Jurnal Teknologi, 49(D) Dis. 2008: 45–60

SERVOMOTOR CONTROL USING DIRECT DIGITAL CONTROL AND STATE-SPACE TECHNIQUE

MOHD FUA'AD RAHMAT[1] & MOHD SYAKIRIN RAMLI[2]

Abstract. This paper presents servomotor position control using direct digital control and state-space technique. A mathematical model of the system is derived and verified by SIMULINK/MATLAB. Full-state feedback controller both with and without integral control are proposed for the controller structures. The performances between these two structures are analyzed. Simulation results are first given. Next, a program is developed via Microsoft Visual Basic 6.0 (VB6.0) and interfaced with the servomotor for experimental evaluation and validation.

Keywords: Full-state feedback control; Bass and Gura's approach; graphical user interface; state-space; servomotor; direct digital control

Abstrak. Kertas kerja penyelidikan ini menyentuh mengenai teknik keadaan ruang dan kawalan digital terus dalam aplikasi sistem kawalan kedudukan motor servo. Permodelan matematik sistem kawalan tersebut diterbitkan dan model tersebut ditentusahkan dengan menggunakan perisian MATLAB/SIMULINK. Pengawal suap balik keadaan tanpa kawalan kamiran dan bersama kawalan kamiran dicadangkan untuk struktur pengawal. Prestasi kedua-dua pengawal dianalisis dan keputusan simulasi disertakan. Seterusnya atur cara komputer direka bentuk dengan menggunakan Microsoft Visual Basic 6.0 dan diantaramukakan dengan motor servo untuk tujuan penilaian dan penentusahan uji kaji.

Kata kunci: Kawalan suap balik keadaan; pendekatan Bass dan Gura's; antaramuka pengguna grafik; keadaan ruang; motor servo; kawalan digital terus

1.0 INTRODUCTION

Automatic control is one of today's most significant areas of science and technology. This attributed to the fact that automation is linked with the development of almost every form of technology. Direct digital control (DDC) is one form of the automatic control where DDC is termed as using a digital computer to interface directly with a plant or system as the control device [1]. The disparity between DDC and indirect digital control or supervisory control is that it does not require any additional hardware

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[1] Control & Instrumentation Department, Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 UTM Skudai, Johor Bahru, Malaysia
Tel: 07-5535900, Fax: 07-5566272. E-mail: fuaad@fke.utm.my

[2] Faculty of Electrical and Electronics Engineering, University College of Engineering and Technical Malaysia, 25200 Gambang, Pahang Darul Makmur
Tel: 017-9710331. E-mail: syakirin@ump.edu.my

to implement the controller. Everything from controller's algorithms or structures in terms of codes and scripts can be programmed using computer software.

The objectives of this project is to create and develop a graphical user interface (GUI) using Microsoft Visual Basic 6.0 that implements DDC and state-space technique in controlling servomotor shaft position. Controller design structure will use full-state feedback with and without integral control [2] where pole-placement design via Bass and Gura's approach [3] is proposed. The full-state feedback controller via pole-placement is chosen since it has the best performance compared to other controllers in terms of oscillation and settling time [4]. Moreover, the pole-placement design could also handle systems with time-varying state space representation [5], or systems with multiple operating conditions [6], as well as systems with multi-input-multi-output (MIMO) signals requirement [7]. Normally, controller design for linear time-varying differential systems is generally a difficult problem, because of the fundamental problems related to the analysis of such systems [5]. For simplicity, the servomotor system will be analyzed as a linear time-invariant (LTI) system, with only single-input and single output condition. This is due to the fact that the method has the properties of the flexibility of shaping the dynamics of the closed-loop system to meet the desired specifications [8].

The paper is organized as follows. In section 2, a mathematical model of servomotor is discussed where the system is considered as a second order system. In section 3, state-space modeling for continuous and discrete time system is presented. Subsequently in section 4, full-state feedback controller using pole-placement design is derived. Three case studies are provided in section 5 where simulation and experimental analysis are thoroughly discussed. Conclusions are given in section 6.

