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EXACT SOLUTIONS FOR MHD NATURAL CONVECTION FLOW NEAR AN OSCILLATING PLATE EMERGED IN A POROUS MEDIUM

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Abstract. Analytical investigation was conducted on the transient natural convection flow past an oscillating infinite vertical plate in present of magnetic field and radiative heat transfer. The classical solution of this problem for impulsively moving plate is given by Seth in [2] and is found to be a special case of the solution to be presented. The governing model equations are solved analytically with the help of Laplace transform technique. The results are expressed in terms of the velocity and temperature profiles as well as the skin-friction and Nusselt number.

Keywords: Natural convection; MHD flow; porous medium; oscillating boundary; ramped wall temperature; Laplace transform

Abstrak. Kajian secara analitik telah dilaksanakan ke atas aliran olakan bebas fana melepasi plat menegak tak terhingga yang berayun dengan kehadiran medan magnetik dan pemindahan haba beradiaktif. Penyelesaian klasik bagi masalah ini bagi gerakan plat secara dedenyut telah diberikan oleh Seth [2] dan penyelesaian ini didapati menjadi kes khas untuk penyelesaian yang akan dibentangkan. Model persamaan menakluk diselesaikan secara analitik dengan menggunakan kaedah penjelmaan Laplace. Keputusan-keputusan yang diperoleh dinyatakan dalam sebutan-sebutan profil halaju dan suhu dan juga dalam sebutan-sebutan geseran kulit dan nombor Nusselt.

Kata kunci: Natural convection; MHD flow; porous medium; oscillating boundary; ramped wall temperature; Laplace transform

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1.0 INTRODUCTION

The equations of natural convection governing the phenomena of heat transfer fluid flow during natural convection tend to be complex because of the present of buoyancy forces and the exact solutions are restricted to a particular combination of simple geometry and boundary. Several authors investigated the solution of such problem both analytically and numerically under different thermal conditions at the bounding plate. In [1] the unsteady natural convection flow of a viscous incompressible fluid near a vertical plate with ramped wall temperature was analysed, where the result of natural convection flow near a ramped temperature plate is compared with constant temperature plate. Theoretical study on the flow through a porous medium have wide variety of applications such as in petroleum technology to study the movement of natural gas, in chemical engineering for filtration and purification process as well as in agricultural engineering to study the underground water. Among the research studies are [4], [5] and [6]. Research works on the MHD natural convection flow in a porous media provide basic knowledge for the development of several MHD devices such as MHD pumps, generators, flow meters, nuclear reactor using liquid metal and geothermal energy extraction. Most problems of natural convection flows occur at a high temperature where the effect of radiation heat transfer on the flow become an important to study and analysed. The knowledge of radiative heat transfer will help in designing pertinent equipment in areas such as furnace design, missiles, nuclear power plants, gas turbines, various propulsion devices for aircraft and satellites. MHD natural convection flow past an impulsively moving vertical plate with ramped wall temperature in the presence of thermal diffusion with heat absorption are studies and analysed in [3] also [7] studied radiation effects on the flow an impulsively vertical infinite plate. Recently, [2] investigated a theoretical study of unsteady MHD natural convection flow of a viscous incompressible fluid with radiative heat transfer near an impulsively moving vertical flat plate with ramped wall temperature in a porous medium.

In this paper, exact solution of natural convection MHD flows of an incompressible viscous fluid past an infinite vertical oscillating plate in porous medium with ramped wall temperature was presented.

2.0 MODEL EQUATIONS

The unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate in porous medium is considered. The fluid is assumed to be non-magnetic passing through uniform magnetic field of strength B_0. The flow is assumed to be in x' -direction. The y' -axis is taken to be normal to the plate. Initially, for time $t' \leq 0,$ both the fluid and the plate are assumed to be at the same temperature T_∞^1. At time $t' > 0,$ the plate starts oscillations along the x' direction with velocity $u_0 \cos(\omega' t')$ or $u_0 \sin(\omega' t')$ and the temperature of the plate is increase or decrease to $T_\infty' + (T_w' - T_\infty')\frac{t'}{t_0}$ when $t' \leq t_0$, and therefore, for $t' > t_0,$ is maintained at uniform temperature T_w'.

