

A New Centroids Method for Ranking of Trapezoid Fuzzy Numbers

Fateen Najwa Azman, Lazim Abdullah*

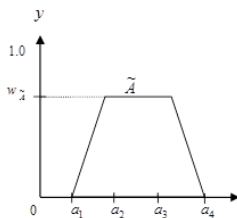
School of Informatics and Applied Mathematics, Universiti Malaysia Terengganu, Malaysia

*Corresponding author: lazim_m@umt.edu.my

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Abstract

Ranking fuzzy numbers has become an important process in decision making. Many ranking methods have been proposed thus far and one of the commonly used is centroid of trapezoid. However, there is still no agreement on the method that can always provide a satisfactory solution to every situation. This paper aims to propose a new method of centroid using the circumcenter. The calculation for the circumcenter is derived from the trapezoidal fuzzy numbers and a series of the proposed steps. The proposed method offers a straightforward calculation by considering the centroid in each part of trapezoid to obtain a new centroid which eventually becomes the circumcenter. The Euclidean distance is used to calculate the ranking function from the circumcenter of centroids and the original point. A numerical example is given to illustrate the proposed method. At the end of this paper, a comparison of centroid method between the proposed method and other methods is presented.

Keywords: Centroid of trapezoid; ranking fuzzy numbers; circumcenter; centroid of triangle; centroid of rectangle

Abstrak

Pemangkatan nombor kabur telah menjadi satu proses yang penting dalam membuat keputusan. Banyak kaedah pemangkatan telah dicadangkan setakat ini dan satu daripada kaedah yang biasa digunakan adalah sentroid bagi trapezium. Walau bagaimanapun, masih tiada persetujuan tentang kaedah yang boleh memberikan satu penyelesaian yang memuaskan untuk setiap situasi. Kertas ini bertujuan untuk mencadangkan satu kaedah baru sentroid dengan menggunakan pusat bulatan yang melalui ketiga-tiga bucu sebuah segitiga. Kaedah yang dicadangkan dapat menawarkan satu pengiraan yang mudah dengan mengambil kira sentroid setiap bahagian trapezium untuk mendapatkan sentroid baru yang akhirnya menjadi bulatan yang melalui ketiga-tiga bucu segitiga. Jarak Euclidean digunakan untuk mengira fungsi pemangkatan dari pusat bulatan yang melalui ketiga-tiga bucu segitiga dan titik asal. Pengiraan untuk pusat bulatan yang melalui ketiga-tiga bucu sebuah segitiga itu diperolehi daripada nombor kabur trapezium dan beberapa siri langkah pengiraan. Satu contoh berangka diberikan untuk menggambarkan kaedah tersebut. Pada pengakhiran kertas ini, satu perbandingan kaedah sentroid antara kaedah yang dicadangkan dan kaedah-kaedah lain turut ditunjukkan.

Kata kunci: Sentroid bagi trapezium; pemangkatan nombor kabur; pusat bulatan yang melalui ketiga-tiga bucu sebuah segitiga; sentroid segitiga; sentroid segi empat tepat

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1.0 INTRODUCTION

Ranking fuzzy numbers has been an indispensable area of research especially for its applications in decision making analysis to represent uncertain value. Many ranking fuzzy number approaches have been suggested in literature for multi attribute fuzzy decision making problems, data analysis and artificial intelligence, thus make ranking fuzzy numbers is imperative because the measurements are imprecise in nature [19]. In order to rank of fuzzy numbers, one fuzzy number needs to be evaluated and compared with other fuzzy numbers. However, this may not be seen as a straight forward process as fuzzy numbers are represented by possibility distributions. They can overlap with each other and it is

difficult to determine clearly whether one fuzzy number is larger or smaller than another [2]. Many authors have proposed different methods for ranking fuzzy numbers since 1976, but most of the methods proposed are nondiscriminating and counterintuitive.

Ranking fuzzy numbers started in 1976 when Jain [4] proposed a ranking fuzzy numbers for decision making in the presence of fuzzy variables by representing the ill-defined quantity as a fuzzy set. Since then, a number of researchers proposed various procedures to rank fuzzy quantities. Bortolan and Degani [5] reviewed some of ranking methods for ranking fuzzy subsets. Chen [6] proposed ranking fuzzy numbers with maximizing set and minimizing set. Delgado *et al.* [7] presented a procedure for ranking fuzzy numbers. Kim and Park [8] presented a method of

ranking fuzzy numbers with index of optimism. Yuan [9] presented a criterion for evaluating fuzzy ranking methods. Choobineh and Li [10] presented the ranking of fuzzy sets based on the concept of existence. Cheng [11] presented a new approach for ranking fuzzy numbers by distance method. Chen and Lu [12] presented an approximate approach for ranking fuzzy numbers based on left and right dominance. Deng, *et al.* [13] presented ranking fuzzy numbers with an area method using radius of gyration. Nejad and Mashinchi [14] presented the ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number.

