*Jurnal Teknologi*, 57 (Sciences & Engineering) Suppl 1, March 2012: 69–81


# FUZZY TIME SERIES AND SARIMA MODEL FOR FORECASTING TOURIST ARRIVALS TO BALI

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**Abstract.** Forecasting is very important in many types of organizations since predictions of future events must be incorporated into the decision-making process. In the case of tourism demand, better forecast would help directors and investors make operational, tactical, and strategic decisions. Generally, in time series we can divide forecasting method into classical method and modern methods. Although recent studies show that the newer and more advanced forecasting techniques tend to result in improved forecast accuracy under certain circumstances, no clear-cut evidence shows that any one model can consistently outperform other models in the forecasting competition [1]. In this study, the forecasting performance between Box–Jenkins approaches of seasonal autoregressive integrated moving average (SARIMA) and four models of fuzzy time series has been compared by using MAPE, MAD and RMSE as the forecast measures of accuracy. The empirical results show that Chen's fuzzy time series model outperforms the SARIMA and the other fuzzy time series models.

*Keywords*: Fuzzy time series; SARIMA

**Abstrak.** Peramalan adalah amat penting dalam kebanyakan jenis organisasi memandangkan peramalan kejadian masa hadapan berkait rapat dengan proses membuat keputusan. Dalam kes permintaan pelancongan, peramalan yang baik dapat membantu pengarah dan pelabur membuat keputusan dalam operasi, tektik dan keputusan strategi. Secara amnya, siri masa dapat dibahagikan kepada kaedah klasik dan kaedah moden. Walaupun kajian terkini menunjukkan teknik peramalan yang lebih baru dan lebih canggih memberikan keputusan peramalan yang lebih tepat, tiada bukti yang kukuh menunjukkan sesuatu model dapat menandingi model yang lain secara konsisten dalam saingan peramalan [1]. Dalam kajian ini, pencapaian peramalan di antara kaedah pendekatan Box-Jenkins *seasonal auto regressive integrated moving average (SARIMA)* dan empat model *fuzzy time series* telah dibandingkan dengan menggunakan *MAPE*, *MAD* dan *RMSE* sebagai pengukuran ketepatan peramalan. Keputusan emperikal menunjukkan model *Chen's fuzzy time series* menandingi SARIMA dan model-model *fuzzy time series* yang lain.

*Kata kunci*: *Fuzzy time series; SARIMA*

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## 1.0 INTRODUCTION

Forecasting is very important in many types of organizations since predictions of future events must be incorporated into the decision-making process. In the case of tourism demand, better forecast would help directors and investors make operational, tactical, and strategic decisions. Besides that, government bodies need accurate tourism demand forecasts to plan required tourism infrastructures, such as accommodation site planning and transportation development, among other needs. There are many types of forecasting methods. Generally, in time series we can divide forecasting method into classical or traditional method and modern methods. Although recent studies show that the newer and more advanced forecasting techniques tend to result in improved forecast accuracy under certain circumstances, no clear-cut evidence shows that any one model can consistently outperform other models in the forecasting competition[1].

The time series forecasting methods have found applications in very wide areas including but not limited to finance and business, computer science, all branches of engineering, medicine, physics, chemistry and many interdisciplinary fields. Conventionally, researchers have employed traditional methods of time series analysis, modeling, and forecasting. Some of mainly been used that will discuss in this paper are Box–Jenkins methods seasonal auto-regressive integrated moving average (SARIMA), Holt Winters and time series regression. The conventional time series modeling methods have served the scientific community for a long time; however, they provide only reasonable accuracy and suffer from the assumptions of stationarity and linearity. Due to these constrains, comes the idea of alternative solution that is fuzzy time series method. In this study, the performance of forecasting between classical Box–Jenkins methods seasonal auto-regressive integrated moving average (SARIMA) and fuzzy time series has been compared

[2, 3]first introduced the definitions of fuzzy time series, and developed their model by using fuzzy relation equations and approximate reasoning. Since that, fuzzy time series has gains much attention from researchers in many fields and the methods have been developed rapidly. In forecasting time series, [4] proposed a first order fuzzy time series used simplified arithmetic operations and fuzzy logical relationship groups to forecast the enrollments of the University of Alabama. Then, [5] developed a high-order fuzzy time series model by extending Chen's first-order model [6]. [7] improved the forecasting accuracy of Chen's model [4]

by properly defining the number of linguistic variables. In order to overcome recurrence and weighting problems in fuzzy time series forecasting, [8] developed the weighted fuzzy time series models. Recently, in tourism demand forecasting [9] developed adaptive fuzzy time series model to forecast Taiwan's tourism demand.

