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IMPROVEMENT ON ESTIMATING MEDIAN FOR FINITE POPULATION USING AUXILIARY VARIABLES IN DOUBLE-SAMPLING

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Abstract

The aim of the study is to suggest a difference-cum-ratio type of median estimator for finite population median using two-auxiliary variables in double sampling. Using simple random sampling without-replacement scheme (SRSWOR) the estimated mean square error (MSE) and BIAS are computed for the new suggested median estimator. The suggested median estimator has a smaller *MSE* than all other median estimators currently in practice, showing a valid contribution to the literature. In addition, some members of the suggested estimator and theoretical comparison of MSE are also computed. Finally, the numerical and graphical comparison of percent relative efficiency (PRE) is also computed for five different real data sets. The PRE of the suggested estimator were 13610.69%, 177.59%, 17626.95%, 204.13% and 181.29% for dataset I, II, III, IV and V respectively which reveals the importance of the new estimator.

Keywords: Auxiliary variables, BIAS, median, mean square error, percentage relative efficiency

Abstrak

Tujuan kajian ini adalah untuk mencadangkan penganggar median bagi nisbah-pembeza-merangkap bagi median populasi terhingga menggunakan dua pemboleh ubah bantu dalam pensampelan berganda. Anggaran min ralat kuasa dua dan BIAS bagi persampelan rawak menggunakan skim persampelan tanpa pengembalian dikira untuk menguji keupayaan penganggar median baru yang dicadangkan. Penganggar median yang dicadangkan mempunyai MSE yang lebih kecil berbanding anggaran median yang sering digunakan dalam kajian, menunjukkan anggaran yang dicadangkan mempunyai sumbangan yang sah dalam kajian literatur. Akhir sekali, perbandingan berangka dan grafik peratus kecekapan relatif (PRE) juga dikira untuk lima set data sebenar yang berbeza. Yang PRE daripada mencadangkan penganggar adalah 13610.69%, 177.59%, 17626.95%, 204.13% dan 181.29% untuk dataset I, II, III, IV dan V yang menunjukkan betapa pentingnya penganggar baru.

Kata kunci: Pemboleh ubah bantu, BIAS, median, min ralat kuasa dua, peratus kecekapan relative

1.0 INTRODUCTION

In this study a new estimator is suggested for finite population median in double sampling using information of two-auxiliary variables. In real life most of the data are non-normal or in other words highly skewed. Thus, in case of highly skewed distribution in survey sampling which is more frequent the preferred average is median because its handle the outliers in the data sets. Considerable amount of literature e.g Gross [1], Kuk and Mak [2], Allen *et al*., [3], Singh *et al*., ([4], [5]), Gupta *et al.*, [6] have studied the estimation of median in double sampling. Most recently the seminal work of Gupta *et al*., [6] has suggested a new median estimator in double sampling having the MSE equal to the estimator suggested by Singh *et al*., [5], the only improvement was in the $BIAS$ factor. However, the study of Gupta *et al*., [6] provided less $BIAS$ median estimator as compared to the Singh *et al*., [5] but not more efficient. Therefore, in this study we suggested a new median estimator which is more efficient.

Let us Consider a finite population $U_1 = (1,2,3,\dots,j,\dots,N_1)$. Suppose that study variable is denoted by A, the first auxiliary variable denoted by B and the second auxiliary variable denoted by C. The sample values of the respective variables are denoted by a_j, b_j and c_j where $(j = 1,2,3,\dots,n_1)$ selected by the method of $SRSWOR$ method from the known finite population. Our supposition is that the variables A and B are strongly connected with each other but no information on population median M_B is available, and information on the second auxiliary-variable C on all units of the population is available (closely connected to the first auxiliary variable B but slightly connected to the study variable A). The double sampling or two phase sampling scheme are as follows:

- The sample in first phase $i.e. s_1$ of size n_1 $(s_1 \subset U_1)$ is selected to observe only B for obtaining an estimate of M_B.
- From the sample s_1 of size n_1 on first phase, the second phase sample of size m_1 $(s_{m_1} \subset s_1)$ is selected to observe A, B, and C.

Let us suppose that M_A, M_B, and M_C are the respective subscripts population medians and the sample medians are denoted by M_a, M_b, and M_c. Let the second phase sample medians are denoted by $\widehat{M}_a$, $\widehat{M}_b$, and $\widehat{M}_c$ while the first phase sample medians are denoted by $\widehat{M}_b$, and $\widehat{M}_c$, respectively . The probability density functions of M_a, M_b and M_c are denoted by $f(M_a)$,$f(M_b)$ and $f(M_c)$ respectively. The correlation coefficient between sampling distribution of $\widehat{M}_a$ and $\widehat{M}_b$ are denoted by ρ_{ab} which is defined as ρ_{ab}=$\rho_{(\widehat{M}_b,\widehat{M}_a)}$= $4\{P_{11}(a,b)-1\}$, where $P_{11}(a,b) = P(B \le M_b \cap A \le M_a)$. The bivariate variables (A,B) distribution tends to a continuous distribution (when $N_1 \to \infty$) with their respective marginal densities for A and B . The same, we describe ρ_{bc} as : $\rho_{bc} = \rho_{(\widehat{M}_b,\widehat{M}_c)} = 4\{P_{11}(b,c)-1\}$, where $P_{11}(b,c)$= $P(B \le M_b \cap C \le M_c)$ and ρ_{ac}= $\rho_{(\widehat{M}_a,\widehat{M}_c)}$ = $4\{P_{11}(a,c)-1\}$, where $P_{11}(a,c)$=$P(A \le M_a \cap C \le M_c)$ respectively. Suppose that $a_{(1)}, a_{(2)},\dots,a_{(n_1)}$ are the values in sample a in ascending or descending order of magnitude. Let t_1 be an integer value such that $A_{(t_1)} \le M_a \le A_{(t_1+1)}$ and let $P_1 = \frac{t_1}{m_1}$ is the subset of A and the sample values that are less than or equal to the value M_a (Singh *et al*., [7]).

2.0 METHODOLOGY

In this section the methodology used for the new suggested median estimator and previous estimators suggested by various authors will be discuss in details.

2.1 Various Estimators of Median Suggested by Different Authors

First of all the following median estimators are discuss and later will be used for comparison before introducing the new suggested estimator. The suggested estimators by numerous authors and their variances, mean squares errors and BIAS expressions are as follows:

(a) Median estimator (per unit)

Gross, S.T. [1] proposed the given estimator:

$$\widehat{M}_{Gross} = \widehat{M}_a \tag{1}$$

The mean square error of $\widehat{M}_{Gross}$ is as follows:

$$MSE(\widehat{M}_{Gross}) = \frac{\left(\frac{1}{n_1}-\frac{1}{N_1}\right)}{4f_a(M_a)^2}. \tag{2}$$

(b) Median estimator (Difference type)

Median estimator (Difference type)given as follows:

$$\widehat{M}_{Diff} = \widehat{M}_a + y_1(\widehat{M}_b{}' - \widehat{M}_b), \tag{3}$$

where y_1 is constant.

