

# Developing Pre-service Secondary Teachers' Skills of Using the Geometer's Sketchpad to Teach Mathematics through Lesson Study

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## Abstract

The purpose of this study was to develop pre-service secondary teachers' skills of using The Geometer's Sketchpad (GSP) to teach mathematics through Lesson Study (LS). GSP is a dynamic geometry software program for constructing and investigating mathematical objects that adds a powerful dimension to the teaching and learning of geometry and many other areas of mathematics. Lesson Study is a Japanese model of teacher professional development in which small groups of teachers collaboratively plan, teach and revise a lesson to improve the quality of their teaching as well as to enrich students' learning experiences. Twenty-three LS groups comprising 2 pre-service secondary teachers who attended a mathematics teaching methods course in a local public university were set up in four tutorial groups each consisting of five or six LS groups. This paper discusses how LS has helped to develop pre-service secondary teachers' skills of using GSP to teach the topic of 'Loci in Two Dimensions' in one of the LS groups. Analysis of their GSP sketches in the first, second and third lessons indicates that the participants of this LS group showed positive changes in their skills of using GSP to teach the topic.

*Keywords:* Pre-service secondary teachers; the Geometer's Sketchpad; lesson study; loci in two dimensions

## Abstrak

Tujuan kajian ini adalah untuk membangunkan kemahiran guru-guru sekolah menengah pra-perkhidmatan untuk menggunakan perisian Geometer's Sketchpad (GSP) bagi mengajar matematik melalui pendekatan Lesson Study (LS). GSP adalah satu perisian yang dinamik bagi membina dan mengkaji objek-objek matematik dengan berkesan. Ini telah memberikan satu dimensi baru yang amat berguna dalam pengajaran dan pembelajaran geometri serta topik-topik lain dalam matematik. Lesson Study adalah satu model pembangunan profesional guru di negara Jepun di mana sekumpulan kecil guru bekerjasama merancang, mengajar dan mengulangkaji pengajaran dalam usaha untuk meningkatkan kualiti pengajaran mereka serta untuk memperkayakan pengalaman pembelajaran pelajar. Dua puluh tiga orang guru yang terdiri daripada mereka yang telah mengikuti kursus kaedah mengajar matematik di sebuah universiti tempatan telah disusun dalam empat kumpulan mengandungi lima atau enam guru dalam setiap kumpulan LS. Kertas ini membincangkan bagaimana model LS telah membantu membangunkan kemahiran guru menggunakan GSP bagi mengajar tajuk Lokus dalam dua dimensi. Analisis lakaran GSP guru dalam sesi pengajaran pertama, kedua dan ketiga menunjukkan bahawa terdapat perubahan yang positif dalam penguasaan kemahiran mereka menggunakan GSP dalam pengajaran tajuk berkenaan.

*Kata kunci:* Guru menengah pra-perkhidmatan; Geometer's Sketchpad; lesson study; lokus dalam dua dimensi

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## 1.0 INTRODUCTION

The Geometer's Sketchpad (GSP) is a dynamic geometry software program for constructing and investigating mathematical objects. It is a dynamic tool for construction, demonstration and exploration that adds a powerful dimension to the learning of geometry and many other areas of mathematics. According to Finzer and Jackiw (1998), GSP "can best foster mathematical inquiry and learning through 'dynamic manipulation' experiments" (p. 2) because it

possesses three main attributes. Firstly, students can directly manipulate mathematical objects represented on the computer screen. Using a mouse, for example, students can point at a vertex of a square and directly drag it from one point to another point. Secondly, the mathematical objects remain coherent and whole at all times as they are dragged. Continuing the example above, as the vertex of the square moves from one point to another point, students can observe that while the orientation and size of the square change continuously the resulting figure will always remain

a square with all its properties intact. Thirdly, students feel that they are involved with the mathematical objects that they are manipulating. That is they can focus on how to achieve their mathematical goals like understanding the properties of squares, for instance, instead of how to use GSP.