In this study we used data of tourist arrivals to Bali in Indonesia are considered as case-study. The data were taken from the Indonesia Central Bureau of Statistics. All the dataset contain monthly data from January 1989 to December 1997. We only consider the data until 1997 to anticipate extreme data. For the estimation (in-sample) purpose, data are taken from January 1989 to December 1996. Meanwhile, data from January 1997 to December 1997 are considered for the testing or evaluation (out-sample) purpose.

This paper was organized as follows. Section 2 contains brief explanation on methodology and application procedure of fuzzy time series. Statistics to evaluate the accuracy of forecasting performance are presented in Section 3 followed by the results in Section 4. Finally, the conclusions contained in Section 5.

## 2.0 METHODS

### 2.1 The SARIMA Model

The Box-Jenkins approach to modelling autoregressive integrated moving average (ARIMA) processes involved an iterative three-stage process of model selection or identification, parameter estimation and model checking.

Since the tourist arrivals data that we used in were measured at regular calendar intervals within a year, it may exhibit periodic behaviour. Hence, the general Box-Jenkins model which allocates seasonality with $P$ seasonal autoregressive terms, $D$ seasonal differences and $Q$ seasonal moving average terms (refer [10]) is given as follows:

$$\phi_p(B)\Phi_P(B^s)\nabla^d\nabla_s^D z_t = \theta_q(B)\Theta_Q(B^s)a_t$$

where

$$\nabla = \nabla_1 = 1 - B$$

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p$$
$$\Phi_P(B^s) = 1 - \Phi_1 B^S - \Phi_2 B^{2S} - \cdots - \Phi_P B^{PS}$$
$$\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q$$
$$\Theta_Q(B^s) = 1 - \Theta_1 B^S - \Theta_2 B^{2S} - \cdots - \Theta_Q B^{QS}$$

## 2.2 Fuzzy Time Series

[11] and [12]first introduced the definitions of fuzzy time series, and developed their model by using fuzzy relation equations and approximate reasoning. General definitions of fuzzy time series are given as follows:

Let $U$ be the universe of discourse, where$U = \{u_1, u_2, \ldots, u_b\}$. A fuzzy set $A_i$ of $U$ is defined as$A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_b)/u_b$, where $f_{A_i}$ is the membership function of the fuzzy set $A_i; f_{A_i}: U \to [0,1]$. $u_a$ is a generic element of fuzzy set $A_i; f_{A_i}(u_a)$ is the degree of belongingness of $u_a$ to $A_i; f_{A_i}(u_a) \in [0,1]$ and $1 \le a \le b$.

**Definition 1.** Fuzzy time series. Let $Y(t)(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers $R$, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \ldots$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

**Definition 2.** If there exists a fuzzy relationship $R(t-1, t)$, such that $F(t) = F(t-1) \circ R(t-1, t)$, where $\circ$ is an arithmetic operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ can be denoted by $F(t-1) \to F(t)$.

**Definition 3.** Suppose $F(t)$ is calculated by $F(t-1)$ only, and $(t) = F(t-1) \circ R(t-1, t)$. For any $t$, if $R(t-1, t)$ is independent of $t$, then $F(t)$ is considered as a time-invariant fuzzy time series. Otherwise, $F(t)$ is time-variant.

**Definition 4.** Suppose $F(t-1) = A_i$and $F(t) = A_j$, a fuzzy logical relationship can be defined as

$A_i \to A_j$ ,

where$A_i$and $A_j$ are called the left-hand side (LHS) and right-hand side (RHS) of the fuzzy logical relationship, respectively.The definition of the first order seasonal fuzzy time series model for forecasting proposed by Song (1999) is given as follows:

**Definition 5.** Let $F(t)$ be a fuzzy time series. Assume there exists seasonality in $\{F(t)\}$, first order seasonal fuzzy time series forecasting model:

$$F(t-m) \rightarrow F(t)$$

where $m$ denotes the period.

The high order fuzzy time series model proposed by[5] is given as follows:

**Definition 6.** Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1), F(t-2), \ldots,$and $F(t-n)$, then this fuzzy logical relationship is represented by

$$F(t-n), \ldots, F(t-2), F(t-1) \rightarrow F(t)$$

and it is called the $n$th order fuzzy time series forecasting model.

