

## MODELS DEVELOPMENT FOR SINGLE-ROW NETWORKS FROM CONNECTED GRAPHS

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**Abstract.** In this paper, we present a collection of models for connected graphs mapping into single-row networks. The collection involves three specific models for perfect binary trees, trees and partially dense graphs, and three general models for connected graphs. These models are compared in terms of their structures, energy values, congestion and number of doglegs in the single-row transformation. The numerical experiments are run by each respective developed program. The transformation is necessary in applications such as in the assignment of telephone channels to caller-receiver pairs roaming in cells in a cellular network on real-time basis.

*Keywords:* Single-row network; transformation; connected graph; tree; simulated annealing

**Abstrak.** Dalam kertas kerja ini, kami membentangkan satu koleksi model-model untuk penjelmaan graf-graf berkait kepada rangkaian-rangkaian baris tunggal. Koleksi tersebut mengadungi tiga model khusus untuk pepohon perduaan sempurna, pepohon dan graf-graf sebahagian padat, serta tiga model am untuk graf-graf berkait. Model-model ini dibandingkan dari segi struktur-struktur, nilai-nilai tenaga, kesesakan dan jumlah lintasan dalam penjelmaan kepada baris tunggal. Eksperimen-eksperimen berangka dijanakan oleh setiap program terbina yang berkenaan. Penjelmaan tersebut diperlukan untuk aplikasi-aplikasi seperti dalam penguntukan saluran telefon kepada pasangan pemanggil dan penerima merayau dalam sel-sel dalam rangkaian telefon atas asas masa nyata.

*Kata kunci:* Rangkaian baris tunggal; penjelmaan; graf berkait; pepohon; simulasi penyepuhlindapan

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## 1.0 INTRODUCTION

Single-row routing is crucial in printed circuit boards (PCBs) design. The optimal routing requires the minimum congestion and doglegs to ensure smooth communication between the components in the PCBs. Hence, many researches have been carried out to solve the single-row routing problem. Single-row routing problem has been shown to be NP-complete problem [1]. Kuh *et al.* had developed the necessary and sufficient condition for optimum single-row routing problem ([1] and [2]). In [3], a partitioning strategy was proposed to group the nets into zones which produces some reasonably good solutions for some restricted models. Bhattacharya *et al.* proposed a new approach based on graph theoretic representation in [4] which relates the intervals of the single-row network with the overlap and interval graphs to solve the single-row routing problem.

The main objective in single-row routing problem is to minimize the congestion. However, the minimization of doglegs or inter-street crossings is also crucial in printed circuit boards as the presence of doglegs increases the system overhead. A model called *Enhanced Simulated Annealing Technique for Single Row Routing* (ESSR) was proposed in 2002 to optimize the network by minimizing both the congestion and number of doglegs [5]. When the total energy value is being minimized, congestion and number of doglegs are minimized as well. Based on the simulated annealing technique [6], the energy function in ESSR is a function of the height of the segments of the nets in the single-row network. The technique has successfully been applied to produce optimal solutions to all net sizes.

Many engineering and science problems can be represented as a problem in graph theory. The graph represents the scenario of the real-life applications where the nodes in the graph can be treated as nodes in a network, and the edges are representing the communication links between the nodes. The relation between a complete graph and its single-row representation was first formulated in [7] and [8]. Both models discuss the technique of transforming a graph into a single-row network where ESSR [5] is applied to produce optimal results. The transformation finds its application, for example, in assigning telephone channels to caller-receiver pairs roaming in cellular regions in a cellular network on real-time basis.

In order to optimize the single-row transformation of connected graphs, a series of researches are carried out to generalize the transformation technique to

an arbitrarily connected graph. Connected graphs can be categorized based on the factors of structure, order and size. Three *specific models* which are Perfect Binary Tree Sequence (PBTS) [9], Tree Sequence Model (TSM) [10] and Connected Graph Sequence (CGS) [11] have been developed for specific cases of the graph in the form of perfect binary tree, tree and partially dense graph, respectively. Meanwhile, another three models, namely *general models* which include Double Simulated Annealing (DSA) [12], Spanning Tree of Connected Graph Model (STCGM) [13] and Graph Partitioning and Transformation Model (GPTM) [14] are developed for different orders and sizes of arbitrary connected graphs.

Apparently, the best model for each of the three specific cases of the graph is none other than their respective specific models. In this paper, the three general models are compared within themselves, including the use of the specific cases of graphs to infer the advantages of each model.

## 2.0 DEVELOPMENT OF TRANSFORMATION MODELS

There are six models which have been developed to address the single-row transformation of connected graphs based on the properties, structure, order and size of graphs. A new algorithm for formation of intervals is developed and it is applied in all developed models. A model named Double Simulated Annealing (DSA) [12] is introduced to transform a connected graph into a single-row network. The DSA applies simulated annealing twice: first on connected graph  $G$  to produce the optimal node labeling, then on single-row network  $S$  to produce the optimal routing, which is also known as the minimization of energy by ESSR [5]. Node labeling is an essential for the single-row transformation technique as it significantly affects the results.