The minimum MSE of $\widehat{M}_{Difference}$ are given as follows:

$$MSE(\widehat{M}_{Diff})_{min} = \frac{1}{4f_a(M_a)^2}\left\{\left(\frac{1}{m_1}-\frac{1}{N_1}\right)-\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\rho_{ab}^2\right\} \tag{4}$$

for

$$y_{1(opt)} = \rho_{ab}\frac{M_b f_b(M_b)}{M_a f_a(M_a)}$$

(c) Median Estimator (Ratio)

Singh, S. *et al*., [4] proposed the ratio estimator for median are as follows:

$$\widehat{M}_{Singh} = \frac{\widehat{M}_a}{\widehat{M}_b}\widehat{M}_b{}' \tag{5}$$

The $BIAS$ and MSE of the estimator $\widehat{M}_{Singh}$ are follows:

$$Bias(\widehat{M}_{Singh}) \cong \frac{M_a}{4\{M_b f_b(M_b)\}^2}\left(\frac{1}{m_1}-\frac{1}{N_1}\right)\left[1-\rho_{ab}\left(\frac{M_b f_b(M_b)}{M_a f_a(M_a)}\right)\right] \tag{6}$$

and

$$MSE(\widehat{M}_{Singh}) \cong \frac{1}{4f_a(M_a)^2}\left[\left(\frac{1}{m_1}-\frac{1}{N_1}\right) + \left(\frac{1}{m_1}-\frac{1}{n_1}\right)\left(\frac{M_a f_a(M_a)}{M_b f_b(M_b)}\right) \times\left\{\frac{M_a f_a(M_a)}{M_b f_b(M_b)}-2\rho_{ab}\right\}\right] \tag{7}$$

(d) Median Estimator (ratio type)

Following Srivastava, S.K. [8], a ratio type median estimator are follows:

$$\widehat{M}_{Sriv} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)^{\delta_1} \tag{8}$$

where δ_1 is constant.

The minimum $BIAS$ and MSE of $\widehat{M}_{Sriv}$ are follows:

$$Bias(\widehat{M}_{Sriv})_{min} \cong \frac{\rho_{ab}}{8M_b\{f_b(M_b)f_a(M_a)\}^2}\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\left[1-\rho_{ab}\left\{\frac{M_b f_b(M_b)}{M_a f_a(M_a)}\right\}\right] \tag{9}$$

and

$$MSE(\widehat{M}_{Sriv})_{min} \cong \frac{1}{4f_a(M_a)^2}\left\{\left(\frac{1}{m_1}-\frac{1}{N_1}\right)-\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\rho_{ab}^2\right\} \quad (10)$$

for

$$\delta_{1(opt)} = \rho_{ab}\frac{M_b f_b(M_b)}{M_a f_a(M_a)}$$

(e) Median estimator (Chain ratio type)

Following Chand, L. [9], a median estimator (chain ratio type) is as follows:

$$\widehat{M}_{Chand} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)\left(\frac{\widehat{M}_c}{\widehat{M}_c{}'}\right) \quad (11)$$

The $BIAS$ and MSE expression respectively of $\widehat{M}_{Chand}$ are follows:

$$Bias(\widehat{M}_{Chand}) \cong \frac{M_a}{4}\left[\frac{1}{\{M_b f_b(M_b)\}^2}\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\left\{1-\rho_{ab}\left(\frac{M_b f_b(M_b)}{M_a f_a(M_a)}\right)\right\}+\frac{1}{\{M_c f_c(M_c)\}^2}\left(\frac{1}{n_1}-\frac{1}{N_1}\right)\left\{1-\rho_{ac}\left(\frac{M_c f_c(M_c)}{M_a f_a(M_a)}\right)\right\}\right] \quad (12)$$

and

$$MSE(\widehat{M}_{Chand}) \cong \frac{1}{4f_a(M_a)^2}\left[\left(\frac{1}{m_1}-\frac{1}{N_1}\right)+\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\left(\frac{M_a f_a(M_a)}{M_b f_b(M_b)}\right)\times\left\{\frac{M_a f_a(M_a)}{M_b f_b(M_b)}-2\rho_{ab}\right\}+\left(\frac{1}{n_1}-\frac{1}{N_1}\right)\left(\frac{M_a f_a(M_a)}{M_c f_c(M_c)}\right)\times\left\{\frac{M_a f_a(M_a)}{M_c f_c(M_c)}-2\rho_{ac}\right\}\right] \quad (13)$$

(f) Median Estimator (Power-chain-type-ratio)

Srivastava, S.K. *et al.*, [10] median estimator (power-chain-type-ratio) is follows:

$$\widehat{M}_{Sriv1} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)^{\theta_1}\left(\frac{\widehat{M}_c}{\widehat{M}_c{}'}\right)^{\theta_2} \quad (14)$$

where $\theta_k(k=1,2)$ are constants. The respective expression of the minimum $BIAS$ and MSE of $\widehat{M}_{Srivastava1}$ are follows:

$$Bias(\widehat{M}_{Sriv1})_{min} \cong \frac{1}{8f_a(M_a)}\left[\frac{\rho_{ab}}{M_b f_b(M_b)}\left(\frac{1}{m_1}-\frac{1}{N_1}\right)\left\{1-\rho_{ab}\left(\frac{M_b f_b(M_b)}{M_a f_a(M_a)}\right)\right\}+\frac{\rho_{ac}}{M_c f_c(M_c)}\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\times\left\{1-\rho_{ac}\left(\frac{M_c f_c(M_c)}{M_a f_a(M_a)}\right)\right\}\right] \quad (15)$$

and

$$MSE(\widehat{M}_{Sriv1})_{min} \cong \frac{1}{4f_a(M_a)^2}\left[\left(\frac{1}{m_1}-\frac{1}{N_1}\right)-\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\rho_{ab}^2-\left(\frac{1}{n_1}-\frac{1}{N_1}\right)\rho_{ac}^2\right] \quad (16)$$

for

$$\theta_{1(opt)} = \rho_{ab}\left(\frac{M_b f_b(M_b)}{M_a f_a(M_a)}\right) \text{ and } \theta_{2(opt)} = \rho_{ac}\left(\frac{M_c f_c(M_c)}{M_a f_a(M_a)}\right)$$

The MSE of the median estimator (power-chain-type-ratio) $\widehat{M}_{Sriv1}$ in (14) is equivalent to the variance of difference-type median estimator in double sampling using two auxiliary variables are as follows:

$$\widehat{M}_{Diff} = \widehat{M}_a + t_1(\widehat{M}_b{}' - \widehat{M}_b) + t_2(\widehat{M}_c - \widehat{M}_c{}'),$$

where $t_k(k=1,2)$ are constants.

(g) Median Estimator by Singh (Ratio-type-estimator)

Singh, S. et al., [5] considered the below median estimator (Ratio-type-estimator):

$$\widehat{M}_{Singh1} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)^{\eta_1}\left(\frac{M_c}{\widehat{M}_c{}'}\right)^{\eta_2}\left(\frac{M_c}{\widehat{M}_c}\right)^{\eta_3} \quad (17)$$

where $\eta_l(l=1,2,3)$ are constants. The respective expression of the minimum $BIAS$ and MSE of $\widehat{M}_S$ are follows:

$$Bias(\widehat{M}_{Singh1})_{min} \cong \frac{M_a}{8\{M_a f_a(M_a)\}^2(1-\rho_{bc}^2)^2}\times[(\frac{1}{m_1}-\frac{1}{n_1})\{(\rho_{ab}-\rho_{ac}\rho_{bc})^2-2\rho_{ab}(\rho_{ab}-\rho_{ac}\rho_{bc})(1-\rho_{bc}^2)+(\frac{M_b f_b(M_b)}{M_a f_a(M_a)})(\rho_{ab}-\rho_{ac}\rho_{bc}))\}(1-\rho_{bc}^2+(\frac{1}{n_1}-\frac{1}{N_1})\{\rho_{bc}^2(\rho_{ab}-\rho_{bc}\rho_{bc})^2-2\rho_{ac}\rho_{bc}(\rho_{ab}-\rho_{ac}\rho_{bc})(1-\rho_{bc}^2)^2+(\frac{M_a f_a(M_a)}{M_c f_c(M_c)})\rho_{bc}\rho_{ab}-\rho_{ac}\rho_{bc}(1-\rho_{bc}^2)^2\}2\rho_{bc}(\rho_{ab}-\rho_{ac}\rho_{bc})(\rho_{ac}-\rho_{ab}\rho_{bc})+(\rho_{ac}-\rho_{ab}\rho_{bc})^2+(\frac{M_a f_a(M_a)}{M_c f_c(M_c)})(1-\rho_{bc}^2)(\rho_{ac}-\rho_{ab}\rho_{bc})(1-\rho_{bc}^2)\}\ ]$$

(18)

and

$$MSE(\widehat{M}_{Singh1})_{min} \cong \frac{1}{4f_a(M_a)^2}\left[\left(\frac{1}{m_1}-\frac{1}{N_1}\right)-\left(\frac{1}{n_1}-\frac{1}{N_1}\right)\rho_{ac}^2-\left(\frac{1}{m_1}-\frac{1}{n_1}\right)R_{a.bc}^2\right] \quad (19)$$

where $R_{a.bc}^2 = \frac{\rho_{ab}^2+\rho_{ac}^2-2\rho_{ab}\rho_{ac}\rho_{bc}}{1-\rho_{bc}^2}$ is the multiple correlation coefficient.

