

$$[B_p^{T}(X_A(t), t) B_p(X_A(t), t)]^{-1} = [M(X_A(t), t)]^2 \quad \text{Eqn. 42}$$

Thus

$$\begin{aligned} B_p^{\dagger}(X_A(t), t) &= [M(X_A(t), t)]^2 [0 \vdots M^{-1}(X_A(t), t)] \\ &= [0 \vdots M(X_A(t), t)]. \end{aligned} \quad \text{Eqn. 43}$$

Since $M(X_A(t), t)$ exists and is unique, therefore, $B_p^{\dagger}(X_A(t), t)$ also exist and is unique. This concludes the proof.

Equation (44) not only provides the proof for the existence and the uniqueness of the matrix $B_p^{\dagger}(X_A(t), t)$, but also provides a simple method of determining the matrix $B_p^{\dagger}(X_A(t), t)$.

* It should be noted that in general, the Penrose-Pseudoinverse of a matrix is not unique.

For the third order actuator model ($n_i = 3$), due to the structures of the F_A and B_A matrices, it is observed that equation (39) are equivalent to the actuators input matrix B_A, which is constant and independent of $X_A(t)$ and t:

$$B(X_A, t) = B_A \qquad \text{Eqn. 44}$$

This can be verified from the structure of the actuator dynamic equations (14) and (15). Equations (14) and (15) are 'independent' of the input voltage $v_i(t)$ and the load torque $T_i(t)$, respectively. Due to the structure of the load distribution matrix F_{Ai}, the components of the load torque $T_i(t)$ (that is, the link inertias, the Coriolis and centrifugal forces, etc.) will coupled with the elements of the second row of the system matrix A_{Bi} only. Thus, equation (15) remains the same when the ith mechanical link dynamic equation $T_i(t)$, which can be obtained from the mechanical link equation (9), is directly substituted into equation (14). Hence, the input term $V_i(t)/L_i$ is unchanged for the integrated model for the ith link. Hence, the input matrix $B_A(X_A, t)$ for the integrated model remains the same as the input matrix of the augmented actuator model B_A.

The method presented above is different from those outlined by Vukobratovic et. al [1985], and Troch [13]. The main difference lies in the choice of the form of the dynamic equation for the mechanical linkage used in the formulation. Here, the formulation of the integrated model is based on the mechanical link dynamic model in state space form (9), while those in the references based their formulation on equation (1). Furthermore, the structure of the integrated dynamic model obtained here is slightly different from that of Vukobratovic et. al [16], Troch [13]. However, as it is shown in the Appendix, the integrated model derived above (equations 36, 37, and 38) is equivalent to those obtained by Vukobratovic et. al [16], Troch [13].

Form B

Here, the integrated robotic model based on equation (28) of the actuator dynamics is presented. The derivation of the integrated model is not as straightforward as the previous one due to the need to find the time derivative of the dynamic equation of the mechanical part of the manipulator.

From equation (2), the derivative of the torque, $T(t)$, may be written as:

$$\dot{T}(t) = M(\theta(t), t)\dddot{\theta}(t) + \tilde{C}(\theta(t), \dot{\theta}(t))\ddot{\theta}(t) + \tilde{D}(\theta(t), \dot{\theta}(t))\dot{\theta}(t) \qquad \text{Eqn. 45}$$

where

$$\tilde{C}(\theta(t), \dot{\theta}(t))\ddot{\theta}(t) = \dot{M}(\theta(t), t)\ddot{\theta}(t) + D(\theta(t))\dot{H}(\dot{\theta}(t)) \qquad \text{Eqn. 46}$$

$$\tilde{D}(\theta(t), \dot{\theta}(t))\dot{\theta}(t) = \dot{D}(\theta(t))H(\dot{\theta}(t)) + \dot{G}(\theta(t)), \qquad \text{Eqn. 47}$$

and $\tilde{C}(\theta(t), \dot{\theta}(t))$ and $\tilde{D}(\theta(t), \dot{\theta}(t))$ are $N \times N$ matrices. Define the following transformations:

$$\begin{aligned} \ddot{\theta}(t) &= Z_B Z_B(t) \\ \dot{\theta}(t) &= Z_{B1} X_B(t) \\ \theta(t) &= Z_{B2} X_B(t) \end{aligned} \qquad \text{Eqn. 48}$$

where Z_B, Z_{B1}, and Z_{B2} have the following form:

$$
Z_B = \begin{bmatrix} 001 & & & 0 \\ & 001 & & \\ & & \ddots & \\ 0 & & & 001 \end{bmatrix}
$$

$$
Z_{B1} = \begin{bmatrix} 010 & & & 0 \\ & 010 & & \\ & & \ddots & \\ 0 & & & 010 \end{bmatrix} \qquad \text{Eqn. 49}
$$

$$
Z_{B2} = \begin{bmatrix} 100 & & & 0 \\ & 100 & & \\ & & \ddots & \\ 0 & & & 100 \end{bmatrix}
$$

