

The Effects of Radiation on Free Convection Flow with Ramped Wall Temperature in Brinkman Type Fluid

Muhamad Najib Zakaria^a, Abid Hussanan^a, Ilyas Khan^a, Sharidan Shafie^{a*}

^aDepartment of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia

*Corresponding author: sharidan@utm.my

Article history

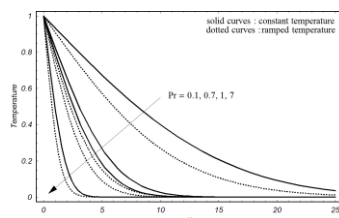
Received :18 March 2013

Received in revised form :

26 April 2013

Accepted :17 May 2013

Graphical abstract



Abstract

The present paper is on study of the influence of radiation on unsteady free convection flow of Brinkman type fluid near a vertical plate containing a ramped temperature profile. Using the appropriate variables, the basic governing equations are reduced to nondimensional equations valid with the imposed initial and boundary conditions. The exact solutions are obtained by using Laplace transform technique. The influence of radiation near a ramped temperature plate is also compared with the flow near a plate with constant temperature. The numerical computations are carried out for various values of the physical parameters such as velocity, temperature, skin friction and Nusselt number and presented graphically.

Keywords: Natural convection; Brinkman type fluids; ramped wall temperature; exact solutions

Abstrak

Kertas kerja ini adalah mengenai kajian kesan radiasi terhadap aliran tak mantap olakan semulajadi bendalir tak mampat jenis Brinkman berhampiran plat menegak dengan profil suhu tajakan. Menggunakan pemboleh ubah yang bersesuaian, persamaan menakluk yang asas diturunkan ke persamaan tak bermatra memenuhi syarat awal dan syarat sempadan yang dikenakan. Penyelesaian tepat diperoleh dengan menggunakan teknik penjelmaan Laplace. Pengaruh kesan radiasi berhampiran plat dengan suhu tajakan juga dibandingkan dengan aliran berhampiran plat suhu malar. Pengiraan berangka dijalankan bagi pelbagai nilai parameter fizikal seperti halaju, suhu, geseran kulit dan nombor Nusselt dan dipaparkan secara grafik.

Kata kunci: Olakan semula jadi; bendalir jenis Brinkman; suhu tajakan; penyelesaian tepat

© 2013 Penerbit UTM Press. All rights reserved.

1.0 INTRODUCTION

Natural convection flows past a vertical plate are vital in solving some industrial and engineering problems such as the filtration and design of processes, the drying of porous materials in textile industries, and solar energy collector. Numerous investigations have been reported in the literature to solve the problems using analytical and numerical methods. The unsteady natural convection flow of an incompressible viscous fluid near a vertical flat plate was analyzed by Chandran *et al.* in [1]. The problem has been solved using analytical Laplace transform method under usual Boussinesq's approximation, and the wall temperature is assumed to have a temporally ramped continuous profile. The fluid convection resulting from a wall temperature is likely to be of relevance in several industrial applications. Free convection flow of an incompressible and viscous fluid past a moving vertical plate with the influence of radiation when it is heated has been studied in [2]. This problem was solved analytically using Laplace transform method. The radiative heat flux parameter in the energy equation was described using Rosseland approximation. The resulting flow from this problem was analyzed in three different

situations of the moving plate which are plate with uniform velocity motion, plate with uniformly accelerated motion and plate with exponentially accelerated motion. Deka and Das [3] studied the effects of radiation on free convection flow near a vertical plate with ramped wall temperature. The problem was solved analytically by using Laplace transform method and the influence of the various parameters entering into the problem was also studied. In 2010, Rajesh [4] studied the effects of thermal radiation on magnetohydrodynamic (MHD) free convection flow near a vertical plate with ramped wall temperature. The exact solutions were obtained by using Laplace transform method. The obtained results were extensively discussed with the help of graphs. The results were discussed based on two values of Prandtl number, Pr which are $Pr=0.71$ for air and $Pr=7$ for water. Rajesh [5] performed a finite difference analysis to study the effects of thermal radiation and chemical reaction on the transient MHD free convection and mass transform flow of a dissipative fluid past an infinite vertical plate with ramped wall temperature. The Crank-Nicolson method was used to obtain the results and the effects of various parameters were discussed graphically. Seth *et al.* [6] have investigated the effects of impulsively moving vertical

plate with ramped wall temperature on natural convection flow with radiative heat transfer. They obtained the exact solutions by using Laplace transform method and conclude that thermal diffusion and radiation tends to enhance the fluid temperature. Natural convection flow near a vertical plate in a porous medium conducting with ramped wall temperature has been solved by Deka and Das in [7]. The solutions of energy and momentum equations have been obtained in closed form by using Laplace transform technique. The resulting temperature profiles for air and water were analyzed and discussed. Deka and Deka [8] were attempted to improve the earlier results based on free convection near a vertical plate which relates the problem with ramped wall temperature and presence the heat source.