The optimum values of $\eta's$ are given as follows:

$$\eta_{1(opt)} = \left(\frac{M_b f_b(M_b)}{M_a f_a(M_a)}\right)\left(\frac{\rho_{ac}\rho_{bc}-\rho_{ab}}{\rho_{bc}^2-1}\right),$$

$$\eta_{2(opt)} = \left(\frac{M_c f_c(M_c)}{M_a f_a(M_a)}\right)\rho_{bc}\left(\frac{\rho_{ac}\rho_{bc}-\rho_{ab}}{\rho_{bc}^2-1}\right)$$

and

$$\eta_{3(opt)} = \left(\frac{M_c f_c(M_c)}{M_a f_a(M_a)}\right)\left(\frac{\rho_{ab}\rho_{bc}-\rho_{ac}}{\rho_{bc}^2-1}\right).$$

(h) Gupta Median estimator

Gupta, S. *et al.*, [6], work out on the median estimator suggested by Singh, S. *et al.*, [5] using two-auxiliary variables in double sampling and included the range of the second auxiliary variable as a transformation. The new suggested median estimator of Gupta as follows:

$$\widehat{M}_{Gupta} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)^{\xi_1}\left(\frac{M_c+R_c}{\widehat{M}_c{}'+R_c}\right)^{\xi_2}\left(\frac{M_c+R_c}{\widehat{M}_c+R_c}\right)^{\xi_3}, \quad (20)$$

where $\xi_j(j=1,2,3)$ are constants.The respective mathematical expression of the minimum $BIAS$ and MSE of $\widehat{M}_{Gupta}$ are follows:

$$Bias(\widehat{M}_{Gupta})_{min} \cong \frac{M_a}{8\{M_a f_a(M_a)\}^2(1-\rho_{bc}^2)^2} \times [(\frac{1}{m_1}-\frac{1}{n_1})\{(\rho_{ab}-\rho_{ac}\rho_{bc})^2 - 2\rho_{ab}(\rho_{ab}-\rho_{ac}\rho_{bc})(1-\rho_{bc}^2) + (\frac{M_b f_b(M_b)}{M_a f_a(M_a)})(\rho_{ab}-\rho_{ac}\rho_{bc})(1-\rho_{bc}^2)\} + (\frac{1}{n_1}-\frac{1}{N_1})\{\rho_{bc}^2(\rho_{ab}-\rho_{ac}\rho_{bc})^2 - 2\rho_{ac}\rho_{bc}(\rho_{ab}-\rho_{ac}\rho_{bc})(1-\rho_{bc}^2)^2 + (\frac{M_a f_a(M_a)}{M_c f_c(M_c)})(\frac{M_c}{M_c+R_c})\rho_{bc}\rho_{ab} - \rho_{ac}\rho_{bc}(1-\rho_{bc}^2)^2\} + (\frac{1}{m_1}-\frac{1}{N_1})\{2\rho_{bc}(\rho_{ab}-\rho_{ac}\rho_{bc})(\rho_{ac}-\rho_{ab}\rho_{bc}) + (\rho_{ac}-\rho_{ab}\rho_{bc})^2 + (\frac{M_a f_a(M_a)}{M_c f_c(M_c)})(\frac{M_c}{M_c+R_c})(1-\rho_{bc}^2)(\rho_{ac}-\rho_{ab}\rho_{bc})(1-\rho_{bc}^2)\}] \quad (21)$$

and

$$MSE(\widehat{M}_{Gupta})_{min} \cong \frac{1}{4 f_a(M_a)^2}\left[\left(\frac{1}{m_1}-\frac{1}{N_1}\right) - \left(\frac{1}{n_1}-\frac{1}{N_1}\right)\rho_{ac}^2 - \left(\frac{1}{m_1}-\frac{1}{n_1}\right)R_{a.bc}^2\right] \quad (22)$$

The optimum $\xi's$ values are follows:

$\xi_{1(opt)} = \eta_{1(opt)}$, $\xi_{2(opt)} = \eta_{2(opt)} \frac{1}{\left(\frac{M_c}{M_c+R_c}\right)}$

and $\xi_{3(opt)} = \eta_{3(opt)} \frac{1}{\left(\frac{M_c}{M_c+R_c}\right)}$

The MSE of the median estimator $\widehat{M}_{Gupta}$ in (20) is equal to Singh,S. *et al.*, [5] median estimator as shown in (17). However, the expressions of the $BIAS$ term of the two median estimators $\widehat{M}_{Gupta}$ and $\widehat{M}_{Singh1}$ are not equal.

(j) Exponential type of Median estimator

An exponential type of median estimator is given as follows Singh, S. [11].

$$\widehat{M}_{Exp} = [\tau_1\widehat{M}_a + \tau_2(\widehat{M}_b' - \widehat{M}_b)]\exp\left(\frac{M_c-\widehat{M}_c'}{M_c+\widehat{M}_c'}\right) \quad (23)$$

where $\tau_k (k = 1,2)$ are the two constants which is to be determined.The minimum mathematical expression of the $BIAS$ and MSE of $\widehat{M}_{Exp}$ are follows:

$$Bias(\widehat{M}_{Exp}) = [(\tau_1 - 1)M_a \frac{\tau_1\rho_{ac}}{8 f_a(M_a)M_c f_c(M_c)}\left(\frac{N_1-n_1}{N_1 n_1}\right) + \frac{3\tau_1 M_a}{32(M_c f_c(M_c))^2}\left(\frac{N_1-n_1}{N_1 n_1}\right)] \quad (24)$$

and

$$MSE(\widehat{M}_{Exp})_{min} \cong M_a^2 - \frac{M_a^4(f_a(M_a))^2 B^2}{4M_a^2 A(f_a(M_a))^2 + \rho_{ab}^2\left(\frac{m_1-n_1}{m_1 n_1}\right)} \quad (25)$$

The two constants optimum values are follows:

$$\tau_{1(opt)} = \frac{-X}{2Y\left[1+\left(\frac{m_1-n_1}{m_1 n_1}\right)\frac{\rho_{ab}^2}{4Y(M_a f_a(M_a))^2}\right]}$$

and

$$\tau_{2(opt)} = \frac{-X\rho_{ab} f_b(M_b)}{2Y f_a(M_a)\left[1+\left(\frac{m_1-n_1}{m_1 n_1}\right)\frac{\rho_{ab}^2}{4Y(M_a f_a(M_a))^2}\right]},$$

where

$$Y = [1 + \left(\frac{N_1-m_1}{4m_1 N_1}\right)\frac{1}{(M_a f_a(M_a))^2} - \left(\frac{N_1-n_1}{2N_1 n_1}\right)\frac{\rho ab}{M_a f_a(M_a)M_c f_c(M_c)} + \left(\frac{N_1-n_1}{4N_1 n_1}\right)\frac{1}{(M_c f_c(M_c))^2}]$$

and

$$X = [\frac{1}{4}\left(\frac{N_1-n_1}{N_1 n_1}\right)\frac{\rho ab}{M_a f_a(M_a)M_c f_c(M_c)} + \left(\frac{N_1-n_1}{16 N_1 n_1}\right)\frac{3}{(M_c f_c(M_c))^2} - 2]$$