Initially, the repeated FLRs were simply ignored when fuzzy relationships were established. In many previous studies, each FLR was treated as if it was of equal importance, which may not have reflected the real world situation ([11], [12], [4], [13], [14]). In this scenario, the occurrences of the same FLRs are regarded as if there were only one occurrence. In other words, the recent identical FLRs are simply ignored. To explain this, suppose there are FLRs in chronological order that have the same LHS, $A_1$, as follows:

$$\begin{aligned} &(t=1) A_1 \rightarrow A_2 \\ &(t=2) A_1 \rightarrow A_1 \\ &(t=3) A_1 \rightarrow A_1 \\ &(t=4) A_1 \rightarrow A_3 \\ &(t=5) A_1 \rightarrow A_1. \end{aligned} \tag{1}$$

Following [6], these FLRs in Eq. (1) are used to establish an FLRG as

$$\mathrm{A}_1 \rightarrow \mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3 \; . \tag{2}$$

The ignoring of recurrence, however, is questionable. [14]argued that the occurrence of a particular FLR represents the number of its appearances in the past. For instance, in Eq. (1), $A_1 \rightarrow A_1$ appears three times, both $A_1 \rightarrow A_2$ and $A_1 \rightarrow A_3$ only once. The recurrence can be used to indicate how the FLR may appear in the future.

Later,[14]proposed the chronological weights to deal with recurrent fuzzy relationships and their importance. To illustrate it, suppose there are FLRs in chronological order as in Eq. (1), and then the weights are as follows:

$(t = 1)A_1 \rightarrow A_2$ with weight 1,

$(t = 2)A_1 \rightarrow A_1$ with weight 2,

$(t = 3)A_1 \rightarrow A_1$ with weight 3,

$(t = 4)A_1 \rightarrow A_3$ with weight 4,

$(t = 5)A_1 \rightarrow A_1$ with weight 5.

As a result, the most recent FLR $(t = 5)$ is assigned the highest weight of 5, which means that the probability of its appearance in the near future is higher than in the case of the others. On the other hand, the most aged FLR $(t = 1)$ is assigned the lowest weight of 1, which means that the probability of its appearance in the near future is lower than in the case of the others.

Recently,[15] proposed the weights focused on the probability of its appearance and their importance of chronological FLR for the same recent identical FLRs. To explain it, suppose there are FLRs in chronological order as in Eq. (1), and then the weights are as follows:

$(t = 1)A_1 \rightarrow A_2$ with weight 1,

$(t = 2)A_1 \rightarrow A_1$ with weight 1,

$(t = 3)A_1 \rightarrow A_1$ with weight 2,

$(t = 4)A_1 \rightarrow A_3$ with weight 1,

$(t = 5)A_1 \rightarrow A_1$ with weight 3.

In this study, we applied the FTS according to the following procedures:

**Step 1**: The data are not stationary, hence data preprocessing has been carried out by taking transformation according to [16]:

$$Z_t = (Y_t^{\lambda} - 1)/\lambda$$

Where $\lambda$ is the coefficient from Box-Cox Transformation.

**Step2:** Fuzzy relationship was determined according to SARIMA model for the data set. This procedure also has been done by [17]in order to select neural network input variable. Due to limitation of considered methods, we have to ignore the lag from MA and SMA. For instance SARIMA model for Bali is $(0,1,1)(0,1,1)^{12}$. Hence the fuzzy relationship can be denoted by:

$F(t-13),\ F(t-12),\ F(t-1) \rightarrow F(t).$

**Step 3**: In order to select input and fuzzy time series order, firstly we try to input all the three input from fuzzy relationship that obtained in step 2. To experiment the selection of input, we try all the possible combination of two input from the three inputs and single input as well. So all possible input are:

1. Lag 1,12, 13
2. Lag 1, 12
3. Lag 1, 13
4. Lag 12, 13
5. Lag 1
6. Lag 12
7. Lag 13

**Step 4:** The optimum length of intervals was calculated following average-based length. Huang (2001)

**Step 5:** Forecast. Four different FTS methods was used:

(a) [5]
(b) [15]
(c) [8]

**Step 6:** Forecast data were transformed back and the forecast accuracy were calculated.

## 3.0 MEASURES OF ACCURACY

In order to evaluate the accuracy of forecast data by the six methods, we computed mean absolute percentage error (MAPE), mean absolute deviation (MAD), mean square error (MSE) and root mean square error (RMSE). For all three measures, the smaller the value, the better the fit of the model. These statistics are compute as follows:

$$\text{MAPE} = \frac{\sum|(y_t - \hat{y}_t)/y_t|}{n}(100) \quad ; \; y_t \neq 0$$

$$\text{MAD} = \frac{\sum|(y_t - \hat{y}_t)|}{n}$$

$$\text{MSE} = \frac{\sum(y_t - \hat{y}_t)^2}{n}$$

$y_t$is the actual value at time *t*, and $\hat{y}_t$ is the fitted value.For in sample,*n* is the number of observations (degree of freedom in case of SARIMA). Meanwhile, for out sample, *n* is the number of forecast data which is 12 in this study.