Besides, using GSP to teach mathematics “based on experimentation, observation, data recording and conjecturing” (Olive, 2000, p. 3) encourages “a process of discovery that more closely reflects the way mathematics is invented” (Bennett, 1999, p. viii). Hence, learning mathematics in an instructional environment using GSP should “give students the opportunity to engage in mathematics as mathematicians, not merely as passive recipients of others mathematical knowledge” (Olive, 2000, pp. 3-4).

Further, research in other countries has also shown that GSP is an important tool for enhancing students’ learning of mathematics. While Elchuck (1992) found that mathematics achievement and time of independent investigation using GSP were significant predictors of conjecture-making ability, Frerking (1995) found that the abilities to conjecture and justify conjectures in a geometry class using GSP were directly related to proof-writing abilities. Choi (1996), Choi-Koh (1999), July (2001), McClintock, Jiang and July (2002) and Thompson (2006) showed that GSP could enhance students’ van Hiele levels of geometric thinking. Driskell (2004) found that the dynamic capability of GSP, inquiry-based tasks, as well as student-student and researcher-student interactions deepened students’ conception of two-dimensional shapes. Additionally, Cory and Garofalo (2011) showed that GSP could enhance pre-service secondary mathematics teachers’ understanding of limits of sequences.

In fact, a number of studies conducted in Malaysia have also shown that GSP is an essential tool for enhancing students’ learning of mathematics. Nurul Hidayah Lucy (2005), Chew and Noraini Idris (2006), Noraini Idris (2007) as well as Chew (2007) found that GSP could enhance secondary students’ geometry achievement and van Hiele levels of geometric thinking. Most of the students also showed positive perceptions of using GSP to learn geometry (Chew and Noraini Idris, 2006; Noraini Idris, 2007). Apart from that, Rosanini Mahmud, Mohd Arif Hj Ismail and Lim (2009) showed that a GSP-based courseware called ‘G-Reflect’ had a significant effect on secondary students’ achievement and motivation in learning the topic of ‘Reflections’. In addition, Chew and Lim (2010) found that GSP could enhance primary pupils’ van Hiele levels of geometric thinking of selected regular polygons.

In view of its importance, the Malaysian Ministry of Education (2003) advocates the integration of GSP into the teaching and learning of mathematics. However, the use of GSP in the teaching and learning of mathematics in Malaysian classrooms is still very much at its infancy state although the Ministry of Education has purchased the GSP license and supplied the GSP software to all secondary schools in Malaysia since 2004. While it is envisaged that this initiative will benefit many students, teachers and teacher educators nationwide, teacher enthusiasm and willingness to use GSP remains an issue to be addressed (Teoh & Fong, 2005). In fact, a survey conducted by Kasmawati (2006) on 151 secondary mathematics teachers in Penang showed that 26% of the teachers had attended GSP training courses but only 2% used GSP to teach mathematics in the classroom. The two main reasons given by the mathematics teachers were firstly lack of time to prepare a GSP sketch, and secondly lack of skills and confidence to use GSP to teach mathematics in the classroom. Hence, there is an urgent need to develop pre-service secondary mathematics teachers’ skills of using GSP to teach mathematics through a collaborative group effort such as Lesson Study which will provide helpful support and sustain the continuous integration of GSP in the

teaching and learning of mathematics as advocated by the Malaysian Ministry of Education.

## ■2.0 LESSON STUDY

Lesson Study (LS) is a direct translation for the Japanese term *jogyokenkyu* (*jogyo* means lesson and *kenkyu* means study or research) and it was already well established in Japan since the 1960s. It is an on-going practice as a form of teacher professional development especially in elementary schools throughout Japan. That is teachers actively engage in a continuous process of improving the quality of their teaching to enrich their students’ learning experiences by participating in LS groups (Fernandez, & Yoshida, 2004). More specifically, LS is a process by which small groups of teachers meet at stipulated time to plan lessons, observe these lessons unfold in actual classrooms, discuss their observations and to revise the lesson plans collaboratively. Basically, LS comprises six main steps: (1) collaboratively planning the lesson plan, (2) seeing the lesson plan in action, (3) discussing the lesson plan, (4) revising the lesson plan, (5) teaching the new version of the lesson, and (6) sharing reflections about the new version of the lesson (Fernandez & Yoshida, 2004).

