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Stochastic Geometry Based Dynamic Fractional Frequency Reuse for OFDMA Systems

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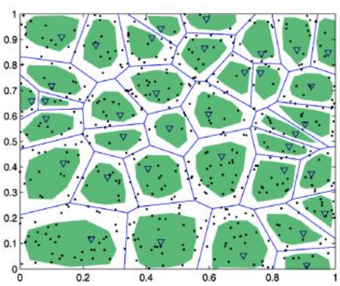

Abstract

Fractional Frequency Reuse (FFR) has been acknowledged as an efficient Interference Management (IM) technique, which offers significant capacity enhancement and improves cell edge coverage with low complexity of implementation. The performance of cellular system greatly depends on the spatial configuration of base stations (BSs). In literature, FFR has been analyzed mostly with cellular networks described by Hexagon Grid Model (HGM). HGM is neither tractable nor scalable to the dense deployment of next generation wireless networks. Moreover, the perfect geometry based HGM tends to overestimate the system's performance and not able to reflect the reality. In this paper, we use the stochastic geometry approach; FFR is analyzed with cellular network modeled by homogeneous Poisson Point Process (PPP). PPP model provides complete randomness in terms of BS deployment, which captures the real network scenario. A dynamic FFR scheme is proposed in this article, which take into account the randomness of the cell coverage area described by Voronoi tessellation. It is shown that the proposed scheme outperforms the traditional fixed frequency allocation schemes in terms of capacity and capacity density.

Keywords: Interference management; fractional frequency reuse; Long Term Evolution (LTE); Poisson Point Process

Abstrak

Guna-Semula Frekuensi Pecahan (FFR) telah diiktiraf sebagai teknik Pengurusan Gangguan (IM yang cekap, serta menawarkan peningkatan kapasiti yang besar dan meningkatkan liputan tepi sel dengan kerumitan pelaksanaan yang rendah. Prestasi sistem selular banyak bergantung kepada konfigurasi ruang stesen pangkalan (BSS). Dalam kajian, FFR telah dianalisis kebanyakannya dengan rangkaian selular diterangkan oleh Model Grid Hexagon (HGM). HGM bukan penurut dan sukar disskala untuk penempatan padat rangkaian tanpa wayar generasi akan datang. Selain itu, HGM berasaskan geometri yang sempurna cenderung untuk menilai terlalu tinggi prestasi sistem dan tidak dapat mencerminkan realiti. Dalam kertas ini, kami menggunakan pendekatan geometri stokastik; FFR dianalisis dengan rangkaian selular dimodelkan oleh homogen Poisson Point Proses (PPP). Model PPP menyediakan rawak lengkap dari segi BS penempatan, yang menangkap senario rangkaian sebenar. Skim FFR dinamik dicadangkan dalam artikel ini, yang mengambil kira rawak kawasan liputan sel yang diterangkan oleh Voronoi tessellation. Ia menunjukkan bahawa skim yang dicadangkan melebihi performa skim peruntukan frekuensi tetap tradisional dari segi kapasiti dan kepadatan kapasiti.

Kata kunci: Pengurusan gangguan; guna-semula frekuensi pecahan; *Long Term Evolution (LTE); Poisson Point Process*

1.0 INTRODUCTION

Future Mobile communication systems are changing rapidly because of the proliferation of smart phones, tablets and bandwidth hungry application services. The exponential growth in the data traffic, which projects about 1000 fold increase from 2007-2016 [1], forcing network operators to significantly increase their capacity in a cost effective way. The deployment of cellular networks is currently in transition phase a paradigm shift, as the number of base stations (BS) increases rapidly particularly in the urban areas. Today cellular systems are more crowded in terms of BS deployment. This trend has triggered the development of new cellular standards, of which LTE and LTE-Advanced by Third Generation Partnership Project (3GPP) are the more promising candidates. These standard offer momentous advantages compare to High Speed Packet Access (HSPA) in term of spectrum efficiency, low latency and throughput. However, LTE Release-8 does not encounter International Mobile Telecommunications (IMT)-Advanced provisions for fourth generation (4G) mobile communication, defined and characterized by International Mobile Telecommunications (ITU) [2]. Thus, in order to meet such requirements (e.g., downlink data rates of up to 100 Mb/s and 1 Gbps for mobile and nomadic users, respectively), LTE-Advanced Release 10 was recently finalized [3].