2.0 MODEL OF SERVOMOTOR SYSTEM

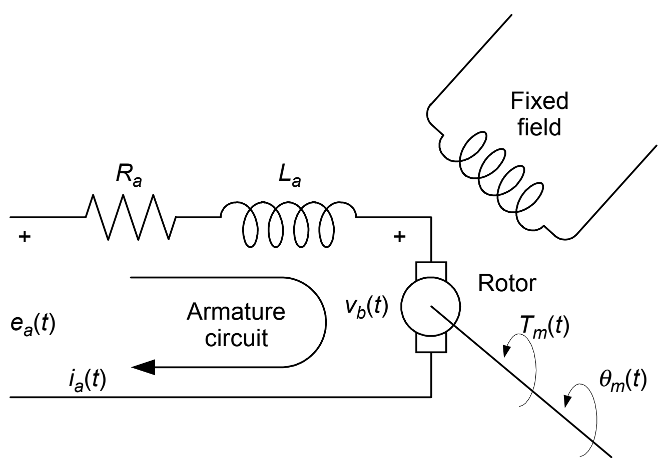


Figure 1 Schematic of servomotor system

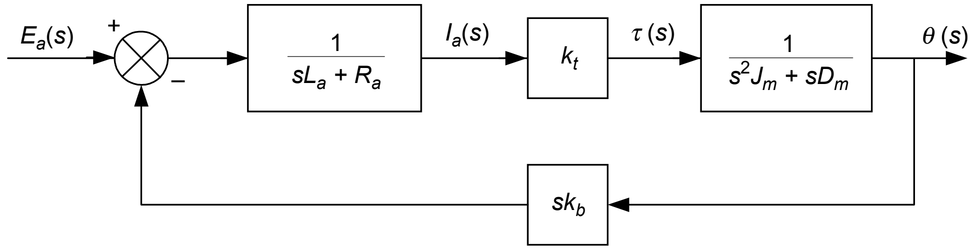


Figure 2 Time domain block diagram representation of servomotor system

Figure 1 shows the schematic of the servo motor, meanwhile Figure 2 is the equivalent block diagram [2]. Based on the block diagram of Figure 2, by ignoring the armature inductance of the system, the open loop transfer function for a second order armature controlled servomotor system is given as follows:

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\left(\dfrac{k_t}{R_a J_m}\right)}{s^2 + s\dfrac{1}{J_m}\left(D_m + \dfrac{k_t k_b}{R_a}\right)} \tag{2.1}$$

where

J_m	= equivalent inertia by the motor	$= 30 \times 10^{-6}$ kgm^2
k_b	= back-emf constant	$= 60 \times 10^{-3}$ Vsrad^{-1}
k_t	= motor torque constant	$= 17 \times 10^{-3}$ NmA^{-1}
R_a	= armature resistance	= 3.2 Ω
D_m	= equivalent viscous density by the motor	= small (cannot be quoted)

The armature inductance is ignored since its value is comparatively small ($L_{m} \approx 8$ mH). By substituting all the parameters into the Equation (2.1), the open loop transfer function for the second order armature controlled servomotor system could be obtained as follows:

$$G(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{177.0833}{s^2 + 10.625s} \tag{2.2}$$

3.0 STATE-SPACE MODELING

3.1 Modeling in Continuous Time

Based on the block diagram of the servomotor as shown in Figure 2, the state variables for second order system are defined as

$x_1(t) = \theta_m(t)$ = angular displacement of the motor shaft

$x_2(t) = \dfrac{d\theta_m(t)}{dt}$ = angular velocity of the motor shaft

Meanwhile, the state input and state output for the second order system are defined as

$u(t) = e_a(t)$ = input signal into the servomotor

$y(t) = x_1(t)$ = output signal from the servomotor

Let

$$\dot{x}_1(t) = \frac{dx_1(t)}{dt} = \frac{d\theta_m(t)}{dt} = x_2(t) \tag{3.1}$$

Now let

$$\dot{x}_2(t) = \frac{dx_2(t)}{dt} = \frac{d^2\theta_m(t)}{dt^2} \tag{3.2}$$

By manipulating equation (2.1), it could be obtained that

$$\dot{x}_2(t) = -\left[\frac{1}{J_m}\left(D_m + \frac{k_t k_b}{R_a}\right)\right] x_2(t) + \frac{k_t}{R_a J_m} e_a(t) \tag{3.3}$$

Therefore, the state-space representation of servomotor in space matrix could be expressed in this form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{J_m}\left(D_m + \dfrac{k_t k_b}{R_a}\right) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{k_t}{R_a J_m} \end{bmatrix} e_a(t) \tag{3.4}$$

and

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{3.5}$$

3.2 Modeling in Discrete Time

From equation (3.4), it could be re-written into

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -M \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ N \end{bmatrix} e_a(t) \tag{3.6}$$

where M and N are defined as:

$$M = \frac{D_m R_a + k_b k_t}{J_m R_a} \tag{3.7}$$

$$N = \frac{k_t}{J_m R_a} \tag{3.8}$$

Let the dynamic of the linear continuous-time servomotor system be represented by the following state and output equations respectively:

$$\dot{x}(t) = Fx(t) + Gr(t) \tag{3.9}$$

$$y(t) = Cx(t) + Dr(t) \tag{3.10}$$

where

$$F = \begin{bmatrix} 0 & 1 \\ 0 & -M \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ N \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

The system matrix $A(T)$ and output matrix $B(T)$ for discrete time servomotor system can be easily determined as follows [9]:

$$A(T) = e^{Ft}\Big|_{t=T} = \left|L^{-1}(SI - F)^{-1}\right|_{t=T} \tag{3.11}$$

$$\begin{aligned} B(T) &= \int_0^T e^{Ft} G d\tau = \int_0^T \left|L^{-1}(SI - F)^{-1}\right|_{t=\tau} G d\tau \\ &= \int_0^T A(\tau) G d\tau \end{aligned} \tag{3.12}$$

Therefore, the state-space representation of servomotor in discrete time space matrix could be expressed in this form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{\left(1 - e^{(-MT)}\right)}{M} \\ 0 & e^{(-MT)} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} \left(\dfrac{N}{M}\right) T - \dfrac{N}{M^2}\left(1 + e^{(-MT)}\right) \\ \dfrac{N}{M}\left(1 - e^{(-MT)}\right) \end{bmatrix} e_a(k) \tag{3.13}$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{3.14}$$

4.0 FULL-STATE FEEDBACK CONTROLLER DESIGN

4.1 State Feedback without Integral Control

The concept of feed-backing all the state variables to the input of the system through a suitable feedback matrix in the control strategy is known as the full-state variable feedback control technique. Using this approach, the desired location of the closed-loop eigenvalues (poles) of the system will be specified. Thus, the aim is to design a feedback controller that will move some or all of the open-loop poles of the measured system to the desired closed-loop pole location as specified. Hence, this approach is often known as the *pole-placement control* design. In this paper, pole-placement technique via Bass and Gura's formula is proposed.