With various approaches of ranking fuzzy numbers have been suggested in literature, Chen and Hwang [1] made a good attempt to classify ranking fuzzy numbers. They classified ranking fuzzy numbers into four classes. The first class is the preference relation, where the techniques involved are degree of optimality, α -cut, Hamming distance and comparison function. The second class is the fuzzy mean and spread which is using the probability and possibility distributions technique. The third class is the fuzzy scoring. This class applied a few techniques such as proportional optimal, left and right scores, centroid index and measurement. The last class is the linguistic expression which contains the intuition and linguistic approximation techniques.

Out of the four classes, the most commonly used technique is the centroid index which is fall under fuzzy scoring class. This method was first proposed by Yager [15] in 1981 with weighting function. Since then, numbers of researchers have been investigated about ranking numbers using centroid method. Cheng [11] improved Yager’s method by presenting the centroid index ranking method that calculates the distance of the centroid point of each fuzzy number and original point. Chu and Tsao [16] pointed out the inconsistent and counter intuition of these two indices and proposed ranking fuzzy numbers with the area between the centroid point and original point. Liang *et al.* [17] also proposed the ranking indices and rules for fuzzy numbers based on gravity center point. Wang and Lee [18] revised the method of ranking fuzzy numbers with an area between the centroid and original points. All of these centroid methods have been successfully tested using triangular fuzzy numbers.

Triangular fuzzy numbers are the special case of trapezoidal fuzzy numbers when the two vertices take the same values. Trapezoidal fuzzy numbers necessitate a generalization process in its ranking. Chen [6,23] for example, presented generalized trapezoidal fuzzy numbers and their operations. In the following years, several researchers have been taking about the similar approaches on generalized trapezoidal fuzzy number. Chen and Chen [24] presented a method for fuzzy risk analysis based on ranking of generalized fuzzy numbers. Abbasbandy and Hajjari [25] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α -levels of trapezoidal fuzzy numbers. Chen and Chen [26] proposed a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Amit Kumar *et al.* [27] presented a procedure on ranking generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread. Bushan Rao and Ravi Shankar [3] proposed a new method based on circumcenter of centroids and uses an index of optimism to reflect the decision maker’s optimistic attitude and also an index of modality that represents the neutrality of the decision maker. Circumcenter is a point of the concurrence of the triangle’s three perpendicular bisectors and the center of the circumcircle [21]. The three sides of the triangle perpendicular bisectors meet in a point called circumcenter.

The ideas of trapezoidal fuzzy numbers, centroids, and circumcenters motivate researchers to propagate a new ranking fuzzy numbers. This paper proposes a new ranking fuzzy numbers method based on circumcenter of centroids. The basic operation in

the proposed method is splitting the area of trapezoidal into three parts. The first, second and third parts consist of a triangle, a rectangle and a triangle respectively. This paper is organized as follows. Section 2 briefly introduces the basic concepts and definitions of fuzzy numbers. Section 3 presents the new proposed method. A numerical example is illustrated in Section 4. The comparison between the new proposed method and other researcher’s methods are presented in Section 5. This paper is concluded in Section 6.

2.0 PRELIMINARIES

As to make this paper self-contained, the following definitions and fuzzy number operations are provided.

Definition 1: Fuzzy Set [20]

A fuzzy set \tilde{A} , defined on the universal set of real numbers \mathfrak{R} , is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : \mathfrak{R} \rightarrow [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$.

Definition 2: Fuzzy Numbers [22]

A fuzzy number is a fuzzy set like $u : \mathfrak{R} \rightarrow [0,1]$ satisfying the following properties:

1. u is upper semi-continuous.
2. $u(x) = 0$ outside of interval $[0,1]$.
3. There are real numbers a, b, c and d such that $a \leq b \leq c \leq d$ and
 - i. $u(x)$ is monotonic increasing on $[a, b]$,
 - ii. $u(x)$ is monotonic decreasing on $[c, d]$,
 - iii. $u(x) = 1, b \leq x \leq c$,

and the membership function u can be express as

$$u(x) = \begin{cases} u_L(x), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ u_R(x), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

where $u_L : [a, b] \rightarrow [0,1]$ and $u_R : [c, d] \rightarrow [0,1]$ are left and right membership function of fuzzy number u , respectively.

