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SOME INTERPRETATIONS OF SELF–TUNING PID CONTROLLERS

RUBIYAH YUSOF & MARZUKI KHALID
Department of Control Engineering
Faculty of Electrical Engineering
Universiti Teknologi Malaysia
Jalan Semarak, 54100, Kuala Lumpur, Malaysia

SIGERU OMATU
Department of Imformation Science and Intelligent Systems
Faculty of Engineering
University of Tokushima
Tokushima,770, Japan

Abstract. One of the most recent development in the theories of adaptive methods in the form of self–tuning algorithms is in the area of self-tuning PID controllers (STPID). These controllers are a class of adaptive controllers but are essentially PID controllers with the capabilities of tuning their parameters automatically online. To this end, the theories of these types of controllers are still in the infancy stage. In this paper, we provide some interpretations of a STPID through some analytical and simulation results, thereby lending way for a better understanding of the algorithms and some insight into the usefullness of the algorithm. The interpretations also serve as an aid in the selection of the tuning parameters of this algorithm which can be a time consuming activity if done dilligently.

1 INTRODUCTION

Although self–tuning controller is more flexible and provide a more systematic way of dealing with uncertainties, non–linearities, and time varying plant parameters, its applications in the process control industries is still not very encouraging. Self–tuning regulators can be viewed as performance oriented in that, the closed loop performance are specified by the user and the algorithm sets out to attain this performance eventhough the plant parameters or the drifts are unknown. Indeed, this implies that the desired performance of the plant can only be achieved given the saturation characteristics of the control actuator and the skill of the plant engineers is crucially important. A progressive step is to provide the plant engineers or users a greater intuitive understanding of the ultimate closed loop performance under self-tuning control. In this light, the theory of self–tuning regulators is moving rapidly towards broadening the range of possible control objectives and to interpret them in classical control engineering terms which are easier to understand.

The possibility of implementing simple fixed structure controllers such as PID controllers for unknown plant can be appraised so easily that control engineers would at least consider using such fixed structure controllers when faced with complex industrial plants before resorting to the self–tuning or adaptive controllers. It is, thus, a natural consequence that

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a combination of a self–tuning and the ever popular PID controller is introduced along the lines.

The main idea of these types of controllers is to combine the ability of self–tuning which can accomodate for changes in the plant parameters online and the simplicity of the PID controllers structures. This type of controller is more popularly known as self–tuning PID(STPID) controller, thus, creating a new era in the field of adaptive or self-tuning control.

There are several schemes of STPID controllers proposed in the literature. For example, [12] proposed STPID controller based on pole placement design. In this design, a PID algorithm is fitted into the control structure which is calculated via pole placement design. The method has some restrictions in that the order of the process which can be controlled is limited. A modification of the method which allows a more general form of processes to be controlled was proposed by [11]. Later, [5] proposed a STPI or STPID controller for continuous systems. The PID algorithm is automatically derived from a reasonable assumptions about the dynamics of the controlled process and also suitable modeling of the non–zero mean disturbances. Other forms of STPID can be found in [1] , [7], [9], [6], [8], [13], [10], etc.

An alternative STPID controller which is of interest in this thesis was introduced by [2]. The algorithm is based on the generalized minimum variance control algorithm ([3] and [4]). The structure of the self–tuning control law is oriented to have a PID structure. This is done by making reasonable assumptions regarding the user defined polynomial functions of the self–tuning control. An integral action which is required to eliminate disturbance of step–wise nature and steady–state error is introduced in the algorithm in an alternative manner. The controller parameters are obtained using parameter estimation scheme. For a PID structure, the controller parameters required is three. This requirement place some limitation in the order of the poles of the open loop systems to be controlled but no limitation is placed on the order of the zeros. However, it can be argued that, for a higher order system, model order reduction technique can be applied.

In most control system designs, some tuning parameters have to be preselected. In the case of STPID controller, the tuning parameters are v, $P_n(z^{-1})$ and $P_d(z^{-1})$. The tuning parameters of the STPID controller play an important role in shaping up the closed loop response of the system under control as it is related to the controller parameters. For PID controllers, the parameters K_p, K_I and K_D have to be selected, although they remain unchanged unless some changes occur in the plant in which retuning of the parameters is required.

In order to assist in the selection of the tuning parameters for STPID such as v, $P_n(z^{-1})$, and $P_d(z^{-1})$, some simulation results are shown in this paper. Some study of the properties and the role of the tuning parameters of STPID is made through the simulation examples. We also perform some simulation examples using PID controller whose parameters are fixed tuned to make some comparisons with the STPID controller. The simulation examples are carried out for a total of 300 sampling instants. The least squares estimate is used to estimate the parameters and the covariance matrix is initially chosen to be ten multiplies by the unit matrix. The forgetting factor λ is chosen to be one.