The mathematical theory of the flow of the viscous fluid through a porous medium has been established by Darcy [9]. Darcy's law describes the flow in the porous medium. Generally, this law is valid for the flows past a porous body with low permeability. Certain flows that pass through bodies with high porosity do not follow the Darcy's law but Brinkman's model is applicable for this type of flows. Brinkman equations represent a viscous fluid flow through a cloud of spherical particles whose size is smaller than the characteristic length scale of the flow, and it occupies a negligible volume. Therefore the viscous fluid flow in a porous medium is accurately described by the Brinkman equations for incompressible flow. Numerous studies have been done in the fluid flow problem in a porous medium using Brinkman model. A study on flow of viscous incompressible fluid through a porous channel using Brinkman model has been presented in [10]. This problem was solved in two cases which are, (1) when both walls are porous and (2) when the upper wall is rigid and the lower wall is porous. The flow through the channel is with high permeability and therefore Brinkman's model has been considered. Brinkman model was used to solve the mixed convection boundary layer flow past a horizontal circular cylinder in a porous medium in [11]. Both cases of a heated (assisting flow) and a cooled (opposing flow) cylinder were considered in this problem and were solved numerically. Analytical solutions of two immiscible viscous fluid have been obtained using Brinkman model in [12]. In this research the convective Couette flow of two viscous, incompressible, immiscible fluids through two straight parallel horizontal walls has been discussed. Recently, the exact solutions corresponding to the Stoke's problems for fluid of Brinkman type have been obtained by Fetecau *et al.* in [13]. The governing equation with appropriate initial and boundary condition was solved using Fourier sine transform method instead of Hankel transform. An analytical solution of free convection flow about a semi-infinite vertical flat plate in a porous medium using Brinkman model has been obtained in [14]. The governing equations based on Brinkman model were solved using the method of matched asymptotic expansions.

In this paper, we consider the radiation effects on unsteady natural convection flow near a vertical plate using Brinkman model. The wall temperature is assumed to has a temporally ramped continuous profile. The exact solutions for the governing equation have been obtained by using the Laplace transform technique.

2.0 GOVERNING EQUATIONS

We consider the unsteady two dimensional flow of Brinkman type fluid near an infinite vertical flat plate. The fluid considered here is a gray, absorbing/emitting but a non-scattering medium. Figure 1 shows the physical configuration of the problem. With respect to an arbitrarily chosen origin O in this plate, the axis Ox' is

taken along the wall in the upward direction while the axis Oy' is taken perpendicular to it into the fluid. Initially, for time $t' \leq 0$ both the fluid and the plate are at rest and at the constant temperature T'_∞ . At time $t' > 0$, the temperature of the plate is

raised or lowered to $T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0}$ when $t' \leq t_0$, and

thereafter, for $t' > t_0$, is maintained at the constant temperature T'_w . The aim of this paper is to provide the exact solutions

corresponding to the unsteady free convection flow resulting from the ramped temperature profile of the bounding plate. We assume that the flow is laminar such that the effects of the convective and pressure gradient terms in the momentum and energy equations can be neglected. Moreover, as a result of the boundary layer approximations, the physical variables in this case become functions of the variable t' and the space variable y' only.

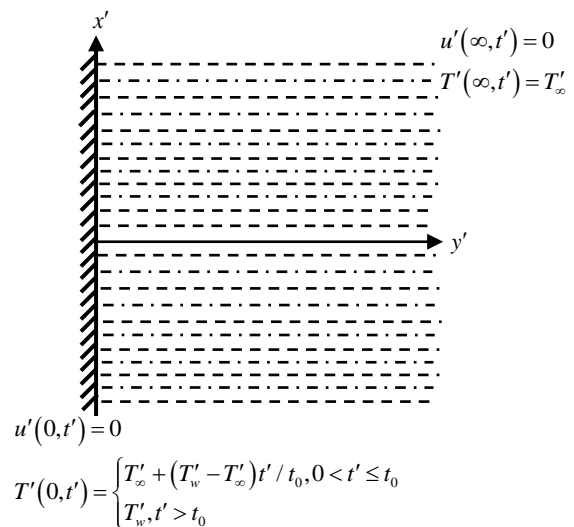


Figure 1 Physical configuration

Applying the Boussinesq approximation, the free convective flow is governed by the equations

$$\frac{\partial u'}{\partial t'} + \beta^* u' = \nu \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T'_\infty), \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q'_r}{\partial y'}, \quad (2)$$

where u' is the velocity in the x' direction, T' is the temperature of the fluid, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, ν is the kinematic viscosity, ρ is the density, k is the thermal conductivity, c_p is the heat of the fluid at constant pressure and β^* is defined as $\beta^* = \alpha/\rho$, where α is referred as a drag coefficient that is usually positive constant.