Definition 3: Generalized Trapezoidal Fuzzy Number [6]

Let \tilde{A} be a generalized trapezoidal fuzzy number, $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ as shown in Figure 1, where a_1, a_2, a_3, a_4 are real values, $w_{\tilde{A}}$ denotes the height of the

generalized trapezoidal fuzzy number \tilde{A} , and $w_{\tilde{A}} \in [0,1]$. If $-1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$, then \tilde{A} is called a standardized generalized fuzzy number.

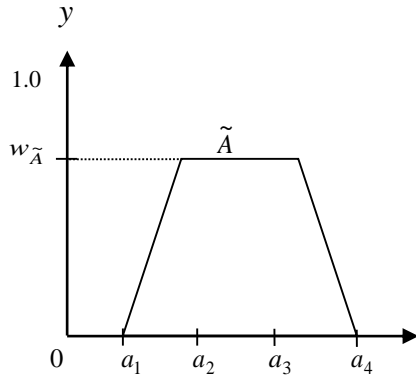


Figure 1 A generalized trapezoidal fuzzy number \tilde{A}

Assume that \tilde{A} and \tilde{B} are two generalized trapezoidal fuzzy numbers, where $\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$, $\tilde{B} = (b_1, b_2, b_3, b_4; w_{\tilde{B}})$, $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are real values, $0 \leq w_{\tilde{A}} \leq 1$ and $0 \leq w_{\tilde{B}} \leq 1$. some arithmetic operations between the generalized fuzzy numbers \tilde{A} and \tilde{B} are shown as follows.

- 1) Generalized fuzzy numbers addition \oplus :

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \oplus (b_1, b_2, b_3, b_4; w_{\tilde{B}}) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(w_{\tilde{A}}, w_{\tilde{B}})) \end{aligned}$$

- 2) Generalized fuzzy numbers subtraction \ominus :

$$\begin{aligned} \tilde{A} \ominus \tilde{B} &= (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \ominus (b_1, b_2, b_3, b_4; w_{\tilde{B}}) \\ &= (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4; \min(w_{\tilde{A}}, w_{\tilde{B}})) \end{aligned}$$

- 3) Generalized fuzzy numbers multiplication \otimes :

$$\begin{aligned} \tilde{A} \otimes \tilde{B} &= (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \otimes (b_1, b_2, b_3, b_4; w_{\tilde{B}}) \\ &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4; \min(w_{\tilde{A}}, w_{\tilde{B}})) \end{aligned}$$

- 4) Generalized fuzzy numbers division \oslash :

$$\begin{aligned} \tilde{A} \oslash \tilde{B} &= (a_1, a_2, a_3, a_4; w_{\tilde{A}}) \oslash (b_1, b_2, b_3, b_4; w_{\tilde{B}}) \\ &= (a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1; \min(w_{\tilde{A}}, w_{\tilde{B}})) \end{aligned}$$

where $b_1 \neq 0, b_2 \neq 0, b_3 \neq 0$ and $b_4 \neq 0$.

Definition 4: Triangular Fuzzy Number [1]

A triangular fuzzy number \tilde{A} is a fuzzy number with a piecewise linear membership function $\mu_{\tilde{A}}$ defined by:

$$\mu_{\tilde{A}} = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise.} \end{cases}$$

which can be denoted as a triplet (a_1, a_2, a_3) .

Let \tilde{X} and \tilde{Y} be two triangular fuzzy numbers parameterized by the triplets (x_1, x_2, x_3) and (y_1, y_2, y_3) respectively. The fuzzy number arithmetic operations between \tilde{X} and \tilde{Y} are as follows:

$$\begin{aligned} \text{Addition operation: } \tilde{X} \oplus \tilde{Y} &= (x_1 + y_1, x_2 + y_2, x_3 + y_3) \\ \text{Subtraction operation: } \tilde{X} \ominus \tilde{Y} &= (x_1 - y_1, x_2 - y_2, x_3 - y_3) \\ \text{Multiplication operation: } \tilde{X} \otimes \tilde{Y} &= (x_1 \times y_1, x_2 \times y_2, x_3 \times y_3) \\ \text{Division operation: } \tilde{X} \oslash \tilde{Y} &= (x_1 / y_1, x_2 / y_2, x_3 / y_3) \end{aligned}$$

The definitions and operations are needed to internalize the proposed method of ranking fuzzy numbers based on circumcenter. The main contribution of this paper is given in Section 3.