(k) Median estimator (proposed by Nursel)

Nursel, K. [12] have suggested a class of estimators for population median are as follows:

$$\widehat{M}_{Nursel} = \pi_1\widehat{M}_a + \pi_2(\widehat{M}_b' - \widehat{M}_b) + \pi_3\left(\frac{M_c}{\widehat{M}_c'}\right)\exp\left[\frac{m(\widehat{M}_c' - M_c)}{M_c + n(\widehat{M}_c' - M_c)}\right] \quad (26)$$

where $\pi_j (j = 1,2,3)$ are constants will be determined and m, n are either function of known parameters of the population of second-auxiliary variable such as skewnes $\beta_{1(c)}$, kurtosis $\beta_{2(c)}$, correlation-coefficient ρ_{bc} etc or constants. Various estimators can be generated by giving suitable values to m and n. The expression of bias and minimum MSE respectively of the estimator $\widehat{M}_{Nursel}$ are given as follows:

$$Bias(\widehat{M}_{Nursel}) \cong (\pi_1 - 1)M_a + \pi_3[1 + \left\{\frac{m(m-2n)}{2} - m + 1\right\} \times \left(\frac{N_1-n_1}{4N_1 n_1}\right)\frac{1}{(M_c f_c(M_c))^2}] \quad (27)$$

and

$$MSE(\widehat{M}_{Nursel})_{min} \cong M_a^2 - \frac{M_a^2 T^2}{4R} - \frac{1}{4R}\frac{(4R+TS)^2 M_a^4(f_a(M_a))^2}{\left[(4QR-S^2)M_a^2(f_a(M_a))^2 + R\rho_{ab}^2\left(\frac{m_1-n_1}{m_1 n_1}\right)\right]} \quad (28)$$

where

$$Q = \left[1 + \left(\frac{N_1-m_1}{4N_1 m_1}\right)(M_a f_a(M_a))^2\right],$$

$$R = \left[1 + (2m^2 - 2mn + 3 - 4)\left(\frac{N_1-n_1}{4N_1 n_1}\right)\frac{1}{(M_c f_c(M_c))^2}\right],$$

$$S = [2 + (2m^2 - 2mn - 2m + 2)\left(\frac{N_1-n_1}{4N_1 n_1}\right)\frac{1}{(M_c f_c(M_c))^2} + (2m-2)\left(\frac{N_1-n_1}{4N_1 n_1}\right)\frac{rho_{ac}}{M_a f_a(M_a)M_c f_c(M_c)}]$$

and

$$T = \left[-2 + (2m - m^2 + 2mn - 2)\left(\frac{N_1-n_1}{4N_1 n_1}\right)\frac{1}{(M_c f_c(M_c))^2}\right].$$

(l) Aamir median estimator using auxiliary information

Aamir, M. *et al*., [13] used the following estimator for estimating population median in two phase sampling using two auxiliary variables:

$$\widehat{M}_{Aamir} = k_1\widehat{M}_a + k_2(\widehat{M}_b' - \widehat{M}_b) + k_3 exp\left[\frac{q(\widehat{M}_c - M_c)}{(2r + q(\widehat{M}_c + M_c))} + \frac{q(\widehat{M}_c' - M_c)}{(2r + q(\widehat{M}_c' + M_c))}\right] \quad (29)$$

where $k_j (j = 1,2,3)$ are constants, r and q are function of known population parameters of second auxiliary variable such as correlation-coefficient ρ_{bc}, skewness, kurtosis, Range or either constants. The respective minimum expression of $BIAS$ and MSE of $\widehat{M}_{Aamir}$ are follows:

$$Bias(\widehat{M}_{Aamir}) = \frac{M_a}{Z_1}\left[Z_2 r M_a f(M_a) - Z_1 - Z_4\left(2{M_c}^2 f(M_c)^2 + r^2\left(\frac{1}{4n} - \frac{1}{4m}\right)\right)\right] \quad (30)$$

and

$$MSE(\widehat{M}_{Aamir}) = \frac{M_a^2}{Z_1^2}[Z_2\gamma M_a f(M_a)\left(Z_2\gamma M_a f(M_a) - 2Z_1 - \gamma^2 Z_4\left(\frac{1}{2n} - \frac{1}{2m}\right)\right) + \gamma^2\{(2Z_1 Z_4 + 2Z_2 Z_3\rho_{ba} - Z_3^2)\left(\frac{1}{4n} - \frac{1}{4m}\right) + Z_2^2\left(\frac{1}{4m} - \frac{1}{4N}\right) - Z_4 M_c f(M_c)\left(Z_2\rho_{ac}\left(\frac{1}{n} + \frac{1}{m} - \frac{2}{N}\right) + Z_3\rho_{bc}\left(\frac{1}{n} - \frac{1}{m}\right)\right)\} + Z_1^2]$$

(31)

The optimum values of $k_j (j = 1,2,3)$ are as follows:

$$k_1 = \frac{Z_2\gamma M_a f(M_a)}{Z_1}, k_2 = \frac{Z_3\gamma M_a f(M_b)}{A_1} and k_3 = \frac{-2Z_4 M_a M_c^2 f(M_z)^2}{Z_1}$$

where

$$\gamma = \frac{qM_z}{2r + qM_z}$$

2.2 The New Proposed Median Estimator

Motivated by the modified versions of median estimators from Singh, S. *et al*., [5], Gupta, S. *et al.*, [6], Nursel, K. [12], Aamir, M. *et al*., [13] and Jhajj and Walia [14], we suggest a generalized difference-cum-ratio type of median estimator in double sampling using information on two auxiliary variables for the finite population median. The new suggested estimator are as follows:

$$\widehat{M}_{Propose} = [\widehat{M}_a + \theta(\widehat{M}'_a - \widehat{M}_a)] \left[\frac{\widehat{M}_{b'}}{\widehat{M}_b + \theta(\widehat{M}_{b'} - \widehat{M}_b)}\right]^{\kappa_1} \left[\frac{\widehat{M}_{c'}}{\widehat{M}_c + \theta(\widehat{M}_{c'} - \widehat{M}_c)}\right]^{\kappa_2} \left[\frac{M_c}{M_c + \theta(\widehat{M}_{c'} - M_c)}\right]^{\kappa_3} \quad (32)$$

where $\kappa_j (j = 1,2,3)$ and θ are constants. The main purpose of using $\widehat{M}_{Propose}$ in (32) is to increase the precision of the median estimator by taking the relevant advantage of the correlation between a and b, a and c, and b and c to calculate the properties of the proposed-estimator $\widehat{M}_{Propose}$ to the first-order of approximation. Let

$$e_{10} = \left(\frac{\widehat{M}_a - M_a}{M_a}\right), e_{11} = \left(\frac{\widehat{M}_a{}' - M_a}{M_a}\right), e_{12} = \left(\frac{\widehat{M}_b - M_b}{M_b}\right),$$

$$e_{13} = \left(\frac{\widehat{M}_b{}' - M_b}{M_b}\right), e_{14} = \left(\frac{\widehat{M}_c - M_c}{M_c}\right), e_{15} = \left(\frac{\widehat{M}_c{}' - M_c}{M_c}\right).$$