Research has shown that LS improves teachers’ learning and supports teachers to grow professionally (Stigler & Hiebert, 1997; Shimahara, 1998; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999; Lewis, 2000; Fernandez, & Yoshida, 2004; Lim, White & Chiew, 2005). In particular, research has also shown that LS is a worthwhile and beneficial learning experience for pre-service teachers. Chiew and Lim (2003) found that LS helped improve the pre-service mathematics teachers’ content knowledge and enhance their confidence to teach mathematics, and they gained much more diverse teaching ideas that helped them improve their pedagogical content knowledge.

Fernandez and Robinson (2006) identified three main categories as central to the pre-service teachers’ learning through Lesson Study, namely connecting theory and practice, collaboration, and reflection. Lim (2006) found that despite facing the problems of time constraint and peer conflict, the majority of pre-service secondary teachers suggested LS as a good way of preparing them to teach mathematics and would like to continue to be involved in LS later in schools.

Moreover, Chew and Lim (2011a) showed that LS could encourage the innovative use of GSP in the teaching and learning of mathematics among secondary school teachers. The teachers showed positive changes in their knowledge and skills of using GSP to teach the topics of “Lines and Planes in Three Dimensions,” “Loci in Two Dimensions” and “Plans and Elevations”. The teachers also showed positive acceptance and feedback about LS such as providing peer support and collaboration. Hence, the teachers had more confidence in using GSP to teach mathematics at the secondary school level after the LS process. Chew and Lim (2011b) also showed that LS could enhance pre-service secondary teachers’ skills of using GSP to teach the topic of ‘Concept of Regular Polygons’ in Form Three Mathematics.

## ■3.0 PURPOSE OF THE STUDY

The purpose of this study was to develop pre-service secondary teachers’ skills of using GSP to teach mathematics through LS. More specifically, this paper aimed to examine the changes in the pre-service secondary mathematics teachers’ skills of using GSP in one of the selected LS groups that used GSP to teach the topic of ‘Loci in Two Dimensions’ in Form Two Mathematics after engaging in LS.

## ■4.0 METHODOLOGY

### 4.1 Research Design and Sample

The researchers employed a case study research design because the foci of this study were to: (1) study pre-service secondary mathematics teachers' learning of GSP skills by focusing on selected LS groups (that is cases); (2) conduct an in-depth study of each selected LS group; and (3) study each selected LS group's learning of GSP skills; and (4) study each selected LS group's skills of using GSP to teach the topics in secondary school mathematics (Gall, Gall & Borg, 2003). The participants of this study comprised 46 pre-service secondary teachers who enrolled in a mathematics teaching methods course in a Malaysian public university. Twenty-three LS groups, each comprising two pre-service secondary mathematics teachers, were set up in four tutorial groups with six LS groups (known as LS Group 1 to LS Group 6) in the first three tutorial groups and five LS groups (known as LS Group 1 to LS Group 5) in the fourth tutorial group. The members of four LS groups (one LS group from each tutorial group) volunteered to serve as case study participants. Purposeful sampling was employed to select the sample as the goal of this case study was not to generalize the results of the study from the sample to the population from which it was drawn, but rather to examine if there were any changes in the pre-service secondary mathematics teachers' skills of using GSP to teach mathematics after engaging in LS.

### 4.2 Research Procedure

In the first two-hour lecture the first author (the course coordinator) explained to all the participants the course outline, the coursework (namely, an individual review of a journal article on teaching mathematics with GSP, a group lesson plan for teaching mathematics with GSP, and an individual simulated teaching of the planned lesson), Fernandez and Yoshida's (2004) LS process as well as the research procedure. At the end of the lecture, the participants were divided into four tutorial groups. Each tutorial group met at a specific tutorial time for one hour every week. For each tutorial group, the course coordinator conducted two GSP workshops during the first two tutorials. The aim of the first GSP workshop was to explain the functions of the Title bar, Menu bar, Sketch plane, and Toolbox of GSP as well as how to use the basic tools of GSP (that is Selection Arrow Tool, Point Tool, Compass Tool, Straightedge Tool, Text Tool, and Custom Tool) to construct mathematical objects such as points, segments, rays, lines, circles, and polygons. The aim of the second GSP workshop was to design GSP sketches for teaching secondary school mathematics. After the workshops, the six main steps of LS were implemented as follows:

**Step 1 (Collaboratively Planning the Lesson Plan):** During the third tutorial, each LS group was allowed to choose a topic in the Malaysian secondary school mathematics syllabus. Next, each LS group discussed and planned collaboratively a 40-minute lesson plan for teaching the chosen topic with GSP. Finally, each LS group planned a schedule for the subsequent meetings to complete their lesson plan and GSP sketches before the fourth tutorial.

**Step 2 (Seeing the Lesson Plan in Action):** During the fourth tutorial, one participant from LS Group 1 in each tutorial group taught the 40-minute lesson as planned to their peers in the Mathematics Teaching Room. The lesson was observed by his/her partner of LS Group 1 and the course coordinator using the lesson plan and GSP sketches to guide their observations.

**Step 3 (Discussing the Lesson Plan):** After the lesson, the peers and the course coordinator provided comments and suggestions to improve the lesson plan and GSP sketches.

**Step 4 (Revising the Lesson Plan):** After the tutorial, the members of LS Group 1 in each tutorial group planned a schedule for the subsequent meetings to revise their lesson plan and GSP sketches based on their peers' as well as the course coordinator's comments and suggestions before the fifth tutorial. The end product of this step would be a revised lesson plan and GSP sketches.

**Step 5 (Teaching the New Version of the Lesson):** During the fifth tutorial, the new version of the lesson based on the revised lesson plan and GSP sketches was then taught by the other partner of LS Group 1 in the other tutorial group to different peers in the Mathematics Teaching Room. The lesson was observed by his/her partner of LS Group 1 (who had taught the first lesson) and the course coordinator using the revised lesson plan and GSP sketches to guide their observations. After the lesson, the peers and the course coordinator provided comments and suggestions to further improve the lesson plan and GSP sketches.

**Step 6 (Sharing Reflections about the New Version of the Lesson):** After the tutorial, the members of LS Group 1 in each tutorial group planned a schedule for the subsequent meetings to revise their lesson plan and GSP sketches for a second time based on their peers' as well as the course coordinator's comments and suggestions before the sixth tutorial. The end product of this step would be a final lesson plan and GSP sketches for submission as their coursework during the sixth tutorial.

Steps 2 to 6 were repeated for LS Groups 2, 3, 4, 5 and 6 in the subsequent tutorials. For each LS group, qualitative data were collected through videotaped lessons and discussions, observations, written lesson plans and reflections, as well as GSP sketches. In this paper, the discussion focuses on the analysis of the GSP sketches in the first, second and third lessons of one of the selected LS groups. The LS group consisted of two female pre-service secondary mathematics teachers. The group members selected a topic in Form 2 Mathematics, namely 'Loci in Two Dimensions'. The learning objective of the lessons was to enable students to understand that the locus of points that are of constant distance from a fixed point is a circle.

## ■5.0 FINDINGS AND DISCUSSION

The changes in the pre-service secondary mathematics teachers' GSP sketches in the first, second and third lessons are presented and discussed as follows:

### 5.1 First Lesson

In the first lesson, the LS group constructed a GSP sketch (see Figure 1) to enable students to understand that the locus of points that are of constant distance from a fixed point is a circle. As shown in Figure 1, the members of the LS group knew how to construct a circle using the Compass Tool. They also knew how to construct segments using the Straightedge Tool and then label the segments using the Text Tool. They were also able to use the Length command in the Measure menu to measure the length of segment AB. In addition, they could construct a point on the circle using the Point Tool and then animate the point using the Animation command of the Action Buttons in the Edit menu.