The initial and boundary conditions are

$$\left. \begin{aligned} u' &= 0, & T' &= T'_\infty & \text{for } y' \geq 0 & \text{and } t' \leq 0 \\ u' &= 0, & & & \text{at } y' = 0 & \text{for } t' > 0 \\ T' &= T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0}, & \text{at } y' = 0 & \text{for } 0 < t' \leq t_0 \\ T' &= T'_w, & \text{at } y' = 0 & \text{for } t' > t_0 \\ u' &\rightarrow 0, & T' &\rightarrow T'_\infty & \text{as } y' \rightarrow \infty & \text{for } t' > 0. \end{aligned} \right\} \quad (3)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^* \sigma (T'^4_\infty - T'^4) \quad (4)$$

It is assumed that the temperature differences with the flow are sufficiently small so that T'^4 may be expressed as a linear function of the temperature, which is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms. Thus,

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (5)$$

The non-dimensional quantities are defined as

$$y = \frac{y'}{\sqrt{\nu t_0}}, \quad t = \frac{t'}{t_0}, \quad u = u' \sqrt{\frac{t_0}{\nu}}, \quad T = \frac{T' - T'_\infty}{T'_w - T'_\infty} \quad (6)$$

where t_0 is characteristic time defined as

$$t_0 = \left[\frac{\nu}{g^2 \beta^2 (T'_w - T'_\infty)} \right]^{1/3}$$

Using the non-dimensional quantities in Eq. (6), Eqs. (1) and (2) can be expressed in the form of

$$\frac{\partial u}{\partial t} + \beta_1 u = \frac{\partial^2 u}{\partial y^2} + T, \quad (7)$$

$$\text{Pr} \frac{\partial T}{\partial t} = N \frac{\partial^2 T}{\partial y^2}, \quad (8)$$

where $\beta_1 = \beta^* t_0$ is arbitrary constant, $N = 1 + \frac{4}{3R}$, $R = \frac{kk'}{4\sigma T'^3_\infty}$,

and $\text{Pr} = \frac{\rho \nu c_p}{k}$.

The initial and boundary condition in non-dimensional form are

$$\left. \begin{aligned} u &= 0, & T &= 0 & \text{for } y \geq 0 & \text{and } t \leq 0 \\ u &= 0, & & & \text{at } y = 0 & \text{for } t > 0 \\ T &= t, & \text{at } y = 0 & \text{for } 0 < t \leq 1 \\ T &= 1, & \text{at } y = 0 & \text{for } t > 1 \\ u &\rightarrow 0, & T &\rightarrow 0 & \text{as } y \rightarrow \infty & \text{for } t > 0. \end{aligned} \right\} \quad (9)$$

3.0 SOLUTIONS

By taking Laplace transform of Eqs. (7) and (8) with respect to t , in conjunction with Eq. (9) and solving the resulting differential equations for the transformed variables $\bar{T}(y, s)$ and $\bar{u}(y, s)$ in the (y, s) -plane, we obtained

$$\bar{T}(y, s) = \frac{1 - \exp(-s)}{s^2} \exp(-y\sqrt{sF}), \quad (10)$$

$$\bar{u}(y, s) = a_1 \left(\frac{1}{s} + \frac{a_0}{s^2} - \frac{1}{(s - a_0)} \right) \left[\frac{\{1 - \exp(-s)\} \exp(-y\sqrt{s + \beta_1})}{s} - \frac{\{1 - \exp(-s)\} \exp(-y\sqrt{sF})}{(s - a_0)} \right]. \quad (11)$$

The exact solutions for the temperature and velocity of the fluid can be obtained from Eqs. (10) and (11), respectively, by taking their inverse transform. After detailed simplifications and shifting on the t -axis, the solutions can be expressed as

$$T(y, t) = G(y, t) - G(y, t-1)H(t-1), \quad (12)$$

$$\begin{aligned} u(y, t) &= a_1 \left[u_1(y, t) + a_0 u_2(y, t) + u_3(y, t) - u_4(y, t) - a_0 u_5(y, t) \right. \\ &\quad \left. + u_6(y, t) - \{u_1(y, t-1) - a_0 u_2(y, t-1) - u_3(y, t-1) \right. \\ &\quad \left. + u_4(y, t-1) + a_0 u_5(y, t-1) - u_6(y, t-1)\} H(t-1) \right], \quad (13) \end{aligned}$$

where

$$a_0 = \frac{\beta_1}{(F-1)},$$

$$a_1 = \frac{1}{a_0^2 (F-1)},$$

$$F = \frac{\text{Pr}}{N},$$

$$G(y, t) = \left(t + \frac{Fy^2}{2} \right) \text{erfc} \left(\frac{y\sqrt{F}}{2\sqrt{t}} \right) - \sqrt{\frac{Ft}{\pi}} y \exp \left(-\frac{Fy^2}{4t} \right),$$

$$u_1(y, t) = \frac{1}{2} \left[\exp(y\sqrt{\beta_1}) \text{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1 t} \right) \right.$$

$$\left. + \exp(-y\sqrt{\beta_1}) \text{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1 t} \right) \right],$$

$$u_2(y, t) = \exp(-y\sqrt{\beta_1}) \text{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1 t} \right) \left(\frac{t}{2} + \frac{y}{4\sqrt{\beta_1}} \right)$$

$$+ \exp(y\sqrt{\beta_1}) \text{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1 t} \right) \left(\frac{t}{2} - \frac{y}{4\sqrt{\beta_1}} \right),$$

$$u_3(y, t) = \frac{\exp(a_0 t)}{2} \left\{ \exp(y\sqrt{a_0 + \beta_1}) \text{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{(a_0 + \beta_1)t} \right) \right.$$