From the numerical experiment, DSA is an efficient tool in transforming a lower order of graph below 50 nodes. Since it is difficult to deal with a graph in general as different graphs have inbuilt properties that make them resistant to mapping, case by case analysis is needed to develop a model which can be applied by an arbitrary graph. The first step in generalizing the transformation technique to an arbitrary graph is to study the graph properties of perfect binary trees.

A perfect binary tree has unique graph properties where each node has two child nodes except for the leaves and all the leaves are at the same depth. A model named Perfect Binary Tree Sequence (PBTS) [9] is developed to optimally

transform a perfect binary tree to a single-row network. In the formation of zones, a new technique called Formation of Spine through Insertion Mechanism is proposed, followed by Expansion of Spine into an optimal order of zones. It is then followed by ESSR to minimize the energy. Based on the samples, numerical experiment shows that PBTS is an efficient model to transform the perfect binary trees into single-row networks and the energy value ( $E$ ) grows linearly with the order of perfect binary tree ( $n$ ) as in Equation (1).

$$E = 0.4553n - 2.6243. \quad (1)$$

Following that, a case by case analysis is furthered from a perfect binary tree to an arbitrary tree. A new model named Tree Sequence Model (TSM) [10] which optimally transforms a tree into a single-row network is introduced based on some preliminary concepts from the study of the relation between a tree and its single-row representation. In the formation of zones, new techniques such as the formation of partitions, followed by the insertion mechanism into an optimal arrangement of zones are developed. In order to enhance the scalability of the new developed model, an idea that stems from the previous work on Perfect Binary Tree Sequence (PBTS) is introduced, namely partition enhancement. Numerical experiment supports the hypothesis that TSM is an efficient model in transforming the trees into single-row networks and the results formulation based on the samples shows the energy value ( $E$ ) grows linearly with the order of tree ( $n$ ) as in Equation (2).

$$E = 0.4556n - 4.5555. \quad (2)$$

The work is then furthered from a tree to a sparsely connected graph. A sparsely connected graph is a connected graph with relatively few edges. The sparsely connected graph in this research is assumed to have one unit of weight for each edge. The relation between the sparsely connected graph with its spanning tree and the spanning tree with its single-row representation are studied and a model was developed based on the finding, namely the Spanning Tree of Connected Graph Model (STCGM) [13]. A new algorithm is developed in the formation of zones, namely Path-Growing Tree-Forming algorithm which is applied with Vertex-Prioritized to produce the spanning tree from the sparsely connected graph. The spanning tree is then transformed into single-row network using TSM [10]. Based on the samples, numerical experiment shows that the STCGM is outstanding in single-row transformation for arbitrary sparsely connected graph with an upper limit as shown in Equation (3)

$$n \times \text{Density} < 0.02n + 3.3. \quad (3)$$

Next, the case by case analysis is furthered from a sparsely connected graph to a partially dense graph. A partially dense graph is defined as a graph where a number of densely connected components, namely subgraphs, are connected by some links. A new model is introduced, namely Connected Graph Sequence (CGS) [11] to transform the partially dense graphs into single-row networks. In order to optimally transform the partially dense graph into single-row, graph partitioning is needed to divide the graph into a number of subgraphs according to the structure and connection of the given graph.

There are two different types of the formation of zones presented in CGS. The models are named CGS by modeling (CGS-M) and CGS by simulated annealing (CGS-SA) where the algorithm is a non-heuristic based modeling and a heuristic based simulated annealing, respectively. Since the partially dense graph is divided into a number of subgraphs, the algorithm of the formation of zones in [12] is modified where each subgraph is treated as a graph before the completion of intervals formation. Numerical experiment shows both of the CGS models produce optimal results for partially dense graphs and CGS-SA works slightly better than the CGS-M.

Since graph partitioning gives a systematic arrangement that leads towards better solutions, graph partitioning is applied in the effort to generalize the transformation technique to an arbitrary graph. A new model is developed, namely Graph Partitioning and Transformation Model (GPTM) [14] to transform an arbitrary graph into a single-row network. The GPTM consists of linear-time heuristic graph partitioning which was introduced [15] where the minimization algorithm of the scheme requires  $O(P)$  time to complete one pass.

The given connected graph is divided into two equally order of subgraphs and the nodes are swapped between the two subgraphs to obtain the min cut of the graph. The two subgraphs are then transformed into an optimal ordering of zones as in CGS-SA before the ESSR is applied to solve the single-row routing problem. From the numerical experiment, GPTM improves the results of DSA for an order of graph larger than 20 nodes even with higher of graph size. Based on the samples, GPTM is an outstanding model in terms of results as the order and density of the graph increase.