Following [2] with above assumption, we have the following governing flow equations

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{K'} u' + g\beta' (T' - T_\infty') \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r'}{\partial y'} \tag{2}$$

where u' is the fluid velocity in x' direction, T' is the temperature of the fluid, g is the acceleration due to gravity, β' is the volumetric coefficient of thermal expansion, ν is the kinematic coefficient of viscosity, σ is the electrical conductivity, ρ the fluid density, k is the thermal conductivity, K' is the permeability of porous medium, c_p is the specific heat at constant pressure and q_r' is the radiative heat flux vector.

With the following initial and boundary conditions

$$
\begin{aligned}
&u' = u_0 \cos(\omega' t') \quad or \quad u_0 \sin(\omega t) \text{ at } y' = 0 \text{ for } t' > 0\\
&T' = F(t') \text{ at } y' = 0\\
&u' \to 0, T' \to T'_\infty \text{ as } y \to \infty \text{ for } t' > 0\\
&u' = 0,\ T' = T'_\infty \text{ for } y' \geq 0 \text{ and } t' \leq 0
\end{aligned} \tag{3}
$$

where u_0 is the maximum amplitude of wall velocity oscillation, ω is the frequency of the wall velocity and

$$F(t') = \begin{cases} T'_\infty + (T'_w - T'_\infty)\frac{t'}{t_0} & 0 < t' \leq t_0 \\ T'_w & t' > t_0 \end{cases}$$ the ramped wall temperature.

The local radiant is given by

$$q'_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{4}$$

where k^* is the mean absorption coefficients and σ^* is the Stefan-Boltzmann constant.

With small temperature difference between fluid temperature T' and free stream temperature T'_∞, T'^4 is expanded in Taylor series about a free stream temperature T'_∞ which after neglecting higher order terms we obtain the following simple form

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \tag{5}$$

By using Eq. (4) and (5), Eq. (2) becomes

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho c_p}\frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{6}$$

Consider the set of non-dimensional variables:

$$y = \frac{y'}{u_0 t_0}, \quad u = \frac{u'}{u_0}, \quad t = \frac{t'}{t_0}, \quad T = \frac{\left(T' - T'_\infty\right)}{\left(T'_w - T'_\infty\right)}$$

$$K_1 = \frac{K' U_0^2}{\upsilon^2}, \quad G_r = \frac{g\beta' \upsilon\left(T'_w - T'_\infty\right)}{u_0^3}, \quad \Pr = \frac{\rho \upsilon c_p}{k}$$

$$\omega = \frac{\upsilon \omega'}{u_0^2}, \quad N = \frac{16\sigma^* T_\infty'^3}{3kk^*}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2} \tag{7}$$

Using (7), Eqs. (1) and (6) becomes:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - Mu - \frac{u}{K_1} + GrT \tag{8}$$

$$\frac{\partial T}{\partial t} = \frac{\left(1 + N\right)}{\Pr} \frac{\partial^2 T}{\partial y^2} \tag{9}$$

where M is the magnetic parameter, K_1 is the permeability parameter, Gr is the Grashof number, $\Pr$ is the Prandtl number and N is the radiation parameter. With the characteristic time t_0 defined by

$$t_0 = \frac{\upsilon}{u_0^2} \tag{10}$$

Eq. (3), using Eq. (7) and (10) becomes

$$u = \cos(\omega t) \quad or \quad u = \sin(\omega t) \text{ at } y = 0 \text{ for } t > 0$$

$$T = \begin{Bmatrix} t & \text{for } 0 < t \le 1 \\ 1 & \text{for } \quad t > 1 \end{Bmatrix} \text{ at } y = 0$$