## 4.0 RESULTS AND DISCUSSIONS

### 4.1 SARIMA

In this study we used MINITAB version 14 to analyze SARIMA model. In model identification stage, firstly we used time series plot to see briefly whether the data have seasonal and trend patterns. Fig. 1 shows clearly that the data sets have trend patterns; hence the assumption of stationary condition in mean is not satisfied. Yet, we validate this assumption by using autocorrelation function (ACF) and partial autocorrelation (PACF) plot and from the results ACF plot for all data only dies down until first differencing both in non-seasonal and seasonal (S=12). From box-cox plot, we found out that both data are not stationary in variance.

Data transformation were made according to [18]. Fig. 2 shows the data has been detrended and deseasonalised after transformation and differencing for both non-seasonal and seasonal order. ACF and PACF were plotted again in order to identify the model and took a few possible models. The best model was chosen among these competitive models base on the smallest RMSE value. The model parameters were estimated by using least squares estimation. Models with insignificant parameters (exclude constant) were eliminated. Remaining models then proceed to the diagnostic checking step to see whether the models adequate by using Ljung-Box statistics.

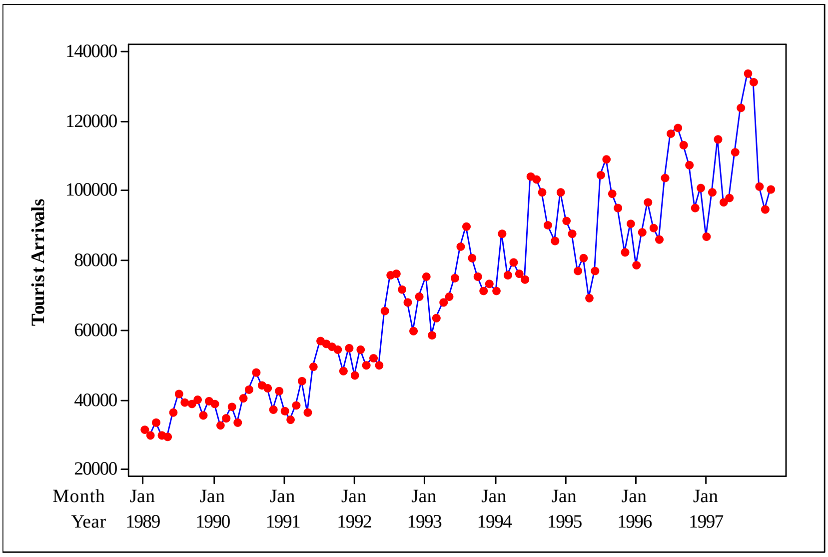


**Figure 1** Time series plot for number of tourist arrivals to Bali

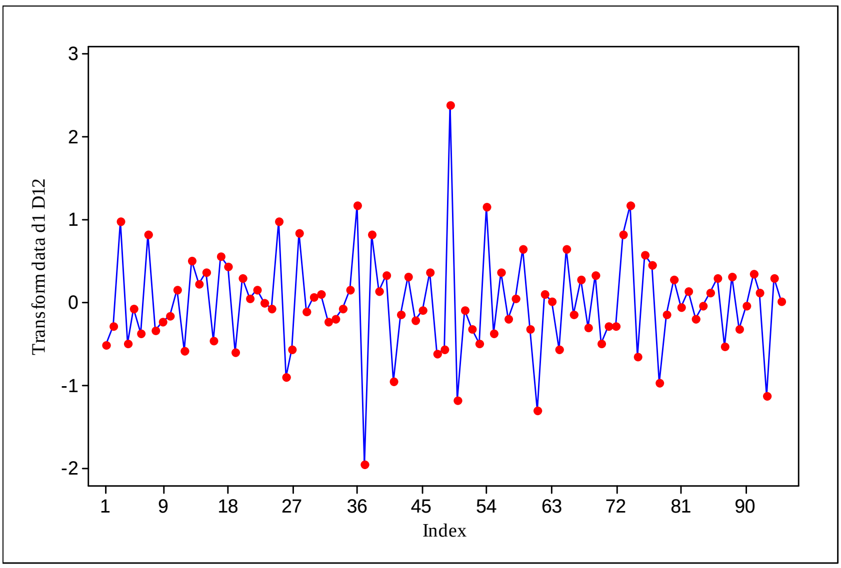


**Figure 2** Time series plot for number of tourist arrivals to Bali after data transformation and differencing

According to RMSE value, the best SARIMA model for Bali is SARIMA$(0,1,1)(0,1,1)^{12}$. Therefore, the models to forecast tourist arrivals to Bali after taking the parameter estimation from MINITAB output are following the below equation:

$$Z_t = Z_{t-1} + Z_{t-12} - Z_{t-13} + a_t - 0.6489a_{t-1} + 0.8160a_{t-12} - 0.5295a_{t-13}$$

where $Z_t = (Y_t^{\lambda} - 1)/\lambda$ .