2 SOME FEATURES OF STPID ALGORITHM

The basic principles of the STPID algorithms are derived from the generalized minimum variance control. A more detail description of this algorithm can be found in [3], [4] and

[14].

2.1 The plant model

A CARMA (controlled auto regressive moving average) model as shown in Fig. 1 is considered in the derivation of STPID algorithm. The system model can be represented in mathematical form as follows:

$$A(z^{-1})y(t) = B(z^{-1})u(t-k) + C(z^{-1})\xi(t) \tag{1}$$

where $y(t)$ is the measured output, $u(t)$ is the control input, $\xi(t)$ is an uncorrelated sequence of random variables with zero mean and covariance σ, k is the time delay, and t is the time in sample instant (integer). $A(z^{-1})$, $B(z^{-1})$, and $C(z^{-1})$ are expressed in terms of the backward shift operator z^{-1}

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_{n_a} z^{-n_a} \tag{2}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_{n_b} z^{-n_b} \tag{3}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + ... + c_{n_c} z^{-n_c} \tag{4}$$

It is assumed that the roots of $C(z^{-1})$ lie within the unit disc. No assumption is made concerning the roots of $A(z^{-1})$ and $B(z^{-1})$ polynomials, i.e the root of $A(z^{-1})$ may be outside the unit disc in which the plant is open loop unstable, or the root of $B(z^{-1})$ may be outside the unit circle which means that the plant is minimum phase.

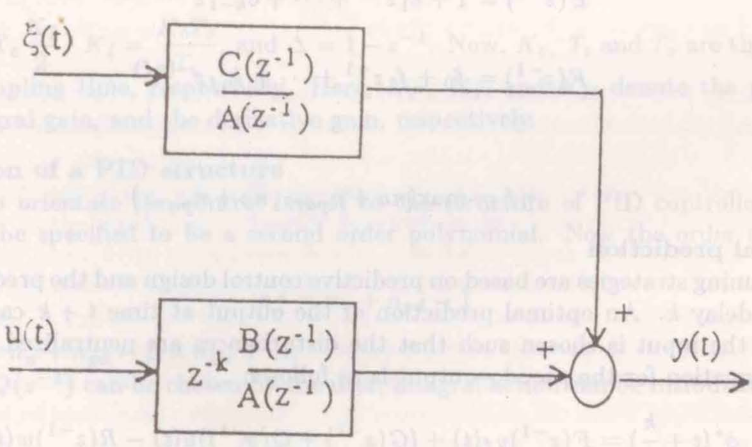

Fig. 1 A representation of a CARMA model with random disturbance

2.2 Cost criterion

The cost criterion adopted in this algorithm is of the following form

$$J = P(z^{-1})y(t) + Q(z^{-1})u(t-k) - R(z^{-1})w(t-k) \tag{5}$$

where $P(z^{-1})$, $R(z^{-1})$ and $Q(z^{-1})$ are user defined polynomials and $w(t)$ is the set–point. $P(z^{-1})$ is a rational transfer function of the form

$$P(z^{-1}) = \frac{P_n(z^{-1})}{P_d(z^{-1})} \tag{6}$$

where $P_n(z^{-1})$ and $P_d(z^{-1})$ are polynomials with degree n_{pn} and n_{pd}, respectively. The cost criterion takes into consideration both the regulatory control and also the servo control. In addition to that, the controller has a costing on the control input which prevents excessive control input being produced in the occurance of some cancellation of zeroes which are outside the stability region.

2.3 Generalized system output

The approach introduces the pseudo–output $\phi(t)$ defined by

$$\phi(t) = P(z^{-1})y(t) + Q(z^{-1})u(t-k) - R(z^{-1})w(t-k) \tag{7}$$

The system can now be considered as a generalized system output with a feedforward term, a filtering action on the output and the set–point.

2.4 Identity equations

To obtain an optimal prediction of the pseudo–output, we need to consider first the identity equation of the form:

$$C(z^{-1})P_n(z^{-1}) = E(z^{-1})A(z^{-1})P_d(z^{-1}) + z^{-k}F(z^{-1}) \tag{8}$$

where

$$E(z^{-1}) = 1 + e_1 z^{-1} + \cdots + e_{k-1}z^{-(k-1)} \tag{9a}$$

$$F(z^{-1}) = f_0 + f_1 z^{-1} + \cdots + f_{nf}z^{-(nf)} \tag{9b}$$

and

$$nf = \max(na + n_{pd-1}, nc + n_{pn-k}) \tag{9c}$$

2.5 Optimal prediction

Many self–tuning strategies are based on predictive control design and the prediction horizon is the time delay k. An optimal prediction of the output at time $t+k$ can be obtained at time t if the input is chosen such that the disturbances are neutralized. The optimal prediction equation for the pseudo–output is as follows:

$$\phi^*(t + \frac{k}{t}) = F(z^{-1})y_f(t) + (G(z^{-1}) + Q(z^{-1}))u(t) - R(z^{-1})w(t) \tag{10}$$

and

$$\tilde{\phi}(t+k) = \phi(t+k) - \phi^*(t + \frac{k}{t})$$

$$= E(z^{-1})\xi(t+k) \tag{11}$$

where $\phi^*(t + \frac{k}{t})$ is the optimum prediction of $\phi(t+k)$ based on the measurement up to time t, and is the prediction error. Here $G(z^{-1}) = E(z^{-1})B(z^{-1})$.

