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ACTUATOR AND SENSOR FAULT COMPENSATION USING PROPORTIONAL-PROPORTIONAL INTEGRAL OBSERVER FOR FUZZY TRACKING CONTROL OF PENDULUM-CART SYSTEM

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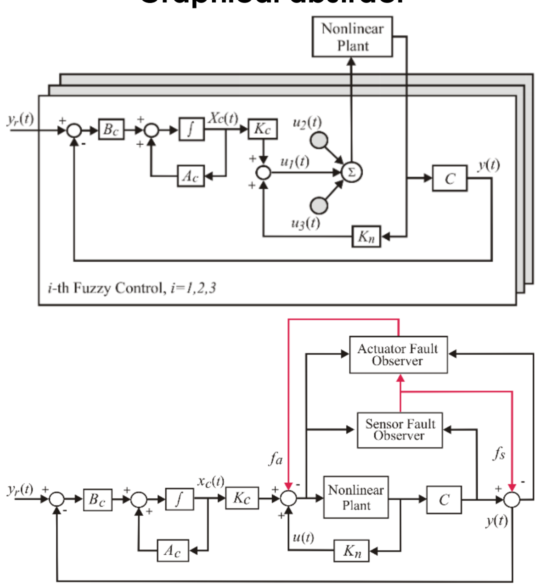
Graphical abstract



Abstract

The pendulum-cart system is a popular system plant as a case study in nonlinear control design and implementation. The controllability and system performance can be influenced by the effectivity of the actuator and sensor. However, actuator and sensor fault sometimes is inevitable and can be occurred during operation. This paper considers fault-tolerant control (FTC) to minimize the actuator and sensor fault. The control objective is to track the sinusoidal reference position of the cart while the pendulum is maintained upright in which the faulty actuator and sensor occurred. Takagi-Sugeno (T-S) fuzzy tracking control is designed based on a compensator scheme where the Proportional-Proportional Integral Observer (PPIO) is utilized for this scheme. The Linear Matrix Inequalities (LMIs) are used to calculate the controller and observer gains. The performance of the proposed controller is verified through simulation and experimental validation. The effectiveness of FTC in the case of actuator and sensor fault is given. The system responses for the compensated and uncompensated controllers (to track the reference signal) are compared. In the case of a sensor fault, only the compensated controller can converge to the reference signal. However, in the case of actuated fault, both compensated and uncompensated controllers converge to the reference signal but the error of the compensated controller is better than the other one.

Keywords: Fault-Tolerant Control (FTC); Proportional-proportional integral observer (PPIO); T-S Fuzzy, LMI, Pendulum-cart system

1.0 INTRODUCTION

Imperfect behavior in the classical pendulum-cart system e.g., loss of effective actuators or sensor failures is an inevitable event that may cause degraded performance. This fault can be classified as an actuator fault, sensor fault, and plant component (process) [1]. To overcome this problem, fault-tolerant control (FTC) can be employed so that the control system becomes more stable [2]. In this paper, we consider the actuator and sensor faults where the faults may result in undesired control signals and cause biased in measurement, respectively. They are not critical failures or complete loss of an actuator or sensor. Hence, the considered faults lie between the control system performances and the severity of failures as described in the classification diagram of FTC [3] in which the performance of the faulty system can be maintained close to the nominal performance. The actuator and sensor faults can be represented as the variation of system parameters or as additional unknown input acting on the dynamics of the system or on the measurements. We adopt active fault-tolerant control (AFTC) to deal with the sensor and actuator faults and implement this method in a real laboratory-scale system, namely a pendulum-cart system for trajectory tracking. We introduce the additional unknown inputs in the form of sinusoidal functions for f_a and f_s which represent the actuator and sensor faults, respectively. In this work, we choose AFTC over the passive fault-tolerant control (PFTC) approach because AFTC actively responds to faulty conditions by reconfiguring control action [2].

Many fault-tolerant control systems of active and passive methods have been used to control system plants. For instance, the use of passive FTC to control the satellite attitude [7] or other applications, namely flight tracking [4], DC motor [5], spacecraft attitude maneuver [6], and inverted pendulum system [9] using active FTC. The control problems were related to actuator failure [4] or sensor failure [11], actuator fault [6],[7], actuator and sensor faults with external disturbance [9], and simultaneous occurrence of actuator/sensor faults [10]. Besides tackling the actuator and sensor faults, FTC was used to handle noise and disturbance [5] or the stator and rotor faults in induction machines [8]. These show the applicability and effectiveness of fault-tolerant controllers in solving engineering problems. For laboratory-scale systems, an inverted pendulum system is well-known as a laboratory testbed for control engineering education and research purposes. The inverted pendulum system consisted of a pole attached to the cart where the pole is stabilized upright from its initial position by controlling the cart. For this system, the challenging problem is to stabilize the pendulum upright and track the trajectory where the system has sensor and actuator faults.

Takagi-Sugeno (T-S) fuzzy model and its representation are employed in fault-tolerant control approaches for the case of uncertain nonlinear systems, nonlinear systems with unknown disturbance, or nonlinear systems with actuator fault as discussed in [10], [12], and [13], respectively. The work presented here is an improvement of the work in [14] and [5]. In [14], the tracking trajectory control is provided using Fuzzy tracking by using an observer-based stabilizing compensator. However, [14] does not consider faulty conditions. In [5], fault-tolerant control was proposed using robust observer-based for simultaneous actuator and sensor faults problem where these faults were described in two auxiliary state vectors. Furthermore, PI structure is used to construct a single robust observer, and then, the proposed observer was formulated using LMI for robust stability. By following the work of [15], the faults are initially estimated using a proportional integral observer and then the observer convergence and the control existence were formulated in linear matrix inequalities (LMI). By combining the works in [5],[14], and [15], in the present paper, we propose fault-tolerant control using T-S Fuzzy controller for a nonlinear system and proportional-proportional integral (PPI) observer to estimate the sensor and actuator fault. This observer is used to estimate the system states and the fault and the observer gains are determined by LMI pole placement [16]. The nominal controller is designed based on a parallel distributed compensation (PDC) scheme while the pole placement method is designed using LMIs for the input-output constraint of control gain. We consider the trajectory tracking control problem of a pendulum-cart system. We use active fault-tolerant control to deal with the sensor and actuator faults while tracking a sinusoidal reference signal.

