Matrix Analytic Methods For Evaluating Loss Performance In An ATM Networks

MAZLAN ABBAS AND ZAINAL ABIDIN AHMAD
Faculty of Electrical Engineering
University Teknologi Malaysia
Jalan Semarak,
54100 Kuala Lumpur.

ABSTRACT

This paper will describe an analytical approach to the solutions of queueing models with finites capacity. The methods is chosen because its abilitiy to model exactly complex non-Markovian model which have correlated, non-renewal input process which is nearly impossible to get exact expressions by using classical methods of generating functions. We applied this methodology to compute the loss performance a of a queueing system specifically in the scenario of Asynchronous Transfer Mode (ATM) networks.

INTRODUCTION

In many stochastic models, tractable analytic or numerical result are usually if certain random varibles are assumed to have a negative exponential distribution. This accounts for the wide use of the Poission process as an arrival process in the analysis of quenus, the birth-and-death assumptions underlying models, and the negative exponential durations applied to service times. The classical memory-less property is the underlying source of all simplification that we owe to the negative exponential distribution.

When assessing the performed of computer and communication system, one often encounters very complication input processes. A typical example is an ATM multiplexer, whose input is a superposition of packetized voice, data and video sources. The number of cell arrivals in adjacent time intervals can be higly correlated, which turns the arrival process into complex non-renewal process. Under such situations, the only alternative to simulations is replacing the input process by an analytically tractable model, which accurately approximates the input process.

Neuts [Neuts 1976, Neuts 1979] introduced a versatile Markovian point process, also called the N-process [Neuts 1976, Neuts 1979], which is also analytically simple and prossess propertise that makes it suitable for the approximation of complicated non-renewal processess. Systematic and details studies on matrix-analytic methods and its related references can be found in [Neuts 81].

This paper will describe an analytical approach to the solutions of queueing models with finites system capacity. The methods is chosen because its ability to model exactly complex non-Markovian model which have correlated, non-renewal input process which is nearly impossible to get exact by using classical methods of generating function [Niu 1992]. Several researchers [Machihara 88a, Zukerman 89, Brochin 1990, Baiocchi 1991, Baiocchi 1991a, Takagi 1991, Yamada 1991, Yamada 1992] have effecitively used the matrix-analytic approach to slove queueing problems. Although, no doubt this matrix manipulations require high-performance computer with big memory, there are also several articles published [Yamada 1992, Niu 1992] on how to compute the matrices more effectively and thus reduces the amount of memory and disk space required. However, in this paper we do not have the chance to use the technique of reducing the complexity of matrix manipulations because of time constraint and time-consuming programming.

Section 2 present the matrix algebra notations used in this paper. Section 3 and 4 review the phase-type and phase-Markov renewal process distributions. Matrix analytic approach is described in Section 5. Section 6 provides a commonly used first-in-first-out queueing discipline as an example with different arrival assumptions. The numerical results are given in Section 7. Finally, Section 8 present the conclusions.

NOTATIONS FOR MATRIX ALGEBRA

Generally, we will avoid declaring dimension of matrices and by adopting the convention. Notably, we write Y × Z only if it is defined, that is, the number of columns of Y is equal to the number of rows of Z. Likewise, Y + Z is defined only if Y and Z are of the same dimensions. The column or row vector whose entry is one is denoted by e, whose dimension will vary in the paper but usually self-evident from the context, and hence suppresses thew notation unless specificaly mentioned. Both row and column vectors are denoted by italic and bold face lower case letters. The identity matrix of order k is always denoted by I_k, is a diagonal matrix all the diagonal entries equal to one. The sysmbol "0" may be a scaler or a matrix according to the context.

