

# Energy Optimization of Brushless DC Motor in Electric Power-Assisted Steering

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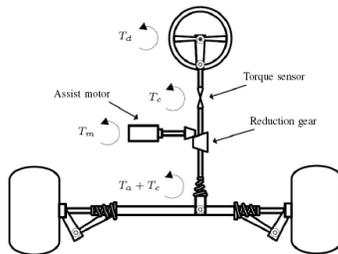
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## Graphical abstract



## Abstract

In an electric power-assisted steering (EPS) an electric motor is controlled to provide assistance in vehicle steering and to enable various steering feels. To optimize energy consumed by a column-type EPS equipped with a brushless dc (BLDC) motor the author designs two controllers as needed. Firstly a controller to generate driver torque is developed based on nonlinear adaptive regulation method using the mathematical model of EPS. The second controller is a PID motor controller that is applied to produce assistance torque for desired energy saving. The trade-off between driver's comfort and energy consumption is demonstrated using Matlab simulation results. In electric vehicles (EVs) where electrical energy is limited the control scheme introduced here is expected to fit perfectly.

**Keywords:** Energy optimization; electric power-assisted steering; brushless DC motor; nonlinear adaptive regulation; PID control

## Abstrak

Dalam sistem bantuan stereng elektrik (BSE) motor elektrik dikawal untuk memberikan kemudahan kepada pemandu dalam pengawalan stereng dan untuk membolehkan pelbagai tahap keselesaan. Untuk mengoptimalkan tenaga yang digunakan oleh BSE jenis lajur yang dilengkapi dengan motor dc tanpa berus, penulis mereka bentuk dua pengawal. Pengawal pertama adalah untuk menjaga tork pemandu yang dibangunkan berdasarkan kaedah kawalan adaptif menggunakan model matematik BSE. Pengawal kedua pula ialah pengawal motor PID yang digunakan untuk menghasilkan tork bantuan untuk merealisasikan penjimatan tenaga yang diinginkan. Perbezaan yang terhasil antara tahap keselesaan pemandu dan penggunaan tenaga elektrik ditunjukkan menggunakan keputusan simulasi Matlab. Dalam kenderaan elektrik di mana tenaga elektrik adalah terhad, skim kawalan yang diperkenalkan di sini adalah amat sesuai.

**Kata kunci:** Penjimatan tenaga; sistem bantuan stereng elektrik; kawalan adaptif; kawalan PID

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## 1.0 INTRODUCTION

In a vehicle EPS assists a human driver by measuring torque exerted by the driver and providing additional torque by means of an electric motor. Conventionally an EPS system consists of a dc motor and relies on vehicle velocity and driver torque measurements to determine target motor current levels from assist characteristic [1] or boost [2] curve.

The assist characteristic curve is a set of graphs of motor current versus driver torque and vehicle velocity that is generated experimentally to produce desired assistance torque and thus predetermined steering comfort [3]. An implication of this is that for a fixed driver torque and vehicle velocity a fixed amount of electrical energy is consumed by an EPS. While in internal combustion engine vehicles provision of maximum steering

comfort to drivers is possible careful considerations are needed to be made in EVs. Given a limited power supply in an EV higher torque assistance would result in a faster battery drain. Therefore the use of the assist characteristic curve to obtain reference assistance torque levels is not applicable in EVs and a better approach enabling energy optimization is justified.

While the use of a traditional brushed dc motor in an EPS requires a low cost and a simple control some disadvantages include high maintenance of the brushes, low overall power density and electromagnetic interference (EMI) problems associated with commutator arcing [4]. These drawbacks motivate the author to apply a BLDC in an EPS system in this paper.

Some previous works on EPS will now be reviewed. In [3] a modified Linear Quadratic Gaussian controller is shown to be able to track the characteristic curve and attenuate external

disturbances for a column-type EPS system. From the simulation results the controller performs well despite the inclusion of nonlinear rotational friction terms even though explicit expressions of the friction are not provided. Chitu *et al.* applied a Linear Quadratic Regulator to derive an optimal controller for an EPS system in [2]. Both simulation and dSPACE ControlDesk real-time application utilizing the assist characteristic curve show stability in frequency, robustness and closed-loop stable step responses during parameter variation. In [5] a Fuzzy PID control strategy is simulated for assist motor current tracking. The simulation model that includes a simple road surface disturbance demonstrates the effectiveness of the proposed controller. While brushed dc motors are considered in the above works an EPS with a BLDC actuator is studied in [6]. Zhu *et al.* propose an adaptive control method to achieve torque control by approximating motor electrical dynamics with back EMF compensation error correction. In another related work Hu *et al.* constructed a test bed and developed a controller for a BLDC EPS system and reported an acceptable system performance in [7].