Equation (2) can be rewritten as:

$$
T(t) = M(\theta(t), t)\theta(t) + \hat{D}(\theta(t), \dot{\theta}(t)), \dot{\theta}(t) + \hat{G}(\theta(t))\theta(t), \qquad \text{Eqn. 50}
$$

where $\hat{D}(\theta(t), \dot{\theta}(t))$ and $\hat{G}(\theta(t))$ are $N \times N$ matrices. By substituting equations (49) and (54) into the augmented actuator dynamic equation (28), and using (52), the integrated dynamic model of the robotic system can be obtained as follows:

$$
\dot{X}_B(t) = A(X_B, t)X_B(t) + B(X_B, t)U(t), \qquad \text{Eqn. 51}
$$

where

$$
\begin{aligned}
A(X_B, t) = {} & [I_{3N} - W_B M(X_B(t), t)Z_B]^{-1}\Big\{A_B + \\
& [F_B M(X_B(t), t) + W_B \tilde{C}(X_B(t), t)]Z_B + \\
& [F_B \hat{D}(X_B(t), t) + W_B \tilde{D}(X_B(t), t)]Z_{B1} + F_B \hat{G}(X_B(t), t)Z_{B2}\Big\} \qquad \text{Eqn. 52} \\
B(X_B, t) = {} & [I_{3N} - W_B M(X_B(t), t)Z_B]^{-1} B_B \qquad \text{Eqn. 53}
\end{aligned}
$$

EXAMPLE APPLICATION OF INTEGRATED ROBOTIC MODEL IN CONTROL SYNTHESIS

The purpose of this section is to demonstrate the application of the derived integrated robotic model for controller synthesis, and to outline the advantages and disadvantages between the two integrated model.

There are many advanced approaches to robot control synthesis [1-5, 8-12, 14-17], but in this paper, only two commonly used control algorithms will be discussed. The control methods considered includes the Model Reference Adaptive Control technique (MRAC) and decentralized control strategies.

MRAC

MRAC technique uses a reference model which specifies the design specifications. The objective of the control system is to minimize the error between the states or outputs of the model and those of the controlled plant, in this case the robot manipulator system, via a suitable adaptation mechanism. In designing a Model Reference Adaptive controller for robot manipulators, it is convenient to write the dynamic equation of the robotic system in state variable form. In order for the states/outputs of the plant to match exactly the states/outputs of the reference model, it is required that in selecting the reference model, a set of 'perfect model-following' conditions (Erzberger conditions) [2, 9] must be satisfied. For a reference model given by the following state equation:

$$\dot{X}_m(t) = A_m X_m(t) + B_m R(t), \qquad \text{Eqn. 54}$$

these conditions can be express as:

$$\begin{aligned} \text{rank}\,[B(X(t), t)] &= \text{rank}\,[B(X(t), t), B_m] \\ &= \text{rank}\,[B(X(t), t), A_m - A(X(t), t)]. \end{aligned} \qquad \text{Eqn. 55}$$

where $A(X(t), t)$ and $B(X(t), t)$ are the plant's system and input matrices respectively, either in Form A or B. However, in view of the structure of the matrices $A(X(t), t)$ and $B(X(t), t)$ for the two forms, the above conditions can be satisfied more easily if a robot manipulator is modeled based on Form B. In other words, it is easier to find a reference model for which the perfect model-following can be satisfied if the integrated dynamic model of the robot manipulator is in Form B.

The next step in the design is then to select an appropriate adaptation mechanism which is driven by the error between the reference model states/outputs and the actual system states/outputs. The adaptation mechanism modifies the feedback gains to the actuators of the robot manipulator. Examples of the adaptation algorithm which have been used in designing robotic controllers are steepest descent method [4], Popov's hyperstability theory [2], and Variable Structure approach [9].

Decentralized Control

Current industrial trend for robot manipulator control design is based on decentralized control strategy or independent joint control technique. The first step in designing such a controller is to decompose the overall integrated model of the robot manipulator and actuators into a set of a set of lower-order subsystem models and their interconnections [10]. Then the control law is completely synthesized on the local sybsystems level.