PHASE-TYPE (PH) DISTRIBUTIONS

In this section, we first give a brief introduction to phase (PH) distributions, which have been discussed in details by Neuts [Neuts 1981]. (Note: The readers are sunggested to refer [Neuts 1981] for further theorems and proofs). Let X(t) be a continuous-time Markov process with state space [1,..., m, m + 1} for which the states {1,....,m} are transient and the state {m + 1} is absorbing. The infinitesimal generator of this Markov chain is given by

$$Q = \begin{bmatrix} T & T^0 \\ 0 & 0 \end{bmatrix} \tag{1}$$

where $Te + T^0 = 0$, T^0 is a non-negative vector, and the matrix T is nonsingular with $T_{ii} < 0$, for $1 \leq i \leq m$, and $T_{ij} \geq 0$, for $i \neq j$. The initial probability vector of X(t) is given by (α, α_{m+1}), with $\alpha e + \alpha_{m+1} = 1$.

The Markov process with the infinitesimal generator generation $Q* = T + T^0A^0$ with $A^0 = \text{dig}(\alpha_1, ..., \alpha_m)$ is now of considerable importance. We any always assume Q* is irreducible, if necessary after deletion of superluous states from the chain Q. The matrix Q* discribes the Markov chain instantaneously using the same initial probabilities, whenever an absorption into the state {m + 1} occurs.

The stationary probility vector π of Q* is obtained by solving the equations $\pi Q^* = 0$, $\pi e = 1$. The times of absorption (and resetting) are readily seen to form a renewal process with the underlying probability distribution F(•). A renewal process in which the inter-renewal times have a PH-distribution is called a PH-renewal process (PH-RP). The pair (α,T) is called a representation of F(•).

We may assume without loss of generality that $\alpha_{m+1} = 0$, so that F(•) does not have a jump at 0. If we model the arrival process as PH-renewal process (also can be modeled as departure process of phase-types), the interarrival time probability distribution function is of the form

$$F(x) = 1 - \alpha \exp(Tx)e \qquad \text{for } x \geq 0, \tag{2}$$

with Laplace Stieltjes transform (LST)

$$F^*(s) = \alpha(sI - T)^{-1}T^0 \tag{3}$$

The PH-distribution are considered versatile probability distribution, which a number of well-known probability distributions can be included as special cases such as:

(i) For the exponential distribution with parameter α, the infinitesimal generation is given by $\begin{bmatrix} -\lambda & \lambda \\ 0 & 0 \end{bmatrix}$ and $\alpha_1 = 1$, $\alpha_2 = 0$ so that F(•) then has the simple representation $(1, -\lambda)$.

(ii) The generalized Erlang distribution obtained by the convolution of k exponential distributions with parameters $\lambda_1, ..., \lambda_k$ has as one of its representations the pair (α,T) given by

$$\alpha = [1\ 0\ ...\ 0], \quad T = \begin{bmatrix} -\lambda_1 & \lambda_2 & ... & ... & 0 \\ 0 & -\lambda_1 & \lambda_2 & ... & 0 \\ ... & ... & ... & ... & ... \\ ... & ... & ... & -\lambda_{k-1} & \lambda_{k-1} \\ 0 & ... & ... & 0 & -\lambda_k \end{bmatrix} \quad \text{with } T^0 = (0, ..., 0, \lambda_k)^t$$

(iii) The hyper-exponential distribution

$$F(x) = \sum_{k}^{i=1} \alpha_i(1 - e^{-\lambda_i x}), \ x \geq 0$$

may be represented by

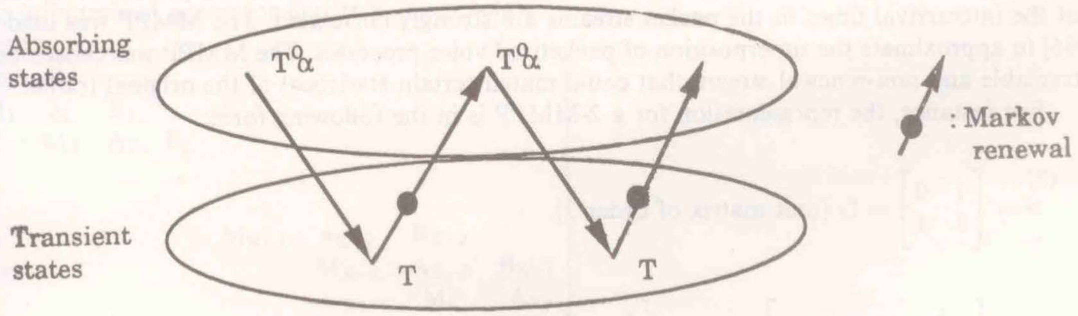
$$\alpha = (\alpha_1, ..., \alpha_k) \text{and} T = \begin{bmatrix} -\lambda_1 & 0 & ... & ... & 0 \\ 0 & -\lambda_2 & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ ... & ... & ... & -\lambda_{k-1} & 0 \\ 0 & ... & ... & 0 & -\lambda_k \end{bmatrix}, \quad T^0 - (\lambda_1, ..., \lambda_k)^t$$