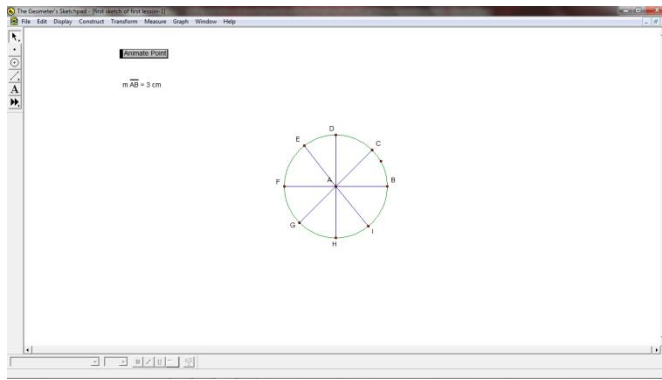


Figure 1 GSP sketch in the first lesson

But, after the first lesson some of their peers commented that the circle, segments, labels, length measurement and action button of the GSP sketch were too small to be seen clearly by the 'students' who were sitting at the back of the class. Some of the peers also commented that the fixed point and moving point ought to be labelled as O and P respectively as used conventionally in the Malaysian Form Two mathematics textbook. In addition, the segments ought to be constructed using dashed lines instead of solid lines so as to emphasise the circle. Apart from that, some of the peers also commented that the 'students' might not understand the symbol  $m \overline{AB}$  as used by GSP to represent the length of segment AB because it is not being used in the Malaysian Form Two mathematics textbook. Besides, some of the 'students' suggested that the phrase 'Animate Point' in the Action Button ought to be changed to 'Move Point' so that 'students' could easily understand the function of the Action Button.

In addition, the coordinator of the course commented that the 'students' might not be able to understand clearly that the locus of points that are of constant distance from a fixed point is a circle because firstly, the moving point was not connected to the fixed point by a segment and secondly, the point moved along the circle that had already been constructed instead of being traced out by the moving point that satisfied the given conditions. Therefore, the coordinator of the course suggested that the sketch could be made more interesting, dynamic and meaningful by constructing a segment to connect the moving point to the fixed point and then tracing the path of the moving point so that 'students' could see clearly the locus of points that are of constant distance from a fixed point is a circle.

After the tutorial, the LS group members were required to make changes to their first GSP sketch based on the comments and suggestions given by their peers and the course coordinator. Further, they were advised to do further readings on GSP.

## 5.2 Second Lesson

Based on the peers' and course coordinator's comments and suggestions, the members of the LS group revised their GSP sketch in the second lesson by referring to GSP books such as *Exploring Geometry with The Geometer's Sketchpad* (Bennett, 1999) and *Geometric Activities for Middle School Students with The Geometer's Sketchpad* (Wyatt, Lawrence, & Foletta, 1999). In addition to the above references, the LS group also sought help and guidance from the course coordinator to revise the GSP sketch (see Figures 2a and 2b).

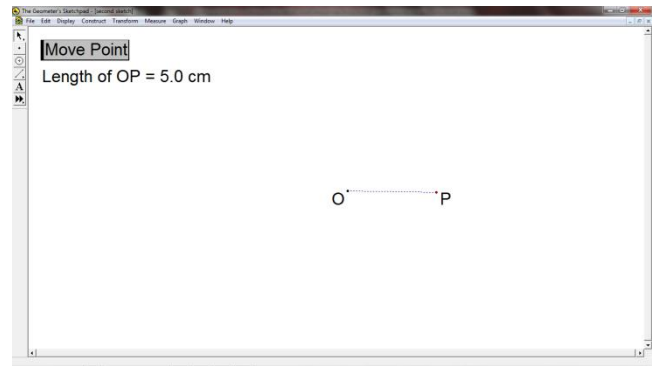


Figure 2a GSP sketch of the second lesson (before point P moved)

As illustrated in Figure 2a, the revised GSP sketch in the second lesson indicated that the members of the LS group were able to enlarge the labels, length measurement and action button of the GSP sketch by pressing the Alt and > buttons simultaneously so that 'students' sitting at the back of the class could see them clearly. They could also change the labels of the fixed point and moving point to O and P respectively as used conventionally in the Malaysian Form Two mathematics textbook. Additionally, the segment joining the fixed point O and the moving point P was constructed using a dashed line instead of a solid line so that students could focus their attention on the circle that was being traced out by the moving point. Apart from that, they were able to change the symbol  $m \overline{AB}$  to 'Length of OP' and the phrase 'Animate Point' to 'Move Point' using the Properties command in the Edit menu so that 'students' could easily understand the meaning of the length measurement and the function of the Action Button respectively.