2.6 Control law

Using the theories of optimal prediction, the control law for generalized minimum variance control is

$$u(t) = \frac{R(z^{-1})w(t) - F(z^{-1})y_f(t)}{G(z^{-1}) + Q(z^{-1})} \tag{12}$$

2.7 Estimation equation

The optimal k step ahead prediction of $\phi_y(t)$, i.e the output which is related only to $y(t)$ is

$$\phi_y^*(t+k) = F(z^{-1})y_f(t) + G(z^{-1})u(t) \tag{13}$$

The elements of $F(z^{-1})$ and $G(z^{-1})$ can be obtained from recursive least squares estimator. The vectors for the parameters and data regression are as follows:

$$\widehat{\theta}(t) = [\widehat{f_0}(t), \widehat{f_1}(t), \ldots, \widehat{f_{nf}}(t), \widehat{g_0}(t), \widehat{g_1}(t), \ldots, \widehat{g_{ng}}(t) \tag{14}$$

$$x^T(t) = [y_f(t-1), y_f(t-2), \ldots, u(t-1), u(t-2), \ldots] \tag{15}$$

where $\widehat{f_0}$, $\widehat{f_1}, \ldots, \widehat{g_0}, \widehat{g_1}$ denote the estimate of the elements of $F(z^{-1})$ and $G(z^{-1})$, respectively.

2.8 Velocity form PID controller

A discrete time velocity form PID controller can be written as follows:

$$\Delta u(t) = K_1 w(t) - [K_P + K_I + K_D]\ y(t) + [K_P + 2K_D]\ y(t-1) - K_D\ y(t-2) \tag{16}$$

where $K_P = K_c \frac{K_I}{2}$, $K_I = \frac{K_c T_s}{T_i}$, and $\Delta = 1 - z^{-1}$. Now, K_c, T_i and T_s are the gain, reset time, and sampling time, respectively. Here, K_P, K_I, and K_D denote the proportional gain, the integral gain, and the derivative gain, respectively.

2.9 Formation of a PID structure

The idea is to orientate the control law of to the structure of PID controller in. Thus, $F(z^{-1})$ must be specified to be a second order polynomial. Now the order of $F(z^{-1})$ is given by

$$nf = n_a + n_{pd} - 1 \tag{17}$$

assuming that $n_a + n_{pd} - 1 > n_{pn} + nc$.

Now since $Q(z^{-1})$ can be chosen by the user, integral action can be introduced by letting

$$\frac{\Delta}{v} = G(z^{-1}) + Q(z^{-1}) \tag{18}$$

The steady–state error can be eliminated by letting

$$R(z^{-1}) = H_0 = \sum_{i=0}^{L} \left(\frac{F_i(z^{-1})}{P_d(z^{-1})} \right)_{z^{-1}=1} \tag{19}$$

Using (18) and (19), (12) can be written as

$$\Delta u(t) = v[R(z^{-1})\, w(t) - (\widehat{f_0} + \widehat{f_1} z^{-1} + \widehat{f_2} z^{-2})\, y(t)] \quad (20)$$

which is the control law for a self-tuning controller with a PID structure. Expressions for the corresponding PID controller parameters are as follows:

$$K_I = \frac{-v[f_0 + f_1 + f_2]}{P_d(1)} \quad (21)$$

$$K_P = \frac{-v[f_1 + 2f_2]}{P_d(1)} \quad (22)$$

$$K_D = \frac{-v f_2}{P_d(1)} \quad (23)$$

3 CLOSED LOOP SYSTEM

The closed loop expression for the STPID controller is as follows:

$$[\Delta A(z-1) - z^{-d}\, B(z^{-1}) \frac{F(z^{-1})}{P_d(z^{-1})}]\, y(t) = z^{-d}\, v\, B(z^{-1})\, H_0 w(k) + \Delta\, \xi(t) \quad (24)$$

where $\xi(t)$ is the disturbance acting on the process. It is obvious that the choice of the prefilter polynomials $P_n(z^{-1})$ and $P_d(z^{-1})$ will have some effect on the closed loop system. It can also be seen that perfect asymptotic tracking is achieved for constant reference signal if

$$H_0 = \frac{F(1)}{P_d(1)} \quad (25)$$

which conforms with (19). Also, note that at steady-state, the disturbance term equals zero, implying that the closed loop system is able to regulate the effect of load disturbance to zero.