PHASE-MARKOV RENEWAL PROCESS (PH-MRP) DISTRIBUTIONS

The phase Markov renewal process (PH-MRP) is a direct **extension** Machihara 1988b, Niu 1992] of the phase type renewal process introduced by neuts [Neuts 1979] that is also similar to general Markov arrival process (also known as MAP). In conjuction with the results reported in [Lucantomi 1990, Lucantomi 1991], Lucantoni suggested a notation better suited for general treatment than that used (Neuts 1979]. However, the notations used in this paper to analyze the PH-MRP (instead of referring it as MAP) is based on [Niu 1992].

Since plural absorbing states are assumed in the PH-MRP, it can represent non-renewal process such as the MMPP, whereas the PH-RP can only represented renewal process due to the assumption of only one absorbing state. Consider a continuous Markov process with state space {1,..,m, m + 1,..,m + n} for which the state {1,..,m} are transient and the states {m + 1,..,m + n} are absorbing. We assume that starting at any transient state, absorption into a state in [m + 1,..,m + n] is almost certain. Then the infinitesimal generation of such a Markov process has the form $\begin{bmatrix} T & T^0 \\ 0 & 0 \end{bmatrix}$, where T is an m × x matrix with $T_{ij} < 0$ and $T_{ij} \geq 0$ for $i \neq j$, such that T^{-1} exists. T^0 is a nonnegative m × m matrix and satisfies $Te + T^0 e_0 = 0$, where e (or e_0) is an m× (or n × 1) column vector with all elements equal to 1.

We will consider the Markov renewal process, which is obtained by instantaneously restarting the Markov process after each absorption. The state transition probability matrix between an epoch visiting the absorbing state and the epoch instantaneously restarted after this epoch is defined as α. (See Figure 1). From this definition, it shows that α is an n × m rectangular matrix.



α: The state transition probability matrix between a visiting states and instantaneously restarted after the epoch.

Figure 1: Markov renewal process [YAMA 91]

MATRIX ANALYTIC APPROACH

A quasi-birth-and-death (QBD) process is a Markov process on the state space $\Omega = \{(i,j): i \geq 0, 1 \leq j \leq m\}$, with infinitesimal generator Q, given by a tridiagonal matrix, in which all the blocks may also be in matrix form, i.e.

$$Q = \begin{bmatrix} A_0 & B_0 & & & & & \\ M_1 & A_1 & B_1 & & & & \\ & M_2 & A_2 & B_2 & & & \\ & & & \ddots & & & \\ & & & M_{i-1} & A_{i-1} & B_{i-1} & \\ & & & & M_i & A_i & B_i \\ & & & & & & \ddots \end{bmatrix} \quad (4)$$

where the matrices M_i, A_i and B_i denote the transition rates from level i to level i-1, i and i+1 respectively. The performance measure of a queueing model can be obtained by solving the system of equations $pQ = 0$ and the normalizing condition $pe = 1$, where the vector p denotes the stationary probabilites of the underlying system. Therefore, as in the above case, $A_0e + B_0e = M_1e + A_1e + B_1e = (M_2 + A_2 + B_2)e = 0$.