$$\left. + \exp(-y\sqrt{a_0 - \beta_1}) \text{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{(a_0 - \beta_1)t} \right) \right\},$$

$$u_4(y, t) = \text{erfc} \left(\frac{y\sqrt{F}}{2\sqrt{t}} \right),$$

$$u_5(y, t) = \left(t + \frac{y^2 F}{2} \right) \text{erfc} \left(\frac{y\sqrt{F}}{2} \right) - \frac{y\sqrt{F}}{\sqrt{\pi}} \exp \left(-\frac{y^2 F}{4t} \right),$$

and

$$u_6(y, t) = \frac{\exp(a_0 t)}{2} \left\{ \exp(y\sqrt{a_0 F}) \text{erfc} \left(\frac{y\sqrt{F}}{2\sqrt{t}} + \sqrt{a_0 t} \right) \right.$$

$$\left. + \exp(-y\sqrt{a_0 F}) \text{erfc} \left(\frac{y\sqrt{F}}{2\sqrt{t}} - \sqrt{a_0 t} \right) \right\}.$$

The complementary error function, $\operatorname{erfc}(x)$ defined as

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x), \quad \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\eta^2) d\eta.$$

Also, $H(t-1)$ is the unit step function defined as

$$H(t-1) = \begin{cases} 1, & t \geq 1 \\ 0, & 0 \leq t < 1 \end{cases}.$$

In order to differentiate the effect of the ramped temperature distribution from the constant wall temperature on the flow, both solutions were compared. The solutions for temperature and velocity of the fluid near an isothermal stationary plate can be expressed as

$$T(y, t) = \operatorname{erfc}\left(\frac{y\sqrt{F}}{2\sqrt{t}}\right), \quad (14)$$

$$u(y, t) = \frac{1}{\beta_1} [u_7(y, t) - u_8(y, t) + u_9(y, t) - u_{10}(y, t)], \quad (15)$$

where

$$u_7(y, t) = \operatorname{erfc}\left(\frac{y\sqrt{F}}{2\sqrt{t}}\right),$$

$$u_8(y, t) = \frac{1}{2} \left[\exp(y\sqrt{\beta_1}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\beta_1}t\right) + \exp(-y\sqrt{\beta_1}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\beta_1}t\right) \right],$$

$$u_9(y, t) = \frac{\exp(a_0 t)}{2} \left[\exp(y\sqrt{a_0 + \beta_1}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_0 + \beta_1)t}\right) + \exp(-y\sqrt{a_0 + \beta_1}) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_0 + \beta_1)t}\right) \right],$$

$$u_{10}(y, t) = \frac{\exp(a_0 t)}{2} \left[\exp(y\sqrt{Fa_0}) \operatorname{erfc}\left(\frac{y\sqrt{F}}{2\sqrt{t}} + \sqrt{a_0}t\right) + \exp(-y\sqrt{Fa_0}) \operatorname{erfc}\left(\frac{y\sqrt{F}}{2\sqrt{t}} - \sqrt{a_0}t\right) \right].$$

We now study the skin friction from velocity field. The non-dimensional form of skin friction is given by

$$\tau = \frac{\partial u}{\partial y} \Big|_{y=0}.$$

Then from Eq. (13), we have expression for the skin friction as

$$\begin{aligned} \tau = \frac{1}{\beta_1^2} & \left[\left(\frac{3\operatorname{Pr}R}{4+3R} - 1 \right) \left(-\frac{\sqrt{3}(4+3R)\sqrt{\frac{\operatorname{Pr}R}{4\pi+3\pi R}}\sqrt{t}\beta_1}{3(\operatorname{Pr}-1)R-4} \{\sqrt{t}+1\} \right. \right. \\ & + \sqrt{\beta_1} \operatorname{erf}(\sqrt{t}\beta_1) + \frac{(4+3R)\sqrt{\beta_1} \operatorname{erf}(\sqrt{t}\beta_1)}{6(\operatorname{Pr}-1)R-8} \\ & \left. \left. + \frac{(4+3R)t\beta_1^{3/2} \operatorname{erf}(\sqrt{t}\beta_1)}{3(\operatorname{Pr}-1)R-4} + \frac{\exp(-t\beta_1)(4+3R)\sqrt{t}\beta_1}{\sqrt{\pi}(3(\operatorname{Pr}-1)R-4)} \right. \right. \end{aligned}$$