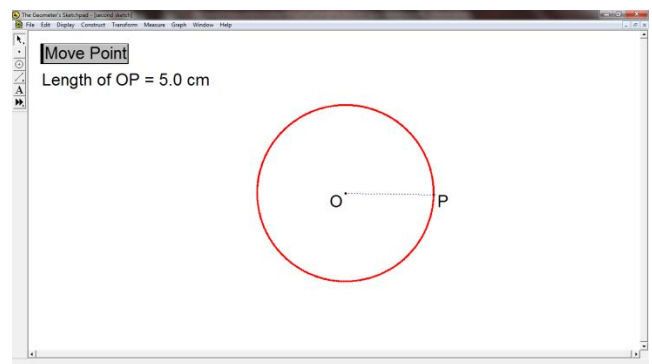


Figure 2b GSP sketch of the second lesson (the circle traced out by point P as it moved)

But, most importantly, the members of the Lesson Study group were able to design an interesting, dynamic and meaningful GSP sketch by constructing a segment to connect the moving point to the fixed point using the Straightedge Tool and then tracing the path of the moving point using the Trace command in the Display menu so that 'students' could see clearly the locus of points that are of constant distance from a fixed point is a circle (see Figure 2b). This is indeed one of the features of an excellent GSP sketch (Bennett, 1999) because it helps to illustrate clearly the meaning of the locus of points that are of constant distance from a fixed point is a circle.

After the second teaching, most of their peers gave positive comments on the revised GSP sketch such as the labels, length measurement and action button were large enough to be seen clearly by the 'students' who were sitting at the back of the class, the labels of the fixed point, moving point, action button and length measurement were suitable, and the Trace feature helped them to visualise clearly the meaning of the locus of points that are of constant distance from a fixed point is a circle. Additionally, some of them provided some helpful comments to further improve the GSP sketch. They suggested that the action button and the length measurement ought to be colourful enough to attract the attention of the 'students' and an action button to move point P back to its original position after it had moved to any positions ought to be provided.

Further, the course coordinator suggested that the title of the topic and the learning objective of the lesson ought to be provided in the sketch. To facilitate whole-class discussion, the sketch could include the question, 'What is the locus of points that are of constant distance from a fixed point?' and an action button to show and hide the answer to the question, 'The locus of points that are of constant distance from a fixed point is a circle.'

After the tutorial, the LS group members were required to make further changes to their second GSP sketch based on the comments and suggestions given by their peers and the course coordinator. They were also advised to do further readings on GSP.

### 5.3 Third Lesson

Based on their peers' and the course coordinator's comments and suggestions, the LS group revised their GSP sketch accordingly by referring to the above GSP books as well as seeking further help and guidance from the course coordinator. As a result, they successfully constructed the revised version of the GSP sketch as shown in Figures 3a and 3b. As depicted in Figure 3a, the members of the LS group were able to construct colourful title of the topic, learning objective, action buttons, and length measurement to attract the attention of the 'students', and colourful constructions were in fact one of the features of an excellent GSP sketch (Bennett, 1999). They could also construct a Reset action button to move point P back to its original position after it had moved to any positions.

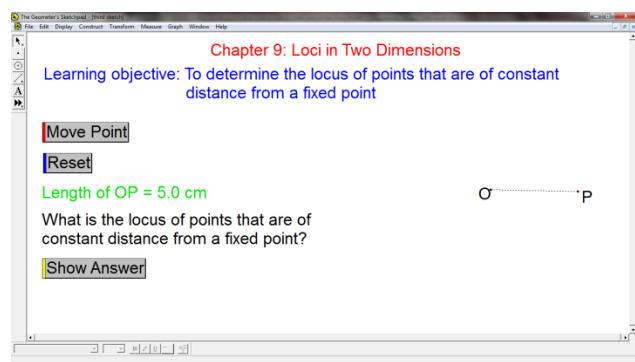


Figure 3a GSP sketch of the third lesson (before point P moved)