The rest of this paper can be organized as follows. In Section 2 we present the proposed fault-tolerant control. The simulation and real-time experimental results are provided in Section 3. Finally, the concluding remark is given in Section 4.

2.0 PENDULUM-CART SYSTEM MODEL

The model of a pendulum-cart system is shown in Figure 1. Let the state variable $x_1 = x$ and $x_2 = \theta$ i.e., the cart position and angular position of pendulum, respectively. Then, $x_3 = \dot{x}$, and $x_4 = \dot{\theta}$ are the respective state variables for cart velocity and pendulum angular velocity. The state-space representation of the pendulum-cart system [17] and [18] can be written as:

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{a(F - T_c - \mu x_4^2 \sin x_2)}{J + \mu l \sin^2 x_2} \\
&\quad + \frac{l \cos x_2 (\mu g \sin x_2 - f_p x_4)}{J + \mu l \sin^2 x_2} \\
\dot{x}_4 &= \frac{l \cos x_2 (F - T_c - \mu x_4^2 \sin x_2)}{J + \mu l \sin^2 x_2} \\
&\quad + \frac{\mu g \sin x_2 - f_p x_4}{J + \mu l \sin^2 x_2}
\end{aligned} \tag{1}$$

where

$$a = l^2 + \frac{J}{(m_c + m_p)}, \quad \mu = (m_c + m_p) l$$

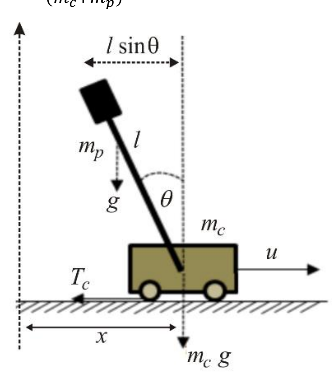


Figure 1 Model of Pendulum-Cart System

l is the length of pendulum, m_c and m_p are the mass of the cart and the mass of the pendulum, respectively. F is the force (control input signal), and T_c is the friction force. Since T_c in (1) is unknown, and hence, we ignore this friction force in design controller. The parameters of pendulum-cart system (Feedback Instrument, 2002) are J = 0.0139 kg.m^2, f_p = 0.0001 kg.m/s, m_c = 1.12 kg, m_p = 0.025 kg, l = 0.402 m, and g = 9.8 m/s^2.

3.0 FAULT-TOLERANT CONTROL APPROACH

3.1 Nominal Fuzzy Tracking Control of Pendulum-Cart System

A fuzzy model has been proposed by Takagi and Sugeno (T-S) to represent the nonlinear system dynamic [19] To build the T-S fuzzy model, the nonlinear pendulum-cart system is linearized in three operating points, i.e., $x^1 = [0\,0\,0\,0]^T$, $x^2 = [0 \pm \frac{\pi}{12}\ 0\,0]^T$ and $x^3 = [0 \pm \frac{\pi}{6}\ 0\,0]^T$. The linear models of the system are obtained as follow:

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$
$$y(t) = C_i x(t), i = 1,2,3 \qquad (1)$$

where:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.253 & 0 & 0 \\ 0 & 15.042 & 0 & -0.008 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0.827 \\ 1.237 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.218 & 0 & 0 \\ 0 & 14.456 & 0 & -0.008 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.826 \\ 1.193 \end{bmatrix} \qquad (2)$$
$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.123 & 0 & 0 \\ 0 & 12.781 & 0 & -0.008 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 0 \\ 0.822 \\ 1.065 \end{bmatrix}$$
$$C_i = C = [1\ \ 0\ \ 0\ \ 0]$$

Let consider a servo-compensator model [20]:

$$\dot{x}_c(t) = A_c x_c(t) + B_c e(t)$$
$$y_c(t) = x_c(t) \qquad (3)$$
$$e(t) = y_r(t) - y(t)$$

where $x_c(t) \in R^{n_c}$ is the compensator states, $y_c(t) \in R^q$ is the reference signals, and $e(t) \in R^q$ is the tracking error. For the sinusoidal reference signal $y_r(t) = 0.1 \sin(0.2\pi t)$, the compensator model is

$$\dot{x}_c(t) = \begin{bmatrix} 0 & 1 \\ -0.395 & 0 \end{bmatrix} x_c(t) + \begin{bmatrix} 0 \\ 0.063 \end{bmatrix} e(t) \qquad (4)$$

The controller is expected to follow the reference signal. The augmented system of the pendulum model and the compensator model is:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_c(t) \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ -B_i C_i & A_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ B_c \end{bmatrix} y_r(t),\ i = 1,2,3 \qquad (5)$$