LTE is based on Orthogonal Frequency Division Multiple Access (OFDMA) to accomplish high throughput and enhanced spectral efficiency. OFDMA divides the total frequency spectrum into group of subcarriers. These sub-carrires are orthogonal to each other, therefore inter-symbol-interference (ISI) is avoided. Furthermore the bandwith of the orthogonal sub-carriers are much lower than the coherence bandwidth of the channel and hence OFDMA provides immunity to frequency selective fading. It Because of OFDMA, Intra-cell interference is no more an issue due to orthogonal carriers for each user in the same cell, however, Inter-Cell Interference (ICI) is one of the prominent limiting factors in performance degradation. ICI problem is more severe at low Signal to Noise plus Interference Ratio (SINR) values especially at the cell edges. To enhance the performance of the cellular network, Inter Cell Interference Coordination (ICIC) is an approach [4] [5], where each cell is allocating its resources in such a way that it minimizes the overall interference experienced in the network and to maximize the spatial reuse. Fractional frequency reuse (FFR) has been anticipated as an attractive ICIC technique in OFDMA based wireless cellular networks due to its low complexity and its offered higher spectral efficiency [3]. Fourth Generation (4G) cellular standards is aiming aggressive spectrum reuse (frequency reuse 1) to accomplish enhanced system capacity and simplifed radio network planning [5]. FFR is the part of 4G standards since Release 8 [2]. The main objective of FFR is to improve the SINR and system throughput by reducing Co-Channel Interference (CCI) and ICI.

In cellular systems, analytical performance modeling is a long standing open issue. Signal power and interference received at typical user; depends mainly on the distance between transmitter and receiver. Thus, the performance of wireless cellular system crucially depends on the spatial configuration of base stations [6]. In literature, most of the performance analysis of FFR has been done with Single Cell Model, Wyner Model and Grid Model. Single Cell Model ignores the other cell interference, whereas Wyner Model assumes equal interference from all neighbors cells [6].

HGM is shown in Figure 1, assumes BSs location at ideal position and complex system level intensive simulations are required for the system design and analysis. Furthermore, these models are highly idealized, often over-simplistic and inaccurate, gives optimistic results and fails to provide meaningful information for unplanned random deployment of BS of next generation wireless cellular systems [7] [8] [9]. These considerations have triggered the research and recently stochastic geometry based models are emerging and gaining much more attentions. These begins with a point process (PP) to model the BSs location statistically [6][8][11]. PPP model provides complete randomness for the BSs distribution, which captures the real network scenario. A view of realistic cellular network is shown in Figure 2. In this article we propose a dynamic FFR scheme for OFDMA cellular network modeled with homogeneous PPP.

