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Hybrid of ARIMA-GARCH Modeling in Rainfall Time Series

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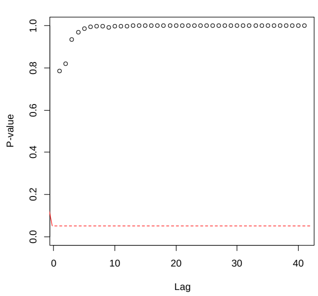
Graphical abstract



Abstract

The dependence structure of rainfall is usually very complex both in time and space. It is shown in this paper that the daily rainfall series of Ipoh and Alorsetar are affected by nonlinear characteristics of the variance often referred to as variance clustering or volatility, where large changes tend to follow large changes and small changes tend to follow small changes. In most empirical modeling of hydrological time series, the focus was on modeling and predicting the mean behavior of the time series through conventional methods of an Autoregressive Moving Average (ARMA) modeling proposed by the Box Jenkins methodology. The conventional models operate under the assumption that the series is stationary that is: constant mean and either constant variance or season-dependent variances, however, does not take into account the second order moment or conditional variance, but they form a good starting point for time series analysis. The residuals from preliminary ARIMA models derived from the daily rainfall time series were tested for ARCH behavior. The autocorrelation structure of the residuals and the squared residuals were inspected, the residuals are uncorrelated but the squared residuals show autocorrelation, the Ljung-Box test confirmed the results. McLeod-Li test and a test based on the Lagrange multiplier (LM) principle were applied to the squared residuals from ARIMA models. The results of these auxiliary tests show clear evidence to reject the null hypothesis of no ARCH effect. Hence indicates that GARCH modeling is necessary. Therefore the composite ARIMA-GARCH model captures the dynamics of the daily rainfall series in study areas more precisely. On the other hand, Seasonal ARIMA model became a suitable model for the monthly average rainfall series of the same locations treated.

Keywords: Volatility clustering; ARIMA; GARCH; autocorrelation function; McLeod-Li test; ARCH LM test; Ljung-Box test

Abstrak

Struktur kebersandaran hujan biasanya sangat kompleks dalam masa dan ruang. Di dalam kertas kerja ini, siri hujan harian di Ipoh dan Alorsetar dipengaruhi oleh ciri-ciri linear varians yang sering dirujuk sebagai kelompok varians atau turun naik, di mana perubahan besar lebih cenderung untuk mengikuti perubahan besar dan perubahan kecil lebih cenderung untuk mengikuti perubahan kecil. Dalam model empirikal siri masa hidrologi, tumpuan utama adalah pada pemodelan dan meramalkan tingkah laku min siri masa melalui kaedah konvensional Model Purata Autoregresi Bergerak (ARMA) yang dicadangkan oleh kaedah Box Jenkins. Model konvensional berfungsi dengan andaian bahawa siri hujan adalah pegun iaitu: min malar dan sama ada varians berterusan atau perbezaan bergantung kepada musim, bagaimanapun, ia tidak mengambil kira masa tertib kedua atau varians bersyarat, tetapi mereka membentuk satu titik permulaan yang baik untuk analisis siri masa. Ralat daripada model ARIMA dari masa siri hujan harian telah diuji untuk tingkah laku ARCH. Struktur autokorelasi ralat dan ralat kuasa dua telah diuji, berdasarkan ujian Ljung-Box didapati ralat tidak berkorelasi tetapi reja kuasa dua menunjukkan autokorelasi. Ujian McLeod-Li dan ujian berdasarkan prinsip pendarab Lagrange (LM) telah digunakan untuk reja kuasa dua daripada model ARIMA. Keputusan ujian menunjukkan bukti yang jelas untuk menolak hipotesis nol iaitu tiada kesan ARCH. Oleh itu, keputusan menunjukkan model GARCH diperlukan. Oleh yang sedemikian, komposit ARIMA-model GARCH digunakan untuk mengesan dinamik siri hujan harian di kawasan kajian dengan lebih tepat. Sebaliknya, model ARIMA bermusim menjadi model yang sesuai untuk purata hujan bulanan siri lokasi yang sama dirawat.

