

FREQUENCY SPECTRUM OF HEAT PULSE PHONONS: A COMPARISON BETWEEN LITTLE MODEL AND PERRIN-BUDD MODEL

ABD. RANI ABD. HAMID

Material Unit

Department of Physics

Universiti Teknologi Malaysia

Sekudai, 80990 Johor Bahru, Malaysia

Abstract. One of the major problems in the study of high frequency phonons in solids has been the calculation of the frequency spectrum of the phonons emitted by a heat pulse generator under specified conditions. This paper compares the calculations using two models; the Little model and the Perrin-Budd model.

Abstrak. Satu daripada masalah utama dalam kajian fonon frekuensi tinggi dalam pepejal adalah penghitungan frekuensi spektrum bagi fonon yang dipancarkan oleh penjana denyut haba pada keadaan tertentu. Kertas ini membandingkan penghitungan dengan menggunakan dua model; model Little dan model Perrin-Budd.

1 INTRODUCTION

A number of techniques have been devised and utilised in attempts to study high frequency phonons in solid. A recent review is given by Wybourne and Wigmore [6]. All have both their advantages and drawbacks. The heat pulse technique has the advantages that it can be easily applied to any material on which thin films can be evaporated, and that it can be used to obtain very high temporal and spatial resolution: polarisation and wavevector. The drawback is that the frequencies of the phonons are not well defined but occupy a broad, approximately thermal distribution of energies. One of the major problems in this area has been the calculation of the frequency spectrum of the phonons emitted by a heat pulse generator under specified conditions.

2 THE LITTLE MODEL

The simplest model is that originally worked out by Little [1]. He made the assumption that, following electrical excitation of the heater film by a short pulse of current, thermal equilibrium is rapidly established between the electrons and phonons in the film at a temperature T_H which is clearly higher than the ambient temperature of the substrate, T_S . Under the conditions, the electrical power supplied to the heater is equal to that radiated from it as phonons, following the Stefan-Boltzmann law. Thus in the long wavelength, Debye, limit the emitted power $P(T_H, T_S)$ is given by the expression

$$P(T_H, T_S) = \frac{A\pi^2 k_B^4}{120\hbar^3} \left\{ \frac{e_L}{\nu_L^2} + \frac{e_{T1} + e_{T2}}{\nu_T^2} \right\} \{T_H^4 - T_S^4\} \quad (1)$$

where A is the area of the heater, e_L and e_{T1} and e_{T2} , the phonon emissivities from the heater material into the substrate averaged over all the angles of incidence, ν_L and ν_T the longitudinal and transverse acoustic velocities in the heater. k_B and h are Boltzmann

constant and Planck constant divided by 2π respectively. This expression is obtained by considering the excess numbers and frequencies of phonons striking the interface from the heater side, and evaluating e_L and e_T on the basis of an acoustic mismatch model, based simply on the differences in acoustic impedance. Full details are given by Weis [3].

Hence the frequency spectrum of the excess phonons of a particular polarisation radiated into the substrate is given by

$$N(\omega) = \frac{A\omega^2}{8\pi^2} \left\{ \frac{e_H}{v_H^2} \frac{1}{e^{\hbar\omega/kT_H} - 1} + \frac{e_S}{v_S^2} \frac{1}{e^{\hbar\omega/kT_S} - 1} \right\} \quad (2)$$

where the suffices H and S refer to heater and substrate respectively. For the system constantan-sapphire, the following values apply (Weis, [3]):

$$e_L = 0.21(\text{constantan} \rightarrow \text{sapphire})$$

$$e_{T1} + e_{T2} = 0.34$$

$$V_L = 5.24 \text{ km.s}^{-1}; \text{ in constantan}$$

$$= 11.1 \text{ km.s}^{-1}; \text{ in sapphire}$$

$$V_L = 2.64 \text{ km.s}^{-1}; \text{ in constantan}$$

$$= 6.05 \text{ km.s}^{-1}; \text{ in sapphire}$$

Using these values, and making the assumption that $T_H \gg T_S$ it can be calculated that

$$\frac{P(T_H)}{A} = 144T_H^4 \text{ Wm}^{-2}$$

Thus, for example, to reach a heater temperature of 10K, it is necessary to supply a heater power flux of 1.44Wmm^{-2} , a modest level. The general validity of the model has been verified by a number of workers, for example Weis [3] and Wigmore [4].