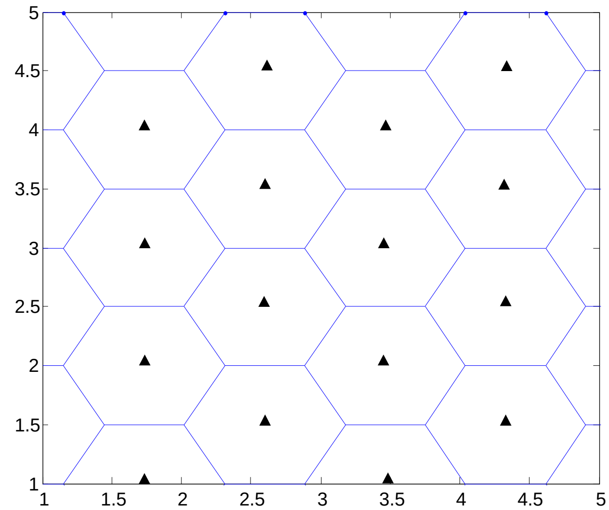

Figure 1 Hexagon grid model

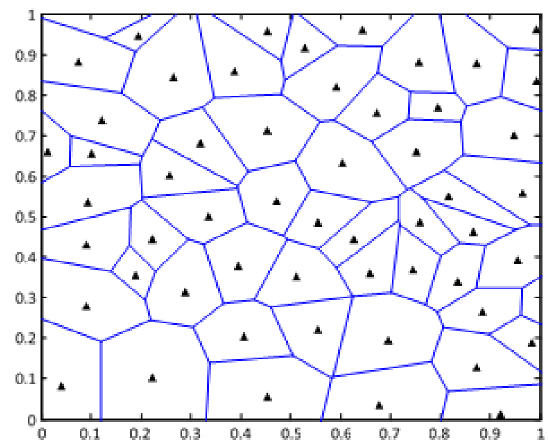

Figure 2 10x10 km view of actual urban area of cellular network

The rest of the paper is organized as; section II presents related work and contribution. Interference management with FFR is explained in section III. Section IV gives model description and proposed scheme. Results and discussion are given at section V, whereas conclusion is presented in section VI.

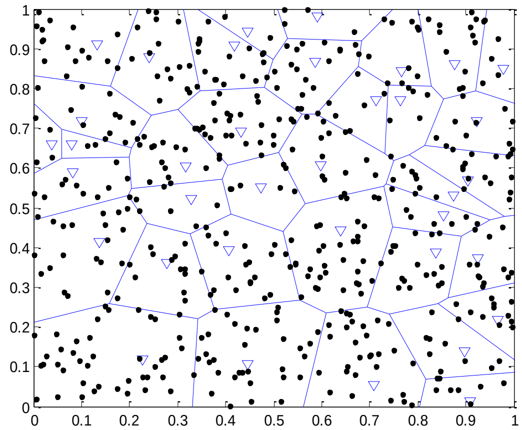

Figure 3 PPP based cellular model

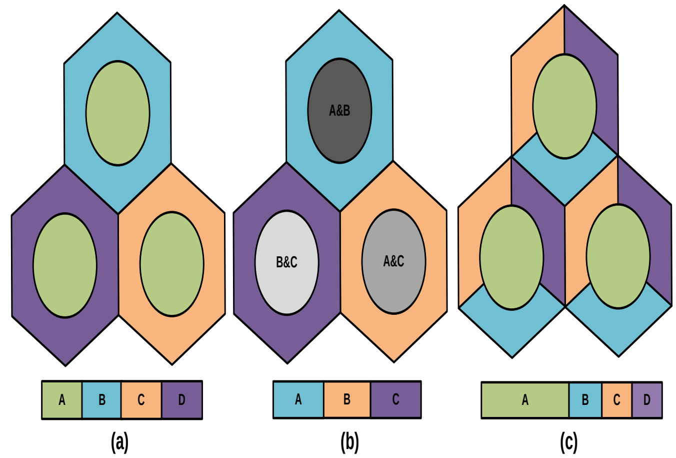

Figure 4 (a) Strict-FFR (b) SFR (c) FFR-3

■2.0 RELATED WORK AND CONTRIBUTION

Analysis of ICI mitigation techniques has become one of the main research focus for achieving dense spectrum reuse. Furthermore it also contributes to enhance the system capacity for the next generation cellular networks. FFR has been proposed as a technique to overcome ICI and CCI problems, since it can efficiently utilize the available spectrum. In literature FFR has been analyzed mostly with perfect geometry model [12] [13] [14] [15]. Perfect geometry models are mostly based on ideal assumptions, therefore unable to capture realistic cellular deployment scenario. Some work along the lines of modeling irregular grids can be found in [16] [17]. The authors used tools from the applied probability and stochastic geometry to analyze the network performance assuming the BSs are placed according to homogeneous PPP.