Kata kunci: Kelompok turun naik; ARIMA; GARCH; fungsi autokorelasi; ujian McLeod-Li; ujian ARCH LM; ujian Ljung-Box

1.0 INTRODUCTION

In most empirical modeling of hydrological time series, the focus was on modeling and predicting the mean behavior of the time series through conventional methods of an Autoregressive Moving Average (ARMA) modeling proposed by the Box Jenkins methodology. The conventional models operate under the assumption that the series is stationary that is: Zero mean and either constant variance or season-dependent variances, however, does not take into account the second order moment or conditional variance. In the field of time series there are many linear processes that can be modeled by the autoregressive moving average (ARMA) models. However, several time series does not exhibit a linear behavior. These processes cannot be well fitted by the common ARMA models. To adequately fit these non-linear time series, other more complicated models that have the ability to capture the dynamics of the series more precisely have to be taken into account.

Much work has been done to capture the observed statistics of rainfall in peninsular Malaysia, among others, (Zalina *et al*., 2002) applied Quantitative goodness of fit tests to determine the probability distribution most appropriate for describing annual maximum rainfall series in Peninsular Malaysia. Suhaila and AbdulAziz (2007) analyzed the statistical distribution of the daily rainfall amount. Suhaila *et al*. (2011) compare rainfall patterns between regions in Peninsular Malaysia via a functional data analysis technique. Hanaish *et al*. (2011) applied a Bartlett Lewis Rectangular Pulses Models to Rainfall data in Peninsular Malaysia. Wong *et al*. (2009) analyzed and quantified the spatial patterns and time-variability of rainfall in Peninsular Malaysia and obtained an overview of rainfall patterns. Juneng (2010) forecast the Malaysian winter monsoon rainfall anomalies using four statistical techniques. Kang and Fadhilah (2012) applied four methods for the homogeneity test to daily rainfall series of three locations in peninsular Malaysia, observed that all stations in those three areas are homogeneous and considered as useful when annual mean and annual maximum rainfall are used as testing variables. However, little or not have been done on the application of Heteroskedastic models to rainfall occurrence in Peninsular Malaysia. The aim of this paper is to model the temporal characteristics, that is, serial dependence and time varying variance (volatility) of daily rainfall series of Ipoh and Alorsetar.

The dependence structure of an observed rainfall is usually very complex both in time and space. It could be shown that rainfall data are affected by nonlinear characteristics of the variance often referred to as variance clustering or volatility, in which large changes tend to follow large changes, and small changes tend to follow small changes (Laux *et al*., 2011)**.** One of the most prominent tools for capturing such changing variance was the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH) models developed by Engle (1982), and later extended by Bollerslev (1986). Two important characteristics within rainfall time series are highly skewed or kurtotic distributions (Villarini *et al*., 2010) and volatility clustering, which can be captured by the GARCH family models. Volatility clustering can be thought of as clustering of the variance of the error term over time: if the regression error has a small variance in one period, its variance tends to be small in the next period, too. In other words, volatility clustering implies that the error exhibits time-varying heteroskedasticity that is, unconditional standard deviations are not constant.

The nature of rainfall in Peninsular Malaysia which consists of heavy rainfall in a short duration and light rainfall in a long duration can be well explained by some stochastic models. In this study, ARIMA/GARCH rainfall models are fitted to the data sets from Ipoh and Alorsetar stations in Peninsular Malaysia for the period from 1968 to 2003.

In a related study (Wang, 2005) verified the existence of conditional heteroskedasticity in the residual series from linear models fitted to the daily and monthly stream flow processes. Szolgayova (2011) investigated whether Heteroskedastic effect can be detected in selected time series of daily discharges in Slovakia. The results show that heteroskedasticity was present in all of the tested time series. However, the GARCH model was applicable only in one case. The EGARCH (1,1) model had to be used otherwise. So far, only few attempts on application of GARCH class models on discharge data were reported in the hydrological modeling literature.

The objective of this paper is to propose a hybrid model that can capture the temporal behavior (serial dependence) and time varying volatility in daily rainfall time series of Ipoh and Alorsetar.

2.0 MATERIALS AND METHODS

The time series can be viewed as the realization of a stochastic process that is, a series of random variables ordered in time. Many problems related to water resources and environmental systems among which rainfall is included deal with temporal data that need to be analyzed by means of time series analysis, which became a major tool in hydrology and meteorology. It is used for building mathematical models to describe hydrological data, forecast hydrologic events, detect trends, provide missing data, etc (Svetlíková *et al*., 2009).