In order to perform the pole-placement design technique, the system must be a "completely state controllable" system. In other words, it must be possible to move all the of system's open-loop poles by state variable feedback, to any arbitrary closed-loop locations. Therefore, before designing the controller, a test has to be performed on the system matrix by checking its rank where the rank must be equal to the number of the column vector. Then it can be concluded that the system is completely state controllable. Otherwise, another controller design has to be performed.

Figure 3 shows detailed block diagram of a system with state feedback control.

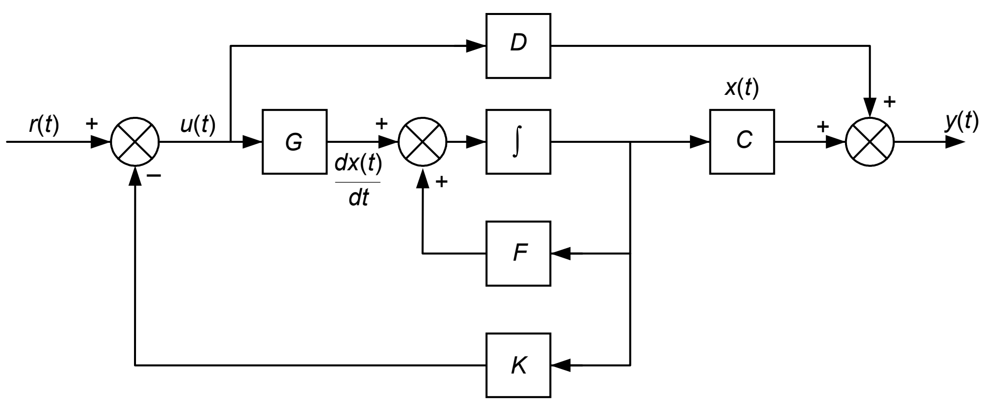


Figure 3 Detailed block diagram of a system with state feedback control

The general state space equation for the block diagram in the Figure 3 is derived as:

$$\dot{x}(t) = [F - GK]x(t) + Gr(t) \tag{4.1}$$

Bass and Gura's formula to determine the state feedback gain matrix is given as follows [3]:

$$K^T = \left[W^T S^T\right]^{-1}(a - \tilde{a}) \tag{4.2}$$

where

K = State feedback gain matrix

S = $\begin{bmatrix} G & FG & \cdots & F_{n-1}G \end{bmatrix}$ = Controllability matrix

a = $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$ = Coefficients of the desired characteristic equation

$\tilde{a}$ = $\begin{bmatrix} \tilde{a}_1 \\ \tilde{a}_2 \\ \vdots \\ \tilde{a}_n \end{bmatrix}$ = Coefficients of the system characteristic equation

W = $\begin{bmatrix} 1 & \tilde{a}_1 & \cdots & \tilde{a}_{n-1} \\ 0 & 1 & \cdots & \tilde{a}_{n-2} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$

4.2 State Feedback with Integral Control

Normally, designing the state feedback controller by using only the pole-placement design will give one major disadvantage where a large steady-state error will be introduced. In order to compensate this problem, an integral control is added where it will eliminate the steady-state error in responding to a step input.

Figure 4 shows the block diagram of the system with the integral control added. Inside the dashed box is the state feedback controller which has been designed before. A feedback path from the output has been added to the error, e, which is fed forward to the controller via an integrator. The main function of adding an integrator is to increase the system type thus reduces the previous finite steady-state error to zero. Therefore, a design for zero steady state error for a step input can be obtained.

From the block diagram in Figure 4, the system matrix with integral control could be given as [2]:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_N(t) \end{bmatrix} = \begin{bmatrix} F & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_N(t) \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \tag{4.3}$$

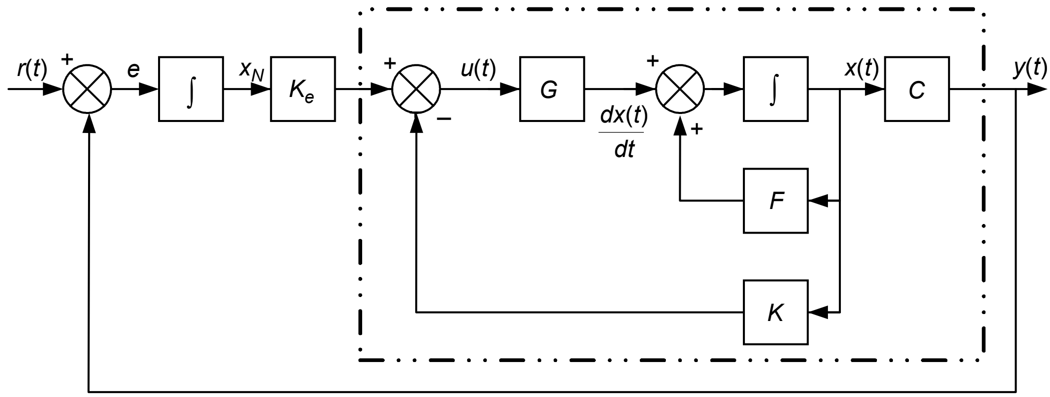


Figure 4 Block diagram of the system with state feedback and integral control

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix} \tag{4.4}$$

However, from Figure 4, we can realize that

$$u(t) = -Kx(t) + K_e x_N = -\begin{bmatrix} K & -K_e \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix} \tag{4.5}$$