Homogeneous PPP model with uniform distribution of users is presented in [11] and a comprehensive framework with tractable expression for the probability of coverage and average rate. Results obtained are compared with that of actual network and it is concluded that PPP is pessimistic whereas, the hexagonal grid model is optimistic. PPP based model of [11] is extended in [16] and [17] and for FFR, where the user are divided into cell center user or cell-edge users, based on threshold SINR. Tractable expression for the coverage probability and average rate are derived for both Strict-FFR and SFR in the single tier deployment of BSs with frequency reuse factor of 3 in the cell-edge users. SINR distribution at an arbitrary location can be calculated while considering the random channel characteristic such as fading and shadowing, which is the main advantage of PPP model.

To get the maximum spectrum efficiency, full frequency reuse or FR(1) is desirable in cellular network deployment. To achieve this target, different modified FFR schemes are proposed in the literature, as discussed in [18]. However, to the best of our knowledge, sectoring with irregular cell geometry such as, Voronoi cell shapes of PPP model has never been addressed. Figure 3 shows a PPP based cellular model for the BSs locations, the coverage area of each BS is a Voronoi tessellation. In this article, we proposed a dynamic FFR scheme for the irregular Voronoi cells. Cell edge users are further divided into three regions by 120^o sectoring.

Unlike a regular hexagon cell, each sector will not have the same structure or area. Hence with uniform user distribution, each sector will not have the same number of users. Allocation equal and fixed amount of frequency to each sector will not be beneficial. Cell sectoring and proposed dynamic frequency allocation is discussed in section IV.

■3.0 FRACTIONAL FREQUENCY REUSE (FFR)

All the users located within the coverage region of a cell are not supposed to be the same from interference point of view. This is because of independent pathloss effects and channel conditions for each user and hence different SINR values. Users located at the cell edges are more sensitive to interference compared to the others at the cell-center. FFR schemes have been proposed in literature as a mean to improve the overall performance of full frequency reuse (reuse-1) OFDMA wireless cellular networks in terms of coverage and capacity.

FFR combines the benefits of both low and high frequency reuse factor by dividing the users within the cell area into two regions. (i) Cell center region, where users are close to the BS (ii) Cell edge region, where the users are more suited to the border of the cell. The inception behind FFR is basically the partition and allocation of cell's bandwidth in such a way that the interference is avoided in adjacent cell for edge users [19]. Whereas, interference received or created by the cell center users is greatly reduced. Moreover, FFR offers higher spectral efficiency as compared to conventional frequency reuse [12] [14] [16] [19] .

As an interference avoidance scheme, FFR ensures to avoid collision of same frequency band at the adjacent cells by allocating different frequencies either in a static way or by scheduling the RBs. Considering the complexity and signaling overhead in the implementation of intelligent scheduler, static method an easy choice for the practical network deployment, and hence is widely adopted [14]. The common modes of deployment for FFR schemes are illustrated in the following subsections.

3.1 Strict FFR

It is the modification of traditional frequency reuse (FR) as illustrated in Figure 4 (a) for a hexagon model with frequency reuse factor of $\delta = 3$. Since a common frequency band is allocated to each cell interior region whereas the bandwidth for the cell edge users is divided across the cell based on the frequency reuse factor of δ, thus in strict FFR total of $\delta + 1$ sub bands are required. Users at the cell-center do not share any spectrum with the cell edge users, thus it significantly reduced the interference for cell-edge and cell-center users.

3.2 Soft Frequency Reuse (SFR)

SFR with frequency ruse factor $\delta = 3$ is illustrated in Figure 4(b), with bandwidth partitioning strategy same as used for Strict FFR. The only difference is that, in SFR the users in the cell-center is allowed to share its allocated bandwidth with the cell edge users at other cells. As no separate band is required for the cell center users thus total of δ sub-bands are required in SFR. In terms of bandwidth efficiency SFR is more efficient as compared to Strict FFR. However, it results more interference to users at cell-center and cell-edge region [16].