2.1 ARIMA Modelling

The autoregressive integrated moving average ARIMA (*p, d, q*) model of the time series { y_t }, $t = 1,2, \dots \dots$ developed by Box & Jenkins (1976) is still very popular time series modelling. Defined as

$$\phi(B)\Delta^d y_t = \theta(B)\varepsilon_t \qquad (1)$$

where y_t and ε_t represent rainfall time series and random error terms at time t respectively. B is the backward shift operator. The $\phi(B)$ and $\theta(B)$ are of order p and q and defined as

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots\dots\dots - \phi_p B^p$$
$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots\dots\dots - \theta_q B^q$$

Where $\phi_1, \phi_2, \dots, \phi_p$ are the autoregressive coefficients that attempt to predict an output of a system based on the previous outputs and $\theta_1, \theta_2, \dots, \theta_q$ are the moving averages coefficients.

In modeling ARIMA (*p, d, q*) processes, the first step is to determine whether the time series is stationary or non-stationary. If it is non-stationary it has to be transformed into a stationary time series by applying the appropriate order of degree of differencing *d*. The appropriate values of autoregressive order *p* and moving average *q* is chosen by examining the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the time series.

2.2 Serial Dependance

The existence of serial correlation complicates statistical inference of time series analysis (Hong, 2006). It is therefore important to check serial correlation for the estimated residuals, which serves as a misspecification test for a linear dynamic regression model.

2.2.1 Ljung-Box Test

Ljung and Box (1978) proposed a *Q*-Test called Ljung–Box test which is commonly used in autoregressive integrated moving average (ARIMA) modeling. It is applied to the residuals of a fitted ARIMA model, not the original series, and in such applications the hypothesis actually being tested is that the residuals from the ARIMA model have no autocorrelation, or it performs a lack-of-fit hypothesis test for model misspecification, which is based on the *Q*-statistic given as:

$$Q = N(N+2)\sum_{j=1}^{L}\frac{\hat{\rho}_j^2}{(N-j)} \quad (2)$$

Where N = sample size, L = number of autocorrelation lags included in the statistic, and $\hat{\rho}_j^2$ is the squared sample autocorrelation at lag *j*. Under the H_0 (no serial correlation), the *Q*-test statistic is asymptotically χ^2 distributed. The p-values above 0.05 indicate the acceptance of the null hypothesis of model accuracy under 95% significant levels.

2.3 Test for Heteroskedasticity

2.3.1 Mcleod-Li Test

This test for ARCH effects was proposed by McLeod and Li (1983) It looks at the autocorrelation function of the squares of the pre whitened data and tests whether corr(x_t^2 , x_{t-j}^2) is non-zero for some *j*. The autocorrelation at lag*j*for the squared residuals { x_t^2 } is estimated by:

$$\hat{\varepsilon}(j) = \frac{\sum_{j=1}^{N}(x_t^2-\hat{\sigma}^2)(x_{t-j}^2-\hat{\sigma}^2)}{\sum_{t=1}^{N}(x_t^2-\hat{\sigma}^2)} \quad (3)$$

Where

$$\hat{\sigma}^2 = \sum_{t-1}^{N}\frac{x_t^2}{N}$$

Under the null hypothesis that x_t is an i.i.d process, McLeod and Li (1983) show that, for fixed L: $\sqrt{N}\hat{\varepsilon} = (\hat{\varepsilon}(1),\ldots,\hat{\varepsilon}(L))$ is asymptotically a multivariate unit normal. Consequently, for L sufficiently large, the usual Box-Ljung statistic

$$Q = N(N+2)\sum_{j=1}^{L}\frac{\hat{\varepsilon}_j^2}{(N-j)} \quad (4)$$

is asymptotically $\chi^2(L)$ under the $H_0$of a linear generating mechanism for the data. Typically L is taken to be around 20 (Ashley and Patterson, 2001).