$$u \to 0, T \to 0 \text{ as } y \to \infty \text{ for } t > 0$$

$$u = 0,\ T = 0 \text{ for } y \ge 0 \text{ and } t \le 0 \tag{11}$$

3.0 SOLUTION FOR TRANSIENT FLOW IN CASE OF VARIABLE TEMPERATURE

3.1 Cosine Oscillation

The non-dimensional governing equations (8) and (9), subject to the boundary conditions (11) are transformed by the Laplace transform method to give

$$\bar{u}(y,s)=\frac{s}{s^2+\omega^2}\exp\left(-y\sqrt{s+\lambda}\right)+\alpha\frac{1-\exp(-s)}{s^2(s-\beta)}\left[\exp\left(-y\sqrt{s+\lambda}\right)-\exp\left(-y\sqrt{bs}\right)\right] \tag{12}$$

$$\bar{T}(y,s)=\frac{1-\exp(-s)}{s^2}\exp\left(-y\sqrt{bs}\right) \tag{13}$$

where

$b=\frac{\Pr}{(1+N)}$, $\alpha=\frac{Gr}{(b-1)}$, $\lambda=M+\frac{1}{K_1}$ and $\beta=\frac{\lambda}{(b-1)}$

By shifting and convolution theorems of Laplace transform we obtained

$$\begin{aligned} u(y,t)=&\frac{\exp(i\omega t)}{4}\left[\begin{array}{l}\exp\left(y\sqrt{(\lambda+i\omega)t}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+i\omega)t}\right)\\ +\exp\left(-y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+i\omega)t}\right)\end{array}\right]\\ &+\frac{\exp(-i\omega t)}{4}\left[\begin{array}{l}\exp\left(y\sqrt{(\lambda-i\omega)t}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda-i\omega)t}\right)\\ +\exp\left(-y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda-i\omega)t}\right)\end{array}\right]\\ &+\alpha\left[P(y,t)-H(t-1)P(y,t-1)\right] \end{aligned} \tag{14}$$

$$T(y,t)=Q(y,t)-H(t-1)Q(y,t-1) \tag{15}$$

where

$$P(y,t)=\frac{\exp(\beta t)}{2\beta^2}\left\{\begin{array}{l}\exp\left(y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta)t}\right)\\+\exp\left(-y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta)t}\right)\end{array}\right\}$$

$$-\left\{\frac{1}{2\beta^2}+\frac{1}{\beta}\left(\frac{t}{2}+\frac{y}{4\sqrt{\lambda}}\right)\right\}\exp\left(y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)$$

$$-\left\{\frac{1}{2\beta^2}+\frac{1}{\beta}\left(\frac{t}{2}-\frac{y}{4\sqrt{\lambda}}\right)\right\}\exp\left(-y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)$$

$$-\frac{\exp(\beta t)}{2\beta^2}\left\{\begin{array}{l}\exp\left(y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y\sqrt{b}}{2\sqrt{t}}+\sqrt{\beta t}\right)\\+\exp\left(-y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y\sqrt{b}}{2\sqrt{t}}-\sqrt{\beta t}\right)\end{array}\right\}$$

$$+\frac{1}{\beta^2}\operatorname{erf}c\left(\frac{y\sqrt{b}}{2\sqrt{t}}\right)$$

$$+\frac{1}{\beta}\left\{\left(\frac{by^2}{2}+t\right)\operatorname{erf}c\left(\frac{y\sqrt{b}}{2\sqrt{t}}\right)-\sqrt{\frac{bt}{\pi}}y\exp\left(\frac{-by^2}{4t}\right)\right\}$$

$$Q(y,t)=\left(\frac{by^2}{2y}+t\right)\operatorname{erf}c\left(\frac{y\sqrt{b}}{2\sqrt{t}}\right)-\sqrt{\frac{bt}{\pi}}y\exp\left(\frac{-by^2}{4t}\right)$$