The control signal of (5) is obtained as follow:

$$u(t) = \sum_{i=1}^{3} [K \quad K_c] \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} \qquad (6)$$

where K is the state feedback gain, and K_c is the compensator gain. Equation (5) and (6) can be written as follow:

$$\dot{\bar{x}}(t) = \bar{A}_i \bar{x}(t) + \bar{B}_i u(t),\ i = 1,2,3 \qquad (7)$$
$$u(t) = \sum_{i=1}^{3} \bar{K}_i \bar{x}(t) \qquad (8)$$

The T–S fuzzy model of the augmented system is described by fuzzy *If-Then* rules and will be employed to deal with the control design problem. The *i*th rules of the fuzzy model [15] are:

Model rule 1:
If $x_2(t)$ is M_1 (about 0 rad)
Then $\dot{\bar{x}}(t) = \bar{A}_1 \bar{x}(t) + \bar{B}_1 u(t)$
$y(t) = C_1(t)$
Model Rule 2:
If $x_2(t)$ is M_2 ($\pm\frac{\pi}{12}$ rad)
Then $\dot{\bar{x}}(t) = \bar{A}_2 \bar{x}(t) + \bar{B}_2 u(t)$
$y(t) = C_2(t)$
Model Rule 3:
If $x_2(t)$ is M_3 ($\pm\frac{\pi}{6}$ rad)
Then $\dot{\bar{x}}(t) = \bar{A}_3 \bar{x}(t) + \bar{B}_3 u(t)$
$y(t) = C_3(t)$

where $C_i = [1\ \ 0\ \ 0\ \ 0]$ for $i = 1,2,3$. M_1, M_2, and M_3 are the triangular fuzzy membership which represent the fuzzy inference function of angular pendulum. Based on the PDC scheme, we design the controller rules as follow:

Control Rule 1:
If $x_2(t)$ is M_1 (about 0 rad)
Then $u(t) = \bar{K}_1 x(t)$
Control Rule 2:
If $x_2(t)$ is M_2 ($\pm\frac{\pi}{12}$ rad)
Then $u(t) = \bar{K}_2 x(t)$
Control Rule 3:
If $x_2(t)$ is M_3 ($\pm\frac{\pi}{6}$ rad)
Then $u(t) = \bar{K}_3 x(t)$

The fuzzy inference is using AND conjunction operator while defuzzification is using the weighted average method. The fuzzy controller based on PDC rule can be written as:

$$u(t) = \sum_{i=1}^{3} h_i \bar{K}_i x(t) \quad (9)$$

where K_i is the gain controller that can be obtained using LMI pole placement.

The block diagram of the overall nominal fuzzy tracking control system can be described in Figure 2. It shown in Figure 2 that $u_1(t)$ is the respective control input nominal system in one operating point. This control input is acquired from the compensator and fuzzy controller. The other control input $u_2(t)$ and $u_3(t)$ are related for nominal system in different operating points.

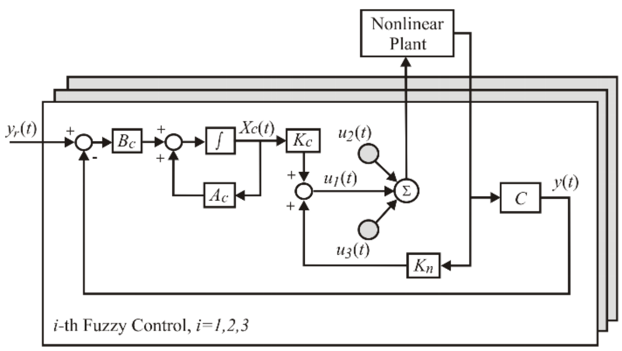


Figure 2 Overall controller for nominal system

As shown in Figure 3, the desired poles of the closed loop system are determined in the left half plane of region D and it can be fulfilled and guaranteed by the use of LMI. This region D is the intersection of pole regions for the prescribed stability degree and prescribed relative damping of continuous-time system [21].

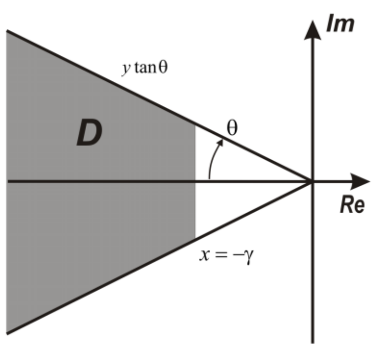


Figure 3 Pole regions for closed loop continuous system

Let the closed-loop system is given as follow:

$$\dot{\bar{x}}(t) = (\bar{A} + \bar{B}\bar{K})\bar{x}(t) \quad (10)$$

The inequality equation to obtain state feedback gain is:

$$P^{-1} > 0$$
$$(\bar{A} + \bar{B}\bar{K})P^{-1}(\bar{A} + \bar{B}\bar{K})^T + 2\gamma P^{-1} < 0 \quad (11)$$
$$\begin{bmatrix} \sin\theta\, H_{11} & \cos\theta\ H_{21} \\ \cos\theta\ H_{12}^T & \sin\theta\, H_{22} \end{bmatrix} < 0$$

where θ is angle between the real axis and upper region D, and γ is the limit line on the left half plane of the desired poles. Furthermore, we have:

$$H_{11} = \bar{A}P^{-1} + P^{-1}\bar{B}^T + \bar{B}\bar{K}P^{-1} + P^{-1}\bar{K}^T\bar{B}^T$$
$$H_{12} = \bar{A}P^{-1} - P^{-1}\bar{A}^T + \bar{B}\bar{K}P^{-1} + P^{-1}\bar{K}^T\bar{B}^T \quad (12)$$
$$H_{22} = H_{11}, H_{21} = H_{12}$$