2.3.2 ARCH LM Test

A methodology to test for the ARCH effect using the Lagrange multiplier test was proposed by Engle (1982). This procedure is as follows: Obtain the squares of the residual from fitted model $\hat{\varepsilon}^2$ and regress them on a constant and *q* lagged values:

$$\hat{\varepsilon}_t^2 = \hat{\alpha}_0 + \sum_{i=1}^{q}\hat{\alpha}_i\hat{\varepsilon}_{t-i}^2 \quad (5)$$

where$q$ is the length of ARCH lags. The null hypothesis is that, in the absence of ARCH components, we have $\alpha_i = 0$ for all i = 1, 2, …q. The alternative hypothesis is that, in the presence of ARCH components, at least one of the estimated α_i coefficients must be significant. In a sample of T residuals under the null hypothesis of no ARCH errors, the test statistic TR^2 follows the χ^2 distribution with *q* degrees of freedom. If TR^2 is greater than the Chi-square table value, we reject the null hypothesis and conclude there is an ARCH effect in the ARMA model. If TR^2 is smaller than the χ^2 table value, we do not reject the null hypothesis.

2.4 GARCH Modelling

The ARIMA(*p*,*d*,*q*) model cannot capture the heteroskedastic effects of a time series process, typically observed in the form of high kurtosis, or clustering of volatilities, and the leverage effect. Engle (1982) introduced the Autoregressive Conditional Heteroskedastic (ARCH) model, later generalized by Bollerslev (1986), named Generalized Autoregressive Conditional Heteroskedastic model (GARCH). The term "conditional" implies the level of association on the past sequence of observations and the "autoregressive" describes the feedback mechanism that incorporates past observations into the present (Laux *et al*., 2011).

The variance equation of the GARCH(*p*,*q*) model can be expressed as;

$$\varepsilon_t = z_t\sigma_t \quad (6)$$

$$z_t \sim \psi_\tau \textbf{(0, 1)}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2$$

$$= \omega + \alpha(\mathrm{B})\varepsilon_{t-i}^2 + \beta(\mathrm{B})\sigma_{t-1}^2 \quad (7)$$

where ψ_τ (0, 1) is the probability density function of the innovations or residuals with zero mean and unit variance.

Optionally, τ are additional distributional parameters to describe the skew and the shape of the distribution. If all the coefficient β is zero the GARCH model is reduced to the ARCH model. Similar to ARMA models a GARCH specification often leads to a more parsimonious representation of the temporal dependencies and thus provides a similar added flexibility over the linear ARCH model when parameterizing the conditional variance. Bolerslev (1986) has shown that the GARCH(*p*, *q*) process is wide-sense stationary if the following conditions hold:

1. $E(\varepsilon_t) = 0$
2. $\mathrm{var}(\varepsilon_t) = \dfrac{\omega}{(1-\alpha(1)-\beta(1))}$
3. $\mathrm{cov}(\varepsilon_t, \varepsilon_s), t \neq s$ if and only if $\alpha(1)+\beta(1)<1$

In most applications, the simple GARCH(1, 1) model has been found to provide a good representation of a wide variety of volatility processes (Bollerslev *et al*., 1992).

3.0 RESULTS AND DISCUSSION

The importance of first modelling the autoregressive integrated moving average (ARIMA) models in rainfall modeling is mainly for their use in explaining whether there is a nonlinear mechanism in the data generating processes that can well be explained by GARCH class models. The generalized autoregressive conditional Heteroskedastic model is a nonlinear model derived from a residual series of an ARIMA model. In order to check and modeled the behavior of rainfall in the study areas, the daily and monthly average rainfall data sets of Ipoh and Alorsetar for the period 01/01/1968 to 31/12/2003 were used. The monthly series were obtained from daily data by taking the average of daily rainfall over a month.

The basic approach to fit a suitable ARIMA model to time series, is to transform a non stationary time series to stationary time series, in this case, the time series(s) were decomposed, the seasonal effects were calculated and subtracted from the series rendering the series free of seasonality. After differencing, the series(s) appears to be quite stable over time. The orders p and q of the ARIMA models were identified and estimated for both series following Box and Jenkins methodology. Table 1 displays the results of fitted ARIMA models. The models were selected based on information criteria.