3.3 FFR-3

FFR-3 scheme is also called sectored-FFR scheme. The cell coverage area is partitioned into cell-center and cell-edge regions, where the cell-edge region is further divided into three sectors as shown in Figure 4(c). The total frequency spectrum is divided into two parts: one part is exclusively assigned to the users at the cell-center (sub-band A) and the other part is partitioned into three sub-bands (B, C, and D) and assigned to the three sectors of the cell-edge region.

■4.0 MODEL DESCRIPTION AND PROPOSED SCHEME

For the system model, consider OFDMA based downlink cellular system, where a mobile user n is served by a BS b. BSs are distributed independently as a homogeneous PPP [20]. The BSs density is represented by ρ_b and each cell is defined by a Voronoi region. Voronoi regions are formed based on received SINR at the mobile station and it defines the cell coverage region.

We assume that the mobile users are uniformly distributed within the coverage area of a cell and each user is served by its nearest BS. $P_{b,n}$ is the power transmitted by the BS b for user n, the path loss exponent is given by α, and σ^2 represent the noise power, I represent the set of all interfering BSs, which uses the same frequency band as by the user n. The distance between the mobile user n and serving BS b is $D_{b,n}$. In consequence of PPP $D_{b,n}$ is assumed to be a random variable with Rayleigh distribution [17]. $P_{i,n}$ is transmitted power of the interferer BS which is located at distance $D_{i,n}$ from user n. $D_{i,n}^{-\alpha}$ is the path loss function that depends on the distance between the interferer's BS and user n. $G_{i,n}$ and $G_{b,n}$ are the random variable that represents the associated channel gain with the cumulative effect of small scale fading and shadowing. Associated SINR of a typical user n is given by

$$SINR_n = \frac{P_{b,n}\, G_{b,n}\, D_{b,n}^{-\alpha}}{\sigma^2 + \sum_{i\in I} P_{i,n} G_{i,n} D_{i,n}^{-\alpha}} \qquad (1)$$

4.1 Cell Partition and Sectoring

One of the crucial design parameters in FFR is to decide the size of the cell partition [21]. Total numbers of users within a cell are divided into cell-edge users and cell-interior users and the available spectrum is then divided among them. It is important to decide the size of the partition carefully and hence the frequency band allocation. This partition also depends upon the geometry of the cell so knowledge of the user's location is also important.