### 3.0 RESULTS COMPARISON ON SPECIFIC CASES OF GRAPHS

In this collected works, there are three specific cases of graphs being studied and each case is designated with a specific model to optimally transform the respective specific case of graphs into single-row networks. The three general models are applied in each of the specific cases together with the respective specific model to study and investigate the superiority of each general model.

#### 3.1 Perfect Binary Trees

Perfect binary tree has the unique graph properties with the lowest density of graph. The specific model Perfect Binary Tree Sequence (PBTS) and the three general models; Spanning Tree of Connected Graph Model (STCGM), Double Simulated Annealing (DSA) and Graph Partitioning and Transformation Model (GPTM) are compared by using perfect binary trees. The simulation results are shown in Table 1 in terms of energy values ( $E$ ), congestion ( $Q$ ) and number of doglegs ( $D$ ).

**Table 1** Results of single-row transformation for some perfect binary trees by PBTS, STCGM, DSA and GPTM

| Height,<br>$h$ | Order<br>of $\tau, n$ | PBTS       |     |     | STCGM      |     |     | DSA         |     |     | GPTM         |     |      |
|----------------|-----------------------|------------|-----|-----|------------|-----|-----|-------------|-----|-----|--------------|-----|------|
|                |                       | $E$        | $Q$ | $D$ | $E$        | $Q$ | $D$ | $E$         | $Q$ | $D$ | $E$          | $Q$ | $D$  |
| 2              | 7                     | <b>2</b>   | 1   | 0   | <b>2</b>   | 1   | 0   | <b>2</b>    | 1   | 0   | <b>2</b>     | 1   | 0    |
| 3              | 15                    | <b>4</b>   | 1   | 0   | <b>5</b>   | 1   | 0   | <b>4</b>    | 1   | 0   | <b>6</b>     | 1   | 0    |
| 4              | 31                    | <b>12</b>  | 2   | 0   | <b>15</b>  | 2   | 0   | <b>21</b>   | 3   | 0   | <b>23</b>    | 3   | 3    |
| 5              | 63                    | <b>24</b>  | 2   | 0   | <b>26</b>  | 2   | 0   | <b>75</b>   | 5   | 12  | <b>116</b>   | 7   | 25   |
| 6              | 127                   | <b>55</b>  | 3   | 1   | <b>81</b>  | 3   | 10  | <b>398</b>  | 14  | 83  | <b>1184</b>  | 15  | 234  |
| 7              | 255                   | <b>114</b> | 4   | 1   | <b>185</b> | 5   | 33  | <b>3724</b> | 24  | 654 | <b>11479</b> | 41  | 1371 |

PBTS is customized to address the single-row transformation of a perfect binary tree. The unique graph properties of perfect binary tree facilitate the establishment of PBTS algorithm in which is essentially an optimal algorithm supported by the simulation results in Table 1.

Among the general models, the STCGM, which is developed to transform the sparsely connected graphs into single-row network, gives the best results fundamentally as the density of trees is close to the sparsely connected graphs. It is then followed by the DSA which is efficient for smaller orders of perfect binary trees below  $n = 50$ .

GPTM gives unsatisfying result when the order of the graph exceeds 200 vertices. In GPTM, the initial solution for graph partitioning is sequentially done. The graph partitioning needs a larger number of pass to complete the min cut searching as the order of graph increases. Hence, the limit for GPTM in this case is same as the DSA, which is below  $n = 50$ .

### 3.2 Trees

The case of trees is similar to the perfect binary trees. A perfect binary tree is considered as a special case of tree. Table 2 shows the results for trees by the specific model Tree Sequence Model (TSM) and the three general models STCGM, DSA and GPTM in terms of energy values ( $E$ ), congestion ( $Q$ ) and number of doglegs ( $D$ ). The trees vary from order of 10 to 80 vertices.

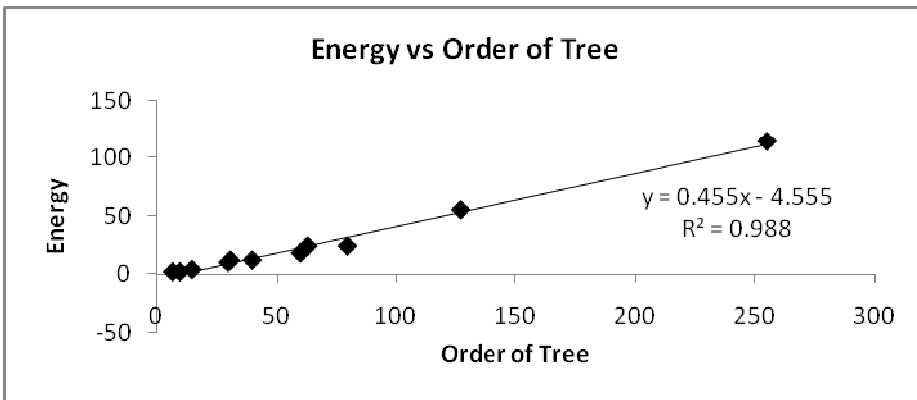
**Table 2** Results of single-row transformation for some trees by TSM, STCGM, DSA and GPTM