Substituting the values of $e_l's$ in (32),and we also assume that $|e_l| < 1$, $\{l$=10,11,12,13,14,15$\}$, therefore we expand $\widehat{M}_{Propose}$, by using second-degree of approximation, we have

$$\widehat{M}_{Propose} = M_a[(1 + e_{10}) + \theta(e_{11} - e_{10})][1 + \kappa_1(1-\theta)(e_{13} - e_{12}) - \kappa_1 e_{12}(e_{13} - e_{12}) + \kappa_1\theta^2(e_{13} - e_{12})^2 + 2\kappa_1\theta e_{12}(e_{13} - e_{12}) - \kappa_1\theta e_{13}(e_{13} - e_{12}) + \frac{\kappa_1(\kappa_1 - 1)}{2} \times (1-\theta)^2 (e_{13} - e_{12})^2][1 + \kappa_2(1-\theta)(e_{15} - e_{14}) - \kappa_2 e_{14}(e_{15} - e_{14}) + \kappa_2\theta^2(e_{15} - e_{14})^2 + 2\kappa_2\theta e_{14}(e_{15} - e_{14}) - \kappa_2\theta e_{15}(e_{15} - e_{14})] + \frac{\kappa_2(\kappa_2-1)}{2} \times (1-\theta)^2(e_{15}e_{14})^2 \left[1 - \kappa_3\theta e_{15} + \frac{\kappa_3(\kappa_3+1)}{2}\theta^2 e_{15}^2\right] \quad (33)$$

The following mathematical expression can be easily obtained from Sukhatme, S. *et al.*, [15] and Dorfman, A.H. [16]. $E(e_l = 0), \{l = 10,11,12,13,14,15\}$

$E(e_{10}^2) = f_{11}K_a^2, E(e_{11}^2) = f_{12}K_a^2, E(e_{12}^2) = f_{11}K_b^2$, $E(e_{13}^2) = f_{12}K_b^2, E(e_{14}^2) = f_{11}K_c^2, E(e_{15}^2) = f_{12}K_c^2, E(e_{10}e_{11}) = f_{12}K_a^2, E(e_{10}e_{12}) f_{11}\rho_{ab}K_bK_a, E(e_{10}e_{13}) f_{12}\rho_{ab}K_bK_a, E(e_{10}e_{14}) = f_{11}\rho_{ac}K_aK_c, E(e_{10}e_{15}) = f_{12}\rho_{ac}K_aK_c, E(e_{11}e_{12}) = f_{12}\rho_{ab}K_aK_b, E(e_{11}e_{13}) = f_{12}\rho_{ab}K_aK_b, E(e_{11}e_{14}) = f_{12}\rho_{ac}K_aK_c, E(e_{11}e_{15}) = f_{12}K_aK_c, E(e_{12}e_{13}) = f_{12}K_b^2, E(e_{12}e_{14}) = f_{11}\rho_{bc}K_bK_c, E(e_{12}e_{15}) = f_{12}K_bK_c, E(e_{13}e_{14}) = f_{12}\rho_{bc}K_bK_c, E(e_{13}e_{15}) = f_{12}\rho_{bc}K_bK_c$, $E(e_{14}e_{15}) = f_{12}K_c^2$.

where

$$f_{11} = \frac{1}{4}\left(\frac{1}{m_1} - \frac{1}{N_1}\right), f_{12} = \frac{1}{4}\left(\frac{1}{n_1} - \frac{1}{N_1}\right), K_b = \frac{1}{M_b f_b(M_b)},$$
$$K_a = \frac{1}{M_a f_a(M_a)}, K_c = \frac{1}{M_c f_c(M_c)}.$$

Subtracting M_a from (33) and taking expectation on both side also Substituting the values of above expectations, we get the bias of the proposed-estimator $\widehat{M}_{Propose}$ to the first degree of approximation as:

$$Bias(\widehat{M}_{Propose}) = M_a[\kappa_1\theta^2K_b^2(f_{11} - f_{12}) + 2\kappa_1\theta K_b^2(f_{12} - f_{11}) + \kappa_1\theta f_{12}K_b^2 + \frac{\kappa_1(\kappa_1-1)}{2} \times (1-\theta)^2K_b^2(f_{11} - f_{12}) - \kappa_2K_c^2(f_{12} - f_{11}) + \kappa_2\theta^2K_c^2(f_{11} - f_{12}) + 2\kappa_2\theta K_c^2(f_{12} - f_{11}) + \frac{\kappa_2(\kappa_2-1)}{2}(1-\theta)^2K_c^2(f_{11} - f_{12}) + \kappa_1\kappa_2(1-\theta)^2\rho_{bc}K_bK_c(f_{11} - f_{12}) + \kappa_1(1-\theta)\rho_{ab}K_aK_b(f_{12} - f_{11}) + \kappa_2(1-\theta)\rho_{ac}K_aK_c(f_{12} - f_{11}) + \kappa_1(1-\theta)\theta\rho_{ab}(f_{11} - f_{12}) + \kappa_2\theta(1-\theta)\rho_{ac}K_aK_c(f_{11} - f_{12}) - \kappa_3\theta f_{12}\rho_{ac}K_aK_c + \frac{\kappa_3(\kappa_3+1)}{2}\theta f_{12}K_c^2]. \quad (34)$$

Now we obtain the expression for the mean square error of the proposed-estimator $\widehat{M}_{Propose}$ to the first-degree of approximation. Subtracting M_a from (33), Taking squaring and apply expectation on both sides, Ignoring the terms of $e_l's$ power equal to three are greater than three and replacing the values of expectations, we ge

$$MSE(\widehat{M}_{Propose}) \cong M_a^2[f_{11}K_a^2 + \theta^2K_a^2(f_{11} - f_{12}) + \kappa_1^2(1-\theta)^2 K_b^2(f_{11} - f_{12}) + \kappa_2^2(1-\theta)^2K_c^2(f_{11} - f_{12}) + 2\theta K_a^2(f_{12} - f_{11}) + (1-\theta)\rho_{ab}K_aK_b(f_{12} - f_{11})2\kappa_1 + 2\kappa_2(1-\theta)\rho_{ac}K_aK_c(f_{12} - f_{11}) + 2(1-\theta)\theta(f_{11} - f_{12})K_a(\kappa_1\rho_{ab}K_b + \kappa_2\rho_{ac}K_c) + 2\kappa_1\kappa_2 (1-\theta)^2\rho_{bc}K_bK_c(f_{11} - f_{12}) + \kappa_3^2\theta^2f_{12}K_z^2 - 2\kappa_3\theta f_{12}\rho_{ac}K_aK_c]. \quad (35)$$

To derive the optimum values of $\kappa_j (j = 1,2,3)$, we differentiate (35) with respect to $\kappa_j (j = 1,2,3)$ and put equal to zero. Thus we get the optimum values of κ_1, κ_2 and κ_3 as follows:

$$\kappa_{1(opt)} = \frac{K_a(\rho_{ab} - \rho_{bc}\rho_{ac})}{(1-\rho_{bc}^2)K_b}, \quad (36)$$

$$\kappa_{2(opt)} = \frac{K_a(\rho_{ac} - \rho_{bc}\rho_{ab})}{(1-\rho_{bc}^2)K_c} \quad (37)$$

and

$$\kappa_{3(opt)} = \frac{K_a\rho_{ac}}{\theta K_c} \quad (38)$$

Substituting the values of (36), (37), (38) and replacing the values of f_{11}, f_{12}, K_a, K_b and K_c in (34) and (35) and also simplify the expression, the optimum $BIAS$ and MSE of the suggested proposed estimator $\widehat{M}_{Propose}$ after simplification are given as follows:

$$Bias(\widehat{M}_{Propose})_{min} \cong (\frac{1}{m_1}-\frac{1}{n_1})\frac{(\rho_{ab}-\rho_{bc}\rho_{ac})}{4f_a(M_a)(1-\rho_{bc}^2)}[\{\frac{\theta(\theta-2)}{M_bf_b(M_b)}+(1-\theta)^2\times(\frac{(\rho_{ab}-\rho_{bc}\rho_{ac})}{2M_af_a(M_a)(1-\rho_{bc}^2)}-\frac{1}{2M_bf_b(M_b)}-\frac{\rho_{ab}}{M_af_a(M_a)})\}$$
$$+\{\frac{\theta(\theta-2)}{M_cf_c(M_c)}+(1-\theta)^2(\frac{(\rho_{ac}-\rho_{bc}\rho_{ab})}{2M_af_a(M_a)(1-\rho_{bc}^2)}-\frac{1}{2M_cf_c(M_c)}-\frac{\rho_{ac}}{M_af_a(M_a)})\}+(1-\theta)^2\rho_{bc}\frac{(\rho_{ac}-\rho_{bc}\rho_{ab})}{M_af_a(M_a)(1-\rho_{bc}^2)}]$$
$$+(\frac{1}{n_1}-\frac{1}{N_1})\frac{\theta}{f_a(M_a)}[\frac{(\rho_{ab}-\rho_{bc}\rho_{ac})}{M_bf_b(M_b)(1-\rho_{bc}^2)}+\frac{(\rho_{ac}-\rho_{bc}\rho_{ab})}{M_cf_c(M_c)(1-\rho_{bc}^2)}-\frac{\rho_{ac}}{\theta}\{\frac{\rho_{ac}}{M_af_a(M_a)}-(\frac{\rho_{ac}}{2M_af_a(M_a)\theta}+\frac{1}{2M_cf_c(M_c)})\}]. \quad (39)$$

and

$$MSE(\widehat{M}_P)_{min} \cong \frac{1}{4f_a(M_a)^2}[\{(\frac{1}{m_1}-\frac{1}{N_1})+(\frac{1}{m_1}-\frac{1}{n_1})\theta(\theta-2)\}+(\frac{1}{m_1}-\frac{1}{n_1})\times\frac{(1-\theta)^2}{(1-\rho_{bc}^2)}[(\rho_{ab}-\rho_{bc}\rho_{ac})\{\frac{(\rho_{ab}-\rho_{bc}\rho_{ac})}{(1-\rho_{bc}^2)}-2\rho_{ab}\}+(\rho_{ac}-\rho_{bc}\rho_{ab})\{\frac{(\rho_{ac}-\rho_{bc}\rho_{ab})}{(1-\rho_{bc}^2)}-2\rho_{ac}\}+\frac{2\rho_{bc}(\rho_{ab}-\rho_{bc}\rho_{ac})(\rho_{ac}-\rho_{bc}\rho_{ab})}{(1-\rho_{bc}^2)}]-(\frac{1}{n_1}-\frac{1}{N_1})\rho_{ac}^2]. \quad (40)$$

For $\theta = 1$, we get

$$MSE(\widehat{M}_{Propose})_{min(\theta=1)} \cong \frac{(\frac{1}{n_1}-\frac{1}{N_1})}{4f_a(M_a)^2}(1-\rho_{ac}^2). \quad (41)$$

Thus the equations (39) and (40) represents the minimum bias and mean square error of the proposed-estimator $\widehat{M}_{Propose}$ and the equation (41) represent the minimum mean square error at the point $\theta = 1$.

2.3 Some Members of the Suggested General Class of Median Estimators

In this section we compare the proposed median estimator with the other existing median estimators .

(a) Put $\kappa_1 = 0$, $\kappa_2 = 0$, $\kappa_3 = 0$ and $\theta = 0$ in (32), we get the following Gross, S.T. [1] estimator:

$$\widehat{M}_{Gross} = \widehat{M}_a \quad (42)$$

(b) Put $\kappa_1 = 0, \kappa_2 = 0, \kappa_3 = 0$ and θ is constant in (32), we get the following difference type of estimator:

$$\widehat{M}_{Diff} = \widehat{M}_a + \theta(\widehat{M}_b{}' - \widehat{M}_b), \quad (43)$$

where θ is constant.

(c) Put $\kappa_1 = 1, \kappa_2 = 0, \kappa_3 = 0$ and $\theta = 0$ in (32), we get the following Singh,S *et al.*, [4] ratio-type median estimator in double sampling:

$$\widehat{M}_{Singh} = \frac{\widehat{M}_a}{\widehat{M}_b}\widehat{M}_b{}' \quad (44)$$

(iv). Put $\kappa_1 = \eta$, $\kappa_2 = 0$, $\kappa_3 = 0$ and $\theta = 0$ in (32), we get the following Srivastava, S.K. [7] median ratio type estimator:

$$\widehat{M}_{Sriv} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)^{\eta}, \quad (45)$$

where η is constant.

(v). Put $\kappa_1 = 1$, $\kappa_2 = -1$, $\kappa_3 = 0$ and $\theta = 0$ in (32), we get the following Chand, L. [8], a chain ratio type estimator:

$$\widehat{M}_{Chand} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)\left(\frac{\widehat{M}_c}{\widehat{M}_{c'}}\right) \quad (46)$$

(vi). Put $\kappa_1 = \eta_1$, $\kappa_2 = -\eta_2$, $\kappa_3 = 0$ and $\theta = 0$ in (32), we get the following Srivastava, S.K. *et al.*, [9], a power chain ratio type estimator:

$$\widehat{M}_{Sriva1} = \widehat{M}_a\left(\frac{\widehat{M}_b{}'}{\widehat{M}_b}\right)^{\eta_1}\left(\frac{\widehat{M}_c}{\widehat{M}_{c'}}\right)^{\eta_2} \quad (47)$$

where $\eta_j (j = 1,2)$ are constants.

3.0 RESULTS AND DISCUSSION

In this section the efficiency conditions of the suggested median estimator compared theoritically as well as numerically computed and comarison will be made for the suggested estimator.

3.1 Efficiency Conditions of the Suggested Median Estimator

The proposed median estimator $\widehat{M}_{Propose}$ is more efficient than the current median estimators if the following conditions are satisfies.

(a) Efficiency Condition (I)
By (2) and (41)

$$MSE(\widehat{M}_{Propose})_{min(\theta=1)} \le MSE(\widehat{M}_{Gross}) \;\; if$$
$$\rho_{ac}^2 \ge 0. \quad (48)$$

The above relationship is always true.

(b) Efficiency Condition (II)
By (4) and (41)

$$MSE(\widehat{M}_{Propose})_{min(\theta=1)} \le MSE(\widehat{M}_{Diff})_{min} \;\; if$$
$$(\frac{1}{m_1}-\frac{1}{n_1})(1-\rho_{ab}^2)+(\frac{1}{n_1}-\frac{1}{N_1})\rho_{ac}^2 \ge 0. \quad (49)$$

The above relationship is always true.

(c) Efficiency Condition (III)
By (7) and (41)

$$MSE(\widehat{M}_{Propose})_{min(\theta=1)} \le MSE(\widehat{M}_{Sriv}) \;\; if$$
$$\left(\frac{1}{m_1}-\frac{1}{n_1}\right)\left(\frac{M_af_a(M_a)}{M_bf_b(M_b)}\right)\left\{\frac{M_af_a(M_a)}{M_bf_b(M_b)}-2\rho_{ab}\right\}+(\frac{1}{m_1}-\frac{1}{n_1})+(\frac{1}{n_1}-\frac{1}{N_1})\rho_{ac}^2 \ge 0. \quad (50)$$

The above relationship is always true.