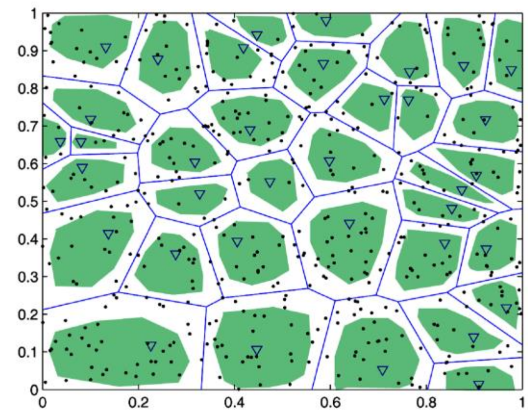

Figure 5 Cell Partition based on average received SINR

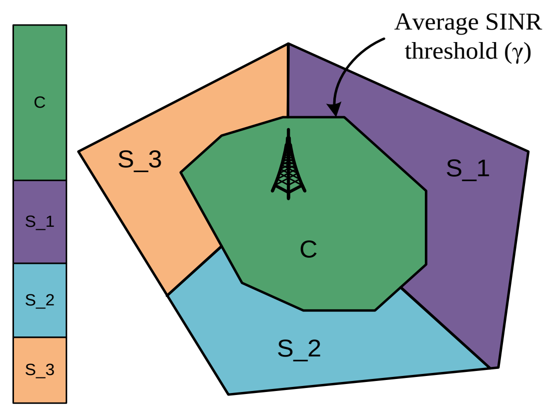

Figure 6 Voronoi Cell Sectoring and Band Allocation

We assume that the users are uniformly distributed in the cell's coverage area defined by Voronoi geometry. As SINR is a good indicator of the distance of the user from the BS [16], we use average received SINR within the cell, to classify the users as cell-edge users or cell-center users. The BS decides the users with average SINR (γ), less than γ are defined as cell-edge users, whereas, users with average SINR greater than γ are defined as cell-interior users as shown in Figure 5. Cell edge users of each cell (Voronoi Region) are further divided into three sectors (120° sectoring). Unlike perfect hexagon, the shape and area of each sector is random depending upon the geometry of the cell as shown in Figure 6.

4.2 Dynamic Frequency Allocation

The users are uniformly distributed within the cell, so each sector/region will have different number of users, larger the area of a sector/region, higher the number of users in that sector/region and hence more is the bandwidth required. It is important to dynamically allocate the spectrum according to the requirement in each sector/region. Fixed and dynamic frequency allocation scheme is illustrated in Figure 7(a), (b).

The proposed dynamic frequency allocation scheme is formulated as; the total available spectrum B_t is divided into two parts for the users at the cell-center region and cell-edge region as presented in[20]. Let B_t is the total available bandwidth, which is divided into B_c bandwidth allocated to the cell-center region and B_e bandwidth allocated to the cell-edge region.

$$B_t = B_c + B_e \quad (2)$$

Total area of the cell A_{cell} given by

$$A_{cell} = A_c + A_e \quad (3)$$

Where, A_c is the area of cell-center region and A_e is the area of cell-edge region. A_e is the sum of the areas of all sectors, which given as

$$A_e = \sum_{i=1}^{3} A_{s\_i} \quad (4)$$

Spectrum allocated to the center region can be calculated as a function of area, which is given by

$$B_c = B_t * \frac{A_c}{A_{cell}} \quad (5)$$

Similarly Spectrum allocated to each sector in the edge region is given by

$$B_{s\_i} = (B_t - B_c)\ \frac{A_{s\_i}}{A_e} \quad (6)$$

4.3 Bandwidth and Capacity Densities

The overall throughput of the cell does not provide information about the individual user performance, thererfore, we aim to maximize the average per user throughput (bit/sec/user) by introducing the concept of capacity density or per area capacity (bit/sec/unit of surface). As per the model description, users are uniformly distributed within the Voronoi cell; therefore the concept of capacity density is fully equivalent to per user capacity.

Normalized bandwidth density (Hz per unit of surface) of a given cell can be calculated by dividing the total bandwidth of the cell by the cell area, we define β as the normalized bandwidth of the given cell

$$\beta = \frac{B_t}{A_{cell}} \quad (7)$$

Normalized bandwidth in the specific region of the cell can be calculated by dividing the allocated bandwidth with the area of that region.

$$\beta_c = \frac{B_c}{A_c} \quad (8)$$

$$\beta_{s\_i} = \frac{B_{s\_i}}{A_{s\_i}}\ ;\ i = 1,2,3 \quad (9)$$

The capacity density γ_{cell} (bit/sec/unit of surface) of a cell is the sum of the capacity density in the cell center region and cell edge region, given us;

$$\gamma_{cell} = \gamma_c + \sum_{i=1}^{3} \gamma_{s\_i} \quad (10)$$

The capacity density of the cell center region γ_c can be calculated as;

$$\gamma_c = \frac{\beta_c}{N_c} \sum_{n=1}^{N_c} \{log_2\,(1 + SINR_n)\} \quad (11)$$

Where N_c represent the total number of users in the cell-center region. Putting Equation (1) in Equation (10), we get