Table 1 Results for estimated model parameters: ARIMA (p, d, q) for daily rainfall and ARIMA (p, d, q)(P, D, Q)[12] for the monthly average rainfall of Ipoh and Alor Setar

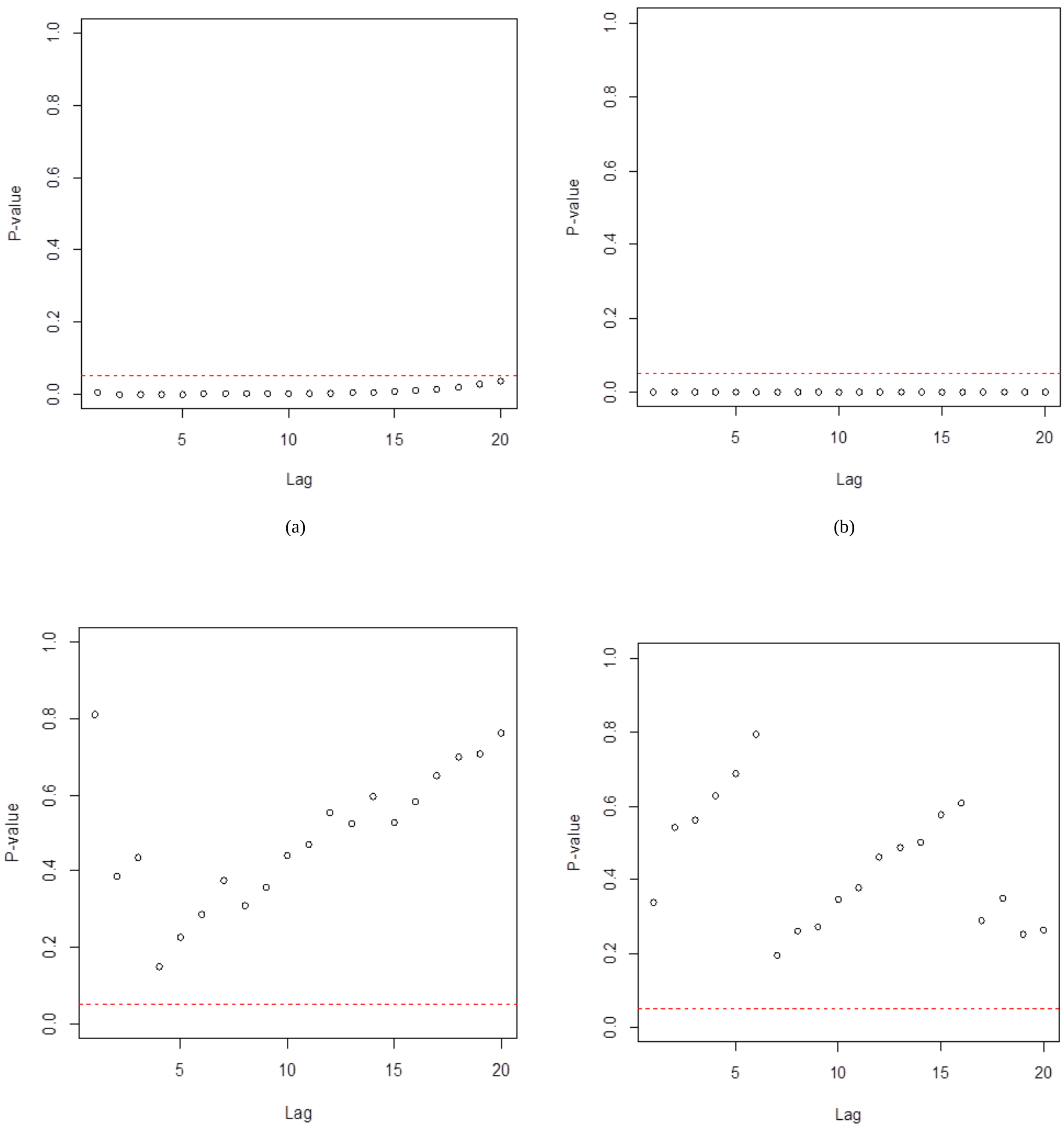
	Ipoh **ARIMA (2, 1 2)**		**Alor Setar** **ARIMA (3, 1, 1)**	
Coefficients	Estimates	S.E	Estimate	S.E
ar1	0.6369	0.0098	0.1075	
ar2	0.0145	0.0087	0.0214	
ar3	-	-	0.0188	
ma1	1.5787	0.0047	-0.9717	
ma2	-0.5804	0.0044	-	
	ARIMA (0, 0, 2)(1, 0, 2)[12]		ARIMA(1, 0, 2)(0, 0, 2)[12]	
Coefficients	Estimates	S.E	Estimate	S.E
ar1	-	-	0.8808	0.0885
sar1	0.4996	0.1008	-	-
ma1	-0.8912	0.0467	-1.8691	0.0860
ma2	-0.0809	0.0461	0.8719	0.0855
sma1	-0.5614	0.1085	-0.0978	0.0499
sma2	-0.1580	0.0511	-0.0733	0.0542

The autocorrelation function of the residuals from the fitted models seems to be uncorrelated and the Ljung-Box Q-test results given in Table 2 confirms the adequacy of the models. This shows that the seasonal ARIMA models can adequately capture the behavior of the data generating process for the monthly series(s).