It is interesting to note that from the corresponding PID controller expressions, (21), (22), and (23), v is expected to give the same effect to the STPID controller as K_c does to the PID controller. A small value of v would result in a more oscillatory control. A reasonable choice for the the filter $P(z^{-1})$ is a lead network. This implies that the closed loop is a low pass filter. The choice of v, $P_n(z^{-1})$ and $P_d(z^{-1})$ is more on a trial and error basis, but a reasonable approach in selecting these parameters is to select a suitable value of v so that the closed loop response is stable and not too oscillatory and then varies the values of $P_n(z^{-1})$ and $P_d(z^{-1})$ to obtain a reasonable performance suited for the application.

4 SOME SIMULATION EXAMPLES

4.1 Simulation example 1

A second order continuous time system with time delay is considered in this example. The system has a transfer function given as follows:

$$Y(s) = \frac{1}{(s+0.1)(s+1.0)}\, e^{-s}\, U(s)$$

A discrete time form of the transfer function can be obtained by means of Z−transform and zero order hold with a sampling time of 0.5s. The discrete time transfer function is given by

$$GH(z) = \frac{z^{-3}(0.105 + 0.087z^{-1})}{1 - 1.55z^{-1} + 0.576z^{-2}}$$

In time domain, the discrete time transfer function can be written as follows:

$$y(t) = 1.557y(t-1) - 0.576y(t-2) + 0.105u(t-3) + 0.087u(t-4)$$

For this system, a STPID controller is realizable only if $P_d(z^{-1})$ is chosen as a first order polynomial. No restriction on the order of $P_n(z^{-1})$ is required and we choose $P_n(z^{-1})$ to be a second order polynomial. Therefore,

$$P_d(z^{-1}) = 1 + p_{d1}z^{-1}$$

and

$$P_n(z^{-1}) = 1 + p_{n1}z^{-1} + p_{n2}z^{-2}$$

The notion of v, the controller gain for STPID having the same function as the controller gain k_c of the PID controller, prompts us into selecting this value first to obtain a somewhat stable, although oscillatory response. The values of p_{d1}, p_{n1}, p_{n2} are then selected to shape up the response. The behaviour of the closed loop response of the system is shown in Figure 2a. It can be seen that the initial transient of the response is rather large. This is because of the initial values of the estimated parameters which are much different from the true values. The response improves considerably in the second transient and even more in the third transient. Looking at the convergence of the estimated parameters in Figure 2b, we can see clearly that during the first transient, the estimated parameters assume rather large values and then converge steadily after a few sampling instants.

We can eliminate the large initial transient by using a conventional PID control in the initial phase of the simulation, in this case the first 20 sampling instants. Figure 3 shows a much reduced initial transient. To make some comparisons of the results, we then use a PID controller to control the system in this example. Figure 4 shows the behaviour of the closed loop response of the system when PID controller whose parameters are fixed tuned is used. Although the initial transient of the response is not so large, the response does not improve in the second nor the third transients.

To make some study on the role of the tuning parameters, we first increase the value of p_{n2} to −0.99 and we obtain a more oscillatory response as shown in Figure 5a. decreasing the value of p_{n1} has a similar effect as illustrated in Figure 5b. As expected, decreasing the value of p_{n1} or p_{n2} results in an underdamped response as shown in Figure 5c. The role of $P_d(z^{-1})$ is also studied by first decreasing p_{d1}. It can be seen from Figure 5d that the rate of change of the response is slow.

The role of p_{d1}, p_{n1} and p_{n2} can be explained in terms of its relation to K_P and K_I, and K_D of the conventional PID controller. From the the identity equation (8), we can see that for the system in this example, the value of pd1 is directly related to f_2 which in turn is directly related to K_D as in (21). The effect of K_D in a conventional PID controller is to provide some anticipation of where the process is heading. It is obvious that the effect