3.2 Sine Oscillaation

Equations (8) and (9), subject to the boundary condition for sine oscillation in (11), are transformed by the Laplace transform method to give

$$\bar{u}(y,s)=\frac{\omega}{s^2+\omega^2}\exp\left(-y\sqrt{s+\lambda}\right)+\alpha\frac{1-\exp(-s)}{s^2(s-\beta)}\left[\exp\left(-y\sqrt{s+\lambda}\right)-\exp\left(-y\sqrt{sb}\right)\right] \tag{16}$$

$$\bar{T}(y,s)=\frac{1-\exp(-s)}{s^2}\exp\left(-y\sqrt{bs}\right) \tag{17}$$

Using shifting and the convolution theorems of the Laplace transform we have

$$u(y,t)=\frac{i\exp(-i\omega t)}{4}\left[\begin{array}{l}\exp\left(-y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda-i\omega)t}\right)\\+\exp\left(y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda-i\omega)t}\right)\end{array}\right]$$

$$-\frac{i\exp(i\omega t)}{4}\left[\begin{array}{l}\exp\left(-y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+i\omega)t}\right)\\+\exp\left(y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+i\omega)t}\right)\end{array}\right]$$

$$+\alpha\left[P(y,t)-H(t-1)P(y,t-1)\right] \qquad (18)$$

$$T(y,t)=Q(y,t)-H(t-1)Q(y,t-1) \qquad (19)$$

4.0 LIMITING CASES FOR VARIABLE TEMPERATURE PLATE

Consider special cases where the plate presents no oscillatory motion. The plate moves with constant longitudinal velocity. This situation is a special case of Eq. (14) and (18) when $w=0$, so that we have $u=1$, limiting solutions are obtained as

$$u(y,t)=\frac{1}{2}\left[\exp\left(y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)+\exp\left(-y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\right]$$

$$+\alpha\left[P(y,t)-H(t-1)P(y,t-1)\right] \qquad (20)$$

$$u(y,t)=\alpha\left[P(y,t)-H(t-1)P(y,t-1)\right] \qquad (21)$$

Eq. (20) is exactly, the same as presented in [2] while Eq. (21) is the first proposed solution of the present problem when sine oscillatory motion was considered in the bounding plate.

5.0 SOLUTION FOR TRANSIENT FLOW IN CASE OF CONSTANT TEMPERATURE

5.1 Cosine Oscillation

The governing equations (8) and (9), subject to the boundary conditions (11), in case of constant temperature are transformed by the Laplace transform method to give

$$\bar{u}(y,s)=\frac{s}{s^2+\omega^2}\exp\left(-y\sqrt{s+\lambda}\right)+\frac{\alpha}{s(s-\beta)}\left[\exp\left(-y\sqrt{s+\lambda}\right)-\exp\left(-y\sqrt{sb}\right)\right] \tag{22}$$

$$\bar{T}(y,s)=\frac{1}{s}\exp\left(-y\sqrt{sb}\right) \tag{23}$$

By shifting and convolution theorems of Laplace transform we obtain

$$\begin{aligned}
u(y,t)=&\frac{\exp(i\omega t)}{4}\left[\begin{aligned}&\exp\left(y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+i\omega)t}\right)\\&+\exp\left(-y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+i\omega)t}\right)\end{aligned}\right]\\
&+\frac{\exp(-i\omega t)}{4}\left[\begin{aligned}&\exp\left(y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda-i\omega)t}\right)\\&+\exp\left(-y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda-i\omega)t}\right)\end{aligned}\right]\\
&-\frac{\alpha}{2\beta}\left[\begin{aligned}&\exp\left(y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\\&+\exp\left(-y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)\end{aligned}\right]\\
&+\frac{\alpha\exp(\beta t)}{2\beta}\left[\begin{aligned}&\exp\left(y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta)t}\right)\\&+\exp\left(-y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta)t}\right)\end{aligned}\right]\\
&+\frac{\alpha}{\beta}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}\right)-\frac{\alpha\exp(\beta t)}{2\beta}\left[\exp\left(y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}+\sqrt{\beta t}\right)\right]\\
&+\exp\left(-y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}-\sqrt{\beta t}\right)
\end{aligned} \tag{24}$$

$$T(y,t)=\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}\right) \tag{25}$$