Table 2 Ljung-Box Q-test results for ARIMA (p, d, q) models

		ARIMA (2, 1, 2)-	**ARIMA (3, 1, 1)-**		
Residuals		Statistic	p value	Statistic	p-value
	Up to lag 10	9.3006	0.1574	5.4503	0.4875
	Up to lag 15	12.6054	0.3199	7.5319	0.7545
	Up to lag 20	18.2709	0.3083	9.5371	0.8897
Squared Residuals	Up to lag 10	26.6501	0.000168	42.0539	1.794e-07
	Up to lag 15	32.0267	0.000755	44.3489	6.313e-06
	Up to lag 20	32.8587	0.007715	48.9038	3.422e-05

However, the autocorrelation function of the squares of the residuals for the daily series(s) shows autocorrelation. The Ljung-Box Q-test given in Table 2 also confirms the results. This is an indication of ARCH effect in the daily series. Formally, McLeod-Li test and a test based on the Lagrange multiplier (LM) principle were applied to the square residuals of the fitted models. The results of the McLeod-Li test for ARCH effect is given in Figure 1 (a & b), clear evidence to reject the null hypothesis of no ARCH effect was established for the daily series(s), and accepted for the monthly series(s) as shown in (c &d). The ARCH LM test results in Table 3 also reject the hypothesis of no ARCH effect. Hence indicates that GARCH modeling is necessary.



(a) (b) (c) (d)

Figure 1 McLeod-Li test for the residuals from (a) ARIMA (2, 1, 2), (b) ARIMA (3, 1, 1), (c) ARIMA(0,0,2)(1,0,2)[12] and (d) ARIMA(1,0,2)(0,0,2)[12] models

The ARIMA-GARCH model is a combination of two models: (1) ARIMA model that take into account the mean behavior of a time series and (2) GARCH model which is used to model the variance behavior (ARCH effect). Using the residual series from the fitted ARIMA models, suitable GARCH models were built. The results of the combined models are given in Table 4.

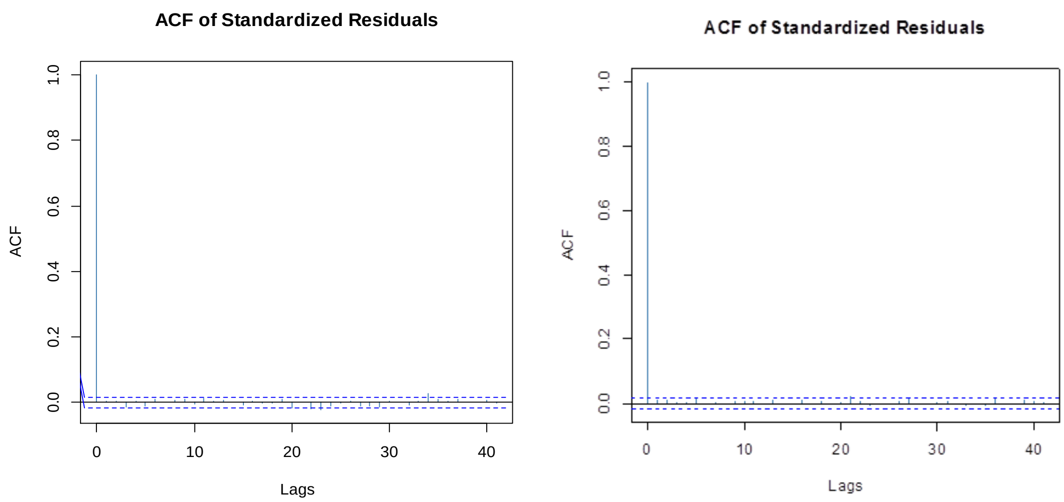
Moreover, Figures 2(a&b) and 3(a&b) shows the resulting standardized autocorrelation functions of residuals and square residuals from the fitted ARIMA-GARCH models for the observed daily rainfall series of Ipoh and Alorsetar respectively. Both autocorrelation functions for the standardized residuals and standardized squared residuals of the ARIMA-GARCH models exhibits no serial dependence.