(d) Efficiency Condition (IV)
By (13) and (41)

$MSE(\hat{M}_{Propose})_{min(\theta=1)} \leq MSE(\hat{M}_{Chand})$ *if*

$$(\frac{1}{m_1}-\frac{1}{n_1})\left(\frac{M_a f_a(M_a)}{M_b f_b(M_b)}\right)\left\{\frac{M_a f_a(M_a)}{M_b f_b(M_b)}-2\rho_{ab}\right\} +\left(\frac{1}{n_1}-\frac{1}{N_1}\right)\left(\frac{M_a f_a(M_a)}{M_c f_c(M_c)}\right)\left\{\frac{M_a f_a(M_a)}{M_c f_c(M_c)}-2\rho_{ac}\right\}+ \left(\frac{1}{m_1}-\frac{1}{n_1}\right)+(\frac{1}{n_1}-\frac{1}{N_1})\rho_{ac}^2 \geq 0. \quad (51)$$

The above relationship is always true.

(e) Efficiency Condition (V)
By (16) and (41)

$MSE(\hat{M}_{Propose})_{min(\theta=1)} \leq MSE(\hat{M}_{Sriv1})_{min}$ *if*

$$1-\rho_{ac}^2 \geq 0. \quad (52)$$

The above relationship is always true.

(f) Efficiency Condition (VI)
By (19) and (41)
$MSE(\hat{M}_{Propose})_{min(\theta=1)} \leq MSE(\hat{M}_{Singh1})_{min}$ *if*

$$R_{a.bc}^2 \leq 1 \quad (53)$$

The above relationship is always true.

(g) Efficiency Condition (VII)
By (25)and (41)

$MSE(\hat{M}_{Propose})_{min(\theta=1)} \leq MSE(\hat{M}_{Exp})_{min}$ *if*

$$M_a^4(Y-\frac{X^2}{4})+\frac{M_a^2\rho_{ab}^2}{4f_a(M_a)^2}(\frac{m_1-n_1}{m_1 n_1}) -\{4M_a^2 Y f_a(M_a)^2+\rho_{ab}^2(\frac{m_1-n_1}{m_1 n_1})\}(\frac{1}{n_1}-\frac{1}{N_1})(1-\rho_{ac}^2) \geq 0. \quad (54)$$

The above relationship is always true iff $Y > \frac{X^2}{4}$.
We observed from equations (48) to (54), that the proposed-median estimator $\hat{M}_{Propose}$ is better than all other median estimators, $i.e.$ $\hat{M}_t$ $(t = Gross, Difference,$ $Singh, Srivastava, Chand, Srivastava1, Singh1, Gupta, Exp,$ $Aamir)$ for all types of finite populations. In our empirical study we use the different values of θ , to check the relative performance of the suggested median estimator over the usual median estimators.

3.2 Description of the Numerical Data Sets

For numerical comparison the following sets of data are being used.

(a) Numerical Data set (I) (Source: Agriculture Department of United States [17])
A: The value of agricultural production in 2009 in million dollars
B: The value of agricultural production of U.S. in 2008 in millions of dollars
C: The value of agricultural production of U.S. in 2007 in millions of dollars

(b) Numerical Data set (II) (Source: Pakistan Ministry of Food and Agriculture [18])
A: In 2003 tomato production district wise (in tonnes)
B: In 2002 tomato production district wise (in tonnes)
C: In 2001 tomato production district wise (in tonnes)

(c) Numerical Data set (III) (Source: Agriculture Department of United States [17])
A: The Soybeans production in 2010 (in million bushels)
B: The Soybeans production in 2009 (in million bushels)
C: The Soybeans production in 2008 (in million bushels)

(d) Numerical Data set (IV) (Source: Horticulture Department India [19])
A: In India State wise major production of spices (in tonnes) 2010-11
B: In India State wise major production of spices (in tonnes) 2009-10
C: In India State wise major production of spices (in tonnes) 2008-09

(e) Numerical Data set (V) (Source: The data taken from Singh [20])
A: In 1995 the marine recreational fisherman caught the number of fish
B: In 1994 the marine recreational fisherman caught the number of fish
C: In 1993 the marine recreational fisherman caught the number of fish

The descriptive statistics of all five data sets are shown in Table 1 the above data sets also used by (Aamir *et al.*, [21])

Table 1 Descriptive statistics from all of the Five data (D) sets

	D.Set I	D.Set II	D.Set III	D.Set IV	D.Set V
N_1	50.0	97.0	31.0	29.0	69.0
n_1	30.0	46.0	15.0	15.0	24.0
m_1	20.0	33.0	10.0	10.0	17.0
$Mean_{(A)}$	6617.60	3134.61	107397.10	184.51	4513.91
$Mean_{(B)}$	7345.49	3051.29	108354.20	139.01	4504.99
$Mean_{(C)}$	6539.96	2744.05	95708.89	143.01	4591.04
ρ_{ab}	0.9988	0.2010	0.9939	0.9989	0.1560
ρ_{ac}	0.9917	0.1229	0.9950	0.0530	0.3170
ρ_{bc}	0.9920	0.1495	0.9039	0.0540	0.1429
$Median(M_a)$	5014.60	1241.99	43711.00	71.42	2067.91
$Median(M_b)$	5652.27	1232.90	64800.00	43.29	2010.95
$Median(M_c)$	5023.68	1207.00	5014	41.70	2306.97
$f_a(M_a)$	0.000081	0.000211	0.000011	0.003679	0.000141
$f_b(M_b)$	0.000070	0.000222	0.000011	0.005279	0.000141
$f_c(M_c)$	0.000081	0.000231	0.000011	0.005219	0.000131
$Range_{(b)}$	41035.0	26406.0	485249.0	1160.00	37975.0
$Range_{(c)}$	38705.0	61547.0	37753.0	1224.00	34025.0

3.3 Percentage Relative Efficiencies of Various Median Estimators

The percentage relative efficiencies are computed as compared to the usual median Gross, S.T. [1] estimator by using the mean square errors of all median estimators for all five numerical data Sets are given as follows:

$$PRE = \frac{MSE(\widehat{M}_{Gross})}{MSE(\widehat{M}_{\xi})} \times 100$$

Where

$$\xi = Gross, Diff, Singh, Sri, Chand, Sri1, Singh1, Gupta, Exp, Aamir, Propose(\theta_j)$$

and $\theta_j = 0.10, 0.50, 1.00, 1.50, 1.90$

Table 2 contains the PRE's of all median estimators over the usual Gross, S.T. [1], median estimator $i.e. \widehat{M}_{Gross}$. Here we discussed that which median estimator is more efficient than the other median estimator in double sampling using two-auxiliary variables. Thus From the Table 2, we observed that the proposed median estimator $i.e.\ \widehat{M}_{Propose}$ are more efficient than all other median estimators in two-phase sampling scheme using auxiliary information for all values of θ.

$i.e.\ \widehat{M}_{\xi}(= Gross, Difference, Singh, Srivastava, Chand, Srivastava1, Singh1, Gupta, Exp, Aamir)$.

