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# Choosing Mathematical Examples: Routine but Not an Easy Task

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**Abstract**

Choosing examples for the purpose of teaching mathematics is routine tasks done by every mathematics teachers. Examples are an important medium used by mathematics teachers as a communication device to discuss mathematical content with their students (Leindhart, 2001). Through examples, students build their understanding about mathematical ideas. In spite of the important role played by mathematical examples, the knowledge about mathematical exemplification is not formally taught to the teachers. It has to be built by the teachers through their teaching experience (Zaslavsky & Zodik, 2007). The purpose of this study is to capture this knowledge. We study six Excellent Mathematics Teachers teaching practices using pre-active notes, observation and interview in order to know the things that influence their choice of examples. Findings show that although choosing examples is a routine task, but it seems that it is not an easy one.

*Keywords*: Examples; exemplification; mathematics teaching; expert teacher; Mathematical knowledge for teaching

**Abstrak**

Memilih contoh untuk tujuan pengajaran matematik adalah tugas rutin bagi semua guru matematik. Contoh adalah medium penting yang digunakan oleh guru sebagai alat komunikasi bagi membincangkan sesuatu kandungan matematik dengan pelajar (Leinhardt, 2001). Pelajar membina kefahaman mengenai idea matematik melalui penggunaan contoh. Walaupun contoh penting dalam pengajaran matematik, namun pengetahuan tentang percontohan matematik tidak didedahkan kepada guru secara formal. Mereka perlu membinanya sendiri melalui pengalaman (Zaslavsky & Zodik, 2007. Kajian ini bertujuan untuk mengetahui mengenai pengetahuan percontohan ini. Untuk itu amalan pengajaran bagi enam orang Guru Cemerlang Matematik dikaji dengan menggunakan catatan pra-aktif, pemerhatian dan temubual untuk mengetahui tentang perkara-perkara yang mempengaruhi mereka apabila memilih contoh. Dapatan memperlihatkan bahawa walaupun pemilihan contoh merupakan tugas rutin, namun ia bukan tugas yang boleh dilakukan dengan mudah.

*Kata kunci:* Contoh; percontohan; pengajaran matematik; guru cemerlang; pengetahuan matematik untuk tujuan pengajaran



## 1.0 INTRODUCTION

Examples are something very common in the teaching and learning of mathematics. It is a communication device used for the purpose of mathematical explanation and discussion (Leinhardt, 2001). Through examples, teachers help their students make generalization and construct their understanding about a mathematical content (Bills *et al*., 2006; Zaslavsky & Zodik, 2007). Zaslavsky and Zodik argue that knowledge about example is an important knowledge needed in mathematics teaching, however, mathematics teachers, either in-service or pre-service, are not fully equipped with this knowledge. It is assumed that all mathematics teachers are able to construct their own knowledge about mathematical examples through their teaching experience. Nevertheless, not all teachers can learn from their experience (Ball & Bass, 2003; Hiebert, Gallimore & Stigler, 2002; Kennedy, 2002).

Examples in mathematics teaching are not only restricted in a form of questions or worked examples as many might think. In their book, Mathematics as a Constructive Activity, Watson and Mason (2005) define examples as anything from which a learner might generalize. This broad definition means that examples can be in almost any form like figures, verbal illustration, question, situation, dynamic image, problems and others. Whatever forms examples are in, they are used to assist students to generalize about something. The extent to which students can understand about a mathematical idea depends on the examples that their teachers used (Bills *et al*., 2006; Watson & Mason, 2002). The content of an example and the way a teacher navigates their students' attention towards this content will have a bearing on their students understanding (Zaslavsky & Zodik, 2007). In short, mathematical examples and how they are used, influenced students' mathematical understanding. Hence, it is the responsibility of a mathematics teacher to choose examples that provide the best learning opportunities and then handle these examples in a manner that best suits their students.

## 2.0 PREVIOUS STUDIES

Previous studies on mathematical examples can be divided into three groups. Firstly, studies that show how examples help students learning especially in gaining initial skills acquisition (Atkinson, Derry, Renkl & Wotham, 2000). Most of these studies were related to worked examples (Pass & Merrienboer, 1994; Quilci & Mayer, 1996; Reed & Bolstad, 1991; Trafton & Reiser, 1993), however, these studies focused on the effects of worked examples towards students' learning, without considering the teachers' factor. In fact, before using any examples in mathematics teaching, the example has to be chosen or constructed by the teacher. Therefore, the first important thing is, a teacher must know how to choose appropriate examples for their teaching.