Therefore, the final derivation for the system matrix with the integral control is as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_N(t) \end{bmatrix} = \begin{bmatrix} (F - GK) & GK_e \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_N(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \tag{4.6}$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_N \end{bmatrix} \tag{4.7}$$

In order to implement the Bass and Gura's formula to find the state feedback gain matrix for the state-space system with integral control, some equation modification has to be performed.

Let

$$F' = \begin{bmatrix} F & 0 \\ -C & 0 \end{bmatrix} \quad \text{and} \quad G' = \begin{bmatrix} G \\ 0 \end{bmatrix} \qquad (4.8)\ \&\ (4.9)$$

The controllability matrix, S', can be obtained using the F' and G' above.

Now, the state feedback gain matrix can be obtained by rewriting equation (4.2) as follows:

$$K'^{T} = \left[W^{T} S'^{T}\right]^{-1} (a - \tilde{a}) \tag{4.10}$$

Therefore,

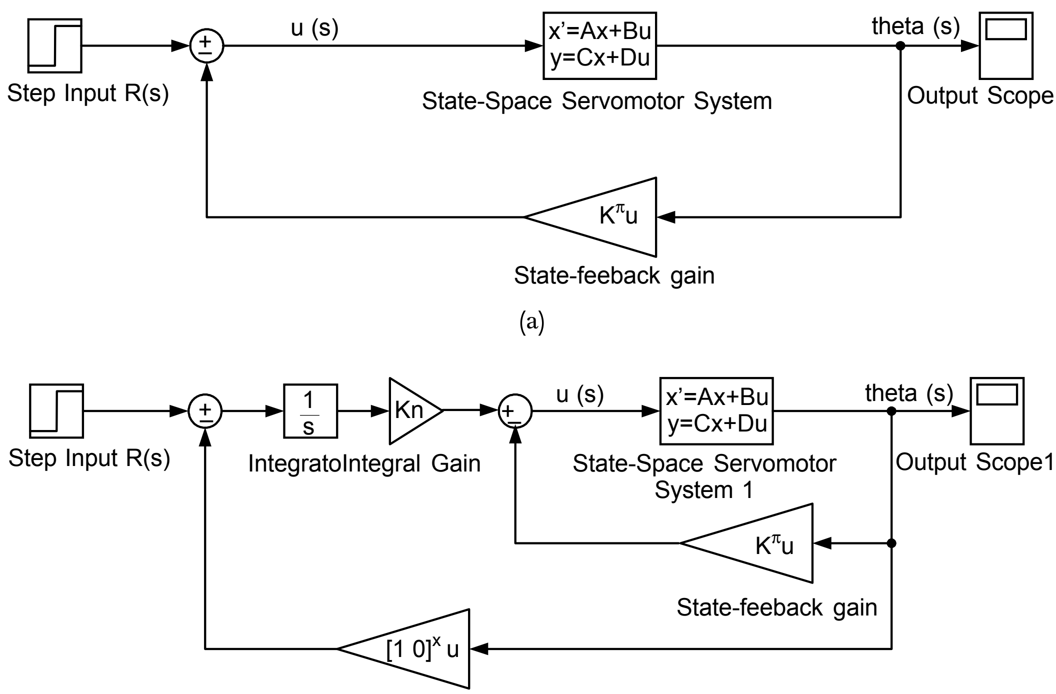
$$K = \left[K_1' \quad K_2' \quad \cdots \quad K_{n-1}'\right] \tag{4.11}$$

and the gain integral can be obtained as:

$$K_e = -K_n' \tag{4.12}$$

5.0 SIMULATION AND EXPERIMENTAL ANALYSIS

Figure 5 shows the continuous time hardware realization for the servomotor position control system with the: (a) state feedback controller without integral control (b) state feedback controller with integral control. Simulations are performed for both controllers' structure where a unit step input (5-volts step) signal is used as the reference signal. To accomplish one of the design requirements, it is desired that the output signal follows the given reference signal. Through simulation, the mathematical modeling for the servomotor is verified and the performances for both controllers' structures are analyzed.