Out of the four control objectives of an EPS system (cf. [8]) the basic function of an EPS i.e. assistance torque control is considered here. To achieve energy optimization the authors develop a mathematical model of a column-type EPS with a BLDC assist motor. Nonlinear rotational friction and LuGre dynamic tire friction are included in the mathematical model for accurate EPS representation. The model is then used to design a controller based on nonlinear adaptive regulation for driver torque generation to track a reference wheel angle trajectory. To enable an option to select torque assist level and consequently amount of energy saving desired eco factor  $E$  is introduced in later sections of this paper. Target assistance torque is then achieved using PID control of motor current with Pulse Width Modulation (PWM) implementation.

The paper is organized as follows; in Section 2.0 the EPS mathematical model is discussed. A brief explanation on controller designs is provided in Section 3.0 after which simulation results and conclusion are given in Sections 4.0 and 5.0 respectively.

## 2.0 MATHEMATICAL MODEL

The following is the mathematical model of a column-type EPS (see Figure 1).

Torque sensor model:

$$T_c = K_s \left( \theta_s - \frac{x_r}{R_s} \right) \quad (1)$$

Steering column model:

$$J_s \ddot{\theta}_s = T_d - T_c - B_s \dot{\theta}_s - T_f^s \quad (2)$$

Assist motor model:

Rack and pinion model:

$$\begin{aligned} T_m &= \frac{n_p}{\theta_m} (E_a i_a + E_b i_b + E_c i_c) \\ V_n &= \frac{1}{3} (V_a + V_b + V_c) - \frac{1}{3} (E_a + E_b + E_c) \\ L \dot{i}_a &= V_a - V_n - R i_a - E_a \\ L \dot{i}_b &= V_b - V_n - R i_b - E_b \\ L \dot{i}_c &= V_c - V_n - R i_c - E_c \\ \left( J_m + \frac{J_G}{G^2} \right) \ddot{\theta}_m &= T_m + \frac{T_c}{G} - \left( B_m + \frac{B_G}{G^2} \right) \dot{\theta}_m - T_f^m \\ T_a &= G T_m \\ M_r \ddot{x}_r &= \frac{T_a + T_c}{R_s} - F_{TR} - B_r \dot{x}_r - K_r x_r \end{aligned} \quad (3)$$

where

- $\theta_s$  - steering wheel angle
- $\theta_m$  - assist motor armature shaft angle
- $T_d$  - driver torque
- $T_c$  - steering column torque sensor measurement
- $T_m$  - assist motor torque
- $T_a$  - assistance torque
- $T_f^s$  - steering column friction
- $T_f^m$  - assist motor and reduction gear friction
- $B_s$  - steering column viscous damping coefficient
- $B_m$  - assist motor viscous damping coefficient
- $B_G$  - reduction gear viscous damping coefficient
- $B_r$  - pinion and rack viscous damping coefficient
- $J_s$  - steering column moment of inertia
- $J_m$  - assist motor moment of inertia
- $J_G$  - reduction gear moment of inertia
- $K_s$  - steering column stiffness
- $K_a$  - assist motor torque coefficient
- $K_b$  - assist motor back emf coefficient
- $K_r$  - rack equivalent spring constant
- $V_{a,b,c}$  - BLDC motor three-phase voltage
- $i_{a,b,c}$  - BLDC motor three-phase current
- $E_{a,b,c}$  - BLDC motor three-phase trapezoidal back EMF
- $V_n$  - BLDC motor neutral voltage
- $n_p$  - BLDC motor number of poles
- $R$  - BLDC motor resistance
- $L$  - BLDC motor inductance
- $G$  - reduction gear ratio
- $x_r$  - horizontal rack displacement
- $R_s$  - pinion radius
- $F_{TR}$  - pinion, rack and wheel equivalent mass
- $M_r$  - dynamic tire friction