| Order of $T$ , $n$ | TSM |     |     | STCGM |     |     | DSA |     |     | GPTM |     |     |
|--------------------|-----|-----|-----|-------|-----|-----|-----|-----|-----|------|-----|-----|
|                    | $E$ | $Q$ | $D$ | $E$   | $Q$ | $D$ | $E$ | $Q$ | $D$ | $E$  | $Q$ | $D$ |
| 10                 | 2   | 1   | 0   | 2     | 1   | 0   | 2   | 1   | 0   | 2    | 1   | 0   |
| 20                 | 5   | 1   | 0   | 5     | 1   | 0   | 6   | 1   | 0   | 11   | 2   | 0   |
| 30                 | 10  | 1   | 0   | 12    | 2   | 0   | 10  | 1   | 0   | 28   | 4   | 1   |
| 40                 | 12  | 1   | 0   | 13    | 2   | 0   | 18  | 3   | 0   | 45   | 5   | 5   |
| 60                 | 18  | 2   | 0   | 18    | 2   | 0   | 62  | 5   | 10  | 84   | 5   | 16  |
| 80                 | 24  | 2   | 0   | 26    | 2   | 0   | 89  | 5   | 18  | 236  | 7   | 51  |

A tree has no cycle; this unique structure leads to the development of TSM which emphasizes on the balancing of the tree, where all children are equally allocated at the both sides of their respective parent nodes for each of the vertex, in the mapping. Compared to perfect binary trees, trees may not have a constant

number of branches, and hence PBTS is generalized into a robust model called TSM for general trees.

From the simulation results, TSM gives the best result as it is developed based on the preliminary concepts for trees and its single-row representation. In addition, the trees in the form of perfect binary trees are also applied, and the results are identical as in the PBTS model. Regression has been tested for the results of the samples in Table 1 (PBTS) together with the Table 2 (TSM) and shown in Figure 1. It shows the energy value grows linearly with the order of tree in TSM by hypothesis testing on the regression line.



**Figure 1** Regression line for energy versus order of tree

For the general models, the results are similar to the result comparison of perfect binary trees as shown in Section 3.1. The STCGM model produces the best result followed by DSA and GPTM. The DSA model obtains a good result for smaller order of graph below 50 vertices and it works better than GPTM in general for trees.

### 3.3 Partially Dense Graphs

Partially dense graph can be partitioned into several subgraphs with less connection between them. The connection of the graph does not disperse among the vertices but it forms a partially dense cluster with several vertices, and a number of connected clusters form a partially dense graph. A model named



Connected Graph Sequence (CGS) is developed to divide a partially dense graph into a number of subgraphs before the zones are formed.

Table 3 presents the results comparison for partially dense graphs by CGS-M and CGS-SA in terms of energy values ( $E$ ), congestion ( $Q$ ) and number of doglegs ( $D$ ). The partially dense graphs are categorized by order varying from 10 to 30 vertices and density of graphs ranging from 0.1 to 0.6.