5.2 Sine Oscillation

The governing equations (8) and (9), subject to the boundary conditions (11), in case of constant temperature plate are transformed by the Laplace transform method to give

$$\bar{u}(y,s)=\frac{\omega}{s^2+\omega^2}\exp\left(-y\sqrt{s+\lambda}\right)+\frac{\alpha}{s(s-\beta)}\left[\exp\left(-y\sqrt{s+\lambda}\right)-\exp\left(-y\sqrt{sb}\right)\right] \tag{26}$$

$$\bar{T}(y,s)=\frac{1}{s}\exp\left(-y\sqrt{sb}\right) \tag{27}$$

By shifting and convolution theorems of Laplace transform we obtained

$$\begin{aligned}
u(y,t)=&\frac{i\exp(-i\omega t)}{4}\begin{bmatrix}\exp\left(-y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda-i\omega)t}\right)\\ +\exp\left(y\sqrt{\lambda-i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda-i\omega)t}\right)\end{bmatrix}\\
&-\frac{i\exp(i\omega t)}{4}\begin{bmatrix}\exp\left(-y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+i\omega)t}\right)\\ +\exp\left(y\sqrt{\lambda+i\omega}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+i\omega)t}\right)\end{bmatrix}\\
&-\frac{\alpha}{2\beta}\left[\exp\left(y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)+\exp\left(-y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)\right]\\
&+\frac{\alpha\exp(\beta t)}{2\beta}\begin{bmatrix}\exp\left(y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta)t}\right)\\ +\exp\left(-y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta)t}\right)\end{bmatrix}\\
&+\frac{\alpha}{\beta}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}\right)-\frac{\alpha\exp(\beta t)}{2\beta}\left[\exp\left(y\sqrt{\beta a}\right)\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}+\sqrt{\beta t}\right)\right]\\
&+\exp\left(-y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}-\sqrt{\beta t}\right)
\end{aligned} \tag{28}$$

$$T(y,t) = \operatorname{erf} c\left(\frac{y}{2}\sqrt{\frac{b}{t}}\right) \tag{29}$$

5.3 Limiting Cases for Constant Temperature

Consider a special cases of Eq. (24) and (28) when $w = 0$, so that $u = 1$, we have the following limiting solutions

$$\begin{aligned} u(y,t) = {} & \frac{1}{2}\left[\begin{aligned} & \exp\left(y\sqrt{\lambda}\right)\operatorname{erf} c\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right) \\ & + \exp\left(-y\sqrt{\lambda}\right)\operatorname{erf} c\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right) \end{aligned}\right] \\ & - \frac{\alpha}{2\beta}\left[\begin{aligned} & \exp\left(y\sqrt{\lambda}\right)\operatorname{erf} c\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right) \\ & + \exp\left(-y\sqrt{\lambda}\right)\operatorname{erf} c\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t}\right) \end{aligned}\right] \\ & + \frac{\alpha\exp(\beta t)}{2\beta}\left[\begin{aligned} & \exp\left(y\sqrt{\lambda+\beta}\right)\operatorname{erf} c\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda+\beta)t}\right) \\ & + \exp\left(-y\sqrt{\lambda+\beta}\right)\operatorname{erf} c\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda+\beta)t}\right) \end{aligned}\right] \\ & + \frac{\alpha}{\beta}\operatorname{erf} c\left(\frac{y}{2}\sqrt{\frac{b}{t}}\right) - \frac{\alpha\exp(\beta t)}{2\beta}\left[\exp\left(y\sqrt{\beta a}\right)\operatorname{erf} c\left(\frac{y}{2}\sqrt{\frac{b}{t}} + \sqrt{\beta t}\right)\right] \\ & + \exp\left(-y\sqrt{\beta b}\right)\operatorname{erf} c\left(\frac{y}{2}\sqrt{\frac{b}{t}} - \sqrt{\beta t}\right) \end{aligned} \tag{30}$$