The nonlinear rotational friction is given by

$$T_f^i = (\alpha_0^i + \alpha_1^i e^{-\alpha_2^i |\dot{\theta}_i|}) \text{sgn}_1(\dot{\theta}_i) + (\alpha_3^i + \alpha_4^i e^{-\alpha_5^i |\dot{\theta}_i|}) \text{sgn}_2(\dot{\theta}_i), \quad i = s, m$$

where

$$\text{sgn}_1(\dot{\theta}_i) = \begin{cases} 1 & \dot{\theta}_i \geq 0 \\ 0 & \dot{\theta}_i < 0 \end{cases}$$

$$\text{sgn}_2(\dot{\theta}_i) = \begin{cases} 0 & \dot{\theta}_i \geq 0 \\ -1 & \dot{\theta}_i < 0 \end{cases}, \quad i = s, m$$

and

$$\alpha_0^i \neq \alpha_3^i, \alpha_1^i \neq \alpha_4^i, \alpha_2^i \neq \alpha_5^i, i = s, m$$

and

$$\alpha_j^i \in \mathbb{R}, \alpha_j^i > 0, j = 0, \dots, 5$$

in general [12].

The LuGre dynamic tire friction can be expressed as

$$\tau_a = M_s + M_{sa}$$

$$F_{TR} = \frac{\tau_a}{r}$$

where is  $r$  the steering arm length and  $M_s, M_{sa}$  are sticking and self-alignment torques respectively. For complete expressions of  $M_s, M_{sa}$  please refer to [13] and [14].

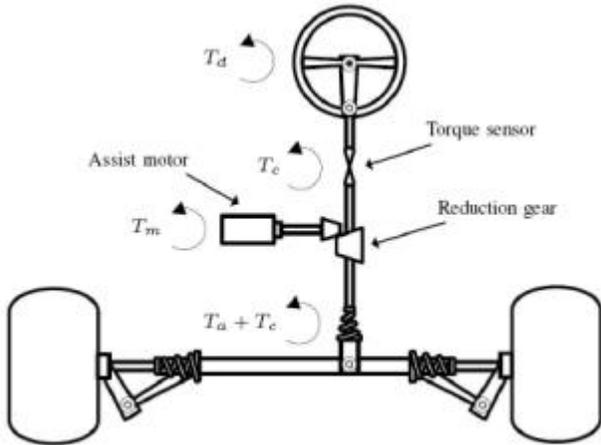


Figure 1 A column-type EPS

### 3.0 CONTROLLER DESIGN

The main objective of this work is to demonstrate the option made available to choose a desired steering comfort. Since a better steering feel is provided by a higher torque assist higher energy consumption would result and the reverse is also true. The amount of assistance torque  $T_a$  to be produced is determined by the *eco factor*  $E$  and driver torque  $T_d$ . Therefore in this section the controller design is done in two stages. In the first part a controller

based on nonlinear adaptive regulation is developed to generate driver torque  $T_d$  to track a reference wheel angle trajectory. Next a reference assistance torque  $T_a^*$  is obtained using the generated  $T_d$  and  $E$ . In the second stage BLDC motor control is carried out using PID and PWM application to track  $T_a^*$ .

### 3.1 Driver Torque Generation

Consider (3) with  $T_a = 0$ ,

$$M_r \ddot{x}_r = \frac{T_c}{R_s} - F_{TR} - B_r \dot{x}_r - K_r x_r \quad (4)$$

and a reference wheel angle trajectory  $\theta_w^*$  given by

$$x_r^* = \sum_{i=1}^N A_i \cos(\Omega_i t + \varphi_i) \quad (5)$$

$$\theta_w^* = \frac{x_r^*}{r},$$

with a fixed  $N$ , unknown amplitudes  $A_i$ , phases  $\varphi_i$  and frequencies  $\Omega_i$ . Then from (4) and (5) a steady state steering column torque  $T_c^*$  needed to track  $\theta_w^*$  is given by

$$T_c^* = (F_{TR} + B_r \dot{x}_r + K_r x_r + M_r \ddot{x}_r^*) R_s \quad (6)$$

From (5) and (6) a reference steering wheel angle is obtained as follows.

$$\theta_s^* = \frac{T_c^*}{K_s} + \frac{x_r}{R_s} \quad (7)$$

By solving for a steady state driver torque  $T_d^*$  from (2) using (7) we have

$$T_d^* = T_c + B_s \dot{\theta}_s + T_f^s + J_s \ddot{\theta}_s^* \quad (8)$$

Note that in steady state  $T_d^*$  will produce  $\theta_s^*, T_c^*$  and consequently  $x_r^*$ .