By substituting $Y = \bar{K}P^{-1}$ and $Q = P^{-1}$ into (11), then we have:

$$Q_i > 0$$
$$\bar{A}_i Q_i + Q_i \bar{A}_i^T + \bar{B}_i Y_i + Y_i^T \bar{B}_i + 2\gamma P < 0 \quad (13)$$
$$\begin{bmatrix} \sin\theta\, G_1 & \cos\theta\ G_2 \\ * & \sin\theta\, G_3^T \end{bmatrix} < 0,\ i = 1,2,3$$

with

$$G_1 = G_3 = \bar{A}_i Q_i + Q_i \bar{A}_i^T + \bar{B}_i Y_i + Y_i^T \bar{B}_i$$
$$G_2 = \bar{A}_i Q_i - Q_i \bar{A}_i^T + \bar{B}_i Y_i - Y_i^T \bar{B}_i \quad (14)$$

i.e., the LMI that ensures the stability of the closed-loop pendulum-cart system. Moreover, as if the closed-loop stability is fulfilled, we are also required to have input and output constraints. This can be obtained by giving constraints for the following equations:

$$\begin{bmatrix} -Q & -Y_i^T \\ * & -\frac{u_{max}^2}{\beta} \end{bmatrix} < 0 \quad (15)$$

$$\begin{bmatrix} -Q & -QC_z^T \\ * & -\frac{u_{max}^2}{\beta} \end{bmatrix} < 0 \quad (16)$$

where β is related to the Lyapunov function:

$$V(x(t)) = x(t)^T P x(t) \le \beta \quad (17)$$

while z_{max} is corresponded to

$$\|C_z x(t)\| \le z_{max} \quad (18)$$

The matrix Q and Y_i are given:

$$Q = P^{-1}, Y_i P^{-1} \quad (19)$$

Then the controller gains can be obtained in the following:

$$K_i = Y_i P,\ i = 1,2,3 \quad (20)$$

3.2 Sensor Fault Observer Design Based on T-S Fuzzy PPIO

Based on augmented system, we develop sensor fault observer using T-S Fuzzy with the premise of state angular pendulum, $x_2(t)$. There are three rules developed in this system as discussed in previous subsection, i.e., $x_2(t)$ is 0 rad, $\pm\pi/12$ rad, and $\pm\pi/6$ rad.

Rule-1:
If is M_1 (around 0 rad)
Then

$$\dot{\hat{\bar{x}}}(t) = \bar{A}_1 \hat{\bar{x}}(t) + \bar{B}_1(u(t) + \hat{f}_a(t))$$

$$+\bar{D}_f\hat{f}_s(t)+\bar{L}_1\bar{C}_c e_x(t))$$

$$\dot{\hat{f}}_s(t)=\bar{F}_1\bar{C}_c(\dot{e}_x(t)+e_x(t))$$

Rule-2:
If $x_2(t)$ is M2 ($\pm\frac{\pi}{12}$ rad)
Then

$$\dot{\hat{x}}(t)=\bar{A}_2\hat{x}(t)+\bar{B}_2(u(t)+\hat{f}_a(t)$$
$$+\bar{D}_f\hat{f}_s(t)+\bar{L}_2\bar{C}_c e_x(t))$$
$$\dot{\hat{f}}_s(t)=\bar{F}_2\bar{C}_c(\dot{e}_x(t)+e_x(t))$$

Rule-3:
If $x_2(t)$ is M3 ($\pm\frac{\pi}{6}$ rad)
Then

$$\dot{\hat{x}}(t)=\bar{A}_3\hat{x}(t)+\bar{B}_3(u(t)+\hat{f}_a(t)$$
$$+\bar{D}_f\hat{f}_s(t)+\bar{L}_3\bar{C}_c e_x(t))$$
$$\dot{\hat{f}}_s(t)=\bar{F}_3\bar{C}_c(\dot{e}_x(t)+e_x(t))$$

The simplified T-S Fuzzy rules are:

$$\begin{aligned}\dot{\hat{x}}(t)&=\bar{A}(p)\hat{x}(t)+\bar{B}(p)(u(t)+\hat{f}_a(t)\\&\quad+\bar{D}_s\hat{f}_s(t)+\bar{L}(p)\bar{C}_c e_x(t)\\ \dot{\hat{f}}_s(t)&=\bar{F}(p)\bar{C}_c(\dot{e}_x(t)+e_x(t))\end{aligned}\qquad(21)$$

where $\dot{\hat{x}}(t)\in\mathbb{R}^n$ is state estimation, $\dot{\hat{f}}_s$ is sensor fault estimation which obtained from observer, $\bar{L}(p)\in\mathbb{R}^{(n+l)\times l}$ and $\bar{F}(p)\in\mathbb{R}^{g\times l}$ are observer gains which designed based on T-S fuzzy model and $e_x(t)$ is error estimation. The error estimation, the proportional gain and observer proportional integral gain are:

$$e_x=\bar{x}(t)-\hat{\bar{x}}(t)\qquad(22)$$

$$\bar{L}(p)=\sum_{i=1}^{r}h_i\bar{L}_i\qquad(23)$$

$$\bar{F}(p)=\sum_{i=1}^{r}h_i\bar{F}_i\qquad(24)$$

It follows from (22),

$$\begin{aligned}\dot{e}_x&=\dot{\bar{x}}(t)-\dot{\hat{\bar{x}}}(t)\\&=(\bar{A}(p)-\bar{L}(p)\bar{C}_e e_x(t)+\bar{D}_f e_{fs}(t)\\&\quad+\bar{B}(p)e_{fs}(t)\end{aligned}\qquad(25)$$

where sensor error estimation e_{fs} and actuator error estimation e_{fa} are given by:

$$\begin{aligned}e_{fs}&=f_s(t)-\hat{f}_s(t)\\e_{fa}(t)&=f_a(t)-\hat{f}_a(t)\end{aligned}\qquad(26)$$

then we have:

$$\begin{aligned}\dot{e}_{fs}(t)&=-\bar{F}(p)\bar{C}(\bar{A}(p)-\bar{L}(p)\bar{C}+I)e_x(t)\\&\quad+\dot{f}_s(t)-\bar{F}(p)\bar{C}\bar{D}_f e_{fs}(t)\\&\quad-\bar{F}(p)\bar{C}\bar{B}(p)e_{fa}(t)\end{aligned}\qquad(27)$$