$$\begin{aligned} & -\sqrt{3} \exp\left(\frac{(4+3R)t\beta_1}{3(\operatorname{Pr}-1)R-4}\right) \sqrt{\frac{\operatorname{Pr}R\beta_1}{3(\operatorname{Pr}-1)R-4}} \{A(t)\} \\ & + \sqrt{3} \exp\left(\frac{(4+3R)(t-1)\beta_1}{3(\operatorname{Pr}-1)R-4}\right) \sqrt{\frac{\operatorname{Pr}R\beta_1}{3(\operatorname{Pr}-1)R-4}} \{A(t-1)\} \\ & + \left\{ (1-t) \left\{ \frac{(4+3R)\beta_1^{3/2} \operatorname{erf}(\sqrt{(t-1)\beta_1})}{3(\operatorname{Pr}-1)R-4} + \frac{\exp(\beta_1-t\beta_1)(4+3R)\beta_1}{\sqrt{\pi}\sqrt{t-1}(3(\operatorname{Pr}-1)R-4)} \right. \right. \\ & \left. \left. - \frac{\sqrt{3}(4+3R)\sqrt{\frac{\operatorname{Pr}R}{4\pi+3\pi R}}\beta_1}{\sqrt{t-1}(3(\operatorname{Pr}-1)R-4)} \right\} - \sqrt{\beta_1} \operatorname{erf}(\sqrt{(t-1)\beta_1}) \right. \\ & \left. + \frac{\sqrt{3}(4+3R)\sqrt{\frac{\operatorname{Pr}R(t-1)}{4+3R}}\beta_1}{3(\operatorname{Pr}-1)R-4} \right. \\ & \left. \left. - \frac{(4+3R)\sqrt{\beta_1} \operatorname{erf}(\sqrt{(t-1)\beta_1})}{6(\operatorname{Pr}-1)R-8} \right\} H(t-1) \right], \quad (16) \end{aligned}$$

$$\text{where } A(t) = \operatorname{erf}\left(\sqrt{3}\sqrt{\frac{\operatorname{Pr}Rt\beta_1}{3(\operatorname{Pr}-1)R-4}}\right) - \operatorname{erf}\left(\sqrt{\frac{(4+3R)t\beta_1}{3(\operatorname{Pr}-1)R-4}}\right).$$

Whereas, the skin friction for the isothermal plate is given by

$$\begin{aligned} \tau = \frac{1}{2\beta_1} & \left[\frac{2\exp(-t\beta_1)}{\sqrt{\pi}\sqrt{t}} - \frac{2\sqrt{3}\sqrt{\frac{\operatorname{Pr}R}{4\pi+3\pi R}}}{\sqrt{t}} + 2\sqrt{\beta_1}(1 - \operatorname{erfc}(\sqrt{t}\beta_1)) \right. \\ & + \frac{1}{\sqrt{\pi}\sqrt{t}} \left(-2\exp\left(\frac{4t\beta_1}{3\operatorname{Pr}R-3R-4}\right) + \exp\left(\frac{4t\beta_1+3Rt\beta_1}{3\operatorname{Pr}R-3R-4}\right) \right. \\ & \left. \left\{ 2\sqrt{3\pi}\sqrt{t}\sqrt{\frac{\operatorname{Pr}R\beta_1}{3\operatorname{Pr}R-3R-4}} \left\{ \operatorname{erfc}\left(\sqrt{3}\sqrt{\frac{\operatorname{Pr}Rt\beta_1}{3\operatorname{Pr}R-3R-4}}\right) - 1 \right\} \right\} \right. \\ & \left. + \frac{1}{\sqrt{t}} \left(2\sqrt{3}\sqrt{\frac{\operatorname{Pr}R}{4\pi+3\pi R}} + \sqrt{3} \exp\left(\frac{4t\beta_1+3Rt\beta_1}{3\operatorname{Pr}R-3R-4}\right) \right. \right. \\ & \left. \left. \left\{ 2\sqrt{t}\sqrt{\frac{\operatorname{Pr}R\beta_1}{3\operatorname{Pr}R-3R-4}} \left\{ 1 - \operatorname{erfc}\left(\sqrt{\frac{(4+3R)t\beta_1}{3\operatorname{Pr}R-3R-4}}\right) \right\} \right\} \right\} \right]. \quad (17) \end{aligned}$$

From Eq. (12), we now study the heat transfer coefficient, i.e. Nusselt number, which is given in dimensional form by

$$\begin{aligned} Nu = -\frac{\partial T}{\partial y} \Big|_{y=0} & = \frac{1}{\sqrt{\pi}\sqrt{t-1}} \left[\sqrt{3}\sqrt{t-1} \left(\sqrt{\frac{\operatorname{Pr}R}{4+3R}}\sqrt{t} + \sqrt{\frac{\operatorname{Pr}Rt}{4+3R}} \right) \right. \\ & \left. + \left(-\sqrt{\frac{\operatorname{Pr}}{1+\frac{4}{3R}}}(t-1) - \sqrt{t-1}\sqrt{\frac{\operatorname{Pr}(t-1)}{1+\frac{4}{3R}}} \right) H(t-1) \right], \quad (18) \end{aligned}$$

while, for the isothermal plate

$$Nu = \sqrt{\frac{3}{t}} \sqrt{\frac{\operatorname{Pr}R}{4\pi+3\pi R}}. \quad (19)$$

4.0 RESULTS AND DISCUSSION

In order to understand the effects of different physical parameters, such as Prandtl number, Pr , radiation, R , and time, t the computations are carried out for temperature and velocity of the fluid. The computed results are presented graphically. In the presented figures, the dotted graph is plotted for ramped temperature case while solid graph is plotted for constant temperature.