Using the nonlinear adaptive regulation method expressions (7) and (8) yield controllers of the forms

$$e_1 = x_r^* - x_r$$

$$u_{st}^1 = k_2^1 (\dot{e}_1 + k_1^1 e_1)$$

$$g_{st}^1 = G u_{st}^1$$

$$\dot{\xi}_1 = (F + G \hat{\Psi}_1) \xi_1 + g_{st}^1$$

$$T_c^u = \hat{\Psi}_1 \xi_1 + u_{st}^1, \quad k_1^1, k_2^1 > 0$$

(9)

and

$$\begin{aligned} \theta_s^u &= \frac{T_c^u}{K_s} + \frac{x_r}{R_s} \\ e_2 &= \theta_s^u - \theta_s \\ u_{st}^2 &= k_2^2(\dot{e}_2 + k_1^2 e_2) \\ g_{st}^2 &= G u_{st}^2 \\ \dot{\xi}_2 &= (F + G\hat{\Psi}_2)\xi_2 + g_{st}^2 \\ T_d &= \hat{\Psi}_2 \xi_2 + u_{st}^2, \quad k_1^2, k_2^2 > 0 \end{aligned} \tag{10}$$

whose performances are subject to the tuning of  $k_1^1, k_2^1, k_1^2, k_2^2$ . For definitions of the control parameters and detailed explanation of control designs (9) and (10) readers are advised to refer to [10] and [11]. The above controllers can be shown to be able to track  $x_r$  in a globally asymptotically and locally exponentially stable manner.

### 3.2 Assist Motor Control

The controller designed in the previous section will generate driver torque  $T_d$  for reference wheel angle  $\theta_w^*$  tracking. From the steering column torque measurement  $T_c$  desired assist motor torque  $T_m^*$  is obtained as follows.

$$\begin{aligned} T_a^* &= E T_c \\ T_m^* &= \frac{T_a^*}{G} \end{aligned} \tag{11}$$

Given the value in (11) reference assist motor current  $i_a^* = T_m^* / K_a$  is then computed for BLDC motor control as depicted in Figure 2. A PID controller is tuned to achieve desired accuracy in  $i_a^*$  tracking of the assist motor that is powered by PWM.

Note that  $E = 1$  would mean that equivalent steering effort has to be put by both a human driver and the assist motor. Even though that results in a comfortable steering it leads to higher battery usage as compared to setting  $E = 0.1$ .

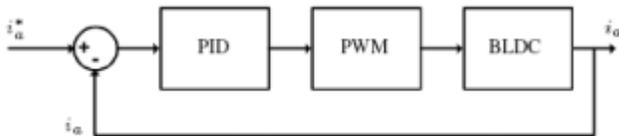


Figure 2 Assist motor control

## 4.0 SIMULATION RESULTS

The mathematical model of a column-type EPS system given in Section 2.0 together with the control designs in Section 3.0 is simulated using Matlab. Parameters of the EPS model are adopted from [7], [6] and [9].

Recall that in Section 3.A a controller is designed to generate driver torque  $T_d$  for reference wheel angle  $\theta_w^*$  tracking. From Figure 3 it could be seen that controllers (9) and (10) performs well since a very close tracking is achieved.

In Figure 4 steering column torque  $T_c$  and assistance torque  $T_a$  are plotted for  $E = 0.6$ . As the values of  $T_a$  are always approximately 60% of  $T_c$  as desired the effectiveness of the PID controller with PWM from Section 3.B is demonstrated.

Figures 5 and 6 show steering column torque  $T_c$  and assist motor torque  $T_m$  respectively for different values of  $E$ . Note the inverse relationship between  $T_c$  and  $T_m$ . As a higher  $T_m$  is desired (by setting a higher value of  $E$ ) a lower  $T_c$  is required for steering.

As expected assist motor phase A (as well as two other phases) current  $i_a$  varies accordingly to changes in  $E$  settings in Figure 7. From Figures 5 and 7 it is clearly shown that a lighter steering feel (a higher  $E$ ) requires a higher current draw from a car battery. However if users are given the option to choose  $E$  as desired a better management between steering comfort and power usage could be achieved for energy optimization.