Secondly, we have studies on the characteristics of examples used by mathematics teachers, either pre-service teachers (Rowland, 2008) or experienced teachers (Zodik & Zaslavsky, 2008). Finally, studies, which suggested that examples teachers used reflected their own knowledge base (Zaslavsky & Zodik, 2007; Zaslavsky, Harel & Manaster, 2006). Although the findings from these studies have provided us with noteworthy knowledge, there seems to be something missing here. We know that examples help students in learning mathematics and for that reason mathematics teachers have to provide their students with examples to enhance their understanding. But what are the factors that influenced teachers' choice of examples? This is the question that we are trying to find the answer.

## 3.0 THEORETICAL FRAMEWORK

In our study, we use Mathematical Knowledge for Teaching (MKT) as our theoretical framework. MKT is a refinement and elaboration of Shulman's (1986) content knowledge for teaching (Ball, Thames & Phelps, 2008). The refinement and elaboration are based on studies done by Ball and her friends (Ball & Bass, 2003; Ball, Hill & Bass, 2005; Ball *et al*., 2008). They analysed mathematics teachers' work in the classroom in order to know the construct of content knowledge needed for teaching mathematics. They named the content knowledge as MKT.

MKT comprises of two main domains, subject-matter knowledge and pedagogical content knowledge. Subject-matter knowledge is further divided into three sub-domains. The first sub-domain is common content knowledge. It is the knowledge about the content of subject matter and its organization (Shulman, 1986). This knowledge is not only required by mathematics teachers, but it is also used in many other professions that need mathematical knowledge (Ball *et al*., 2008). Common content knowledge enables mathematics teacher to identify, verify and justify things that related to mathematical ideas.

The second sub-domain in subject-matter knowledge is specialized content knowledge. Unlike common content knowledge, specialized content knowledge is knowledge uniquely used by mathematics teachers (Ball *et al*., 2008). According to Ball *et al*., this knowledge demands unique mathematical understanding and reasoning to help teachers make particular mathematical content visible and learnable by their students (p. 400). The last sub-domain in subject-matter knowledge is horizon knowledge. Ball & Bass (2009) define this knowledge as an awareness of the large mathematical landscape in which the present experience and instruction are situated. This knowledge helps the teacher to make sure that the present teaching will not contradict or obstruct their students' future learning.

The second domain in MKT is pedagogical content knowledge. It contains another three sub-domains which are knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum. As its names suggest, knowledge of content and students is the combination of knowledge about mathematical content and knowledge about students (Ball *et al*., 2008). According to Ball and her friends, this knowledge is an interaction between teacher's mathematical understanding and familiarity with students and their mathematical thinking (p. 401). The second sub-domain in pedagogical content knowledge is knowledge of content and teaching. This knowledge is an integration of knowing about mathematical content and knowing about teaching (Ball *et al*., 2008). Just like the knowledge of content and students, Ball *et al*., defines this knowledge as an interaction between teacher's specific mathematical understanding and an understanding of pedagogical issues that affect students' learning (p. 401). Finally, the last sub-domain is knowledge of content and curriculum. It is the knowledge on how topics are arranged within one level and across different level over the years and ways of using available materials to coordinate programme of study to the students (Hills, Rowen & Ball, 2005; Shulman, 1986).

The reason for using MKT as our theoretical framework is because it is "a mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (Ball *et al*., 2008, p. 399). Choosing, using and improving examples are some of the recurrent tasks in the teaching of mathematics. Studies have shown that examples that the teachers choose are the reflection of their knowledge base, but some of the questions still remain unanswered. How do teachers choose examples? How do they use examples? How do teachers evaluate and improve the examples? These are the questions that we are trying to answer through this research. However, this paper only discusses a small part of the study that is the things that influence teacher's choice of examples.

## 4.0 METHODOLOGY

This study used a qualitative approach with multiple case studies design. According to Zaslavsky and Zodik (2007), knowledge about mathematical examples is acquired through teaching experience; therefore, it is considered as craft knowledge. One important element about craft knowledge is, experience. For that reason, we used six experienced mathematics teachers as our samples. These teachers are known as Excellent Mathematics Teacher (EMT).