This paper evaluates the performance of a statistical multiplexer in fast packet networks in ATM networks. Since the cell is length of every kind of cell fixed in an ATM network system, the service time of a statistical multiplexer is a unit distribution. In all our analytical model throughout the whole paper, it is necessary that the service time distribution has the phase-type structure. We approximate the unit distribution by the Erlang distribution (β, S, S^0), defined by

$$\beta = [1\ 0\ \dots\ 0], S = \begin{bmatrix} -\mu & \mu & \dots & \dots & 0 \\ 0 & -\mu & \mu & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -\mu & \mu \\ 0 & \dots & \dots & 0 & -\mu \end{bmatrix}$$

with $S^0 = (0, \dots, 0, \mu)^t$ (5)

If the Erlang distribution has an appropriate number of phases, k, the Erlang distribution is a good approximation to a unit distribution.

More recently, modeling of packetized voice and data traffic has required consideration of more complicated arrival processes than the Poisson process. It is now well known [Heffes 1986, Sriram 1986] that the interarrival times in the packet streams are strongly correlated. The MMPP was used in [Heffes 1986] to approximate the superposition of packetized voice processes. The MMPP was chosen because it is a tractable and non-renewal stream that could match certain statistical of the original traffic.

For instance, the representation for a 2-MMPP is in the following form:

$$\alpha = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \text{ (unit matrix of order 2).}$$

$$T = \begin{bmatrix} -\lambda_{1-q1} & q_1 \\ q_2 & -\lambda_{2-q2} \end{bmatrix}, \quad T^0 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (6)$$

The superposition of several PH-MRPs constructs a new PH-MRP. For example, if we consider superposed voice (denoted by (α_v, T_v, T_v^0)) and data (denoted by (α_d, T_d, T_d^0)) ad two 2-phase MMPPs whose representations are given as above, contructs a new PH-MRP with the representation given by

$$\alpha_s = \begin{bmatrix} \alpha_v \otimes I_2 \\ I_2 \otimes \alpha_d \end{bmatrix}, \quad T_s = T_v \otimes I_2 + I_2 \otimes T_d,$$

$$T_s^0 = [T_v^0 \otimes I_2, I_2 \otimes T_d^0] \tag{7}$$

FOFO EXAMPLES

Queueing models with infine buffer using matrix-approach have been investigated widely by [Ramaswami 1980, Nui 1990, Nui 1991b]. The model where the queue has only finite number of waiting places available, seems realistic, especially in systen where buffering memory is limited and where loss probability is an important performance measure of the system. Esaki, [Esaki 1992] has made a study and shown that the cell loss rate is the dominant parameter compared to average queueing delay. This assumption have also been in investigated in several paper, such as [Blondia 1989, Blondia 1991].

To make comparisons between different models, it is necessary to model an uncontrolled queueing system such as with no-priority control by which all classes of traffic are treated equally. Such uncontrolled system is known as a queue model with first-out (FIFO) discipline. These models are frequently analyzed by many researchers and can be found in many textbooks such as [Kleinrock 1975, Allen 1990].

However, the analysis that will be shown below differs from the others because we used matrix-analytic methods to evaluate its performance. Denote by i the number of customers (both Class-1 and Class-2) in the queueing system. The state-transition diagram of its birth-and-death (BD) process is shown in Figure 2.