## 4.2 Fuzzy Time Series

All the four methods of FTS were implemented by using Matlab 2008a. The selection of the best FTS input is according RMSE. We choose only lag 13 for Bali and in case of Chen's method for Bali, since only one lag was chosen, hence it follows Chen's first order method (refer [6]). The number of intervals for Bali is 26 (calculated following average-based length procedure).

## 4.3 Comparison of Forecasting Performance

The forecasting of tourist arrivals to Bali gate in testing period are done using four different approaches (total seven methods). From the results in Table 1, it is clearly shows that FTS Chen's method outperformed the Box-Jenkins method and all others FTS methods. FTS Chen's method gives the most accurate forecast according to MAPE, MAD and RMSE. It seems that all the three accuracy measurement give consistent ranking, except for RMSE for second, third and fourth ranking.

**Table 1** Comparison of forecast performance by different forecasting methods

| Method | MAPE | rank | MAD | rank | RMSE | rank |
|---|---|---|---|---|---|---|
| F.T.S Chen's | 7.265 | (1) | 8100.7 | (1) | 10136.0 | (1) |
| F.T.S Yu's | 7.518 | (2) | 8329.2 | (2) | 10203.3 | (2) |
| F.T.S Cheng's | 7.677 | (3) | 8473.6 | (3) | 10262.8 | (3) |
| SARIMA | 8.395 | (4) | 8684.5 | (4) | 10601.6 | (4) |

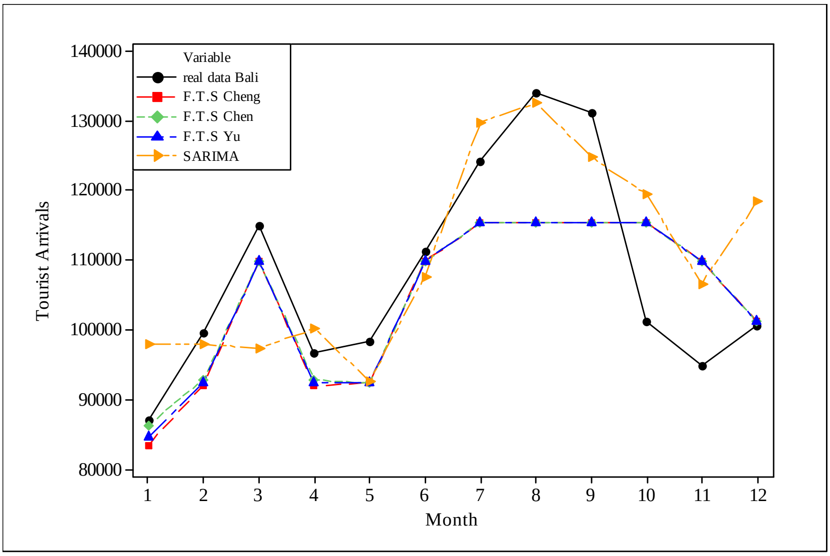


**Figure 3** Forecasts of tourist arrivals in Bali (1997)

## 5.0 CONCLUSIONS

It is found that the best method to forecast the tourist arrivals to Bali is to be the FTS i.e [6]. Although this method known to be the simplest or conventional methods of FTS, yet this result should not be odd since several previous studies also have shown that simple method could outperform more advance or complicated methods (see e.g [20], [21], [22]). According to [22] many simple methods, such as a random-walk model, for example, offer adaptability to structural change, in that the model immediately adapts to the latest level of the series.

Generally, we can conclude that FTS (especially Chen's method) is good to predict fluctuating series as tourist arrivals. Though, future research could be done by using FTS that can consider MA terms in input lag in order to improve the forecast accuracy. More advance accuracy measurement such as statistical test also can be applied to evaluate the forecast accuracy among competition models. Besides, turning points and directional change errors also will be useful since they can give better information on tourism growth cycles instead of just forecast error magnitude.

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