3.0 THE PROPOSED METHOD

In this section, a new approach for ranking fuzzy numbers based on the circumcenter of trapezoidal fuzzy numbers is presented. A trapezoid from Bushan Rao and Ravi Shankar [3] is presented to explain the proposed method. Figure 2 shows a trapezoid ABCSR which are split into three parts. The first part is a triangle (ABR), the second part is a rectangle (BCSR) and the third part is a triangle (CDS).

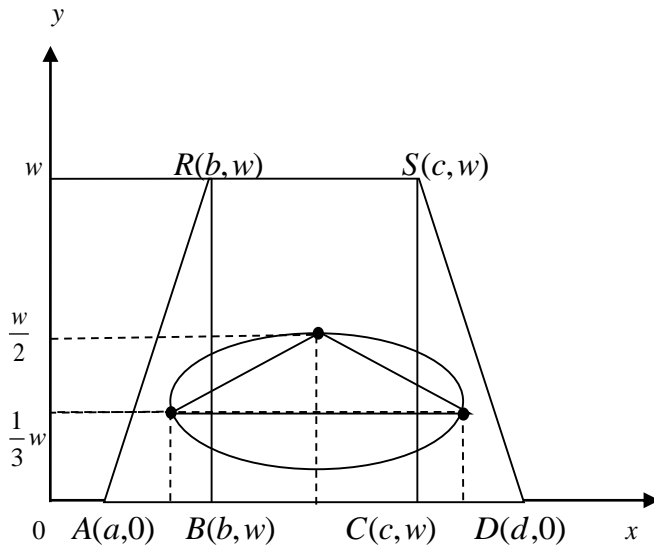


Figure 2 Circumcenter of centroids

The centroid of the trapezoid is considered as a point of balance for the trapezoid. The area and the centroid of each part are calculated using the simple area and centroid calculation. The combination of the centroid for each part will produce a triangle. The next step is to determine the distance from each centroid of each part to the reference axes which is x and y axis. Multiply the distance to the area of each part and sum the products to get the total value for x and y axis. The area of each part is then summed. The summed product of the area and distances is divided to the summed of total area to get the centroid of circumcenter represented by x and y point. Finally, a ranking function is calculated which is the Euclidean distance from the circumcenter of the centroids and the original point.

The proposed procedure can be expressed in the following steps.

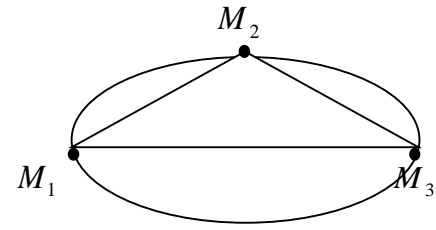
Step 1:

Calculate the area of each part. The area for triangle is equal to width*height, while the area for rectangle is equal to $\frac{1}{2}$ *base*height. With reference to the Figure 2, areas of the three parts are

$$\begin{aligned}
 A_{\Delta ARB} &= \frac{1}{2}(b-a)w \\
 A_{\square BCSR} &= (c-b)w \\
 A_{\Delta CSD} &= \frac{1}{2}(d-c)w
 \end{aligned}
 \tag{1}$$

Step2:

Determine the centroid of each part. Each centroid point is selecting from each part as a point of reference because the centroid points are the balancing points of each part. Thus, the circumcenter will be equidistant from each vertex and it would be a better reference point than the centroid point of trapezoidal [3].



- Centroid of a triangle is located at a distance of 1/3 its height and 1/3 its base.

$$\text{Centroid for } \Delta_{ARB}, M_1 = \left(\frac{1}{3}(b-a), \frac{1}{3}w\right)$$

$$\text{Centroid for } \Delta_{CSD}, M_3 = \left(\frac{1}{3}(d-c), \frac{1}{3}w\right)
 \tag{2}$$

- Centroid of a rectangle is located at a distance of $\frac{1}{2}$ its height and $\frac{1}{2}$ its base.

$$\text{Centroid for } \square_{BCSR}, M_2 = \left(\frac{1}{2}(c-b), \frac{1}{2}w\right)$$

Step 3:

Determine the distance from each simple shape’s centroid to the reference axes (x and y).

$$M_1 = \left(\frac{1}{3}(b-a) + a, \frac{1}{3}w\right)$$

$$M_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)
 \tag{3}$$

$$M_3 = \left(\frac{1}{3}(d-c) + c, \frac{1}{3}w\right)$$

Step 4:

Multiply each simple shape’s area by its distance from centroid to reference axes. This step can be summarized in Table 1.