**Table 3** Results of single-row transformation for some partially dense graphs by CGS-M and CGS-SA

| #  | Order of $\psi$ , $n$ | Density of graph, $Density$ | Size of $\psi$ , $m$ | Results     |     |     |             |     |     |
|----|-----------------------|-----------------------------|----------------------|-------------|-----|-----|-------------|-----|-----|
|    |                       |                             |                      | CGS-M       |     |     | CGS-SA      |     |     |
|    |                       |                             |                      | $E$         | $Q$ | $D$ | $E$         | $Q$ | $D$ |
| 1  | 10                    | 0.2                         | 9                    | <b>8</b>    | 2   | 0   | <b>5</b>    | 1   | 0   |
| 2  | 10                    | 0.3                         | 14                   | <b>6</b>    | 2   | 0   | <b>6</b>    | 2   | 0   |
| 3  | 10                    | 0.4                         | 18                   | <b>12</b>   | 2   | 0   | <b>13</b>   | 2   | 0   |
| 4  | 10                    | 0.5                         | 23                   | <b>32</b>   | 4   | 4   | <b>26</b>   | 3   | 2   |
| 5  | 10                    | 0.6                         | 27                   | <b>53</b>   | 5   | 9   | <b>46</b>   | 5   | 6   |
| 6  | 20                    | 0.1                         | 19                   | <b>8</b>    | 2   | 0   | <b>6</b>    | 1   | 0   |
| 7  | 20                    | 0.2                         | 38                   | <b>38</b>   | 4   | 3   | <b>39</b>   | 4   | 1   |
| 8  | 20                    | 0.3                         | 57                   | <b>184</b>  | 9   | 32  | <b>147</b>  | 7   | 23  |
| 9  | 20                    | 0.4                         | 76                   | <b>598</b>  | 19  | 86  | <b>577</b>  | 18  | 82  |
| 10 | 20                    | 0.5                         | 95                   | <b>884</b>  | 17  | 137 | <b>855</b>  | 16  | 143 |
| 11 | 30                    | 0.1                         | 44                   | <b>23</b>   | 2   | 0   | <b>25</b>   | 2   | 0   |
| 12 | 30                    | 0.2                         | 87                   | <b>297</b>  | 9   | 53  | <b>263</b>  | 8   | 48  |
| 13 | 30                    | 0.3                         | 131                  | <b>1254</b> | 20  | 190 | <b>1351</b> | 24  | 194 |
| 14 | 30                    | 0.4                         | 174                  | <b>4371</b> | 37  | 484 | <b>4167</b> | 38  | 493 |

The flexibility of CGS by Simulated Annealing (CGS-SA) successfully addresses single-row transformation of partially dense graph as shown in Table 3. Both of the CGS models perform differently in the formation of zones and terminals. In overall, model of CGS-SA with heuristic approach algorithm gives a slightly lower energy value compared to CGS-M with non-heuristic approach.

Table 4 presents the results comparison for partially dense graphs by STCGM, DSA and GPTM in terms of energy values ( $E$ ), congestion ( $Q$ ) and number of

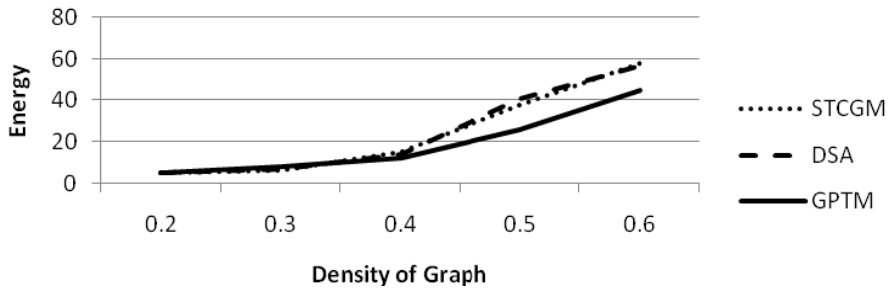
doglegs ( $D$ ). The partially dense graphs are categorized by order varying from 10 to 30 vertices and density of graphs ranging from 0.1 to 0.6.

**Table 4** Results of single-row transformation for some partially dense graphs by STCGM, DSA and GPTM

| #  | Order of $\psi$ , $n$ | Density of graph, $Density$ | Size of $\psi$ , $m$ | Results     |     |     |             |     |     |             |     |     |
|----|-----------------------|-----------------------------|----------------------|-------------|-----|-----|-------------|-----|-----|-------------|-----|-----|
|    |                       |                             |                      | STCGM       |     |     | DSA         |     |     | GPTM        |     |     |
|    |                       |                             |                      | $E$         | $Q$ | $D$ | $E$         | $Q$ | $D$ | $E$         | $Q$ | $D$ |
| 1  | 10                    | 0.2                         | 9                    | <b>5</b>    | 1   | 0   | <b>5</b>    | 1   | 0   | <b>5</b>    | 1   | 0   |
| 2  | 10                    | 0.3                         | 14                   | <b>6</b>    | 2   | 0   | <b>7</b>    | 2   | 0   | <b>8</b>    | 2   | 0   |
| 3  | 10                    | 0.4                         | 18                   | <b>15</b>   | 2   | 2   | <b>14</b>   | 2   | 2   | <b>12</b>   | 2   | 0   |
| 4  | 10                    | 0.5                         | 23                   | <b>38</b>   | 4   | 8   | <b>41</b>   | 5   | 5   | <b>26</b>   | 3   | 2   |
| 5  | 10                    | 0.6                         | 27                   | <b>58</b>   | 5   | 11  | <b>57</b>   | 5   | 10  | <b>45</b>   | 5   | 5   |
| 6  | 20                    | 0.1                         | 19                   | <b>5</b>    | 1   | 0   | <b>5</b>    | 1   | 0   | <b>16</b>   | 3   | 0   |
| 7  | 20                    | 0.2                         | 38                   | <b>48</b>   | 5   | 3   | <b>48</b>   | 4   | 10  | <b>47</b>   | 4   | 2   |
| 8  | 20                    | 0.3                         | 57                   | <b>187</b>  | 8   | 42  | <b>209</b>  | 11  | 38  | <b>185</b>  | 10  | 33  |
| 9  | 20                    | 0.4                         | 76                   | <b>770</b>  | 18  | 115 | <b>785</b>  | 22  | 113 | <b>606</b>  | 17  | 86  |
| 10 | 20                    | 0.5                         | 95                   | <b>1079</b> | 20  | 153 | <b>1046</b> | 19  | 155 | <b>899</b>  | 17  | 153 |
| 11 | 30                    | 0.1                         | 44                   | <b>25</b>   | 2   | 3   | <b>26</b>   | 3   | 0   | <b>34</b>   | 3   | 4   |
| 12 | 30                    | 0.2                         | 87                   | <b>350</b>  | 11  | 71  | <b>406</b>  | 11  | 78  | <b>351</b>  | 10  | 67  |
| 13 | 30                    | 0.3                         | 131                  | <b>1420</b> | 20  | 223 | <b>1647</b> | 27  | 257 | <b>1373</b> | 24  | 181 |
| 14 | 30                    | 0.4                         | 174                  | <b>5118</b> | 39  | 532 | <b>6162</b> | 45  | 627 | <b>4546</b> | 39  | 503 |