(b)

Figure 5 State feedback controller (a) without integral control (b) with integral control

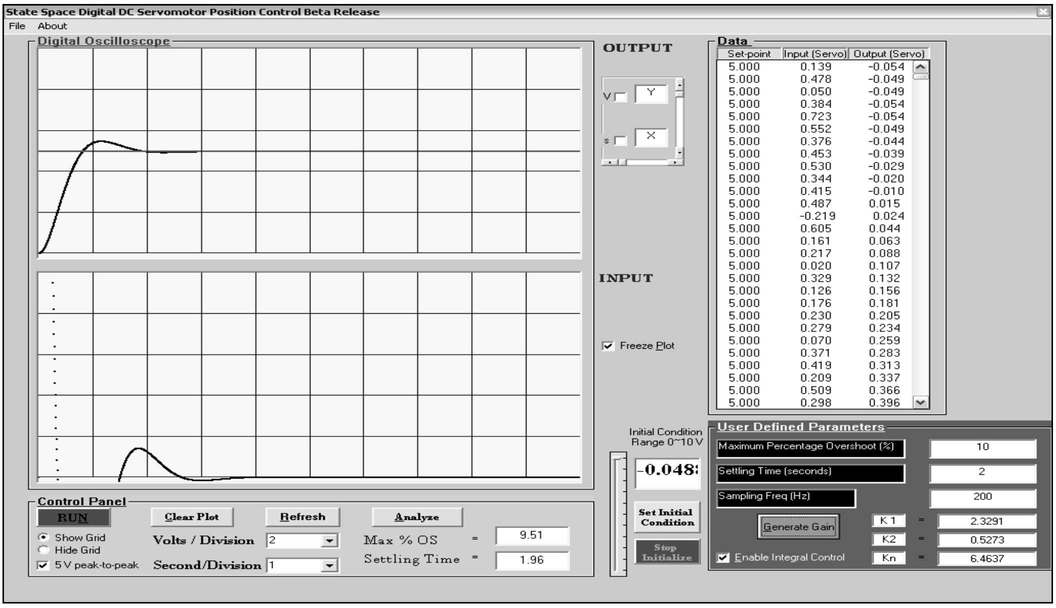


Figure 6 Input and output plot from GUI of servomotor position control

Figures 6 and 7 show the developed GUIs for the servomotor position control system where the state feedback controller structures are implemented. Basically, users will have the ability to specify the parameters for their desired maximum percentage overshoot and settling time. Thus this program provides more freedom and enables users to control the servomotor so that it will perform just the way they want it to be. Besides that, users are also able to observe the responses of all signals online since they are plotted immediately as the system starts running. One example on how these signals are plotted is shown in Figure 7.

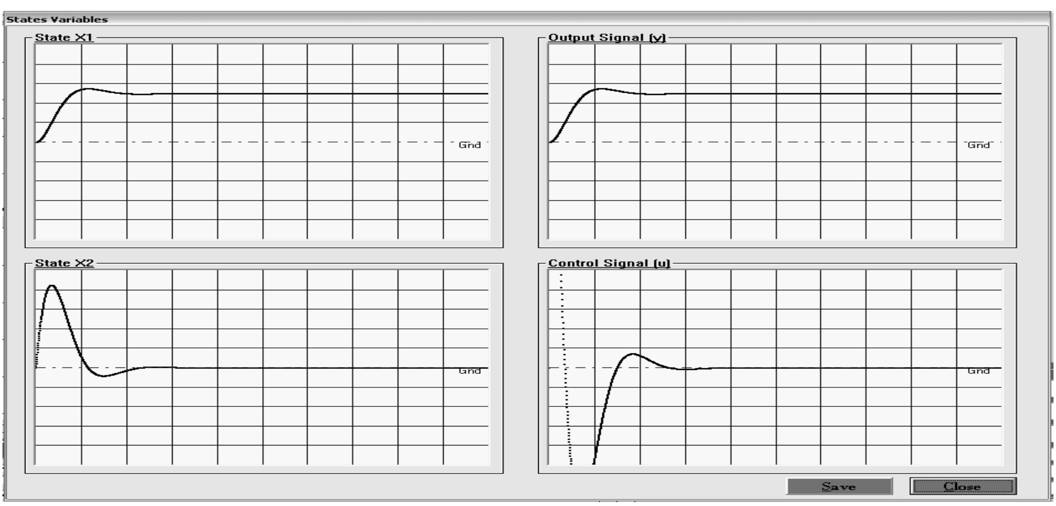


Figure 7 State variable responses from GUI of servomotor position control

5.1 Simulation and Experimental Results

In order to analyze the performances of the proposed controllers, the system is simulated using MATLAB/SIMULINK. Next, a computer program is developed using Microsoft Visual Basic 6.0 for experiment and results validation. For this paper, there are three different case studies presented for discussion.

(a) State feedback controller with integral control

Case study 1: 10% maximum overshoot, 2 seconds settling time, 0 volt initial condition