3 THE PERRIN-BUDD MODEL

Unfortunately, the Little thermalisation model is an over-simplification which is seen to be inadequate on closer inspection. The problem is that the mean free path of phonons with energy 10K is considerably longer than the thickness of a typical heat pulse generator. Thus, it is likely that many of the phonons emitted initially by the excited electrons will be lost from the heater before thermalisation has time to occur. Perrin and Budd [2] considered this model in detail. Energy incident on the heater, whether as electrical or optical pulses, appears initially as an increase in the electron temperature T_e , of the electron distribution since the electron-electron scattering rate is much greater than the electron-phonon rate. As T_e rises, the electrons begin to emit phonons rapidly reaching a steady state in which the power incident is equal to the phonons emitted. But no thermal equilibrium is established. On the (questionable) assumptions of a spherical Fermi surface, no disorder, and an elastically isotropic metal film, only longitudinal phonons are emitted.

For the situation, Perrin and Budd write down the rate equation describing the evolution of the phonon distribution, N_q , referring to a wavevector q ,

$$\frac{\partial N_q}{\partial t} = \frac{\overline{N}_q(T_e) - N_q}{\tau_{ep}} - \frac{N_q - \overline{N}_q(T_o)}{\tau_b} \quad (3)$$

where $N_s(T_e)$ is the Planck distribution corresponding to temperature T_e , τ_{ep} is the electron-phonon relaxation time and τ_b the time for loss the substrate with temperature T_o . In the deformation potential model for a spherical Fermi surface, τ_{ep} can be written.

$$\frac{1}{\tau_{ep}} = \frac{\varepsilon^2 m^2}{2\hbar^3 \pi \rho v} \omega_q \quad (4)$$

where ε is the deformation potential, m is the electron mass, ρ the density, v the phonon velocity and ω_q the phonon frequency.

Finally, the electron temperature T_e , is determined by the energy balance equation

$$C_e \frac{dT}{dt} = \sigma E^2 - \sum_q \hbar \omega_q \left[\frac{\partial N_q}{\partial t} \right]_{ep} \quad (5)$$

where C_e is the electronic specific heat, E the electrical field, and σ the conductivity of the film.

Equations (3) and (5) are solved numerically by considering a small time interval Δt , during which N_q and T_e undergo small changes. Having computed the new N_q and T_e , these are re-substituted back into (3) and (5), and the procedure repeated. Perrin and Budd showed that typically a steady state situation was reached in a time less than 10^{-9} s. The steady state phonon distribution is given by

$$N_q = \frac{\overline{N}_q(T_e)\tau_b + \overline{N}_q(T_o)\tau_{ep}}{\tau_b + \tau_{ep}}$$

and the electron temperature obtained from

$$\sigma E^2 = \sum_q \frac{\hbar \omega_q}{\tau_b + \tau_{ep}} \{ \overline{N}_q(T_e) + \overline{N}_q(T_o) \}$$

4 RESULTS AND DISCUSSIONS

Figure 1 illustrates the phonon distribution calculated using the Perrin-Budd model for a typical constantan heater. The conditions relating to the heater are

$$\text{resistivity} = 52 \times 10^{-6} \text{ } \Omega \text{ cm}$$

$$\text{thickness} = 11.7 \text{ } \mu \text{ m}$$

$$\text{dimensions} = 120 \mu \text{ m} \times \mu \text{ m}$$

$$\text{electric field, } E = 960, 680, 480, 280, 120 \text{ V cm}^{-1}$$

It can be seen that the highest electric field leads to a value for T_e of 48 K, with the peak of the phonon distribution occurring at $5.5 \times 10^9 \text{ m}^{-1}$, approximately one third of the way to the Brillouin zone boundary. For the same value of the electric field, the Little