In Malaysia, EMT is an experienced teacher who is recognised by the Ministry of Education (KPM) as an expert in teaching and learning of mathematics (KPM, 2006). By studying the exemplification process of these EMTs, we hope that we will be able to portray how these excellence teachers choose, use and improve their mathematical examples. All these EMTs, EMT1, EMT2, EMT3, EMT4, EMT5 and EMT6, are chosen by using snowball sampling technique. They are teachers who teach Mathematics or Additional Mathematics to students in the upper secondary school.

We used three data collection methods, pre-active note, observation and interviews. Pre-active note is a modified version of methods used by Artzt, Armour-Thomas and Crucio (2008) to determine teacher's pre-lesson thought. In pre-active note, the subjects were asked to write the learning objectives, learning outcomes, the things that influence their choice of examples, and things that they expect will interfere their students' learning. This pre-active note was then submitted to the first author before she conducted observation of every EMT's teaching. The pre-active note was used to help the first author to understand the EMTs action while teaching. After the observation was done, the first author conducted an interview in order to know if any problem arises during the lesson due to the use of the examples. This data collection process was repeated four times, in four different lessons for each sample. All the data were then analysed using constant comparative methods.

## ■5.0 THINGS THAT INFLUENCED TEACHERS' CHOICE OF EXAMPLES

Findings show that there are four things that influenced these EMTs' choice of examples. The four things are learning outcomes that need to be achieved by their students, the students' previous knowledge, the learning problems that usually occur and the examination questions.

### 5.1 Learning Outcomes That Need to Be Achieved by Students

Mathematical learning outcome is a statement about what students will know or able to do after they learn about a mathematical idea. All the EMTs in this study choose their examples according to the learning outcomes. In Malaysia, the statement of the learning outcomes for every topic in every subject is given in the curriculum specification (CS) of that particular subject. The CS is issued by the Curriculum Development Centre (CDC) of the Ministry of Education. Thus, to determine the learning outcomes that need to be achieved by students, these EMTs refer to the CS. However, not all the time they will use the given learning outcome. There are times when they modify the learning outcome or rearrange its sequence.

The EMTs in this study used two methods to modify the learning outcomes given in the CS. The first method was by combining several learning outcomes, given in the CS, to form one 'new' learning outcome. This method was used by four EMTs, EMT1, EMT3, EMT4 and EMT6. For instance, to teach the topic 'Motion along a Straight Line', EMT1 combined five learning outcomes given in the CS to form one 'new' learning outcome. The five learning outcomes were:

(i) Students will be able to determine velocity (*v*) function of a particle by differentiation,
(ii) Students will be able to determine displacement (*s*) of a particle from velocity (*v*) function by integration,
(iii) Students will be able to determine the acceleration (*a*) function of a particle by differentiation,
(iv) Students will be able to determine instantaneous velocity (*v*) of a particle from acceleration (*v*) function by integration,
(v) Students will be able to determine displacement (*s*) of a particle from acceleration (*a*) function by integration (CDC, 2006, pp. 21-22).

EMT1 combined all these five learning outcomes to formulate one single 'new' learning outcome, which was, 'Students will be able to produce all the function needed (*s, v, a*) when one of the functions (*s, v, a*) is given'.

Another way of modifying learning outcomes is by making one learning outcome becomes more specific. This method was used by EMT1 and EMT5. For example, in the topic of 'Linear Law', one of the learning outcomes in the CS is 'Students will be able to reduce non-linear relations to linear form' (CDC, 2006, p.4). EMT5 separated this learning outcome into two specific learning outcomes which were:

(i) Students will be able to reduce non-linear algebraic relations to linear form.
(ii) Students will be able to reduce index relation to linear form.

These two learning outcomes stated the specific forms of non-linear relation which were algebraic and index.

Other than modifying learning outcomes, these EMTs also rearrange the sequence of the learning outcomes that their students' need to achieve. This method was used by EMT3 and EMT6. For example, when EMT3 teaches the topic 'Probability', the last learning outcome in the CS for this topic is, 'students will be able to predict the occurrence of an outcome'. So, when EMT3 taught this topic, he purposely brought this learning outcome forward and made it as the first learning outcome to be achieved by his students.