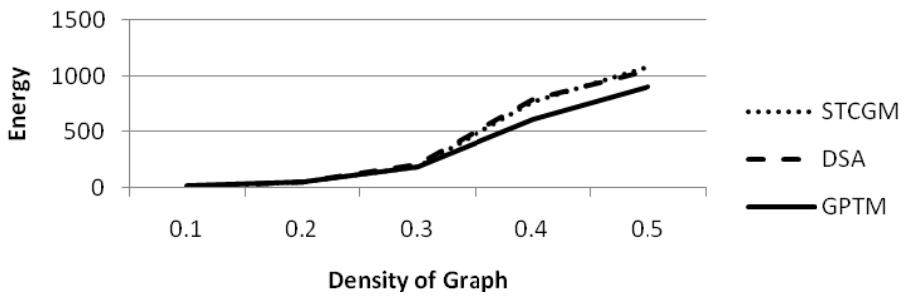
The results from the three general models on partially dense graphs are compared and shown in Figure 2 for  $n = 10$  to 30.

**Results of Single-Row Transformation for STCGM, DSA and GPTM when  $n = 10$**



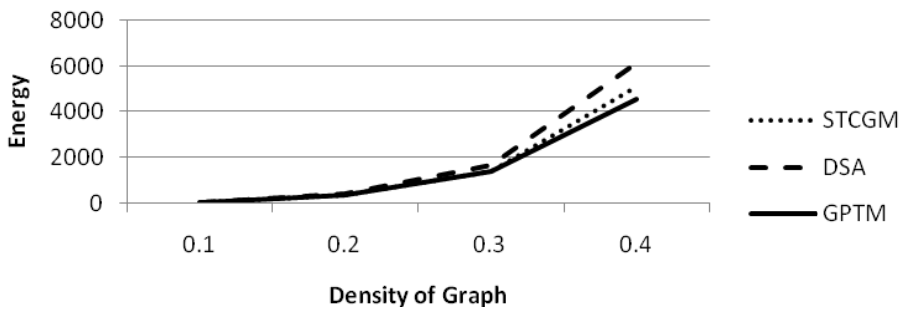
(a)

**Results of Single-Row Transformation for STCGM, DSA and GPTM when  $n = 20$**



(b)

**Results of Single-Row Transformation for STCGM, DSA and GPTM when  $n = 30$**



(c)

**Figure 2** Results of single-row transformation for STCGM, DSA and GPTM models for partially dense graphs a)  $n = 10$  , b)  $n = 20$  , and c)  $n = 30$

Figure 2 shows that GPTM gives the best results among the general models compared to STCGM and DSA models. Similar to CGS, the GPTM also equipped with partitioning technique and hence, it produces better results. The significant difference between CGS and GPTM is; CGS divides the graph based on the structure and connection of the given graph and thus, the number and the order of subgraphs formed by CGS is at random while GPTM separates the graph into two equally order of subgraphs. For example, if the partially dense graph has two similar orders of clusters, GPTM may produce the optimal result as CGS.

For partially dense graph, STCGM and DSA give similar results for graphs with order below 20 vertices. STCGM produces optimal results to the sparsely connected graphs for all given orders and the energy values increase with the density of graphs. On the other hand, DSA an undeniably efficient model for smaller orders of partially dense graphs below 20 vertices. According to Figure 2(c), STCGM produces better results than DSA because a spanning tree is produced from the given graph and the spanning tree acts as the backbone of the order of zones. This leads to the zones corresponding to each of respective vertices from the same cluster or subgraph stay close to each other on the node axis.