The first form of the integrated model developed can be easily decomposed into input decentralized form since the input matrix $B_A(X_A(t), t)$ is in block diagonal form with appropriate dimensions. In input decentralized form, each subsystem can be represented as follows [10]:

$$\dot{X}_{Ai}(t) = A_i(X_A(t), t)X_{Ai}(t) + B_i(X_A(t), t)U_i(t) + \sum_{\substack{j \neq 1 \\ j=1}}^{N} A_{ij}(X_A(t), t)X_{Aij}(t) \qquad \text{Eqn. 56}$$

where the matrices $A_i(X_A(t), t)$ and $A_{ij}(X_A(t), t)$ are the *ii*th and *ij*th component of the matrix $A(X_A, t)$ with appropriate dimension, respectively. However, the nonlinearities and coupling terms in $A_i(X_A(t), t)$ and $A_{ij}(X_A(t), t)$ lie outside the range space of the input matrix $B_i(X_A(t), t)$, thus the number of robust design schemes that can be employed to stabilize each individual subsystem and the overall system is limited.

The second form of the integrated model, equation (55), can be decomposed as follows [10]:

$$\begin{aligned} \dot{X}_{Bi}(t) = A_i(X_B(t), t)X_{Bi}(t) + B_i(X_B(t), t)U_i(t) &+ \sum_{\substack{j=i \\ j=1}}^{N} A_{ij}(X_B(t), t)X_{Bij}(t) \\ &+ \sum_{\substack{J \neq i \\ j=1}}^{N} B_{ij}(X_B(t), t)U_{ij}(t). \end{aligned} \qquad \text{Eqn. 57}$$

The presence of the last term on the right hand side (RHS) of the equation above is depending on the mechanical structure of the manipulator considered. Normally, for cylindrical robot, the input matrix $B(X_B(t), t)$ is in block diagonal form, hence the submatrices $B_{ij}(X_B(t), t)$ are null matrices. However, for non-direct drive robot manipulator, the magnitudes of the non-zero element of $B_{ij}(X_B(t), t)$ submatrices are often very small compared to the non-zero element of $B_i(X_B(t), t)$ matrices. Thus $B_{ij}(X_B(t), t)$ can often be assumed to be negligible and can be ignored. The main advantage of this integrated model is that the nonlinearities, uncertainties and coupling term present in each of the subsystem, as well as the interconnection functions, lie in the range space of the input matrix $B_i(X_B(t), t)$. Thus a great number of advanced decentralized control techniques can be applied to design a robust controller for the robotic system.

For less demanding path control applications, a simple linear feedback controller of the form

$$U_i(t) = -K_i X_i(t), \qquad \text{Eqn. 58}$$

where K_i is the appropriate feedback matrix gain for each subsystem, can be applied to stabilized the system and will produce satisfactory result. Either form of the integrated dynamic model can be used in the control synthesis. The decentralized liner control law will renders the nonlinear robot manipulator system practically stable and tracks a desired trajectory asymptotically if the feedback gain is designed such that a given sufficient condition is satisfied. The sufficient condition is different for each from of the integrated model used.

CONCLUSION

Two methods of deriving a more realistic dynamic mathematical model of a robot manipulator have been described in this paper. The model of the integrated system derived comprises the mechanical part of the system as well as the actuators and the gear trains. The methods are simple to use and provide a more efficient approach to the real situation. The resulting model in state variable form leads to a very convenient approach for the synthesis of advanced control algorithms for controlling the robot arm, for example adaptive model following control techniques, decentralized and hierarchical control methods.

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APPENDIX

ROBOT MANIPULATOR COMPLETE MODEL – A SURVEY

In this appendix, the complete model of the robot manipulator as given by Vukobratovic and Potkonjak [15], Vukobratovic et. al. [16], and Troch [13] are presented for comparison purposes.

In the following, the manipulator link dynamics (equation 2) and the actuator dynamics (equation 21) are reintroduced for convenience. The dynamic model of the mechanical links of an N dof robot manipulator is as follows:

$$M(\theta)(t),t)\ddot{\theta}(t) + D(\theta(t),\dot{\theta}(t)) + G(\theta(t)) = T(t), \qquad \text{Eqn. A.1}$$
$$\theta(t) = [\theta_1(t), \theta_2(t), .., \theta_N(t)]^T$$
$$\theta(t) \in \Re^N,\ \dot{\theta}(t) \in \Re^N,\ \ddot{\theta}(t) \in \Re^N.$$

For N actuators (N dof robot manipulator), the augmented dynamic equation of the actuators can be written in compact form as follows:

$$\dot{X}_A(t) = A_A X_A(t) + B_A U(t) + F_A T(t), \qquad \text{Eqn. A.2}$$
$$X_A(t) = [X_{A1}^T(t), X_{A2}^T(t), .., X_{AN}^T(t)]^T$$
$$X_{Ai}^T(t) = [\theta_i(t), \dot{\theta}_i(t), i_{ai}(t)] \qquad \text{Eqn. A.3}$$
$$X_A(t_o) = X_{Ao},$$
$$X_{Ai}(t) \in \Re^N,\ X_A(t) \in \Re^{3N},\ i \in \jmath$$