Table 1 Multiplication of areas with distance

Shape	Area (A)	x	Ax
<i>ARB</i>	$\frac{1}{2}(b-a)w$	$\frac{1}{3}(b-a) + a$	$\left[\frac{(b-a)w}{2}\right]\left[\frac{(b-a)+a}{3}\right]$
<i>BCSR</i>	$(c-b)w$	$\frac{b+c}{2}$	$[(c-b)w]\left[\frac{b+c}{2}\right]$
<i>CSD</i>	$\frac{1}{2}(d-c)w$	$\frac{1}{3}(d-c) + c$	$\left[\frac{(d-c)w}{2}\right]\left[\frac{(d-c)+c}{3}\right]$

Shape	Area (A)	y	Ay
<i>ARB</i>	$\frac{1}{2}(b-a)w$	$\frac{1}{3}w$	$\left[\frac{(b-a)w}{2}\right]\left[\frac{w}{3}\right]$
<i>BCSR</i>	$(c-b)w$	$\frac{w}{2}$	$[(c-b)w]\left[\frac{w}{2}\right]$
<i>CSD</i>	$\frac{1}{2}(d-c)w$	$\frac{1}{3}w$	$\left[\frac{(d-c)w}{2}\right]\left[\frac{w}{3}\right]$

Step 5:

Sum the products of each simple shape's area and their distances from the centroid to the reference axes.

$$\begin{aligned}
 \sum_{Ax} &= \left[\left(\frac{(b-a)w}{2}\right)\left(\frac{(b-a)+a}{3}\right)\right] + \left[(c-b)w\left(\frac{b+c}{2}\right)\right] + \left[\left(\frac{(d-c)w}{2}\right)\left(\frac{(d-c)+c}{3}\right)\right] \\
 &= \left[\frac{(b^2-ab)w}{6}\right] + \left[\frac{(c^2-b^2)w}{2}\right] + \left[\frac{(d^2-cd)w}{6}\right] \\
 &= \frac{(d^2-cd+3c^2-2b^2-ab)w}{6}
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 \sum_{Ay} &= \left[\left(\frac{(b-a)w}{2}\right)\left(\frac{w}{3}\right)\right] + \left[\frac{(c-b)w^2}{2}\right] + \left[\left(\frac{(d-c)w}{2}\right)\left(\frac{w}{3}\right)\right] \\
 &= \left[\frac{(b-a)w^2}{6}\right] + \left[\frac{(c-b)w^2}{2}\right] + \left[\frac{(d-c)w^2}{6}\right] \\
 &= \frac{(d+2c-2b-a)w^2}{6}
 \end{aligned} \tag{5}$$

Step 6:

Sum the individual simple shape's area to determine total shape area.

$$\begin{aligned} \sum A &= \left[\frac{1}{2}(b-a)w \right] + [(c-b)w] + \left[\frac{1}{2}(d-c)w \right] \\ &= \frac{(d+c-b-a)w}{2} \end{aligned} \tag{6}$$

Step 7:

Divide the summed product of areas and distances by the summed object total area.

$$\begin{aligned} \bar{x} &= \frac{\left[\frac{(d^2 - cd + 3c^2 - 2b^2 - ab)w}{6} \right]}{\left[\frac{(d+c-b-a)w}{2} \right]} \\ &= \frac{d^2 - cd + 3c^2 - 2b^2 - ab}{3d + 3c - 3b - 3a} \tag{7} \\ \bar{y} &= \frac{\left[\frac{(d+2c-2b-a)w^2}{6} \right]}{\left[\frac{(d+c-b-a)w}{2} \right]} \\ &= \frac{(d+2c-2b-a)w}{3d + 3c - 3b - 3a} \end{aligned} \tag{8}$$

Therefore, the circumcenter of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ is defined as

$$S(\bar{x}_0, \bar{y}_0) = \left(\frac{d^2 - cd + 3c^2 - 2b^2 - ab}{3d + 3c - 3b - 3a}, \frac{(d+2c-2b-a)w}{3d + 3c - 3b - 3a} \right) \tag{9}$$

The ranking function of the trapezoidal fuzzy number

$\tilde{A} = (a, b, c, d; w)$ is defined as $R(\tilde{A}) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$ which is the Euclidean distance from the circumcenter of the centroids and the original points.