Figure 2 displays the effect of Prandtl number, Pr on the temperature profile correspond to ramped and constant temperature for $t = 1.3$ and $R = 0.3$. It is observed that temperature decrease with an increase of Pr . It is also shown that the temperature of the fluid is greater in the case of isothermal than in the case of ramped temperature at the wall. This is due to the heating of the fluid more gradually than in the isothermal case. Figure 3 shows the effect of time, t on the temperature profile for fixed values of $R = 0.5$ and $Pr = 0.7$. From the figure, it is demonstrated that the temperature increase gradually in time, t . It is also to be noted that the temperature for both cases decreases with increasing in y to its free stream value. Figure 4 represents the temperature profiles for different values of radiation, R at time, $t = 0.5$ and $Pr = 0.7$. It is found that the temperature decreases slightly with an increase in R .

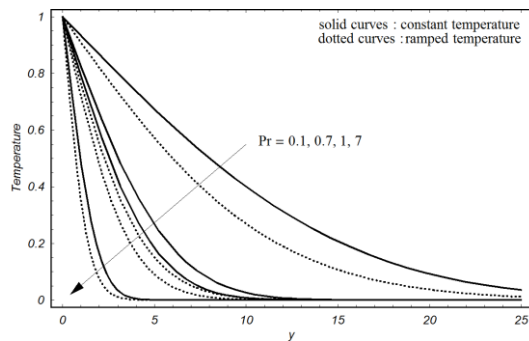


Figure 2 Temperature profiles when $R = 0.3$ and $t = 1.3$

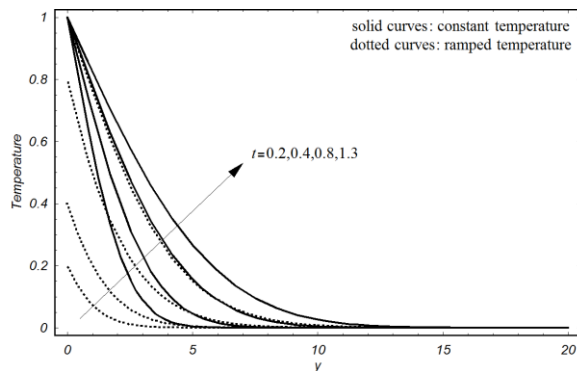


Figure 3 Temperature profiles when $R = 0.3$ and $Pr = 0.7$

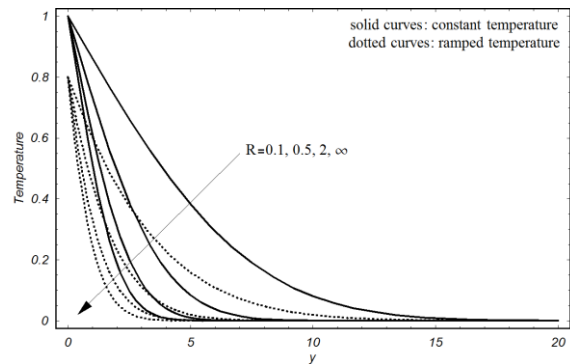


Figure 4 Temperature profiles when $Pr = 0.7$ and $t = 0.8$.

In Figures 5 – 8, the velocity profiles are shown for different value of physical parameters involved for both ramped and isothermal cases. Figure 5 reveals velocity variations with Pr . It demonstrates that the velocity decreases with increasing Prandtl number. In the Figure 6, the graph illustrates the influence of Brinkman fluid parameter, β_1 on the velocity profile. It is observed that the fluid velocity decreases on increasing Brinkman fluid parameter in the boundary region. Figure 7 illustrates the velocity profile decrease with increasing radiation parameter, R . Lastly, Figure 8 shows that the velocity of the fluid increase with increase in time, t . It is also to be noted that for very small values of t , the velocity profiles are nearly flat, but assume parabolic shapes near the plate as t increases.