### 5.2 Students' Previous Knowledge

The second thing that influences these EMTs choice of examples is their students' previous knowledge. There are two types of students' previous knowledge that affected the examples that they choose. The first type is previous knowledge that is needed to learn new topic. Without this knowledge, it is difficult for students to learn the mathematical idea in the new topic. Out of six EMTs, four of them, EMT1, EMT3, EMT5, EMT6, chose examples based on this characteristic. For instance, EMT1 chose examples about differentiation and integration and used it before he taught the topic 'Motion along a Straight line'. Students had learned about integration and differentiation long before the topic 'Motion along a Straight Line'. The knowledge on how to differentiate and integrate is needed in order to find the function of displacement (*s*), velocity (*v*) or acceleration (*a*) which are the main mathematical idea being taught in the topic. If students do not know how to differentiate and integrate, they will be having difficulty to learn this topic.

The second type of students' previous knowledge is knowledge that is related but not needed to learn the new topic. EMT2 and EMT4 chose some examples based on this type of students' previous knowledge. For instance, EMT2 used examples that show a bar chart, a line graph, a pie chart and a pictogram before she taught about histogram. Students had learned about all this when they were in lower secondary, but they do not need this knowledge to learn about histogram. Nevertheless, all these are methods of representing data, just like histogram.

### 5.3 Learning Problems

Another source of influence that affected some of these EMTs choice of examples is the learning problems that are usually faced by their students while learning about certain mathematical idea. EMT1, EMT2 and EMT4, purposely chose specific examples because, from their experience, they knew that students would have problems with certain mathematical ideas that they are about to learn. For instance, EMT 4 purposely chose quadratic graph as examples to explain about the turning point. From her experience, students usually do not understand why at the maximum or minimum point $\frac{dy}{dx}$ is equal to zero. For that reason, she chose the following two graphs in Figure 1 as her examples. These examples showed that, at the minimum and maximum point, the gradient of the tangent line is equal to zero and because of that $\frac{dy}{dx} = 0$.

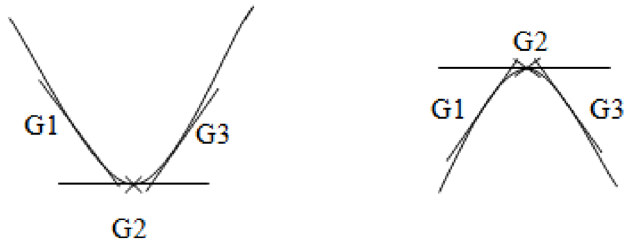


**Figure 1** Quadratic graph used by EMT4 to show that at the turning point $\frac{dy}{dx} = 0$

### 5.4 Examination Question

The fourth thing that influences these EMTs while choosing examples is the examination question. The influence of the exam questions towards teachers' selection of examples is seen in every EMT's teaching. These teachers purposely chose examples that were similar to the real exams questions. For every mathematical idea that they were about to teach they would choose some examples that were similar to the exam question.

## ■6.0 DISCUSSION

The statement of learning outcomes is the first things that influence these teachers choice of examples. The statement of learning outcomes gives information to the teacher about the mathematical idea that they should teach and what students should be able to do with this mathematical idea after they have learned about it. Looking from this point of view, the statement of learning outcome serve as guidance to assist these EMTs to choose examples that will help them teach the mathematical idea and help the students achieve the intended outcome. Thus, all the examples chosen based on the learning outcomes are examples that represent the mathematical idea in it.

Before they can choose examples based on the statement of learning outcomes, the learning outcome has to be formulated. It seems an easy task because the CS provides the learning outcomes for every topic. There were times when these EMTs used the learning outcomes provided by the CS. However, as shown in the findings, there were also times when the EMTs modified the suggested learning outcomes given in the CS or rearranged its sequence. Although they modify the learning outcomes, it still covers the same mathematical ideas. What makes them modify the learning outcomes suggested in the CS? All of these actions spring from the intention to help their students' learning.

The formation of a 'new' learning outcome from a combination of several learning outcomes happens when the mathematical ideas in these several learning outcomes have connections to one another. If the learning outcomes are not combined, each mathematical idea in it will have to be taught separately. In this condition, these EMTs think that it will be difficult for the students to 'see' the connection between the mathematical ideas. Therefore, they combine the learning outcomes and formulate a new statement of learning outcome. The new learning outcome contains a specific statement of what students will know and be able to do after they learn about the connection. This new learning outcome is used as guidance for the teacher to choose examples that portray the relationship between the mathematical ideas. Students can straight away 'see' the relationship between the mathematical ideas through these examples.