By choosing Lyapunov candidate as:

$$V(\tilde{e}_{as}(t))=\tilde{e}_{as}^T(t)\bar{P}\tilde{e}_{as}(t)\qquad(28)$$

where $\tilde{e}_{as}(t)=[e_x(t)\quad e_{fs}(t)]^T$ and

$$\bar{P}=\sum_{i=1}^{3}h_i(x)P_i,\ \ i=1,2,3\qquad(29)$$

It follows from (28) that the derivative of Lyapunov function is:

$$\begin{aligned}\dot{V}(\tilde{e}_{as}(t))&=\tilde{e}_{as}^T(t)(\tilde{A}_s^T\bar{P}+\bar{P}\tilde{A}_s)\tilde{e}_{as}(t)\\&\quad+\tilde{e}_{as}^T(t)\bar{P}\bar{N}\tilde{z}+\tilde{z}^T\bar{N}^T\bar{P}\tilde{e}_{as}(t)\end{aligned}\qquad(30)$$

where

$$\tilde{A}_s=\begin{bmatrix}\bar{A}(p)-\bar{L}(p)\bar{C}_c & \bar{D}_f\\ -\bar{F}(p)\bar{C}(\bar{A}(p)-\bar{L}(p)\bar{C}_c+I & -\bar{F}(p)\bar{C}\bar{D}_f\end{bmatrix}$$

$$\bar{N}=\begin{bmatrix}\bar{B}(p) & 0\\ -\bar{F}(p)\bar{C}\bar{B}(p) & I\end{bmatrix},\ \tilde{z}=\begin{bmatrix}e_{fa}(t)\\ \dot{f}_s\end{bmatrix}\qquad(31)$$

and by calculating Equation (30) such that $\dot{V}(\tilde{e}_{as}(t))<0$, and with Schur complement method, then the LMI can be obtained. Furthermore, by adding some design criterion e.g. (a) to improve observer performance that ensures fast fault estimation, and (b) to assign observer poles in a certain region that may improve the performance (like overshoot), then we have LMI as follow:

$$\min\,(\gamma+\mu)$$

$$\begin{bmatrix}w_{11} & w_{12} & w_{13} & 0 & C_{p1}^T & 0\\ * & w_{22} & w_{23} & I & 0 & C_{p2}^T\\ * & * & -\gamma I & 0 & 0 & 0\\ * & * & * & -\gamma I & 0 & 0\\ * & * & * & * & -\gamma I & 0\\ * & * & * & * & * & -\gamma I\end{bmatrix}<0$$

$$\begin{bmatrix}\mu I & \bar{D}_f P_1-\bar{F}(p)\bar{C}_c\\ * & \mu I\end{bmatrix}>0\qquad(32)$$

$$\Sigma_i+\Sigma_i^T+2\rho\bar{P}<0$$

$$\begin{bmatrix}sin\theta[\Sigma_i+\Sigma_i^T] & cos\theta[\Sigma_i+\Sigma_i^T]\\ * & sin\theta[\Sigma_i+\Sigma_i^T]\end{bmatrix}<0$$

where

$w_{13}=P_1\bar{B}(p)$, $w_{22}=-2\bar{D}_f^T P_1\bar{D}_f$

$w_{23}=-2\bar{D}_f^T P_1\bar{B}(p)$,

$$w_{11}=P_1\bar{A}(p)+(P_1\bar{A}(p))^T-\bar{H}(p)\bar{C}_c-(\bar{H}(p)C_c)^T$$
$$w_{12}=-(\bar{A}^T(p)P_1 D_f-\bar{C}_c^T\bar{H}^T(p)\bar{D}_f$$

$$\Sigma_i=\bar{P}A_s(p,p)$$
$$=\begin{bmatrix}P_1\bar{A}(p)-\bar{H}(p)\bar{C}_c & P_1\bar{D}_f\\ -(\bar{A}^T(p)P_1\bar{D}_f-\bar{C}_c^T\bar{H}^T\bar{D}_f+P_1\bar{D}_f) & -\bar{D}_f^T P_1\bar{D}_f\end{bmatrix}$$

The observer gains of proportional and proportional-integral in Equation (32) can be obtained by:

$$\begin{aligned}\bar{L}(p)&=P_1^{-1}\bar{H}(p)\\&\bar{F}(p)\end{aligned}\qquad(33)$$