$$u(y,t) = -\frac{\alpha}{2\beta}\left[\begin{aligned}&\exp\left(y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\\&+\exp\left(-y\sqrt{\lambda}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)\end{aligned}\right]$$

$$+\frac{\alpha\exp(\beta t)}{2\beta}\left[\begin{aligned}&\exp\left(y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta)t}\right)\\&+\exp\left(-y\sqrt{\lambda+\beta}\right)\operatorname{erf}c\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta)t}\right)\end{aligned}\right]$$

$$+\frac{\alpha}{\beta}\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}\right)-\frac{\alpha\exp(\beta t)}{2\beta}\left[\exp\left(y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}+\sqrt{\beta t}\right)\right]$$

$$+\exp\left(-y\sqrt{\beta b}\right)\operatorname{erf}c\left(\frac{y}{2}\sqrt{\frac{b}{t}}-\sqrt{\beta t}\right) \tag{31}$$

The result (30) is in agreement with the result presented in [2] while in the absence of magnetic field and radiative heat transfer, the result has the same form with the one obtained in [1] in non-porous medium. The result (31) is the exact solutions for the fluid behaviour in a case of sine oscillation for constant temperature plate.

6.0 SKIN FRICTION AND NUSSELT NUMBER

We now study Skin friction denoted by τ and Nusselt number denoted by Nu associated with cosine and sine oscillation for both variable and constant bounding plate temperature.

6.1 Variable Temperature Plate

Considering result of Eqs. (14), (18) and (19) respectively we have:

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\exp(i\omega t)}{2}\left[\sqrt{\lambda+i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda+i\omega t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}\exp(-(\lambda+i\omega t))\right]$$
$$+\frac{\exp(-i\omega t)}{2}\left[\sqrt{\lambda-i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda-i\omega t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}\exp(-(\lambda-i\omega t))\right]$$
$$+\alpha[W(t)-H(t-1)W(t-1)] \tag{32}$$

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{i\exp(-i\omega t)}{2}\left[\sqrt{\lambda-i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda-i\omega t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}\exp(-(\lambda-i\omega t))\right]$$
$$-\frac{i\exp(i\omega t)}{2}\left[\sqrt{\lambda+i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda+i\omega t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}\exp(-(\lambda+i\omega t))\right]$$
$$+\alpha[W(t)-H(t-1)W(t-1)] \tag{33}$$

and

$$Nu = \left.\frac{\partial T}{\partial y}\right|_{y=0} = 2\sqrt{\frac{b}{\pi}}\left[\sqrt{t}-\sqrt{t-1}H(t-1)\right] \tag{34}$$

where

$$W(t)=\tfrac{\exp(\beta t)}{\beta^2}\left\{\sqrt{(\lambda+\beta)}\left(\operatorname{erf} c\left(\sqrt{(\lambda+\beta)t}\right)-1\right)-\tfrac{1}{\sqrt{\pi t}}\exp(-(\lambda+\beta)t)\right\}$$
$$+\tfrac{1}{2\beta\sqrt{\lambda}}\left(1-\operatorname{erf} c\left(\sqrt{\lambda t}\right)\right)+\tfrac{1}{\beta}\left(t+\tfrac{1}{\beta}\right)\left\{\sqrt{\lambda}\left(1-\operatorname{erf} c\left(\sqrt{\lambda t}\right)\right)+\tfrac{1}{\pi t}\exp(-\lambda t)\right\}$$
$$-\tfrac{\exp(\beta t)}{\beta^2}\left\{\sqrt{\beta b}\left(\operatorname{erf} c\left(\sqrt{\beta t}\right)-1\right)-\sqrt{\tfrac{b}{\pi t}}\exp(-\beta t)-\tfrac{1}{\beta}\sqrt{\tfrac{b}{\pi}}\left(2\sqrt{t}+\tfrac{1}{\beta\sqrt{t}}\right)\right\}$$

Equations (32) and (33) represent Skin friction for cosine and sine oscillation respectively, while (34) represent Nusselt number.