The task of formulating a 'new' statement of the learning outcome by combining several mathematical ideas requires them to use some of the knowledge in MKT. Before combining the learning outcomes, the teacher themselves, have to know which of the mathematical ideas are connected and the reason behind the connection. Knowing and understanding about the relationship between mathematical ideas is a form of common content knowledge. Meanwhile, the ability to anticipate how their students might think about the connection between mathematical ideas is a form of knowledge of content and students. When these teachers make the decision to combine several learning outcomes to make their students' learning easier, it shows that these EMTs are using their knowledge of content and teaching.

Other than combining, they also refine certain learning outcomes. This happens when the mathematical idea in the learning outcomes can be specified into more specific form and each of this form needs different skills. For that reason, the EMTs refine the suggested learning outcomes to formulate a more specific learning outcome according to the form. In their opinion, their students may easily be confused if they do not separate the idea according to the specific forms. The following statements given by EMT1 and EMT5 reflected this opinion. EMT1 refined the mathematical idea about permutation to two different forms, namely permutation of numbers and permutation of alphabet. Meanwhile, EMT5 refined the non-linear relation to two different forms which were algebraic relation and index relation.

*Because I don't want them to get confused. Separate alphabet from numbers. When you teach students, don't make it a burden to them. Numbers and alphabet are different things (EMT1).*

*I separate algebraic non-linear relation from index. Students usually have problems with index. You have to break it down (the learning outcome), otherwise they will get confused (EMT5).*

By refining the mathematical idea to a more specific form, the EMTs will then choose examples according to these specific forms. The examples chosen to represent each of this form will be able to show the students specific skills for each form.

Knowing about the different forms of a mathematical idea is common content knowledge. To be able to foresee students' difficulties in understanding a mathematical idea is a form of knowledge of content and students. The decision to refine a mathematical idea to a more specific form to facilitate students understanding is a sign of knowledge of content and teaching. These are the knowledge used by these teachers when they refine the statement of the learning outcome.

Besides modifying the learning outcomes, these EMTs also rearrange the sequence of the learning outcomes given in the CS. This sequence of learning outcomes is actually a sequence of mathematical ideas to be taught in a topic. When the EMTs rearrange the sequence of the learning outcomes, it means that they are rearranging the sequence in teaching these mathematical ideas. According to these EMTs, by doing so, the flow of their teaching will be better than following the sequence given in the CS. The new sequence will help their students to understand better the mathematical idea they are learning. This reason can clearly be seen through the following statements given by EMT3.

*Sometimes I start from the second part, it is better than starting from the first part. I introduce the second part first than it will be much easier to explain the first part. The flow is better. Because the flow is important. It will be meaningful to students (EMT3).*

The reason why these teachers change the sequence of the learning outcomes is actually to ensure that the teaching of a mathematical idea is started from something that their students can understand. This situation suggests that the examples chosen based on the new sequence might be able to help students build their understanding cumulatively. It means that students understanding of the earlier mathematical idea will help them to understand the next mathematical idea that they are about to learn.

To rearrange the content of a teaching, these EMTs evaluate the appropriateness of the sequence of content arrangement given in the CS with the way their students think. It shows that these teachers are using their knowledge of the content and the students. When they decide to rearrange the sequence of the mathematical ideas to be taught for the purpose of improving the flow of their teaching and their students' learning, this situation implies that they are using their knowledge of content and teaching.

Besides learning outcomes, students' previous knowledge also influences these EMTs choice of examples. This influence comes from two types of students' previous knowledge. Previous knowledge needed to learn new mathematical idea or previous knowledge that is related to the new mathematical idea. These teachers know the foundation of every topic that they are about to teach. Because of that they know which previous knowledge is needed to learn new mathematical idea in the topic. Based on this, they choose examples that will help their students to remember what they have learnt before. These examples are also used as a tool to assess their students' understanding of their previous knowledge; either they still remember it or they have forgotten it. In other words, the examples chosen based on the needed previous knowledge help the students to refresh their memory about that knowledge and be prepared to use this knowledge in order to learn a new thing.