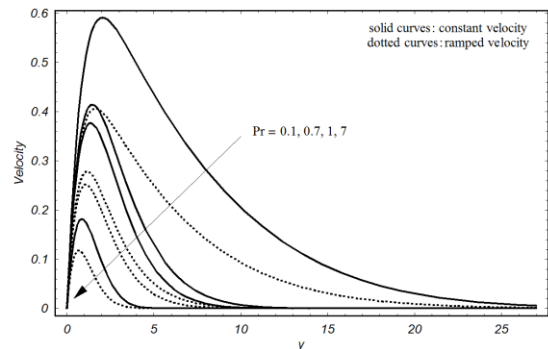


Figure 5 Velocity profiles when $R = 0.3$, $t = 1.3$ and $\beta_1 = 0.8$

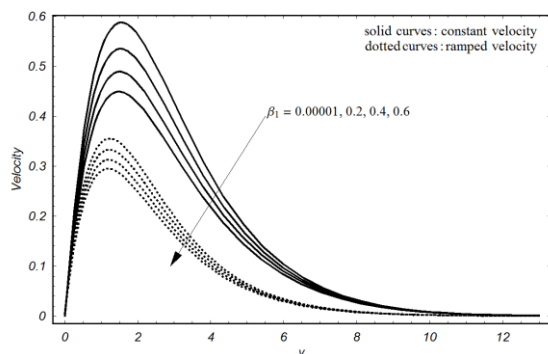


Figure 6 Velocity profiles when $Pr = 0.7$, $R = 0.3$, and $t = 1.3$

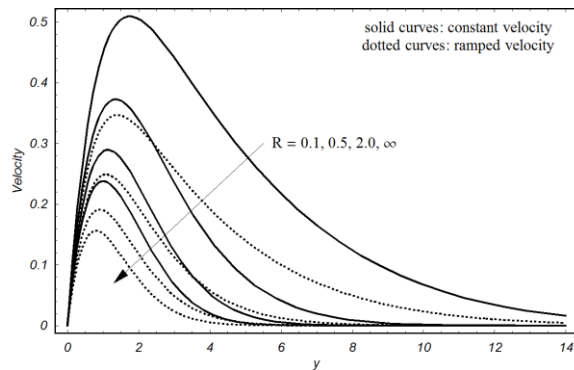


Figure 7 Velocity profiles when $Pr = 0.7$, $\beta_1 = 0.8$, and $t = 1.3$

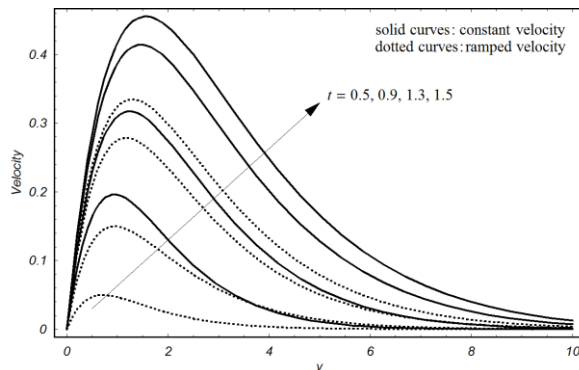


Figure 8 Velocity profiles when $Pr = 0.7$, $R = 0.3$ and $\beta_1 = 0.8$.

The behavior of the skin friction coefficient, τ with change in Pr and R are shown graphically in Figure 9 and 10. From Figure 9, the value of skin friction decrease with an increase in Pr . It is observed that for fluid with high Pr , the skin friction shows marginal variation with t , while for the fluid with small Pr ($Pr < 1$), the skin friction profiles are quite sensitive to small values of t . Besides, it can be noted that the skin friction for the isothermal is greater than the case in ramped. The variation of the skin friction with changes in R is presented in Figure 10. It is observed that the skin friction decrease with increase in R . Finally, Figure 11 and 12 shows the variation of Nusselt number, Nu for some values of Pr and R , respectively in the case of ramped temperature at the bounding plate. It is observed from the graph that the Nu increases for $0 < t < 1$, and decrease for $t > 1$, for all values of Pr and R . On the other hand, Nu is a decreasing function of t in the case of constant wall temperature, as can be seen from Eq. (19).