With fault sensor estimation:

$$\hat{f}_s(t)=\bar{F}(p)\bar{C}_c\int(\dot{e}_x(t)+e_x(t))dt\qquad(34)$$

3.3 Actuator Fault Observer Design Based on T-S Fuzzy PPIO

In a similar manner as in sensor fault procedure, the observer gain for actuator fault can be obtained and it is given by:

$$\min\,(\gamma_a+\mu_a)$$

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & 0 & C_{p1}^T & 0 \\ * & w_{22} & w_{23} & I & 0 & C_{p2}^T \\ * & * & -\gamma_a I & 0 & 0 & 0 \\ * & * & * & -\gamma_a I & 0 & 0 \\ * & * & * & * & -\gamma_a I & 0 \\ * & * & * & * & * & -\gamma_a I \end{bmatrix} < 0$$

$$\begin{bmatrix} \mu_a I & B(p)^T P_a - F_a(p) C_c \\ * & \mu_a I \end{bmatrix} > 0 \qquad (35)$$

$$\Sigma_{ai} + \Sigma_{ai}^T + 2\rho P_a < 0 \begin{bmatrix} sin\theta[\Sigma_{ai} + \Sigma_{ai}^T] & cos\theta[\Sigma_{ai} + \Sigma_{ai}^T] \\ * & sin\theta[\Sigma_{ai} + \Sigma_{ai}^T] \end{bmatrix} < 0$$

with

$$w_{11} = P_a A(p) + (P_a A(p))^T - H(p) C_c - (H(p) C_c)^T$$

$$w_{12} = -(A^T(p) P_a B(p) - C_c^T H^T(p) B(p))$$

$$w_{13} = -H(p) D_f,\ w_{22} = -(B(p)^T P_a B(p) + B(p) P_a B(p)^T)$$

$$w_{23} = -B(p)^T H(p) D_f$$

$$\Sigma_{ai} = P_{a1} A_s(p, p) = \begin{bmatrix} P_{a1} A(p) - H(p) C_c & P_{a1} B(p) \\ -A_{21} & -B(p)^T P_{a1} B(p) \end{bmatrix}$$

$$A_{21} = (A(p)^T P_{a1} B(p) - C_c^T H^T B(p) + B(p)^T P_{a1}(t)$$

The observer gains for actuator fault can be obtained from:

$$L_a(p) = P_a^{-1} H(p)$$

$$F_a(p) \qquad (36)$$

with actuator fault estimation:

$$\hat{f}_a(t) = F_a(p) C_c \int (\dot{e}_x(t) + e_x(t)) dt \qquad (37)$$

The overall structure for sensor and actuator fault can be illustrated in Figure 4. In this figure, the actuator and sensor fault observers are shown in red lines and they are fed to control input and output node, respectively.

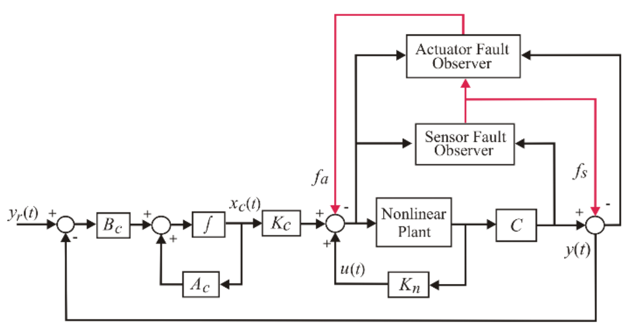


Figure 4 Overall control structure with actuator and sensor fault observers

3.0 RESULTS AND DISCUSSION

3.1 Simulation Results

To verify the effectiveness of the proposed fault-tolerant control for trajectory tracking problems in the pendulum-cart system, we conduct simulation and experimental validation. In the simulation, we use numerical simulation using Matlab/Simulink.

A. Simulation Results without Sensor or Actuator Fault

The simulation results for the nominal system are given in Figure 5, 6, and 7 for the pendulum position, the cart position, and the control input, respectively. We compare the simulation of the nominal system for two cases. In the first case, the initial pendulum velocity is 0.2 rad/s and the initial cart velocity is -0.77 rad/s, while for the second case, the initial pendulum velocity is 0.4 rad/s and initial cart velocity is -1.43 rad/s. From both cases, all the responses for the pendulum position and the cart position converge to zeros and to reference signals, respectively. However, the position of the pendulum in the first case converges faster than the second one, while for the position of the cart, both responses converge at the same time around 3 seconds.

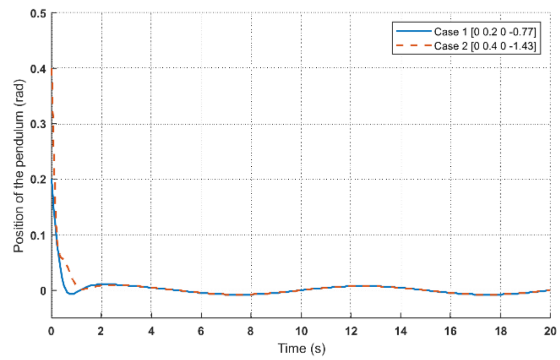


Figure 5 Pendulum position responses for nominal system with different initial conditions

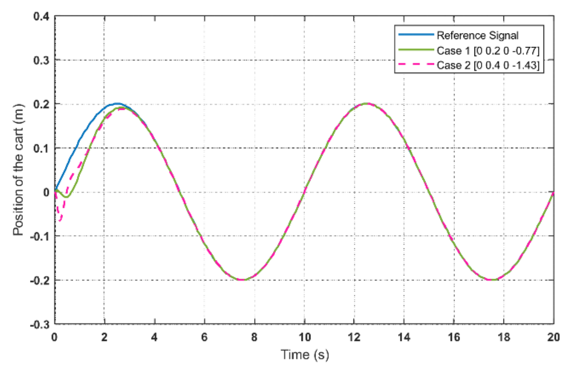


Figure 6 Cart position responses for nominal system with different initial conditions

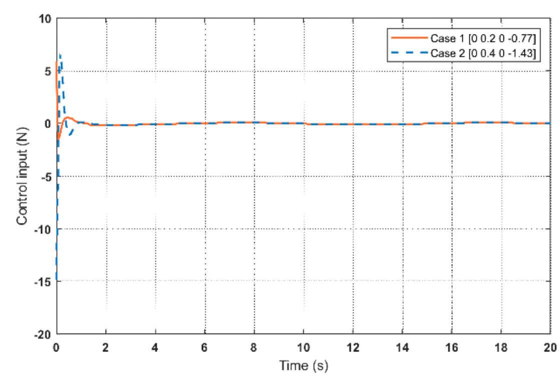


Figure 7 Control input for nominal system with different initial conditions

B. Simulation Results with Sensor and Actuator Faults

We conduct the simulation to observe the system response in the situation when there are faulty conditions in the sensor and actuator as given in Figure 8 and 9, respectively. For each faulty condition, we compare the system response with uncompensated and compensated controllers.