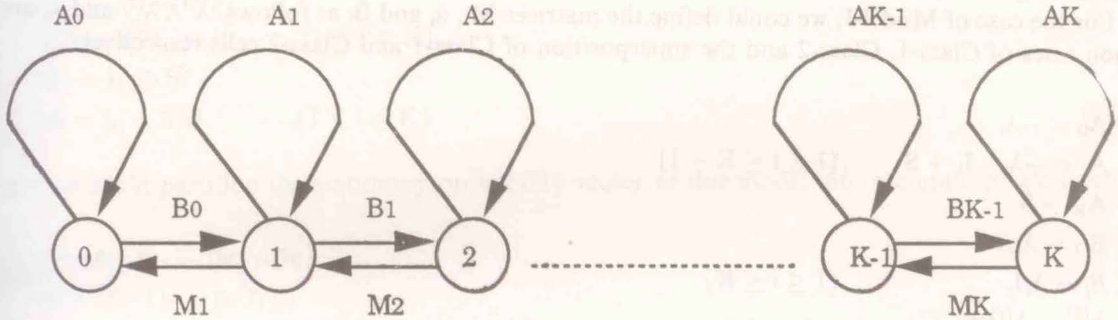


Figure 2: State Transition Diagram for Non-Priority (EIFO) Queueing Discipline

Then, the infinitesimal generator matrix is given below,

$$Q = \begin{bmatrix} A_0 & B_0 & & & & & \\ M_1 & a_1 & B_1 & & & & \\ & M_2 & A_2 & B_2 & & & \\ & & \cdot & \cdot & \cdot & & \\ & & & \cdot & \cdot & \cdot & \\ & & & M_{K-2} & a_{K-2} & B_{K-2} & \\ & & & & M_{K-1} & A_{K-1} & B_{K-1} \\ & & & & & M_k & A_K \end{bmatrix} \tag{8}$$

The generator Q can be interpreted as fallows: matrix B_l denote the arrival rate matrix of the superposed cells (Class-1 and Class-2 cells). The superposed cell arrival increase one level ($i \to i + 1$). On the other hand, the matrix M_i denotes the departure rate of the cell in service at the level i, i.e., the transition matrix from level i to $(i - 1)$.

In order for the process to have no transition from level i, neither cell arrival from Class-1 and Class-2 cells nor departure from the queue is allowed. In other words, the only possible transitions are the changes of the phases during the level i. This can be represented by the matrix A_i which satisfies

$$
\begin{aligned}
&A_0 e + B_0 e = 0 \\
&M_i e + A_i e + B_i e = 0, \qquad (1 \le i \le K-1) \\
&M_K e + A_K e = 0
\end{aligned} \tag{9}
$$

Queueing Model-I: $M_1 + M_2/PH/1/K$ (FIFO)

Define i as the number of cells (both-1 and Class-2) in the queueing system and h the phase of PH service time at an arbitrary time. Then, (i,h) constructs a continuous-time Markov chain (CTMC) on the state space

$$\Omega = \{(0) \cup (i,h) : 1 \le i \le K, 1 \le h \le k\} \tag{10}$$

Then by partitioning the state space into following set of levels:

$$
\begin{aligned}
&\text{level } 0 = \{(0)\}, \\
&\text{Level } i = \{(i,1)\ldots,(i,k)\}, \qquad (1 \le i \le K)
\end{aligned} \tag{11}
$$

we can characterize the underlying queueing models as a modified QBD process whose infintesimal generator is given above.

For the case of Model-I, we could define the matrices M_i, a_i and B_i as follows. $\lambda^{(1)}$, $\lambda^{(2)}$ and λ_s are the Poisson rates of Class-1, Class-2 and the superposition of Class-1 and Class-2 cells respectively.

$$
\begin{aligned}
&A_0 = -\lambda_s \\
&A_i = -\lambda_s \quad I_k + S \qquad ,(1 \le i \le K-1) \\
&A_K = S \\
&B_O = \lambda_s \beta \\
&B_i = \lambda_s I_s \qquad ,(1 \le i \le K) \\
&A_0^{(1)} = \lambda^{(1)} \beta \\
&A_i^{(1)} = \lambda^{(1)} I_k \qquad ,(1 \le i \le K) \\
&A_0^{(2)} = \lambda^{(2)} \beta \\
&A_i^{(2)} = \lambda^{(2)} I_k \qquad ,(1 \le i \le K) \\
&M_1 = S^0 \\
&M_i S^0 \beta \qquad ,(1 \le i \le K)
\end{aligned} \tag{12}
$$

We could then partition the stationary probability vector p according to the levels into

$$p = (p_O, p_1, .., p_{K-1}, p_K) \tag{13}$$

$p_)$ is a scalar but p_i is a vector given by

$$p_i = (p_i(1), p_i(2), ..., p_i(k)) \qquad ,(1 \le i \le K) \tag{14}$$

where $p_i(h)$ denotes the stationary probability that there are i cells in the system and in service phase h at an arbitrary time

Queueing Model-II: $PH - MRP_1 + M_2/PH/1/K$ (FIFO)