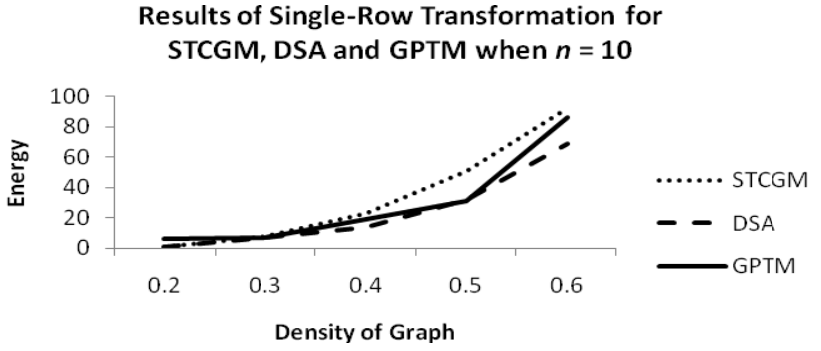
#### 4.0 ARBITRARY CONNECTED GRAPHS

The three general models are compared by arbitrary connected graphs. Double Simulated Annealing (DSA), Spanning Tree of Connected Graph Model (STCGM) and Graph Partitioning and Transformation Model (GPTM) are applied to arbitrary connected graphs with the order of graphs varies from 10 to 30 vertices, each with different densities of graphs. The simulation results are shown in Table 5 in terms of energy values ( $E$ ), congestion ( $Q$ ) and number of doglegs ( $D$ ).

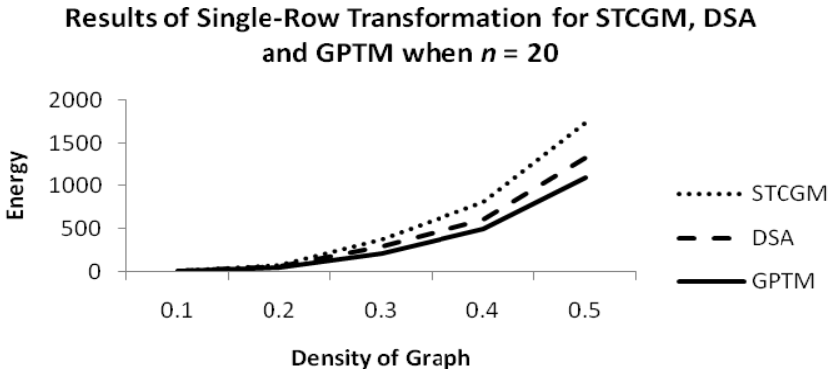
**Table 5** Results of single-row transformation for some general connected graphs by STCGM, DSA and GPTM

| #  | Order of $G$ , $n$ | Density of graph, $Density$ | Size of $G$ , $m$ | Results     |     |     |             |     |     |             |     |     |
|----|--------------------|-----------------------------|-------------------|-------------|-----|-----|-------------|-----|-----|-------------|-----|-----|
|    |                    |                             |                   | STCGM       |     |     | DSA         |     |     | GPTM        |     |     |
|    |                    |                             |                   | $E$         | $Q$ | $D$ | $E$         | $Q$ | $D$ | $E$         | $Q$ | $D$ |
| 1  | 10                 | 0.2                         | 9                 | <b>1</b>    | 1   | 0   | <b>1</b>    | 1   | 0   | <b>6</b>    | 2   | 0   |
| 2  | 10                 | 0.3                         | 14                | <b>8</b>    | 2   | 1   | <b>7</b>    | 2   | 0   | <b>7</b>    | 2   | 0   |
| 3  | 10                 | 0.4                         | 18                | <b>23</b>   | 3   | 6   | <b>14</b>   | 3   | 0   | <b>19</b>   | 4   | 0   |
| 4  | 10                 | 0.5                         | 23                | <b>51</b>   | 6   | 9   | <b>32</b>   | 5   | 4   | <b>31</b>   | 4   | 4   |
| 5  | 10                 | 0.6                         | 27                | <b>92</b>   | 7   | 14  | <b>69</b>   | 6   | 12  | <b>86</b>   | 7   | 13  |
| 6  | 20                 | 0.1                         | 19                | <b>0</b>    | 0   | 0   | <b>2</b>    | 1   | 0   | <b>8</b>    | 2   | 1   |
| 7  | 20                 | 0.2                         | 38                | <b>65</b>   | 5   | 13  | <b>53</b>   | 5   | 4   | <b>49</b>   | 4   | 6   |
| 8  | 20                 | 0.3                         | 57                | <b>376</b>  | 12  | 72  | <b>283</b>  | 15  | 42  | <b>213</b>  | 9   | 42  |
| 9  | 20                 | 0.4                         | 76                | <b>806</b>  | 21  | 117 | <b>602</b>  | 15  | 107 | <b>495</b>  | 15  | 86  |
| 10 | 20                 | 0.5                         | 95                | <b>1745</b> | 21  | 244 | <b>1327</b> | 26  | 200 | <b>1099</b> | 18  | 180 |
| 11 | 30                 | 0.1                         | 44                | <b>44</b>   | 5   | 8   | <b>42</b>   | 5   | 3   | <b>48</b>   | 5   | 4   |
| 12 | 30                 | 0.2                         | 87                | <b>599</b>  | 15  | 105 | <b>508</b>  | 14  | 98  | <b>342</b>  | 11  | 60  |
| 13 | 30                 | 0.3                         | 131               | <b>1837</b> | 25  | 279 | <b>2349</b> | 35  | 321 | <b>2097</b> | 30  | 305 |
| 14 | 30                 | 0.4                         | 174               | <b>7177</b> | 42  | 767 | <b>5673</b> | 50  | 562 | <b>4792</b> | 44  | 541 |