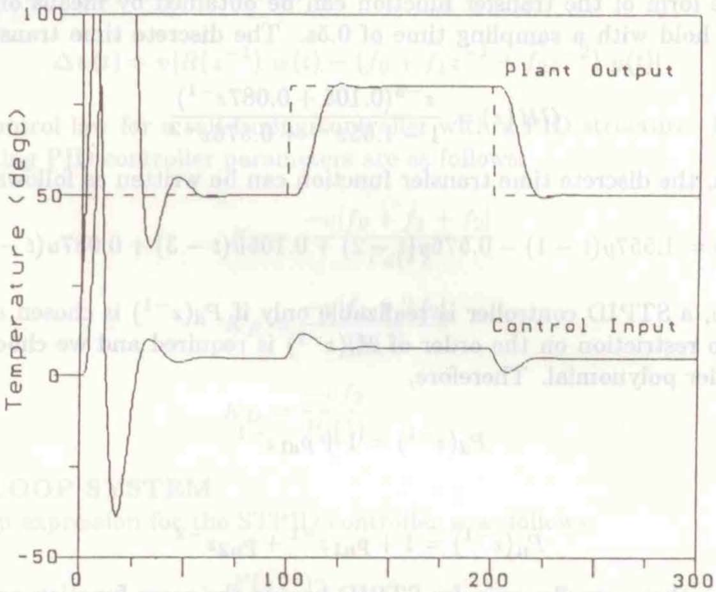

Fig. 2(a) A simulation result of STPID controller for a second order system

of p_{d1} is similar to K_D from the above simulation results. The tuning parameters p_{n1} and p_{n2} have some influences on the corresponding K_P and K_I referring to (8), (21), and (22).

4.2 Simulation example 2

In the second simulation example, we consider a third order continuous time system given by the following transfer function

$$Y(s) = \frac{0.00267}{(s+0.1)(s+0.13)(s+0.2)} U(s)$$

The equivalent discrete time system for a sampling interval of 4s is given as follows:

$$GH(z) = \frac{z^{-1}(0.0186 + 0.0486z^{-1} + 0.0078z^{-2}}{1 - 1.7063z^{-1} + 0.958z^{-2} - 0.1767z^{-3}}$$

The above transfer function can also be written in linear difference equation as follows:

$$y(t) = 1.7063y(t-1) - 0.958y(t-2) + 0.1767y(t-3)$$
$$+ 0.0186u(t-1) + 0.0486u(t-2) + 0.0078u(t-3)$$

A third order system presents an interesting example in the application of STPID control algorithm, not so much on the graphical but on the analytical aspects. As have *been mentioned previously, the order of the controller parameters* $F(z^{-1})$ is restricted to be a maximum of two for a PID structure. For this example, since the system is a third order

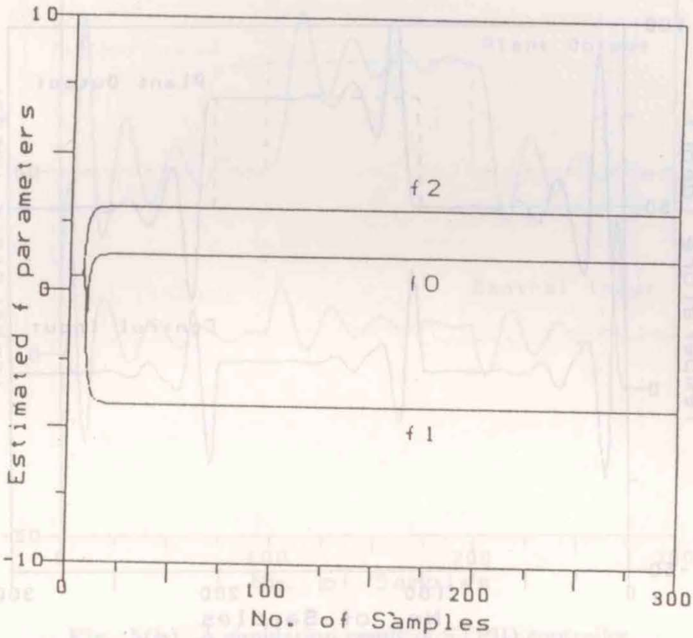

Fig. 2(b) The estimated parameters of the STPID controller

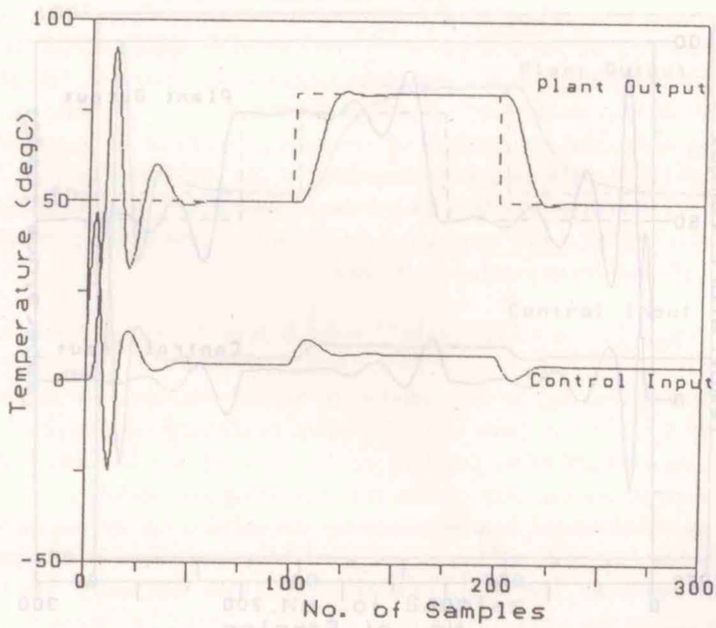

Fig. 3 A simulation result of STPID controller for a second order system with PI controller as a start up control