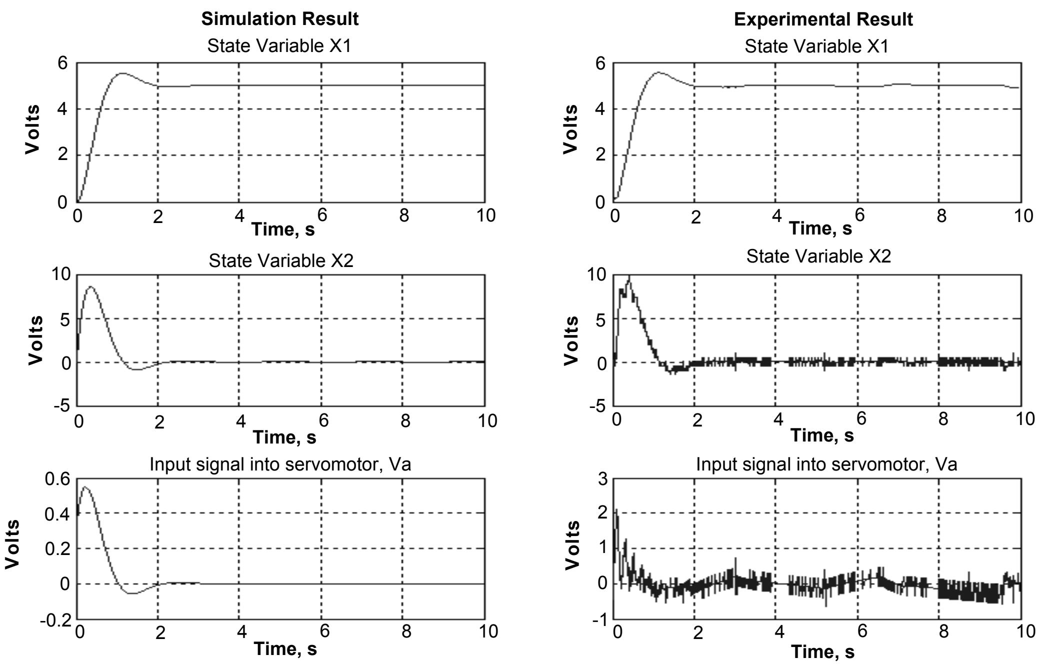


Figure 8 Comparison between simulation and experimental results for case study 1

For the first case study, it is desired to obtain the output signal with 10 percent maximum overshoot and 2 seconds settling time. In this case, the initial condition is set to be 0 volt. Plots on the left-hand side of Figure 8 are the simulation results while on the right-hand side shows the plots of experimental result. Notice that the x_1 signal from experiment approximately resembles with the x_1 signal from simulation. The same observation can be seen for x_2 and the input signals. However, there are some noises associated with the signals from the experiment.

Case study 2: 15% maximum overshoot, 3 seconds settling time, 9 volts initial condition

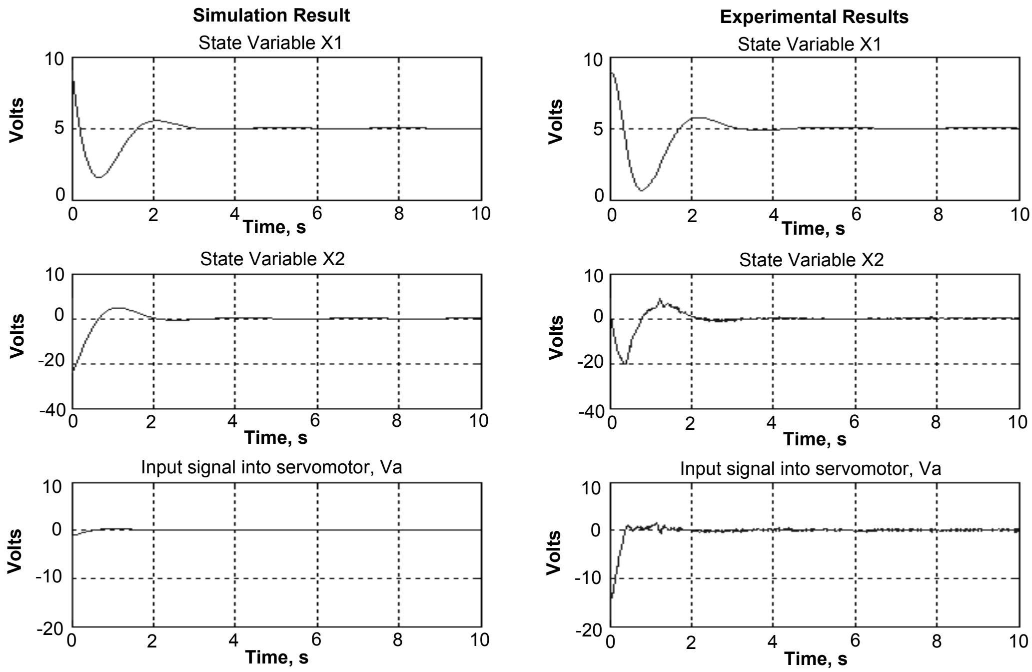


Figure 9 Comparison between simulation and experimental results for case study 2

In the second case study, it is desired to obtain the response signal with 15 percent maximum overshoot and 3 seconds settling time. In order to observe how the system responds to initial condition higher than the reference value, the initial condition is set to 9 volts. The same observation as that in the previous case is obtained where the x_1 signal from experiment approximately resembles the signal from the simulation. However, it can be observed that time delays and noises also exist in the experimental result as can be clearly seen in x_2 and the input signals.

Case study 3: 20% maximum overshoot, 4 seconds settling time, 3 volts initial condition

In the third case study, it is desired to obtain the response signal with 20 percent maximum overshoot and 4 seconds settling time. In this case study, the initial condition is set to 3 volts. By comparing the simulation and experimental results, it can be observed that the x_1 signal from experiment is almost identical to the simulation result but has a small "hump". As observed in previous cases, the x_2 and input signals also resemble the simulation result with the existence of noise.