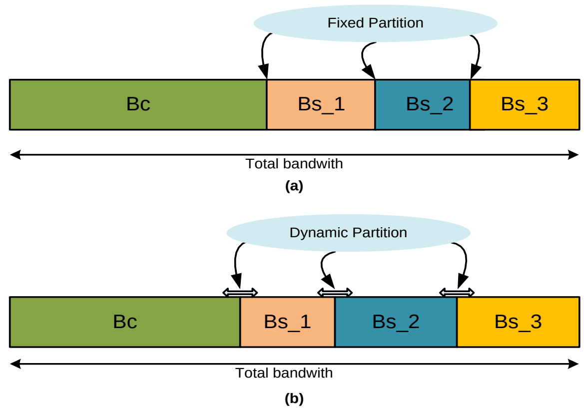

Figure 7 (a) Fixed Frequency Reuse (b) Dynamic Frequency Reuse

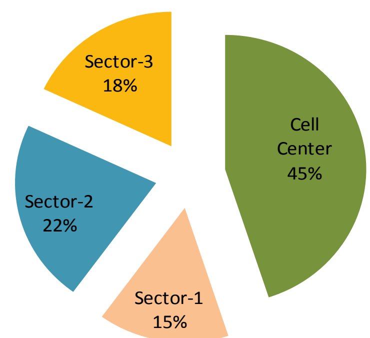

Figure 8 Bandwidth allocated based on the proposed scheme

$$\gamma_c = \frac{\beta_c}{N_c} \sum_{i=1}^{N_c} log_2 \left\{1 + \left(\frac{P_{b,n}\, G_{b,n}\, D_{b,n}^{-\alpha}}{\sigma^2 + \sum_{i\in I} P_{i,n} G_{i,n} D_{i,n}^{-\alpha}}\right)\right\} \quad (12)$$

Similarly the capacity density of any sector at cell edge region can be calculated as;

$$\gamma_{s_i} = \frac{\beta_{s_i}}{N_{s_i}} \sum_{n=1}^{N_{s_i}} \{log_2(1 + SINR_n)\} \quad (13)$$

Where N_{s_i} represent the total number of users in the cell-sector region. Putting Equation (1) and (5) in Equation (9), we get

$$\gamma_{s\_i} = \frac{\beta_{s\_i}}{N_{s\_i}} \sum_{i=1}^{N_{s\_i}} log_2 \left\{1 + \left(\frac{P_{b,n}\, G_{b,n}\, D_{b,n}^{-\alpha}}{\sigma^2 + \sum_{i\in I} P_{i,n} G_{i,n} D_{i,n}^{-\alpha}}\right)\right\} \quad (14)$$

■5.0 RESULTS AND DISCUSSIONS

The proposed scheme allocates bandwidth to each sector/region dynamically. From Equation (5) it is clear that the bandwidth allocated to cell-center region is the function of the ratio of the area of cell-center region to the area of the cell. The high value of the ratio $(A_c\,/A_{cell})$ corresponds to the larger area of the cell-center region, which means, more the mobile users as they are uniformly distributed, and hence more is the bandwidth required. The same is true for Equation (6), where each sector is allocated bandwidth accordingly. Figure 8 shows the allocated bandwidth of a typical Vorornoi cell based on dynamic proposed scheme.