Table 3 ARCH LM test results for ARIMA models

		ARIMA (2, 1, 2)		ARIMA (3, 1, 1)
Statistic	**p-value**	**Statistic**	**p-value**	
Residuals				
Up to lag 10	25.1211	0.0051	39.4692.101e-05	
Up to lag 15	29.7672	0.0128	41.386	0.0003
Up to lag 20	30.5759	0.0310	45.236	0.0010
Squared Residuals				
Up to lag 10	1.9495	0.9967	1.383	0.9993
Up to lag 15	2.6618	0.9998	1.651	1
Up to lag 20	3.3169	1	1.8	1

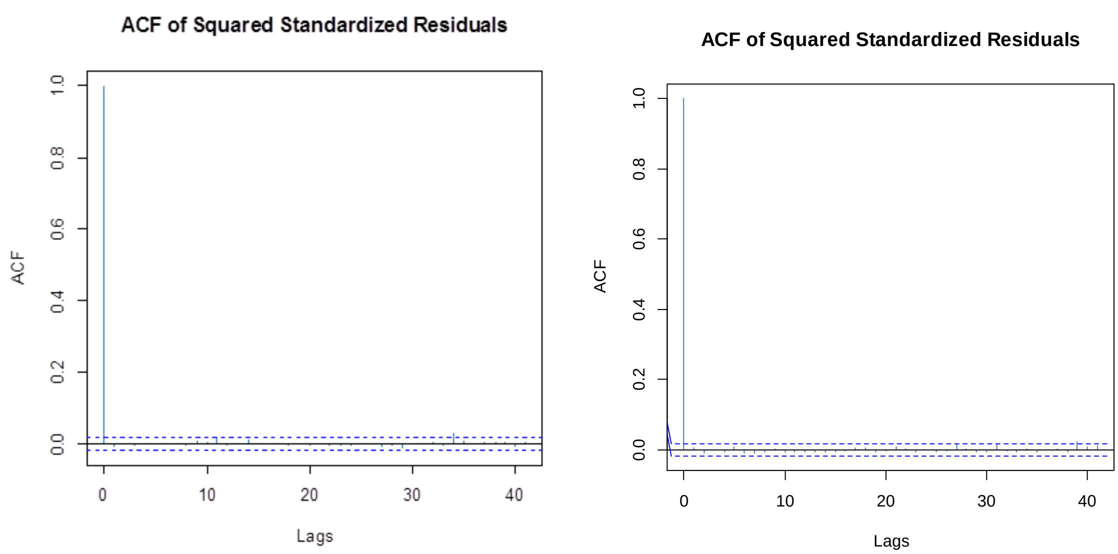
Figure 4 The results of the estimated ARIMA-GARCH model for Ipoh and Alor Setar

	Ipoh			Alor Setar		
Parameter	**Estimate**	**Std. Error**	**t value**	**Estimate**	**Std. Error**	**t value**
μ	0.141677	0.116191	1.219	0.013686	0.104685	0.131
ω	65.97217	10.42572	6.328	65.97217	10.42572	6.328
α	0.053076	0.008384	6.331	0.047572	0.004666	10.196
β	0.587048	0.062261	9.429	0.892369	0.010182	87.645



(a) (b)

Figure 2 Autocorrelation function for standardized residuals from (a) ARIMA(2, 1, 2)-GARCH(1, 1) and (b) ARIMA (3, 1, 1)-GARCH(1, 1) model



(a) (b)

Figure 3 (a) and (b) Autocorrelation function for standardized squared residuals from ARIMA(2, 1, 2)-GARCH(1, 1) and ARIMA (3, 1, 1)-GARCH(1, 1) models