Let Z_C be an $N \times 3N$ transformation matrix such that

$$\dot{\theta}(t) = Z_C \dot{X}_A(t). \qquad \text{Eqn. A.4}$$

The transformation matrix has the following form:

$$Z_C = \begin{bmatrix} 010 & & & \\ & 010 & & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & & & 010 \end{bmatrix} \qquad \text{Eqn. A.5}$$

A1. Method Of Vukobratovic And Potkonjak [15] :

By substituting equation (A.4) into (A.1), the torque can be obtained as follows:

$$T = M(X_A(t),t) Z_C \dot{X}_A + D(X_A(t),t) + G(X_A(t)). \qquad \text{Eqn. A.6}$$

Then, equation (A.2) is substituted into equation (A.6), to give :

$$T = [I_N - M(X_A(t),t) Z_C F_A]^{-1} \Big\{ M(X_A(t),t) Z_C [A_A X_A(t) + B_A U(t)] + D(X_A(t),t) + G(X_A(t)) \Big\}. \qquad \text{Eqn. A.7}$$

Finally, the complete robot manipulator dynamic model is obtained by substituting T in equation (A.7) back into equation (A.2) as follows:

$$\dot{X}_A(t) = A_A(X_A(t), t) + B_A(X_A(t), t)U(t), \qquad \text{Eqn. A.8}$$

$$A_A(X_A(t), t) = A_A X_A(t) + F_A[I_N - M(X_A(t), t)Z_C F_A]^{-1}$$
$$\left\{M(X_A(t), t)Z_C A_A X_A + D(X_A(t), t) + G(X_A(t))\right\} \qquad \text{Eqn. A.9}$$

$$B_A(X_A(t), t) = B_A + F_A[I_N - M(X_A(t), t)Z_C F_A]^{-1} M(X_A(t), t)Z_C B_A. \qquad \text{Eqn. A.10}$$

A2. Method Of Vukobratovic et. al. [16] and Troch [13] :

By substituting equation (A.6) into the actuators state equation (A.2), and after a simple manipulation, the state equation of the robot manipulator consisting the actuators as well as the mechanical links dynamics is obtained as follows:

$$\dot{X}_A(t) = A(X_a(t), t) + B(X_A(t), t)U(t), \qquad \text{Eqn. A.11}$$

where

$$A(X_A(t), t) = [I_{3N} - F_A M(X_A(t), t)Z_C]^{-1}$$
$$\left\{A_A X_A(t) + F_A[D(X_A(t), t) + G(X_A(t))]\right\} \qquad \text{Eqn. A.12}$$

$$B(X_A(t), t) = [I_{3N} - F_A M(X_A(t), t)Z_C]^{-1} B_A. \qquad \text{Eqn. A.13}$$

Clearly, the method of deriving the integrated model of the robot manipulator presented in this section is much simpler than the method outlined in section A1 above. In the following, it will be shown that the state equation of the integrated model of the robot manipulator presented in section 4 (equations 36, 37, and 38), is equivalent to the state equation of the robot manipulator presented in this section (equations A.11, A.12 and A.13).

From section 4, since $B_p^{\dagger}(X_A(t), t) = [O_{NN} \vdots M(X_A(t), t)]$, and bv the fact that the $2N \times 3N$ transformation matrix Z_A has the following structure:

$$Z_A = \left[\begin{array}{cccc} 100 & & & \\ & 100 & & 0 \\ & & \ddots & \\ 0 & & & 100 \\ \hline & & Z_C & \end{array}\right] \qquad \text{Eqn. A.14}$$

by a simple mathematical manipulation, it can be shown that

$$I_{3N} - F_A B_p^{\dagger}(X_A(t), t)Z_A = I_{3N} - F_A M(X_A(t), t)Z_C, \qquad \text{Eqn. A.15}$$

and

$$\left[A_A - F_A B_p^{\dagger}(X_A(t), t)A_p(X_A(t), t)Z_A\right]X_A(t) = A_A X_A(t) +$$
$$F_A[D(X_A(t), t) + G(X_A(t))], \qquad \text{Eqn. A.16}$$

then equation (36) in section 4 can be rewritten as

$$\dot{\mathbf{X}}_A(t) = [\mathbf{I}_{3N} - \mathbf{F}_A\mathbf{M}(\mathbf{X}_A(t), t)\mathbf{Z}_C]^{-1} \left\{\mathbf{A}_A\mathbf{X}_A(t) + \mathbf{F}_A[\mathbf{D}(\mathbf{X}_A(t), t) + \mathbf{G}(\mathbf{X}_A(t))] + \mathbf{B}_A\mathbf{U}(t)\right\}, \qquad \text{Eqn. A.17}$$

which is similar to the integrated model represented by equations A.11 to A.14 above.