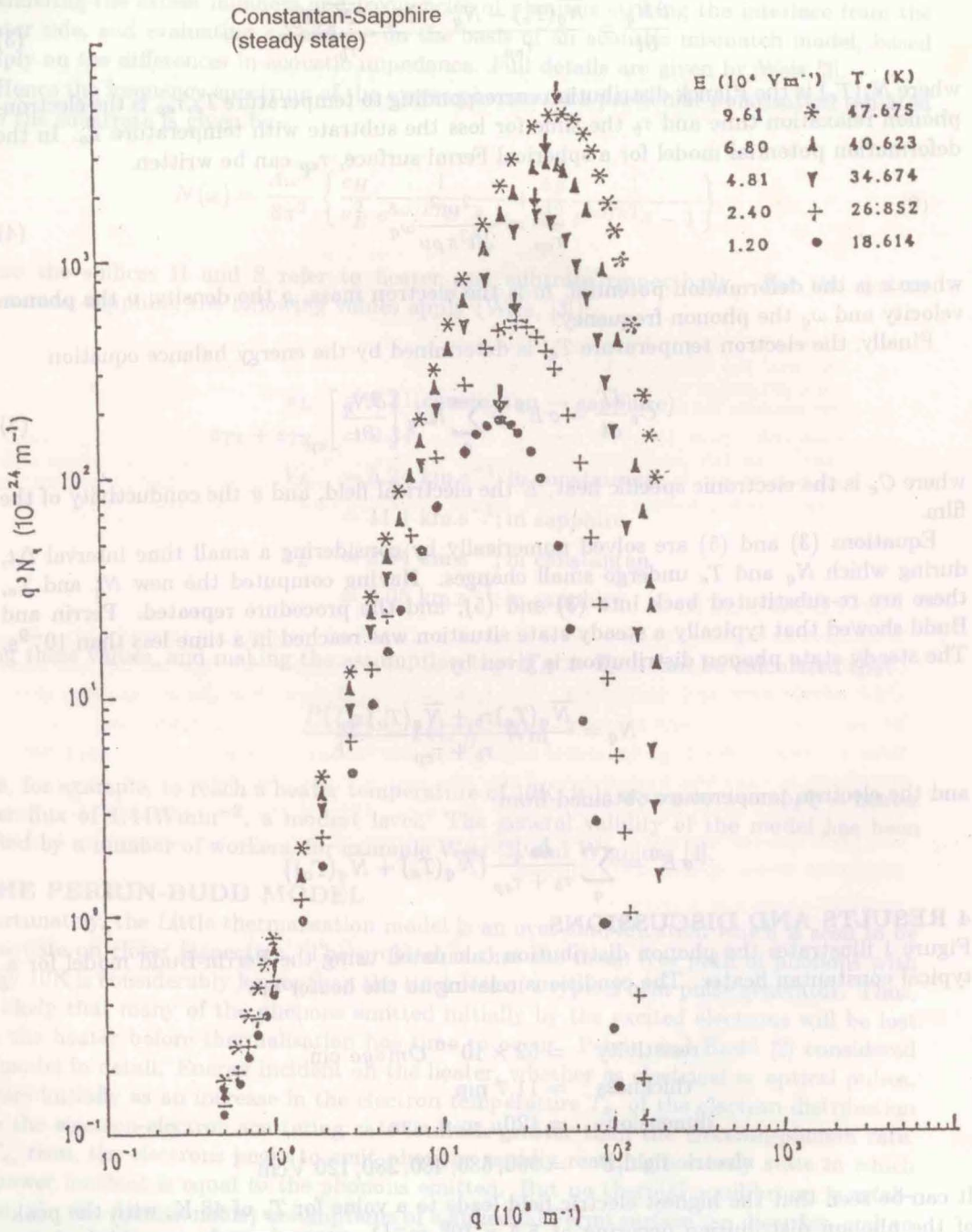


Fig. 1 Illustrating the phonon distribution calculated using the Perrin-Budd model for a typical constantan heater

thermalisation model leads to a phonon Planck distribution of temperature 34 K, peaked at approximately $4.0 \times 10^9 \text{ m}^{-1}$.

5 CONCLUSION

It is generally accepted that the Perrin-Budd model presents a more accurate picture. However, there are still a number of obvious flaws, which so far no one has attempted to address. Firstly, and most obviously, only longitudinal phonons are generated directly, so that any observed transverse modes must be the result of mode conversion. This can occur in a number of ways. Elastic scattering, both diffuse and specular may occur at the interfaces, and if the film is polycrystalline and dirty at the grain boundaries. Inelastic scattering may also be present, and this can result in polarisation as well as frequency conversion. Evidence has been found for frequency down-conversion occurring in thin metal heater films. If this process was important for polarisation conversion, then the ratio of longitudinal-transverse phonons emitted would be expected to vary with heater power. This has indeed been observed in Wybourne et al [5].

ACKNOWLEDGEMENT

Many thanks to N. Perrin (the University of Paris) for the calculation of phonon spectrum and also to J. K. Wigmore for the useful discussion.

REFERENCES

- [1] W. A. Little, Can. J. Phys. **37** (1959), 334.
- [2] N. Perrin & H. Budd, Phys. Rev. Lett. **28** (1972), 1701.
- [3] O. J. Weis, Physique Coll. **33** (1972), 4-48.
- [4] J. K. Wigmore, Phys. Rev. B **5** (1972), 700.
- [5] M. N. Wybourne, C. G. Eddison & J. K. Wigmore, Solid State Commun. **56** (1985), 755.
- [6] M. N. Wybourne & J. K. Wigmore, Rep. Prog. Phys. **51** no 7 (1988), 923-987.