6.2 Constant Temperature Plate

To compute the Skin friction and Nusselt number for cosine and sine oscillation in the bounding plate we consider Eqs. (24), (28) and (29) and we obtain

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\exp(i\omega t)}{2}\begin{bmatrix} \sqrt{\lambda + i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda + i\omega t}\right) - 1\right) \\ -\frac{1}{\sqrt{\pi t}}\exp(-(\lambda + i\omega t)) \end{bmatrix}$$
$$+\frac{\exp(-i\omega t)}{2}\begin{bmatrix} \sqrt{\lambda - i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda - i\omega t}\right) - 1\right) \\ -\frac{1}{\sqrt{\pi t}}\exp(-(\lambda - i\omega t)) \end{bmatrix}$$
$$-\frac{\alpha}{\beta}\left\{\sqrt{\lambda}\left(\operatorname{erf} c\left(\sqrt{\lambda t}\right) - 1\right) - \frac{1}{\sqrt{\pi t}}\exp(-\lambda t)\right\}$$
$$+\frac{\alpha}{\beta}\exp(\beta t)\left\{\begin{aligned} &\sqrt{(\lambda + \beta)}\left(\operatorname{erf} c\left(\sqrt{(\lambda + \beta)t}\right) - 1\right) \\ &-\frac{1}{\sqrt{\pi t}}\exp(-(\lambda + \beta)t) - \sqrt{\beta a}\left(\operatorname{erf} c\left(\sqrt{\beta t}\right) - 1\right) \end{aligned}\right\} \tag{35}$$

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{i\exp(-i\omega t)}{2}\begin{bmatrix} \sqrt{\lambda - i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda - i\omega t}\right) - 1\right) \\ -\frac{1}{\sqrt{\pi t}}\exp(-(\lambda - i\omega t)) \end{bmatrix}$$
$$-\frac{i\exp(i\omega t)}{2}\begin{bmatrix} \sqrt{\lambda + i\omega}\left(\operatorname{erf} c\left(\sqrt{\lambda + i\omega t}\right) - 1\right) \\ -\frac{1}{\sqrt{\pi t}}\exp(-(\lambda + i\omega t)) \end{bmatrix}$$
$$-\frac{\alpha}{\beta}\left\{\sqrt{\lambda}\left(\operatorname{erf} c\left(\sqrt{\lambda t}\right) - 1\right) - \frac{1}{\sqrt{\pi t}}\exp(-\lambda t)\right\}$$
$$+\frac{\alpha}{\beta}\exp(\beta t)\left\{\begin{aligned} &\sqrt{(\lambda + \beta)}\left(\operatorname{erf} c\left(\sqrt{(\lambda + \beta)t}\right) - 1\right) \\ &-\frac{1}{\sqrt{\pi t}}\exp(-(\lambda + \beta)t) - \sqrt{\beta a}\left(\operatorname{erf} c\left(\sqrt{\beta t}\right) - 1\right) \end{aligned}\right\} \tag{36}$$

$$Nu = \left.\frac{\partial T}{\partial y}\right|_{y=0} = \sqrt{\frac{b}{\pi t}} \tag{37}$$

Equations (35) and (36) represent Skin friction for cosine and sine oscillation respectively, while (37) represent Nusselt number.

7.0 SUMMARY AND CONCLUSION

In this paper we presented the exact solutions for the transient MHD natural convection flow exposed in a porous medium with oscillating boundary. The bounding plate has a variable and constant temperature. The governing flow models equations are solved using Laplace transform technique. The results are presented in terms of velocity and temperature fields as well as the Shear stress and Nusselt number are expressed for two cases, variable temperature and constant wall temperature respectively. Limiting cases are considered for no oscillation, absence of magnetic field and radiative heat transfer. The solution agreed favourably with related published articles.

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