In the case of Model-II, we denote i as the number of cells (both Class-1 and Class-2) in the queueing system, j1 the phase of PH-MRP1 represented by (α_1, T_1, T_1^0) and by h the phase of PH service time at an arbitrary time. The state space of this CTMC can be define as

$$\Omega' = \{(0,j_1) \cup (i,j_1,h); 1 \le i \le K, 1 \le j_1 \le r_1, 1 \le h \le k\} \tag{15}$$

We could then partition the state space into the following levels:

level 0 = { (0,1),.., (0, r_1) }.

level i = { (i,1,1),..,(i,1,k),..,(i, r_1, 1),...,(i, r_1, k)}

$(1 \le i \le K)$ (16)

$$\begin{aligned}
&A_O = T_1 - \lambda^{(2)} I_{r_1} \\
&A_i = (T_1 - \lambda^{(2)} I_{r_1}) \otimes I_k + I_{r_1} \otimes S, \quad ,(1 \le i \le K-1) \\
&A_k = (T_1 + T^0) \otimes I_k + I_{r_1} \otimes S \\
&B_O = (T_1^0 \alpha_1 + \lambda^{(2)} I_{r_1} \otimes \beta \\
&B_i = (T_1^0 \alpha_1 + \lambda^{(2)} I_{r_1}) \otimes I_k \quad ,(1 \le i \le K) \\
&\Lambda_0^{(1)} = T_1^0 \alpha_1 \otimes \beta \\
&\Lambda_0^{(1)} = T_1^0 \alpha_1 \otimes I_k \quad ,(1 \le i \le K) \\
&\Lambda_0^{(2)} = \lambda^{(2)} I_{r_1} \otimes \beta \\
&\Lambda_0^{(2)} = \lambda_1^{(2)} I_{r_1} \otimes I_k \quad ,(1 \le i \le K) \\
&M_1 = I_{I_1} \otimes S^0 \\
&M_i = I_{r_1} \otimes S^0 \beta \quad ,(1 \le i \le K)
\end{aligned} \tag{17}$$

Again we could partition the stationary probability vector of this model into the appropriate levels

$$\begin{aligned}
&p = (p_O, p_1, ..., p_{K-1}, p_K) \\
&p_0 = (p_0(1), ..., p_0(r_1)), \\
&p_i = (p_i(1,1), ..., P_i(1,k), ..., p_i(r_1,k)), \quad ,(1 \le i \le K)
\end{aligned} \tag{18}$$

Queueing Model-III PH-MRP$_1$ + PH − MRP$_2$/PH/1/K (FIFO)

Denote by i the number of customers (both Class-1 and Class-2) in the queueing system, by j_n the phase of PH-MRPn (n = 1,2) represented by (α_n, T_n, T_n^0) and by h the phase of PH service time at an arbitrary time. Then we could construct the CTMC of Model-III with the state space

$$\Omega = \{(0,j_1,j_2) \cup (i,j_1,j_2,h); 1 \le i \le k, 1 \le J_1 \le r_1, 1 \le j_2 \le r_2, 1 \le h \le k\} \tag{19}$$

Partition the state space into the following set of levels:

level 0 = { (0,1,1), ...,(0,1,r_2), ...,(0,r_1, 1), ..., (0, r_1, r_2)) }

level i = { (i,1,1,1), ..., (i,1,1,k), ..., (i,1,r_2,1), ...,(i,1,r_2,k), ..., (i,r_1,1,1), ..., (i,r_1,1,k), ..., (i,r_1,r_2,1), ..., (i,r_1, r_2,k) } $(1 \le i \le K)$ (20)