Moreover, as shown in Figure 3b, the members of the LS group were able to provide the question, 'What is the locus of points that are of constant distance from a fixed point?' using the Text tool and an action button to show and hide the answer to the question, 'The locus of points that are of constant distance from a fixed point is a circle' using the Show/Hide command of the Action Buttons in the Edit menu and chose the option of 'Select Objects After

Showing' as well to highlight the answer which was another feature of an excellent GSP sketch (Bennett, 1999).

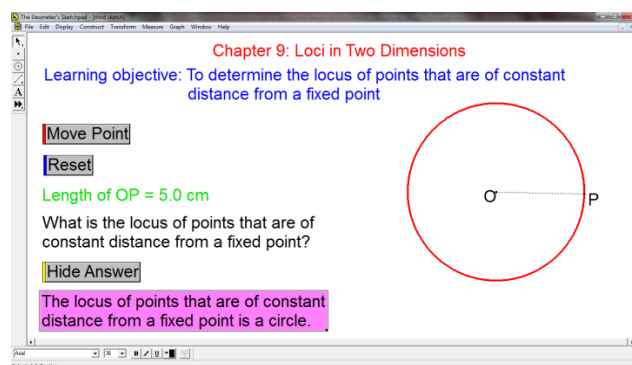


Figure 3b GSP sketch of the third lesson (the circle traced out by point P as it moved and the answer showed by the teacher after discussion)

## 6.0 CONCLUSION

Analysis of the GSP sketches in the first, second and third lessons indicates that the members of this LS group showed significant changes in their skills of using GSP to teach the topic of 'Loci in Two Dimensions'. In the first lesson, they knew how to: (a) construct a circle using the Compass Tool; (b) construct segments using the Straightedge Tool and then label the segments using the Text Tool; (c) measure the length of segment using the Length command in the Measure menu; and (d) construct a point on the circle using the Point Tool and then animate the point using the Animation command of the Action Buttons in the Edit menu. In the second lesson, they were able to: (a) enlarge the labels, length measurement and action button of the GSP sketch by pressing the Alt and > buttons simultaneously; (b) change the labels of the fixed point and moving point; (c) construct a segment joining the fixed point and the moving point using a dashed line instead of a solid line; (d) change the symbol  $m \overline{AB}$  to 'Length of OP' and the phrase 'Animate Point' to 'Move Point' using the Properties command in the Edit menu; and (e) design an interesting, dynamic and meaningful GSP sketch by constructing a segment to connect the moving point to the fixed point using the Straightedge Tool and then tracing the path of the moving point using the Trace command in the Display menu which was indeed one of the features of an excellent GSP sketch (Bennett, 1999) as it helps to illustrate clearly the meaning of the locus of points that are of constant distance from a fixed point is a circle. Lastly, in the third lesson, they successfully (a) constructed the colourful title of the topic, learning objective, action buttons, and length measurement; and (b) created the question using the Text tool and an action button to show and hide the answer to the question using the Show/Hide command of the Action Buttons in the Edit menu and chose the option of 'Select Objects After Showing' as well to highlight the answer which were also features of an excellent GSP sketch (Bennett, 1999).

However, in this paper we only managed to share the positive changes in the pre-service secondary mathematics teachers' skills of using GSP to teach the topic of 'Loci in Two Dimensions' of one selected LS group after engaging in LS. We acknowledge the limitations of observing the positive changes in all the pre-service secondary mathematics teachers' skills of using GSP after engaging in LS. Nevertheless, we were very much encouraged by the positive attitude and commitment of the participants in constructing and re-constructing the GSP sketches several times as

revealed in their GSP sketches in the first, second and third lessons as well as their numerous consultations with the course coordinator.

In conclusion, LS provided an alternative way of enhancing the pre-service secondary mathematics teachers' skills of using GSP to teach mathematics in general and geometry in particular which, in turn, developed their pedagogical content knowledge and confidence in using GSP to teach secondary school geometry.

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