In the case of sensor fault, as shown in Figure 8, the cart position's response for the compensated controller converges to the reference signal after some time, while the response of the uncompensated controller does not converge (to reference signal) even though the pendulum is still stable. In the second situation when there is actuator fault as given in Figure 9, the cart position's response for both compensated and uncompensated converge to the reference signal. However, the error for the uncompensated controller is larger than the compensated controller.

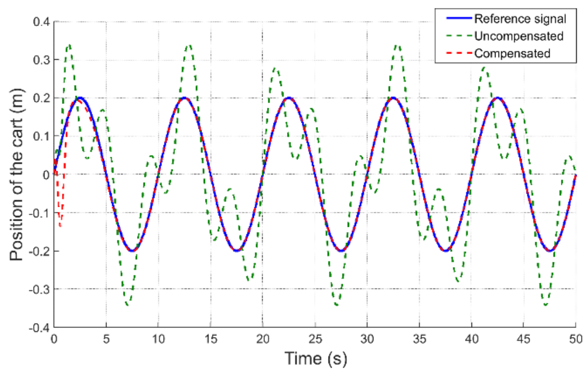


Figure 8 Cart position responses for simulated sensor fault with $f_s = 0.15\sin(0.5\pi)$

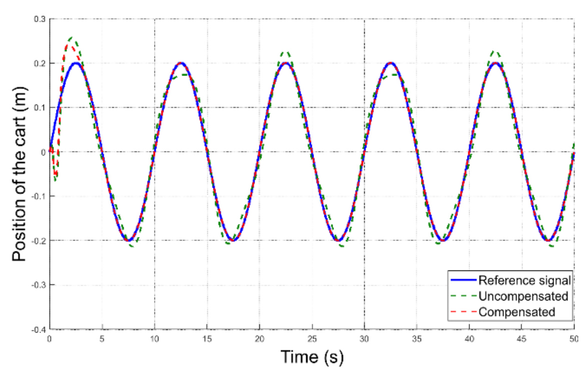


Figure 9 Cart position responses for simulated actuator fault with $f_a = 30\sin(0.5\pi)$

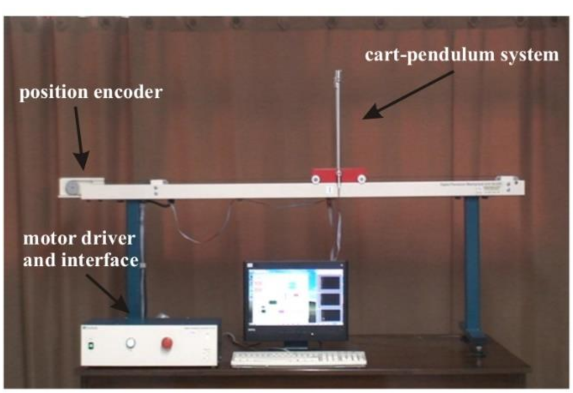


Figure 10 A testbed of pendulum-cart system

3.2 Experimental Results

The procedure to conduct the real-time pendulum-cart experiment can be described as follow. First, the initial position of the pendulum is at the bottom. We run the program and then manually bring up the pendulum to its upper position. Immediately after the pendulum is closer to the equilibrium point, the system responses to stabilize the pendulum upright and to track the commanded reference signal. Consequently, the initial positions of the pendulum shown in Figure 11 and Figure 14 are larger compared to simulation results. Furthermore, in Figure 12, 15, and 16, the cart positions are starting from zeros because the cart is held during positioning the pendulum to the upper position.

We conduct real-time implementation using a pendulum-cart system testbed in Figure 10. First, we run the experiment for the nominal system, and the results are shown in Figure 11, 12, and 13 for the pendulum, cart, and control input, respectively. Secondly, we run the experiment with the situation when there are sensor or actuator faults.