Let \tilde{A}_i and \tilde{A}_j be two fuzzy numbers, then

- i. if $R(\tilde{A}_i) > R(\tilde{A}_j)$, then $\tilde{A}_i \prec \tilde{A}_j$,
- ii. if $R(\tilde{A}_i) < R(\tilde{A}_j)$, then $\tilde{A}_i \prec \tilde{A}_j$,
- iii. if $R(\tilde{A}_i) = R(\tilde{A}_j)$ then in this case, the discrimination of fuzzy numbers is not possible.

To present the rationality and necessity, the proposed method is illustrated in the following example and compared with other methods.

4.0 NUMERICAL EXAMPLE

In this section, a numerical example from Chen and Chen [26] is used to illustrate the steps of computation in the proposed

method. Chen and Chen [26] used the numerical example in their proposed method by considering the defuzzified values, the heights and the spreads for ranking generalized fuzzy numbers. Besides, they also proposed a new algorithm which provides a useful way to deal with fuzzy risk analysis problems.

Let's reconsider a numerical example by Chen and Chen [26] by using the new centroid of trapezoid in ranking fuzzy numbers.

Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$. With reference to Figure 2 and using the proposed method, the calculations are as follows:

Step 1: The calculations for areas \tilde{A} and \tilde{B} using Equation (1) gives the following results:

\tilde{A}	\tilde{B}
$A_{\Delta} = 0.05$	$A_{\Delta} = 0.1$
$A_{\square} = 0.2$	$A_{\square} = 0$
$A_{\Delta} = 0.05$	$A_{\Delta} = 0.1$

Step 2: The centroid for each part is determined based on Equation (2) and the results are obtained as below:




Centroid	\tilde{A}	\tilde{B}
M_1	(0.0333, 0.3333)	(0.0667, 0.3333)
M_2	(0.1, 0.5)	(0, 0.5)
M_3	(0.0333, 0.3333)	(0.0667, 0.3333)

Step 3: The distance from the Centroid in Step 2 to the reference axes is calculated as follow:




	\tilde{A}	\tilde{B}
M_1	(0.1333, 0.3333)	(0.1667, 0.3333)
M_2	(0.3, 0.5)	(0.3, 0.5)
M_3	(0.4333, 0.3333)	(0.3667, 0.3333)

Step 4: A multiplication of areas (Step 1) with distance (Step 3) is calculated based on Table 1. The results are obtained as:

For fuzzy numbers A

Shape	A	x	y	Ax	Ay
	0.0	0.133	0.333	0.00666	0.01666
	5	3	3	5	5
	0.2	0.3	0.5	0.06	0.1
	0.0	0.433	0.333	0.02166	0.01666
	5	3	3	5	5

For fuzzy numbers B

Shape	A	x	y	Ax	Ay
	0.1	0.166	0.333	0.0166	0.0333
	0	0.3	0.5	0	0
	0.1	0.366	0.333	0.0366	0.0333

Step 5: Using Equation (4) and (5), the multiplication is calculated and the results are obtained as:

	\tilde{A}	\tilde{B}
$\sum Ax$	0.08833	0.05334
$\sum Ay$	0.13333	0.06666

Step 6: The summation of individual area is calculated using Equation (6). The results for the total shape areas are:

$$\sum A_{\tilde{A}} = 0.3$$

$$\sum A_{\tilde{B}} = 0.2$$

Step 7: Using the Equation (7) and (8), circumcenter of centroid is given as:

$$\bar{x}_{\tilde{A}} = 0.2944$$

$$\bar{y}_{\tilde{A}} = 0.4444$$

$$\bar{x}_{\tilde{B}} = 0.2667$$

$$\bar{x}_{\tilde{B}} = 0.3333$$

Therefore, by using Equation (9), the circumcenter of the generalized trapezoidal fuzzy number for \tilde{A} and \tilde{B} are:

$$S(\bar{x}_{\tilde{A}}, \bar{y}_{\tilde{A}}) = (0.2944, 0.4444)$$

$$S(\bar{x}_{\tilde{B}}, \bar{y}_{\tilde{B}}) = (0.2667, 0.3333)$$

The ranking function of the trapezoidal fuzzy number for \tilde{A} and

\tilde{B} are calculated using $R = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$ and the results are given as below:

$$R(\tilde{A}) = \sqrt{(0.2944)^2 + (0.4444)^2} = 0.5331$$

$$R(\tilde{B}) = \sqrt{(0.2667)^2 + (0.3333)^2} = 0.4269$$

Therefore, the ranking order is $\tilde{A} \succ \tilde{B}$.