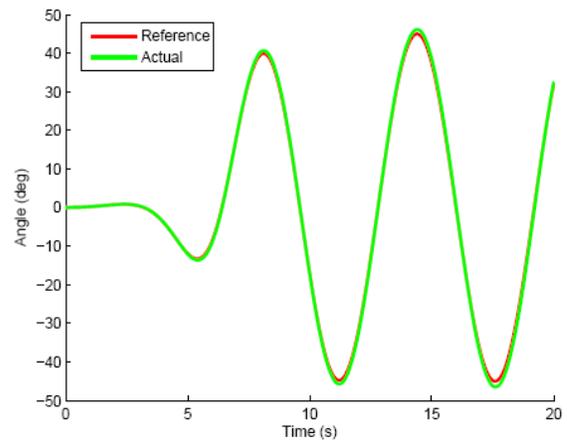


Fig. 3. Reference wheel angle  $\theta_w^*$  and actual wheel angle  $\theta_w$

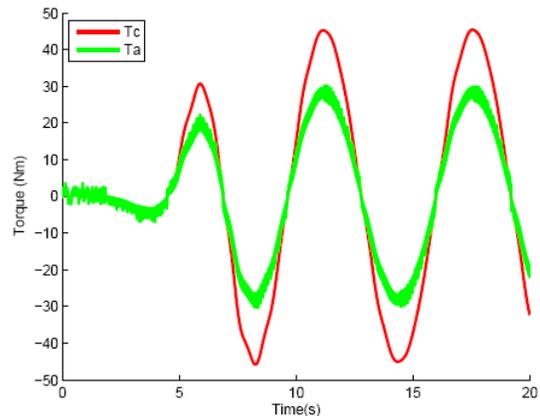
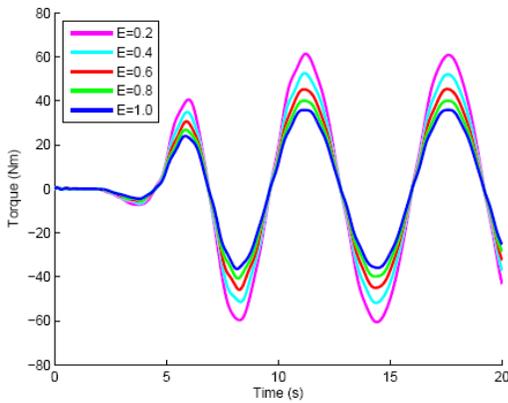
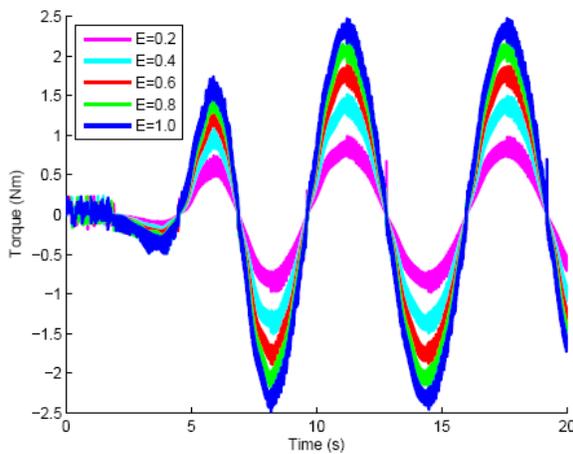
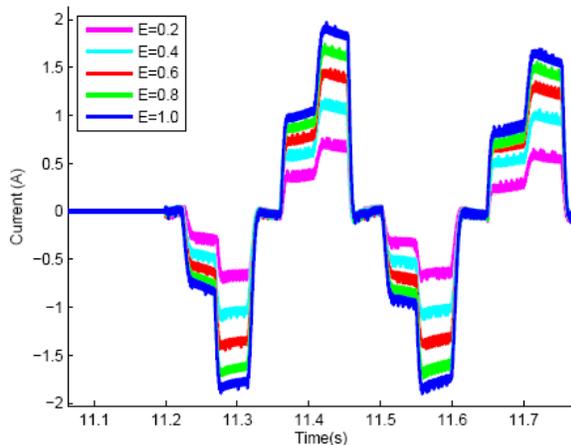


Fig. 4. Steering column torque  $T_c$  and assistance torque  $T_a$  for  $E = 0.6$

Fig. 5. Steering column torque  $T_c$ Fig. 6. Assist motor torque  $T_m$ Fig. 7. Assist motor current  $i_a$ 

## 5.0 CONCLUSION

Conventionally target assist motor current is obtained from a lookup table that is obtained experimentally given vehicle speed and steering column torque. Since the approach generates fixed

values of reference assist motor current to generate required assistance torque energy saving of battery is almost impossible. In this work given the motivation to optimize the energy consumed by EPS in EVs the author introduced the *eco factor E* as an option to set the level of steering comfort as desired. Consequently drivers could choose to save battery energy given the inverse relationship between steering comfort and power consumed by the assist motor. Based on the simulation results produced using Matlab feasibility of the proposed control methods in achieving energy optimization in EPS is verified.

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