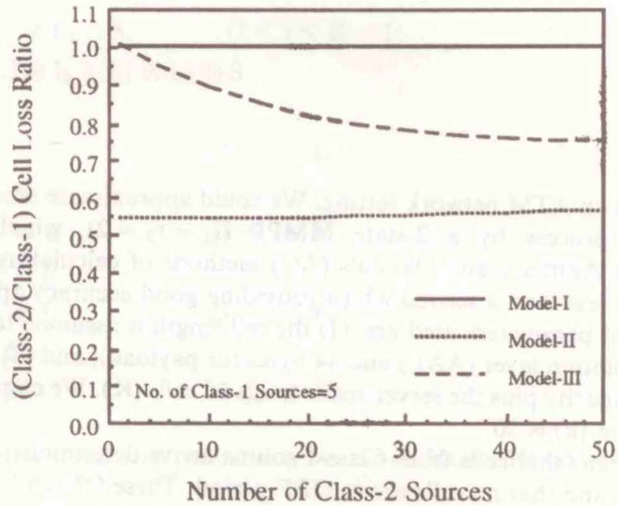


Figure 4: A comparison of cell loss ratio with different models

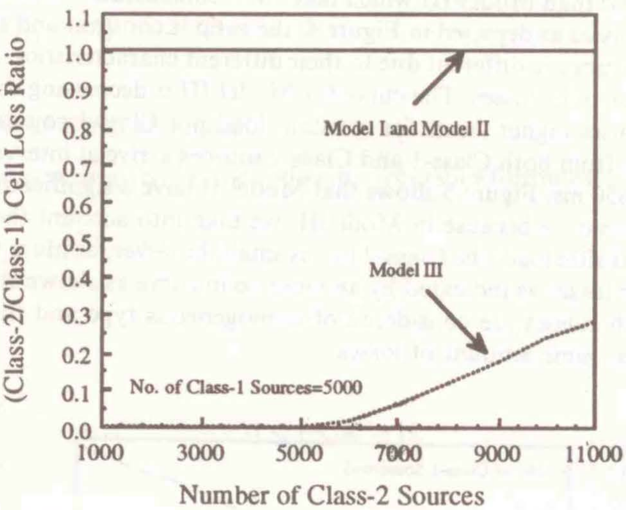


Figure 5: Comparison of cell loss rates for Class-2 with same traffic characteristics but different queueing models

CONCLUSIONS

We have proposed a method of evaluating loss performance in an ATM networks with mixed traffic inputs. The numerical examples in this paper shows that the correlated effects of the traffic had a significant effect on its performance. A system which handles which handles bursty input traffic had higher actual loss rates compared to a simple Poisson arrival model. This method had been successfully applied to priority control in an ATM networks to handle multiple classes [Abbas 1992a, Abbas 1992b].

ACKNOWLEDGEMENTS

The first author would like to thank Prof. H. Akimaru and T. Okuda of Toyohashi University of Technology (TUT), Dr. Z. Niu of Fujitsu Lab. Ltd, and Dr. N Yunus of Celcom for their valuable discussion on this work. Part of this work is done at TUT with the support of Universiti Teknologi Malaysia.