The proposed method need to be validated with other prominent methods. A comparison of ranking fuzzy numbers under different methods is given in the subsequent section.

5.0 COMPARATIVE STUDY

A comparative study between the proposed method and other researcher’s method is shown as below using an example extracted from Rao and Shankar’s [3].

Consider $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (-0.5, -0.3, -0.3, -0.1; 1)$.

The fuzzy numbers are ranked according to the Cheng’s method [11], Chu and Tsao’s method [16], Rao and Shankar’s [3] and the proposed method. The comparison results are given in Table 2.

Table 2 Comparison of various ranking methods

Method	$S_{\tilde{A}} = (\bar{x}_0, \bar{y}_0)$	$S_{\tilde{B}} = (\bar{x}_0, \bar{y}_0)$	$R_{\tilde{A}}$	$R_{\tilde{B}}$	Ranking Order
Cheng [11]	(0.3,0.5)	(-0.3,0.5)	0.5831	0.5831	$R_{\tilde{A}} = R_{\tilde{B}}$
Chu and Tsao[16]	(0.3,0.5)	(-0.3,0.5)	0.5831	0.5831	$R_{\tilde{A}} = R_{\tilde{B}}$
Rao and Shankar [3]	(0.3, 0.4033)	(-0.3,0.4033)	0.5026	0.5026	$R_{\tilde{A}} = R_{\tilde{B}}$
The Proposed Method	(0.1333,0.333)	(-0.0667,0.3333)	0.3589	0.3399	$R_{\tilde{A}} \succ R_{\tilde{B}}$

It is very clear that Cheng’s, Chu and Tsao’s and Rao and Shankar methods lead to indiscriminate ranking order which is $R_{\tilde{A}} = R_{\tilde{B}}$. While the proposed method gives a reasonable and discriminate result which is $R_{\tilde{A}} \succ R_{\tilde{B}}$. The comparison indicates that the proposed method offers a discriminating result and potentially could be used in ranking fuzzy numbers that may involve more than two trapezoidal fuzzy numbers.

6.0 CONCLUSION

Centroid method is one of the commonly used methods in ranking fuzzy numbers. Many centroid methods have been proposed to rank fuzzy numbers but none of the method gives a satisfactory result as it is indiscriminative. This paper has proposed a ranking fuzzy numbers using circumcenter centroid of trapezoid. The seven-steps computation was presented to reduce the

computational risks in obtaining centroid. A numerical example was also given to show the feasibility in executing the proposed method. The discriminative property of two fuzzy numbers using the new proposed method was highlighted in a comparison between the proposed method and other methods. With a straight forward computation, the proposed method could be extended to other trapezoidal fuzzy numbers thereby could overcome the variability and possibility distributions of fuzzy numbers. In order to strengthen the proposed method, a real case study shall be extended in future research.