Table 2 Percentage relative efficiencies of all data (D) sets

Estimator	D.Set I	D.Set II	D.Set III	D.Set IV	D.Set V
$\widehat{M}_{Gross}$	100.00	100.00	100.00	100.00	100.00
$\widehat{M}_{Diff}$	224.30	101.94	194.70	203.18	100.90
$\widehat{M}_{Singh}$	63.90	80.59	66.99	66.30	78.09
$\widehat{M}_{Sriv}$	224.30	101.94	194.70	203.18	100.90
$\widehat{M}_{Cchand}$	12.49	61.98	12.49	12.60	68.20
$\widehat{M}_{Sriv1}$	11375.69	102.79	8723.94	203.71	107.56
$\widehat{M}_{Singh1}$	11429.00	106.09	7584.31	203.71	111.69
$\widehat{M}_{Gupta}$	11429.00	106.09	7584.31	203.71	111.69
$\widehat{M}_{Exp}$	859.03	105.80	48.60	203.60	115.96
$\widehat{M}_{Aamir}$	11728.38	110.44	9231.29	203.90	116.88
$\widehat{M}_{Propose}(0.1)$	11787.90	112.20	10414.99	203.81	120.50
$\widehat{M}_{Propose}(0.5)$	12990.59	149.50	14288.54	204.13	157.01
$\widehat{M}_{Propose}(1.0)$	13610.69	177.59	17626.95	204.24	181.29
$\widehat{M}_{Propose}(1.5)$	12990.59	149.50	14288.54	204.13	157.01
$\widehat{M}_{Propose}(1.9)$	11787.90	112.20	10414.99	203.81	120.50

3.4 Graphical Representation of All Median Estimators with respect to Percentage Relative Efficiency

In this section, PRE are presented graphically for all data sets from Figure 1 to Figure 5 respectively. From the graphical representation of percentage relative efficiency it is clear that the the suggested median estimator is more efficient than all other existing median estimators for all five different data sets. From the graphical representation of PRE clearly stated that the suggested median estimator in two-phase sampling scheme using two auxiliary variables is more efficient for the range of values of θ. $i.e.$ $0.10 \leq \theta \leq 1.90$ and more efficient at the point $\theta = 1.00$.

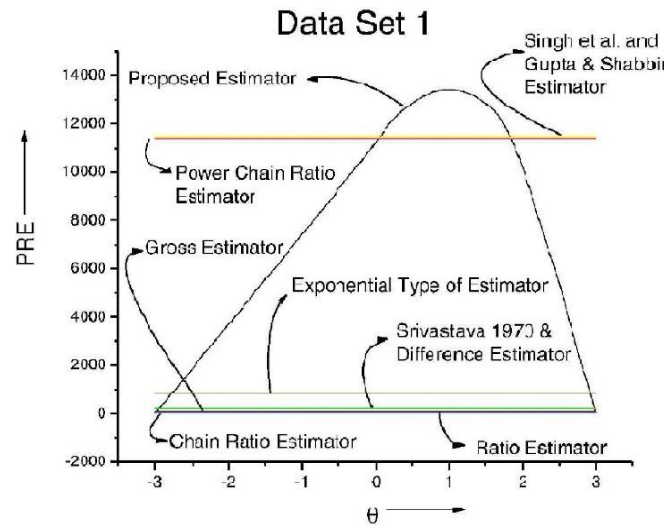

Figure 1 Graph of PRE of Data Set 1

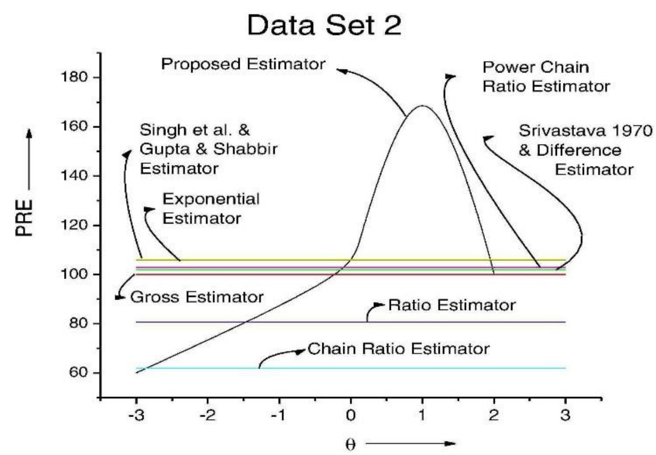

Figure 2 Graph of PRE of Data Set 2

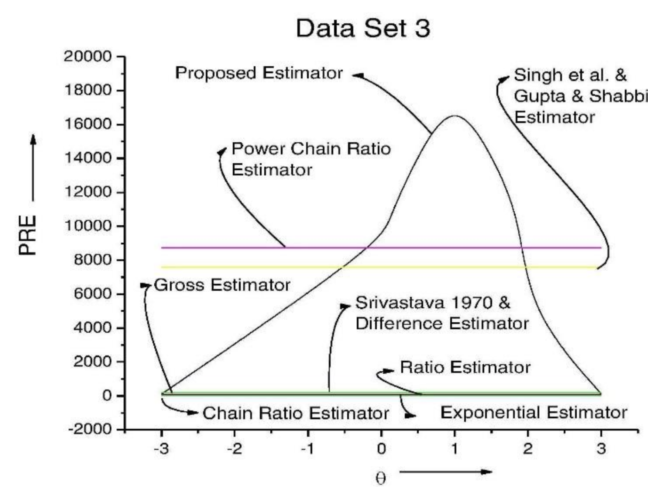

Figure 3 Graph of PRE of Data Set 3

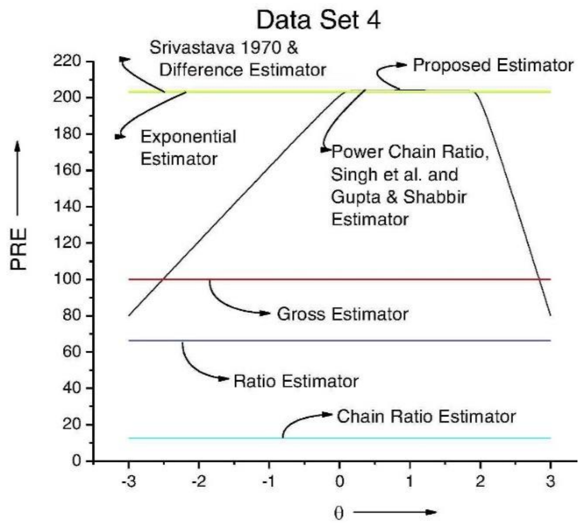

Figure 4 Graph of PRE of Data Set 4

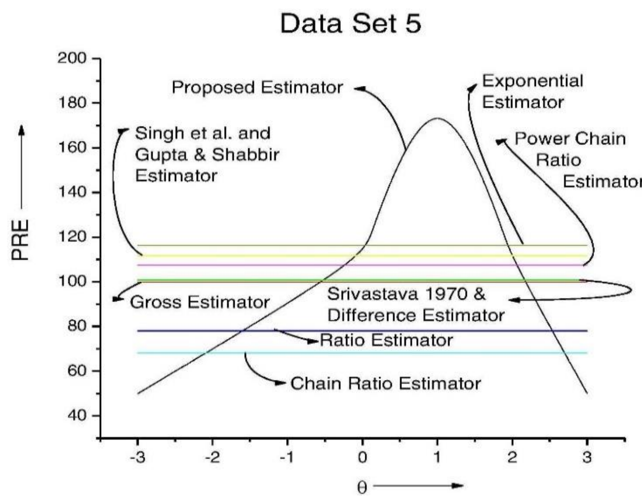

Figure 5 Graph of PRE of Data Set 5

4.0 CONCLUSION

In this study we proved mathematically, empirically as well as graphically that our new suggested estimator $\hat{M}_{Propose}$ is more efficient than all other existing median estimators. The MSEof the suggested estimator is smaller than all other existing median estimators for all data sets. For checking the relative performance of MSE of the suggested estimator, we substitute different values of θ $i.e.$ $(0.10 \leq \theta \leq 1.90)$ to get the minimum MSE for the suggested-estimator and we got the minimum MSE at the point $\theta = 1.00$. Thus we observed that the suggested median estimator produce the minimum MSE at the point $\theta = 1.00$.

Hence, based on this study it is concluded that the suggested median estimator is more efficient than other median estimators in double-sampling. It is recommended that our new suggested estimator for estimating finite population median $\hat{M}_{Propose}$ can be used in practice for obtaining more efficient results.

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