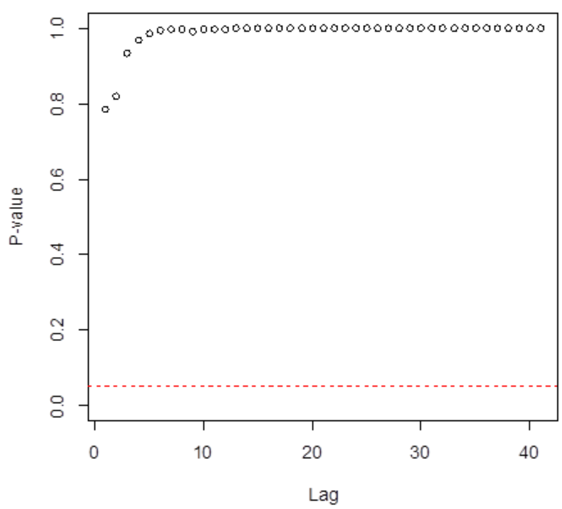
The Ljung-Box Q-test is applied to the residuals and squared residuals of the fitted ARIMA-GARCH models, the results are presented in Table 5.

Table 5 Ljung-Box Q-test results for ARIMA-GARCH model's goodness of fit

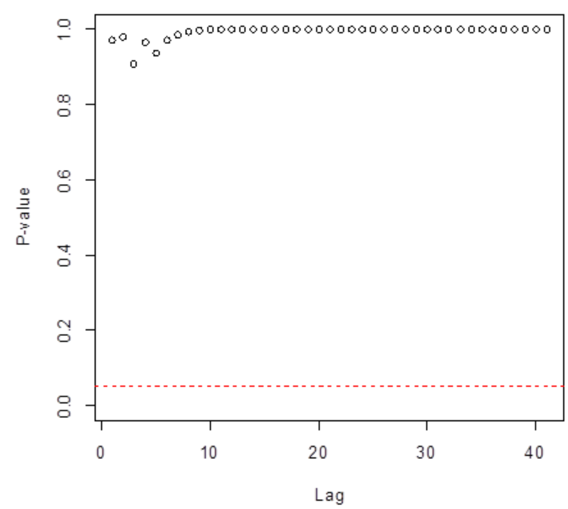
		ARIMA (2, 1, 2)-GARCH (1, 1)		**ARIMA (3, 1, 2)-GARCH (1, 1)**	
Std. Res		Statistic	p-value	Statistic	p-value
	Up to lag 10	8.78889	0.55225	12.78526	0.23593
	Up to lag 15	13.0454	0.59879	16.12719	0.37365
	Up to lag 20	19.0083	0.52129	19.23229	0.50678
Std. Sqr. Res	Up to lag 10	3.75911	0.95756	5.92739	0.82133
	Up to lag 15	10.1500	0.81020	8.94127	0.88057
	Up to lag 20	11.2073	0.94067	10.7504	0.95244

Std. Res = Standardized Residuals; Std. Sqr. Res = Standardized Squared Residuals

According to the results, all models were specified correctly. Also McLeod-Li test for ARCH effect was applied to the residuals of the fitted ARIMA-GARCH models shown in Figure 4(a&b) visual inspection clearly shows that there is no any heteroskedastic effect left in both series, therefore, the models fits the daily rainfall data sets well.



(a)



(b)

Figure 4 (a) and (b) McLeod-Li test for the residuals from fitted ARIMA (2, 1, 2)-GARCH(1,1) and ARIMA (3, 1, 1)-GARCH(1,1) models respectively

■4.0 CONCLUSION

It is shown in this paper that daily rainfall series of Ipoh and Alorsetar are affected by nonlinear characteristics of the variance often referred to as variance clustering or volatility, in which large changes often follow large changes, and small changes often follow small changes. In this work, a hybrid ARIMA-GARCH models were developed to take into accounts both the serial dependence and volatility in the daily rainfall series of Ipoh and Alorsetar. On the other hand, seasonal ARIMA models were developed and proved to be adequate for modelingthe monthly average rainfall time series of the stations considered. Thesemodelscan provideinformationthat will helpdecisionmakersin understandingthedynamicbehaviorof therainfall of Ipoh and Alorster for Agricultural and other hydrological issues. The methodology in the paper presupposes that trend analysis is not a factor in the modelling process; this puts a limit on the scope of the study. On the other hand incorporating it does not make the problem amendable. Modeling the mean and volatility of rainfall time series is not satisfactory to capture the whole mechanism that generates the rainfall due to the complex nature of its dependence structure. Therefore, further research based on long memory volatility modeling is suggested for better results.

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