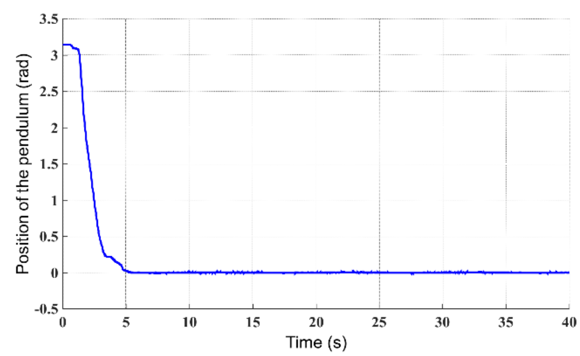


Figure 11 Pendulum position response for nominal system

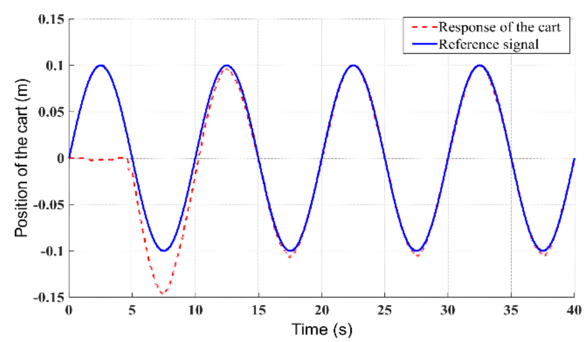


Figure 12 Cart position response for nominal system

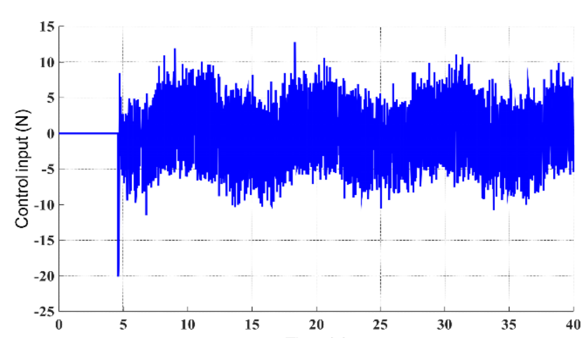


Figure 13 Control input response for nominal system

A. Experiment Results without Sensor or Actuator Fault

For the nominal system as given in Figure 11, 12, and 13, there is no faulty condition occurred. Both the pendulum position and cart position converge to the equilibrium point and the reference signal. The control input is shown in Figure 13 where the signal starts from zero and then after some time the control input stabilizes the pendulum upright and tracks the reference signal.

B. Experiment Results with Sensor and Actuator Faults

To observe the performance of the proposed controller in a real-time experiment, we conduct experiments for sensor fault and actuator fault conditions. The results are shown in Figure 14 through Figure 16.

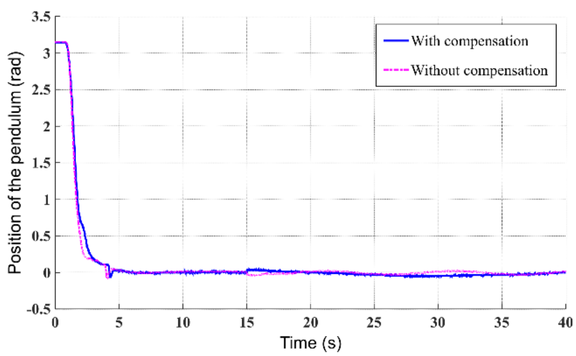


Figure 14 Pendulum position responses for sensor fault

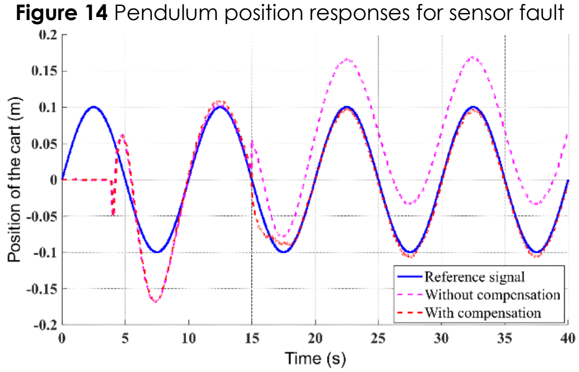


Figure 15. Cart position responses for sensor fault

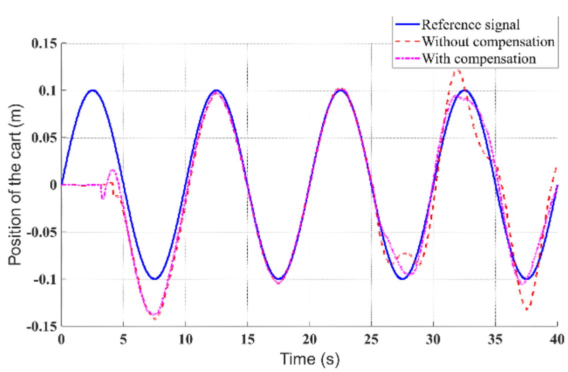


Figure 16. Cart position responses for actuator fault

In Figure 14 and Figure 15, we set the simulated sensor fault model as sinusoidal function with f_s = 0.5 sin(0.5π)u(t-15). Figure 14 shows the pendulum positions for uncompensated and compensated controllers that converge to equilibrium points. Furthermore, there are substantial differences in the cart positions shown in Figure 15.

In the case of the actuator fault shown in Figure 16, the actuator fault model is given by the sinusoidal function of f_a = 4 sin(0.5π)u(t-25). It is shown in this figure that the compensated and uncompensated controllers both converge to the reference signal. The cart position for the uncompensated controller has a larger error compared to the cart position for the compensated controller.

For the given actuator and sensor fault models on the cart-pendulum system, we found that the sensor fault has a significant effect on the system performance compared to actuator faults as given in Figure 8 and Figure 15.

4.0 CONCLUSION

Fault-tolerant control for sensor and actuator fault of the pendulum-cart system is proposed. T-S Fuzzy controller is used for nonlinear pendulum-cart system and proportional-proportional integral observer is employed to estimate the sensor and actuator fault. Based on the augmented system, the sensor fault observer is developed using T-S Fuzzy with the premise of state angular pendulum and by some design criterion, the control gain and observer gain are obtained using LMI. The proposed controller is implemented and the effectiveness of the controller is verified in simulation and experiment. It is shown that the proposed fault-tolerant control can compensate the sensor and actuator fault for different scenarios. We conclude that the fault-tolerant control for fuzzy tracking of the pendulum-cart system has satisfactory results with good tracking performance.

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