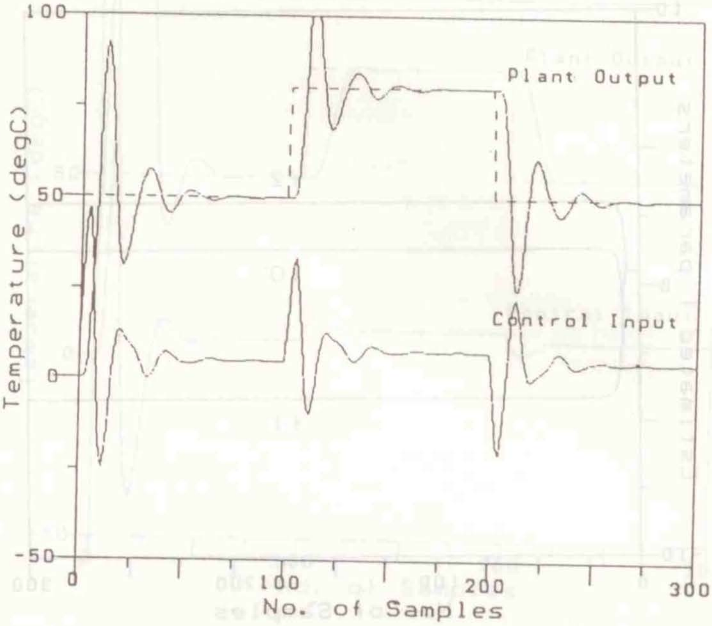

Fig. 4 A simulation result of fixed tuned controller for a second order system

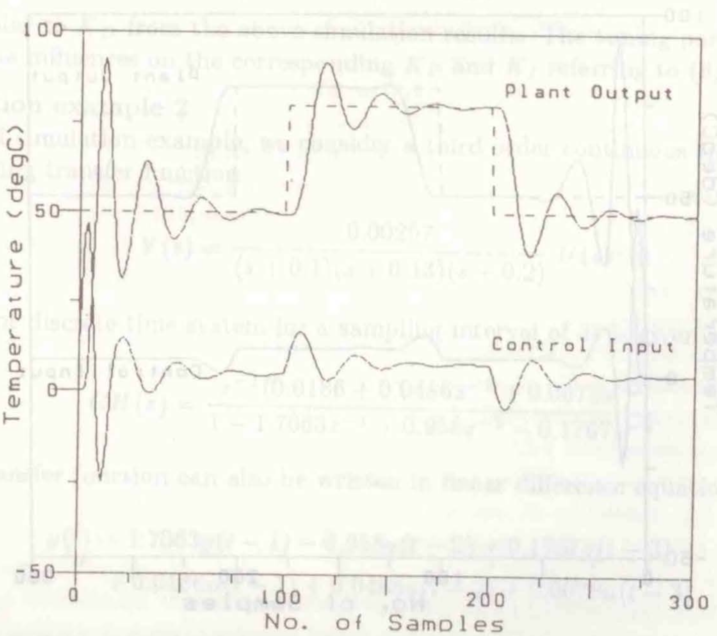

Fig. 5(a) A simulation result of STPID controller with an increase in p_{n2} value

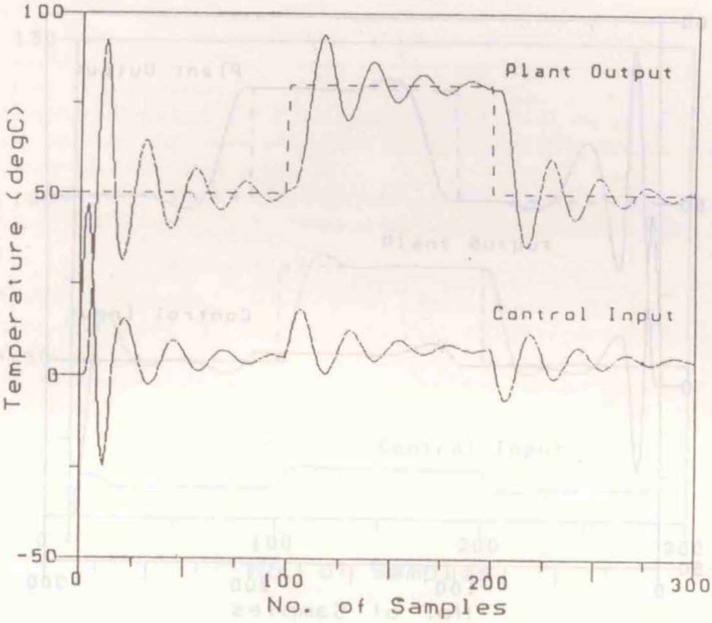

Fig. 5(b) A simulation result of STPID controller with an increase in p_{n1} value