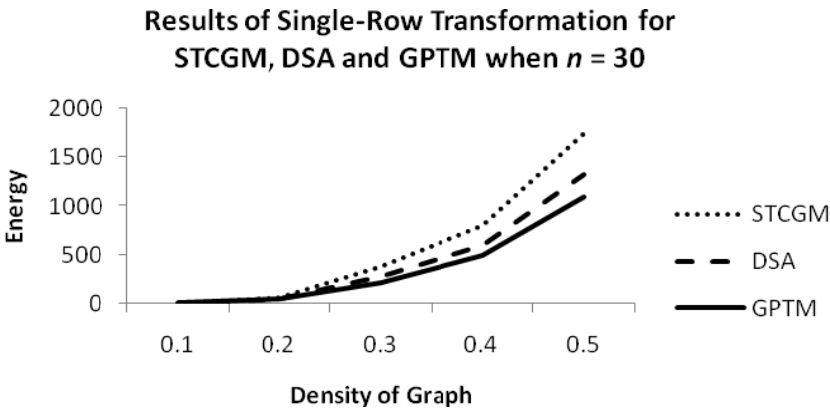
The results are plotted into graphs and shown in Figure 3 for  $n=10$  to 30. From Figure 3(a), DSA produces the best results compared to STCGM and GPTM models. In general, the DSA is an efficient model to transform a smaller order of graph below 50 vertices into a single-row. As the order of graph increases from 10 to 20 vertices, GPTM improves the results from DSA to be the best model in transforming an arbitrary connected graph into a single-row network as shown in Figure 3(b) followed by DSA and STCGM.



(a)



(b)



(c)

**Figure 3** Results of single-row transformation for STCGM, DSA and GPTM models for arbitrary connected graphs a)  $n = 10$  , b)  $n = 20$  , and c)  $n = 30$

DSA and GPTM are similar in applying simulated annealing twice for the best order of terminals and intervals. The only difference is, GPTM is enhanced by graph partitioning technique which is not equipped in DSA. The DSA is a powerful and highly robust model for a smaller order of graph. Nevertheless the result may not be as promising as the order of graph which grows larger than 50 vertices. To address this problem, GPTM is developed where the given graph is partitioned into two similar orders of denser subgraphs before the single-row transformation. The GPTM improves the results of DSA especially for larger order with higher density of graph as shown in Figure 3(c).

In general, STCGM gives the best results for the lowest density of graph in each given order of graph. The STCGM is an outstanding model for low density of graphs or sparsely connected graphs. The concern for the way adjacent vertices to be mapped into the optimal zone ordering is greater for a lower density of graph. A low density of graph may be perfectly represented by an appropriate spanning tree in order to be optimally transformed into a single-row. As shown in Section 3.2, TSM is an efficient model to transform a tree into a single-row network. Hence, STCGM which consists of TSM is introduced to produce a desired spanning tree from the given graph and before it is transformed into single-row network by TSM.

## 5.0 CONCLUSION

In this paper, three general models which are Double Simulated Annealing (DSA), Spanning Tree of Connected Graph Model (STCGM), and Graph Partitioning and Transformation Model (GPTM) have been applied to perfect binary trees, trees, partially dense graphs and arbitrary connected graphs. Differences between the three models are discussed in detail and each model has significant contribution in generalizing the single-row transformation technique to an arbitrary graph.

For a lower order of graph, the DSA is efficient in transforming optimally a connected graph with  $n < 50$  into a single-row network. Meanwhile, for a lower size of graph, STCGM appears to be the best model for a sparsely connected graph which satisfies the upper limit as shown in Equation (3). The GPTM which is generalized from DSA improves the results of DSA for a graph with  $n > 20$  except for trees.

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