REFERENCES

1. A. Baiocchi, N.B. Melazzi and A. Roveri, "Buffer Dimensioning Criteria for an ATM Multiplexer Loaded with Homogeneous ON-OFF" Sources, *Queueing Performance and Control in ATM (ITC-13)*, Elsevies Science Publishers (North-Holland), 1991.
2. A.O Allen, Probability, Statistics, and Queueing Theory with Computer Science Applications, 2nd. Edition, *Academic Press*, Inc., 1990.
3. C.Blondia, "Finite Capacity Vacation Models with Non-Renewal Input", *J. Appl. Prob.*, Vol. 28 1991.
4. C. Blondia, "The N/G/1 Finite Capacity Queue", *Commun. Statist-Stochastic Models*, Vol,5, No.2, 1989.
5. D.M. Lucantoni, "New Result on the Single Server Queue with a Batch Markovian Arrival Process", *Commun. in Statist-Stochastic Models*, Vol.7, No.1, 1991.
6. D.M. Lucantomi, K.S. Meier-Hellstern and M/F. Neuts, "A Single-Server Queue with Server Vacations and a Class of Non-Renewal Arrival Process", *Adv. Appl, Prob.*, Vol. 22, 1990.
7. F.Machihara, "Completion Time of Service Unit Interrupted by PH-Markov Renewal Customers and its Applications", *12th Int. Teletraffic Congress*, Torino, Paper 5.4B.5.
8. F. Machihara, "On Overflow Processes form the PH1 + PH2/M/S/K Queue with Two Independent PH-Renewal Inputs", *Performance Evaluation*, Vol.8, 1988.
9. F.M. Brochin, "The Lossy and Lossless Periods in an ATM Switching Node', *IEICE Tech Report*, IN90-53, 1990.
10. H. Esaki, "Call Admission Control Method in ATM Networks", *Proc. ICC '92*, Paper 354.4, 1992.
11. H. Heffes and D.M. Lucantomi, "A Markov Modulated Characterization of Packetized Voice an Data Traffic Related Statistical Performance', *IEEE JSAC*, Vol.4, No 6, Sept. 1986.
12. H. Takagi and Y. Takashashi, "Priority Queues with Batch Poisson Arrivals", *Operations Research Letters*, Vol. 10, 1991.
13. H. Yamada, "Cell Loss Behavior in a Statistical Multiplexer with Bursty Input'', *IEICE Tech Report*, SSE91-2, 1992.
14. H. Yamada and F. Machihara, "Performance Analysis of a Statistical Multiplexer with Control on Input and/or Service Process", *Performance Evaluation*, Vol. 14, 1991.
15. K. Sriram and W. Whitt, "Characterizing Superposition Arrival Processes in Packet Multiplexers for Voice and Data", *IEEE JSAC*, Vol. 4, No. 6, Sept. 1986.
16. L. Kleinrock, Queueing Systems. Vol. 1: Theory, *John Wiley & Sons*, New York, 1975.
17. M. Abbas and Z.A. Ahmad, "Analysis of a Partial Buffer Sharing Control with Bursty Traffic in ATM Network". to appear in *Proc, 1990 IEEE Region 10 Int. Conf. (TENCON '92)*, Melbourne, Nov. 1992.
18. M. Abbas and Z.A. Ahmad, "Performance Evaluation of Selective Cell Discarding Control in ATM Network", to appear in *1992 Singapore Int. Conf. on Communication Systems and Int. Sym. on Information Theory and its Applications (ICCS/ISITA '92)*, Singapore, Nov. 1992
19. M.F.Neuts, "A Versatile Markovian Points Process", *J. Appl Prob.*, Vol.16 1979.
20. M.F. Neuts, Matrix-Geometric Solution in Stochastic Models: An Algoritmic Approach, *Baltimore, MD: The John Hopkins University Press*, 1981.
21. M.F. Neuts, "Renewal Process of Phase type", *Naval Research Log. Quart.*, Vol.5. 25, 1976.
22. M. Zukerman, "Application of Matrix-Geometric Solutions for Queueing Performance Evaluation of a Hybrid Switching System", *J. Austral. Math. SOC. Ser.*, Vol. B31, 1989.
23. T. Okuda and H. Akimaru, "A Simplified Perfomance Evaluation for Packetized Voice Systems", *IEICE Trans*, Vol. E 73, No.6, 1990.
24. V. Ramaswami, "The N/G/1 Queue and its Detailed Analysis", *Adv. Appl. Prob*, Vol. 12, 1980.
25. Z. Niu and H. Akimaru, "Matrix-Gometric Solution for Mixed Delay and Nondelay Systems with Priorities in ATM Networks", *IEICE Tech. Report*, SSE90-132, 1990.
26. Z. Niu and H. Akimaru, "Analysis od Statistical Multiplexer with Selectivel Cell Discarding Control in ATM System", *IEICE Trans*, Vol, E74, No. 12 Dec. 1992.
27. Z. Niu and H. Akimaru, "Studies on Mixed Delay and Nondelay Systems in ATM Network", *Teletraffic and Datatraffic in a Period of Change, ITC-13*, A. Jensen and V.B. Iversen (Eds.) Elsevier Science Publishers B.V. (North-Holland). 1991.
28. Z. Niu, "Studies on Mixed Loss and Delay System in Telecommunications Networks". *Ph.D. Thesis*, Toyohashi University of Technology, 1992.