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References

- [1] Chen, S. J. and C. L. Hwang. 1992. *Fuzzy Multiple Attribute Decision Making, Methods and Applications*. Berlin: Springer.
- [2] Lee, J. H. and K. H. You. 2003. A Fuzzy Ranking Method for Fuzzy Numbers. *IEICE Trans Fundam Electrom Commun Comput Sci*. E86-A(10): 2650–2658.
- [3] Rao, P. P. B. and N. R. Shankar. 2011. Ranking Fuzzy Numbers with a Distance Method using Circumcenter of Centroids and Index of Modality. *Advances in Fuzzy Systems*.
- [4] Jain, R. 1976. Decision-making in the Presence of Fuzzy Variables. *IEEE Transactions on Systems, Man, and Cybernetics*. 6(10): 698–703.
- [5] Bortolan, G. and R. Degani. 1985. A Review of Some Methods for Ranking Fuzzy Subsets. *Fuzzy Sets and Systems*. 15(1): 1–19.
- [6] Chen, S. H. 1985. Ranking Fuzzy Numbers with Maximizing Set and Minimizing Set. *Fuzzy Sets and Systems*. 17(2): 113–129.
- [7] Delgado, M., J. L. Verdegay and M. A. Vila. 1988. A Procedure for Ranking Fuzzy Numbers Using Fuzzy Relations. *Fuzzy Sets and Systems*. 26(1): 49–62.
- [8] Kim, K. and K. S. Park. 1990. Ranking Fuzzy Numbers with Index of Optimism. *Fuzzy Sets and Systems*. 35(2): 143–150.
- [9] Yuan, Y. 1991. Criteria for Evaluating Fuzzy Ranking Methods. *Fuzzy Sets and Systems*. 43(2): 139–157.
- [10] Choobineh, F. and H. Li. 1993. An Index for Ordering Fuzzy Numbers. *Fuzzy Sets and Systems*. 54(3): 287–294.
- [11] Cheng, C.H. 1998. A New Approach for Ranking Fuzzy Numbers by Distance Method. *Fuzzy Sets and Systems*. 95(3): 307–317.
- [12] Chen, L. H. and H. W. Lu. 2001. An Approximate Approach for Ranking Fuzzy Numbers Based on Left and Right Dominance. *Computers and Mathematics with Applications*. 41(12): 1589–1602.
- [13] Deng, Y., Z. Zhenfu and L. Qi. 2006. Ranking Fuzzy Numbers with an Area Method Using Radius of Gyration. *Computers and Mathematics with Applications*. 51(6–7): 1127–1136.
- [14] Nejad, A. M. and M. Mashinchi. 2010. Ranking Fuzzy Numbers Based on The Areas on The Left and The Right Sides of Fuzzy Number. *Computers and Mathematics with Applications*. 61(2): 431–442.
- [15] Yager, R. R. 1981. Fuzzy Decision Making Including Unequal Objectives. *Fuzzy Sets and Systems*. 1(2): 87–95.
- [16] Chu, T. C. and C. T. Tsao. 2002. Ranking Fuzzy Numbers with an Area between the Centroid Point and Original Point. *Computers and Mathematics with Applications*. 43(1): 111–117.
- [17] Liang, C., J. Wu and J. Zhang. 2006. Ranking Indices and Rules for Fuzzy Numbers Based on Gravity Center Point. *Intelligent Control and Automation*. 1: 3159–3163.
- [18] Wang, Y. J. and H. S. Lee. 2008. The Revised Method of Ranking Fuzzy Numbers with an Area between the Centroid and Original Points. *Computers and Mathematics with Applications*. 55: 2033–2042.
- [19] Lian, Z., B. Jiao and X. Gu. 2006. A Novel Method of Ranking Fuzzy Numbers for Decision-making Problems. *Intelligent Systems Design and Application*. 1: 354–360.
- [20] Kaufmann, A. and M. M. Gupta. 1988. *Fuzzy Mathematical Models in Engineering and Management Science*. Amsterdam: North-Holland.
- [21] Loy, J. 2011. The Centers of Triangle. <http://www.jimloy.com/geometry/centers.htm>. Retrieved on 12 March 2012.
- [22] Goetschel, R. and W. Voxman. 1986. Elementary Fuzzy Calculus. *Fuzzy Sets and Systems*. 18(1): 31–43.
- [23] Chen, S.H.. 1999. Ranking Generalized Fuzzy Number with Graded Mean Integration. *Proceedings of the Eighth International Fuzzy Systems Association World Congress*. 2:899–902.
- [24] Chen, S.J. and S.M. Chen. 2007. Fuzzy Risk Analysis Based on the Ranking of Generalized Trapezoidal Fuzzy Numbers. *Applied Intelligence*. 26(1): 1–11.
- [25] Abbasbandy, S. and T. Hajjari. 2009. A New Approach for Ranking of Trapezoidal Fuzzy Numbers. *Computers and Mathematics with Applications*. 57(3): 413–419.
- [26] Chen, S.M. and J.H. Chen. 2009. Fuzzy Risk Analysis Based on Ranking Generalized Fuzzy Numbers with Different Heights and Different Spreads. *Expert Systems with Applications*. 36(3): 6833–6842.
- [27] Kumar, A., P. Singh, A. Kaur and P. Kaur. 2010. Ranking of Generalized Trapezoidal Fuzzy Numbers Based on Rank, Mode, Divergence and Spread. *Turkish Journal of Fuzzy Systems*. 1(2): 141–152.