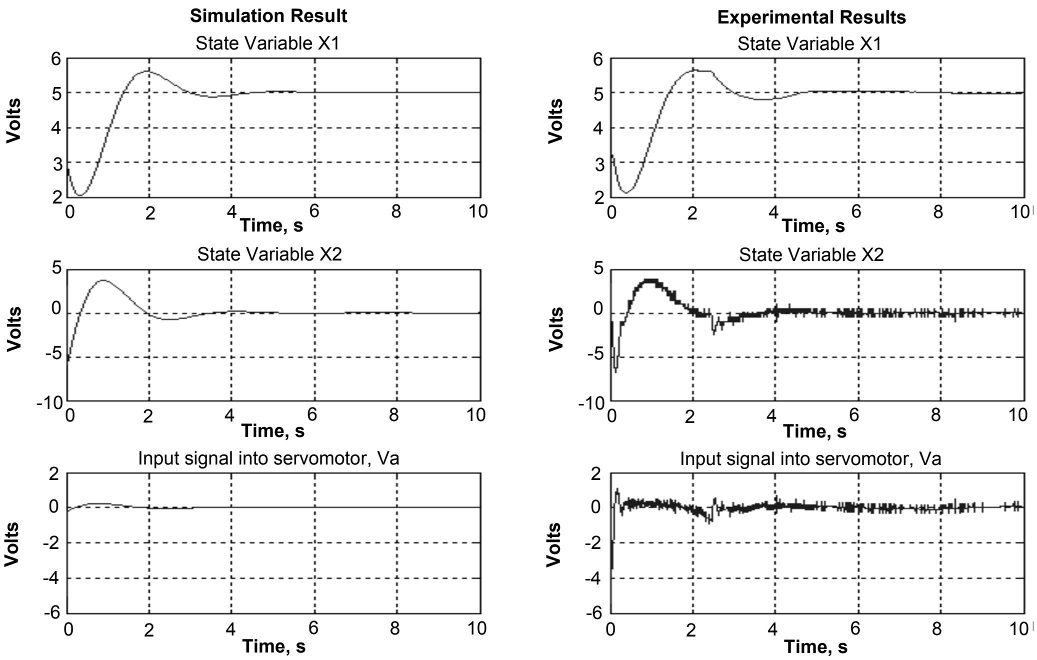


Figure 10 Comparison between simulation and experimental results for case study 3

(b) State feedback controller without integral control

Case study 1: 10% maximum overshoot, 2 seconds settling time, 0 volt initial condition

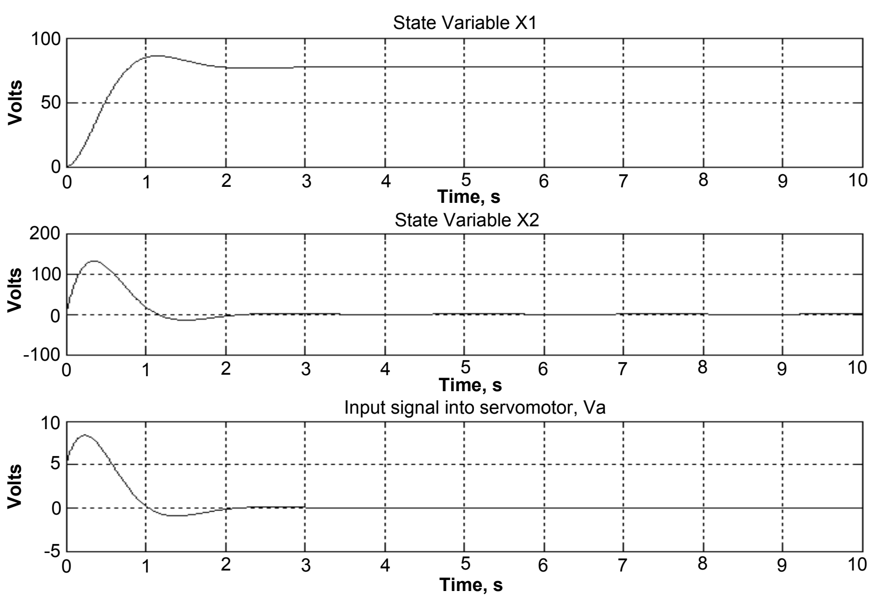


Figure 11 Simulation result for case study 1

In the first case study, it is desired to obtain the response signal with 10 percent maximum overshoot and 2 seconds settling time with 0 volt initial condition. By referring to x_1 signal, it can be observed that the steady-state error of the output response is approximately 72.36 volts.