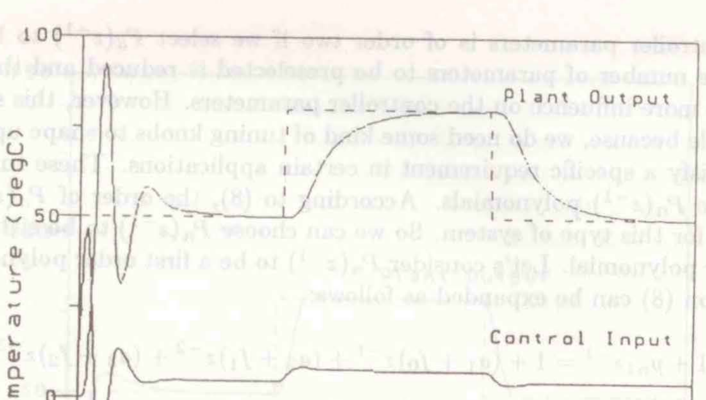

Fig. 5(c) A simulation result of STPID controller with a decrease in p_{n1} and p_{n2} value

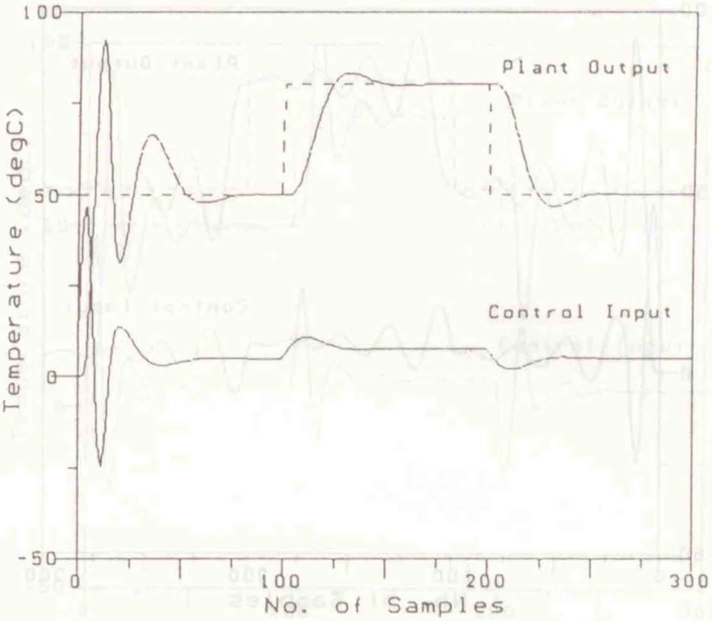

Fig. 5(d) A simulation result of STPID controller with an increase in p_{d1}

system, the controller parameters is of order two if we select $P_d(z^{-1})$ to be unity. This implies that the number of parameters to be preselected is reduced and the system open loop poles have more influence on the controller parameters. However, this situation is not entirely desirable because, we do need some kind of tuning knobs to shape up the close loop response to satisfy a specific requirement in certain applications. These tuning knobs are provided by the $P_n(z^{-1})$ polynomials. According to (8), the order of $P_n(z^{-1})$ cannot be more than two for this type of system. So we can choose $P_n(z^{-1})$ to be either a first order or second order polynomial. Let's consider $P_n(z^{-1})$ to be a first order polynomial, then the identity equation (8) can be expanded as follows:

$$1 + p_{n1}z^{-1} = 1 + (a_1 + f_0)z^{-1} + (a_2 + f_1)z^{-2} + (a_3 + f_2)z^{-3}$$

Clearly, we can see that p_{n1} is only related to fo and consequently has some effect on the corresponding K_I. We can have more flexibility in shaping up the close loop response if we let $P_n(z^{-1})$ be a second order polynomial. In this case, p_{n2} has some influence on the controller parameter f_1, and in turn on the corresponding K_P and K_I.

The behaviours of the closed loop response for the third order system are illustrated in the Figures 6 - 8. In Figure 6, we let $P_n(z^{-1})$ be unity and vary the controller gain v to get an acceptable response. The response is however rather oscillatory. We can reduce the oscillation by including the term pn1 in $P_n(z^{-1})$ as illustrated in Figure 7. In Figure 8, it is shown that a further reduction in the oscillation and a faster response can be obtained if we include the term p_{n2} in $P_n(z^{-1})$.