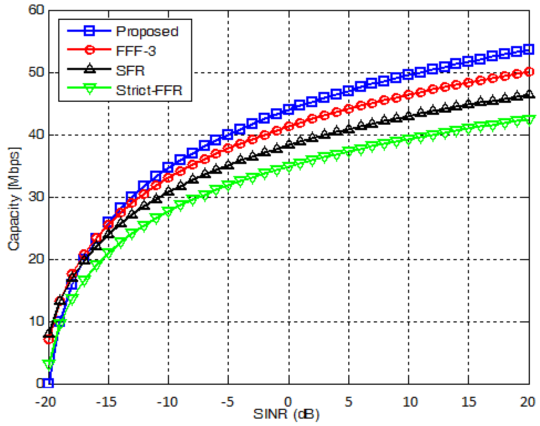

Figure 9 Capacity of a cell

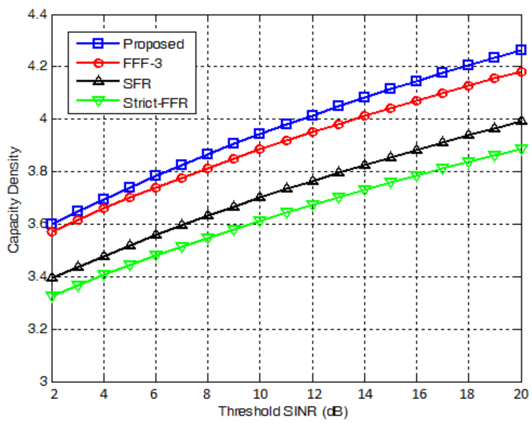

Figure 10 Capacity density at cell-center region

Equation (8) and (9) give the normalized bandwidth for the cell center and sector-i of cell edge regions respectively, where B_c is the bandwidth allocated to the cell center region and B_{s_i} is the bandwidth allocated to each sector, as calculated in Equation (5) and (6).The proposed scheme further devides the cell-edge region into three different sectors. Each sub-region or sector is allocated frequency spectrum based on its coverage area. The total available frequency spectrum is divied into four sub-bands $(B_c, B_{s\_1}, B_{s\_2}, B_{s\_3})$. The total available frequency spectrum is orthogonally deployed within a cell, which improves spectrum efficiency. The total capacity of a cell based on proposed FFR scheme is plotted with respect to SINR in Figure 9 and compared with strict-FFR, SFR and FFR-3 schemes. Results show that the proposed scheme improve the capacity by 6%, 14% and 20% compared to FFR-3, SFR and Strict-FFR schemes respectively. Normalized bandwidth densities for the cell-center and cell-edge regions are the same because the bandwidth allocation is not fixed. Thus the proposed dynamic frequency allocation scheme provides uniform bandwidth density throughout the cell. The capacity density of a cell can be calculated by Equation (10), whereas the capacity density for the cell-center region and cell-sector region are derived in Equation (12) and (14) respectively.

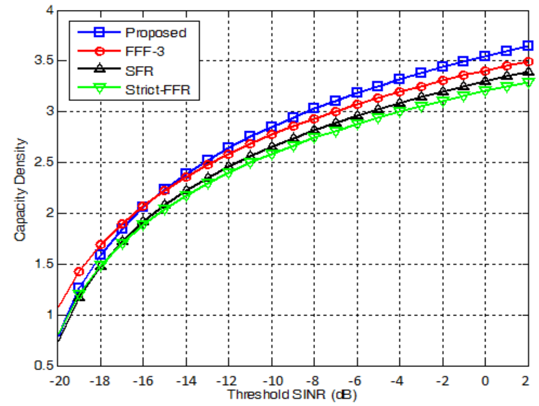

Figure11 Capacity Density at cell-edge region

Capacity densities for cell-center and cell-edge regions are presented with respect to SINR in Figure 10 and Figure 11 respectively. The obtained results are compared with strict-FFR, SFR and FFR-3 schemes. It is clear that the proposed scheme provides better results in term capacity density and hence per user capacity.

■6.0 CONCLUSION

Dynamic fractional frequency reuse scheme is proposed in this article for a stochastic geometry based PPP cellular model, with the uniform user distribution. The proposed scheme divides the total number of users into cell-center and cell-edge users, based on average received SINR (γ). Cell edge region of a cell is further divided into three sectors. Unlike perfect geometry cell, each sector has different coverage area and hence different number of users. Fixed frequency allocation strategy is not feasible for the cells with coverage areas define by Voronoi tessellation. The proposed dynamic frequency allocation scheme follows the geometry of the cell model. Bandwidth allocated to each sector/region is according to the requirement of that sector/region in terms of bandwidth per user. The capacity and capacity density of a typical cell based on proposed scheme are calculated and plotted with respect to SINR values. Results are compared and it is shown that the proposed scheme outperforms the traditional fixed frequency allocation schemes in terms of capacity and capacity density.

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