Case study 2: 15% maximum overshoot, 3 seconds settling time, 9 volts initial condition

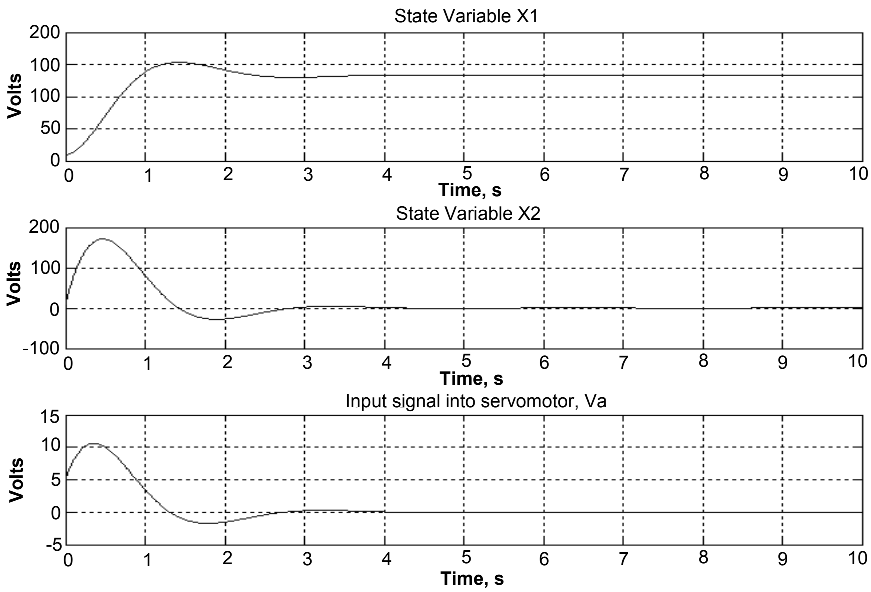


Figure 12 Simulation result for case study 2

For the second case study, it is desired to obtain the response signal with 15 percent maximum overshoot and 3 seconds settling time. The initial condition is set to 9 volts. The same observation can be obtained for x_1 signal where large steady-state error (approximately 128.1 volts) is produced by the output response.

Case study 3: 20% maximum overshoot, 4 seconds settling time, 3 volts initial condition

In the third case study, it is desired to obtain the response signal with 20 percent maximum overshoot and 4 seconds settling time. For this case study, the initial condition is set to 3 volts. By referring to x_1 signal in the Figure 13, the steady-state error of the output response is approximated to be 179.1 volts.

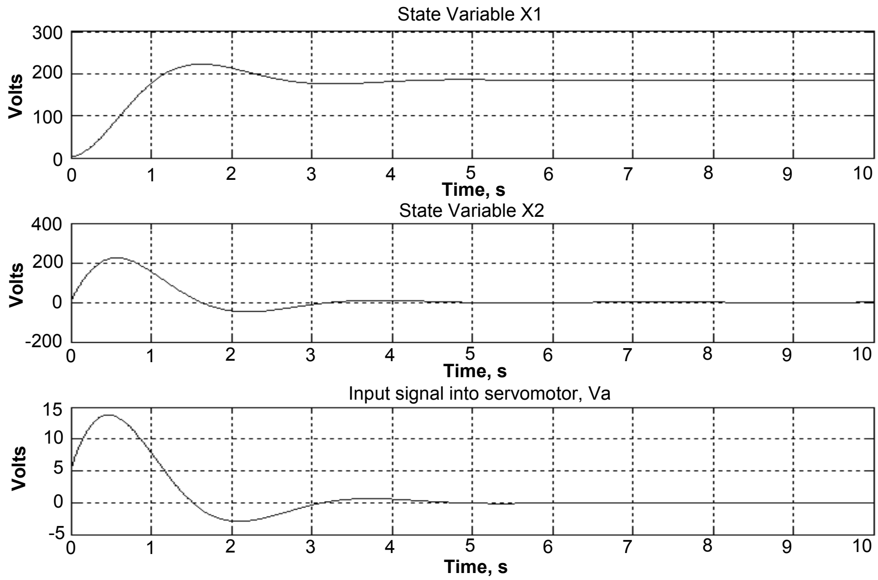


Figure 13 Simulation result for case study 3

5.2 Discussion

Figures 8 to 10 show the simulation and experimental results for the performances of three different case studies for state-feedback controller with the integral control. While Figures 11 to 13 show the performances of three different case studies for state-feedback controller without the integral control.

From Figures 8 to 10, it can be observed that the experimental results for all three cases equivalent with the simulation results. It can clearly be shown that the state variable x_1 signals (which are the desired output responses) from experiments is approximately same with the state variable x_1 signals from the simulation. The state variable x_2 and the input signals, on the contrary, resembled the simulation results but in the presence of acceptable range of noise. The existence of noise is practically due to the characteristics and dynamics of a real system.

Figures 11 to 13 show only the simulation results for state-feedback controller without the integral control. From these plots, it can be observed that the steady-state values were far off than the reference input (5 volts unit step input) thus lead to unacceptably large steady-state errors. In engineering point of view, by having very large steady-state error does reflect to instability of the designed system. Practically, the experimental procedures can not be performed using the state feedback controller design without the integral control due to signal limitation of the hardware components. Furthermore, expected instability system from simulation has already proven that there will not be possible to obtain the desired output response by experiment.

6.0 CONCLUSION

A graphical user interface (GUI) using Microsoft Visual Basic 6.0 is successfully developed where state-feedback controller with and without integral controller structures have been implemented. Through experiment, it can be observed that the experimental results for the state-feedback controller with integral control agreed with the simulation results. While, the simulation results for the state-feedback controller without the integral control show that this design leads to unacceptably large steady-state errors on the output responses (x_1). Therefore, experiment cannot be performed using the GUI to implement this controller structure.

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