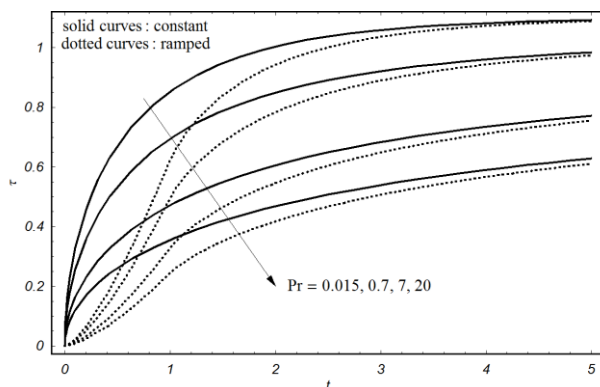


Figure 9 Skin friction when $R = 0.3$ and $\beta_1 = 0.8$

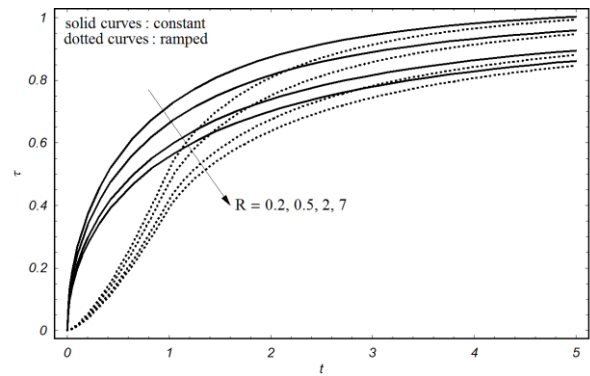


Figure 10 Skin friction when $Pr = 0.7$ and $\beta_1 = 0.8$

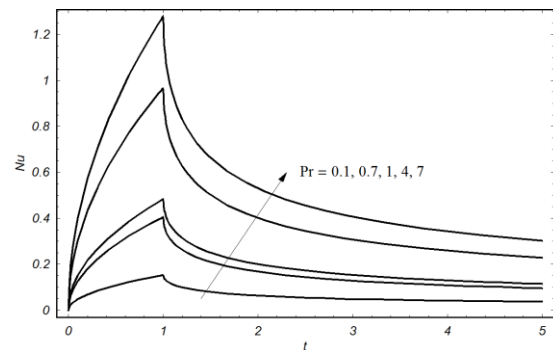


Figure 11 Nusselt number with $R = 0.3$

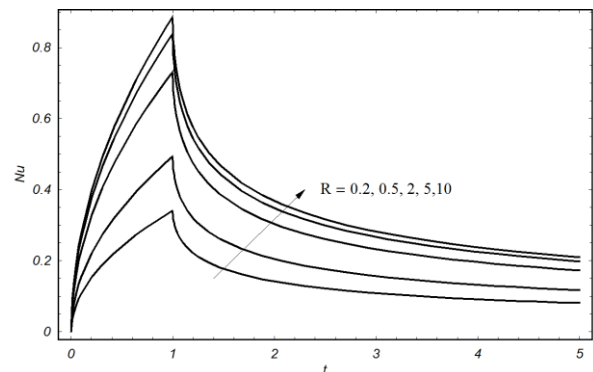


Figure 12 Nusselt number with $Pr = 0.7$.

5.0 CONCLUSION

In this paper, the behaviour of radiation effects on free convection flow of Brinkman type fluid in the presence of ramped wall temperature is studied. Some important conclusions that can be obtained from the graphical results are:

- Velocity decreases with increasing values of Brinkman fluid parameter, β_1 .
- Velocity decreases with increasing values of radiation, R .
- Temperature decreases with increasing radiation.
- Skin friction is reduced with increasing radiation.
- Nusselt number is greater with increasing radiation.

The physical parameters in the present case have also been compared with the case of constant temperature plate. For this

scenario our results indicate that all physical parameters are greater in the case of constant temperature than the ramped temperature plate. The present results have immediate relevance in industrial thermofluid dynamics, transient energy system and atmospheric vertical flows.

Acknowledgement

The authors would like to acknowledge the Research Management Centre – UTM and Ministry of Higher Education Malaysia (MOHE) for the financial support through research grant (Vot FRGS No.4F109 & RUG:04H27) for this research.

References

- [1] Chandran, P., N. C. Sacheti and A. K. Singh. 2005. Natural Convection Near a Vertical Plate with Ramped Wall Temperature. *Heat Mass Transfer*. 41: 459–464.
- [2] Narahari, M., and A. Ishak. 2011. Radiation Effects on Free Convection Flow Near a Moving Vertical Plate with Newtonian Heating. *Journal of Applied Science*. 11(7): 1096–1104.
- [3] Deka, R. K. and S. K. Das. 2011. Radiation Effects on Free Convection Flow Near a Vertical Plate with Ramped Wall Temperature. *Engineering*. 3: 1197–1206.
- [4] Rajesh, V. 2010. Radiation Effects on MHD Free Convection Flow Near a Vertical Plate with Ramped Wall Temperature. *Int. J. of Appl. Math and Mech*. 6(21): 60–77.
- [5] Rajesh, V. 2011. Chemical Reaction and Radiation Effects on the Transient MHD Free Convection Flow of Dissipative Fluid Past an Infinite Vertical Porous Plate with Ramped Wall Temperature. *Chemical Industry & Chemical Engineering Quarterly*. 17(2): 189–198.
- [6] Seth, G. S., Md. S. Ansari and R. Nandkeolyar. 2011. MHD Natural Convection Flow with Radiative Heat Transfer Past an Impulsively Moving Plate with Ramped Wall Temperature. *Heat Mass Transfer*. 47: 551–561.
- [7] Deka, R. K. and S. K. Das. 2011. Natural Convection Near a Vertical Plate in a Porous Medium with Ramped Wall Temperature. *International Journal of Mathematical Archive*. 2(7): 1080–1086.
- [8] Deka, D., and R. K. Deka. 2010. Natural Convection Near a Vertical Plate with Ramped Wall Temperature and Heat Source. *Far East Journal of Applied Mathematics*. 41(1): 21–32.
- [9] Darcy, H. 1971. *Les Fontaines Publique*. De La Ville De Dijon, Dalmont, Paris.
- [10] Varma, S. V. and M. S. Babu. 1985. A Brinkman for MHD Viscous Incompressible Flow Through a Porous Channel. *Indian Journal Pure Applied Mathematics*. 16(7): 796–806.
- [11] Nazar, R., N. Amin, D. Filip, and I. Pop. 2003. The Brinkman Model for the Mixed Convection Boundary Layer Flow Past a Horizontal Circular Cylinder in a Porous Medium. *International Journal of Heat and Mass Transfer*. 46(17): 3167–3178.
- [12] Singh, A. K. 2005. Convective Flow of Two Immiscible Viscous Fluids Using Brinkman Model. *Indian Journal of Pure Applied Physics*. 43: 415–421.
- [13] Fetecau, C., C. Fetecau and M. A. Imran. 2011. On Stokes problem for fluids of Brinkman type. *Math. Reports*. 13(63): 15–26.
- [14] Hsu, C. T. and P. Cheng. 1985. The Brinkman Model for Natural Convection About Semi Infinite Vertical Flat Plate in a Porous Medium. *International Journal of Heat and Mass Transfer*. 28(3): 683–697.