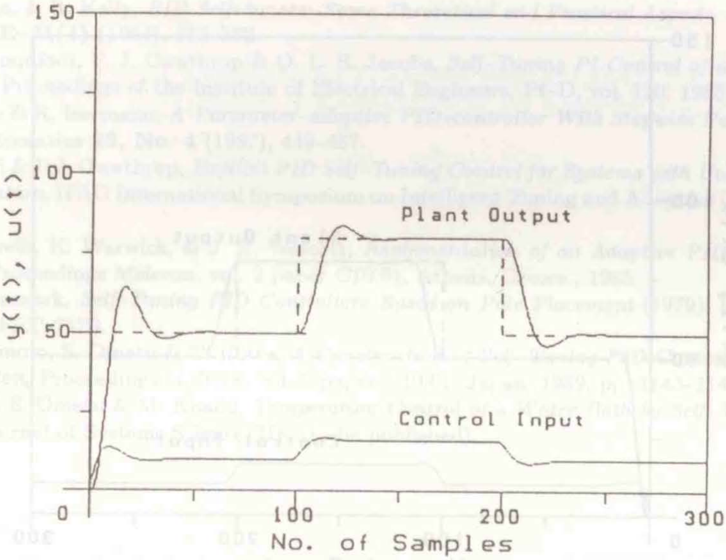

Fig. 6 A simulation result of STPID controller for a third order system with $p_n(z^{-1})$ equals 1

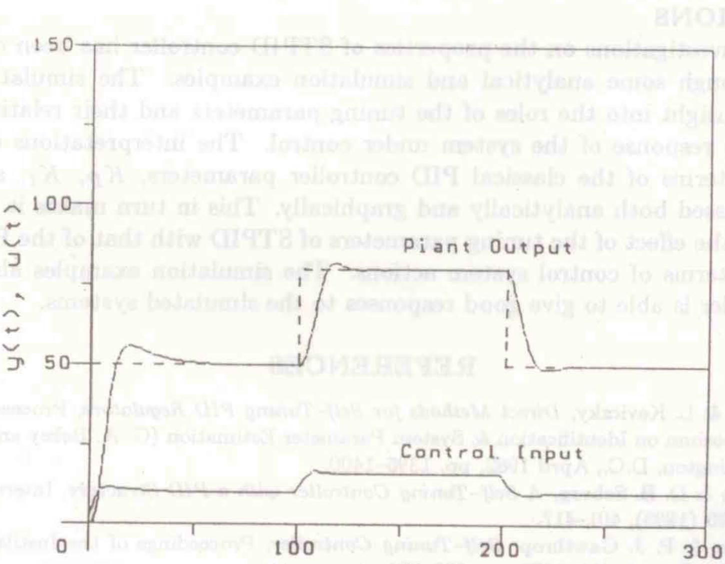

Fig. 7 A simulation result of STPID controller for a third order system with p_{n1} included $p_n(z^{-1})$

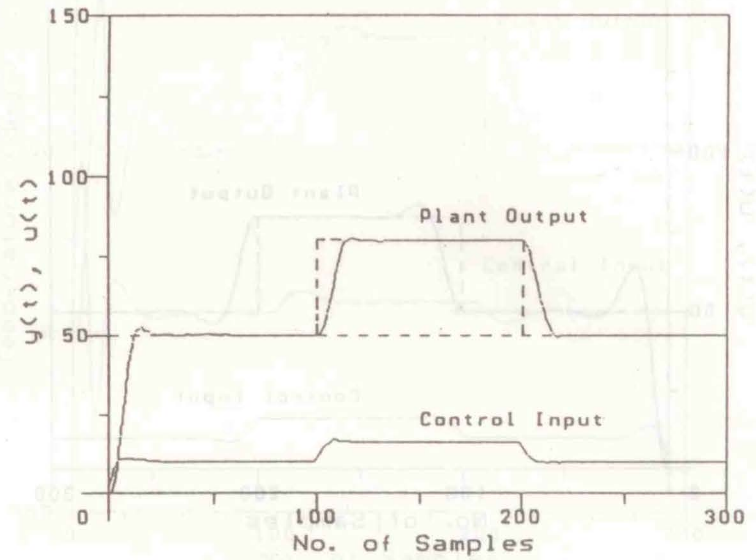

Fig. 8 A simulation result of STPID controller for a third order system with p_{n1} and p_{n2} included in $p_n(z^{-1})$

5 CONCLUSIONS

An extensive investigations on the properties of STPID controller has been carried out in this paper through some analytical and simulation examples. The simulation examples provide some insight into the roles of the tuning parameters and their relatives effects on the closed loop response of the system under control. The interpretations of the tuning parameters in terms of the classical PID controller parameters, K_P, K_I, and K_D have also been discussed both analytically and graphically. This in turn makes it easier for the users to relate the effect of the tuning parameters of STPID with that of the PID controller parameters in terms of control system actions. The simulation examples also prove that STPID controller is able to give good responses to the simulated systems.

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