Another type of students' previous knowledge is knowledge that is related to the mathematical idea that they are about to learn, but this knowledge is not needed, it is just related to the new mathematical idea. Even though this knowledge is not needed in the present learning, these EMTs still choose examples based on it. This situation suggests that the teachers are using the examples to show the relationship between what the students have learnt with what they are going to learn. In other words, they are using examples as tools to help the students to be aware of the relationship between the mathematical ideas.

The influence of students' previous knowledge towards these EMTs choice of examples shows that these teachers have the knowledge about mathematical curriculum. Knowing which previous knowledge is needed and which previous knowledge is related to a topic is a trait of knowledge of content and curriculum. The existence of this knowledge is clear because these EMTs know what their students' previous knowledge is and how it is connected to their current learning. These teachers make use of their knowledge of content and curriculum to choose examples that can help their students to remember what they have learnt, prepare them to make use of that knowledge in present learning and to show relationship between their previous knowledge and the present learning.

These EMTs are also influenced by learning problems that usually happen to their former students while learning about certain mathematical idea. Worried that their current students will be facing the same problems, they choose examples that can prevent the same problems from happening. These examples are chosen as prevention tools in order to help their currents students. The tendency among these teachers to choose examples that can avoid their students from facing the same problem as their former students shows that these EMTs are using their knowledge of content and student. They know what kind of problems their students will be facing and by knowing this, they choose examples that can help them curb the problems.

The last thing that influences the EMTs' choice of examples is the form of exam questions for every mathematical idea they teach. For these teachers, besides for the purpose of learning, the examples they choose should also give experience to the students about the kind of question that they have to answer in examination. By doing this, students will be familiar with the form of examination questions that they have to answer for every mathematical idea they learn. In order to choose examples which are similar to the real examination questions, the teachers need to know the possible pattern and structure of the questions that would be asked in order to assess students understanding about certain mathematical idea. This is the type of knowledge that perhaps only needed in the teaching profession and that is why this kind of knowledge can be considered as specialized content knowledge.

## ■7.0 CONCLUSION

Choosing examples to be used in a lesson is a routine that needs to be done by any mathematics teachers. However, the findings show that it is not an easy task. The findings indicate that to choose examples these EMTs have to consider the need of the curriculum, the need of their students and the need of the examination. They try to meet the needs of these three aspects. To meet the needs of the curriculum, these EMTs try to ensure that the examples they choose for their teaching will help their students achieve the intended learning outcomes stated in the CS. For that reason, they use the statement of learning outcomes as guidance to help them choose the examples. Although some of the learning outcomes or the sequences are modified, the things that students should learn and what they should be able to do with it are pretty much the same. This modification is done to cater for their students' need. These teachers also choose examples that contain their students' previous knowledge to show the relation between what they have learned and what they are going to learn. This is also one of the curriculum needs. At the same time, since examination is important to their students' future, they give them a glimpse of experience on how the questions in real examination might look like through examples. This situation shows how these teachers try to meet the need of the curriculum, the students and the examination when they choose examples.

Besides that, the task of choosing examples requires these teachers to use not only subject matter knowledge but also their pedagogical content knowledge. Rationales behind the things that influenced these EMTs choice of examples show that they are using almost every knowledge components in MKT which are common content knowledge, specialized content knowledge, knowledge of content and students, knowledge of content and curriculum and knowledge of content and teaching. There are interactions between these knowledge components in every rationale and action taken by these teachers. Clearly all these show that, although choosing examples is routine, but it is not an easy task.

## References

Artzt, A. F., Armour-Thomas, E. & Curcio, F. R. 2008. *Becoming a Reflective Mathematics Teacher*. 2nd ed. New York: Lawrence Erlbaum Associates.

Atkinson, R. K., Derry, S. J., Renkl, A., & Wortham, D. 2000. Learning from Examples: Instructional Principles From The Worked Examples Research. *Review of Educational Research*. 70(1): 181–214.

Ball, D. L. & Bass, H. 2003. Towards a Practice-Based Theory Of Mathematical Knowledge for Teaching. In B. Davis & E. Simmt (Eds.). *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group* Edmonton: Canadian Mathematics Education Study Group. 3–14.

Ball, D. L., & Bass, H. (2009, February). *With an Eye on the Mathematical Horizon: Knowing Mathematics for Teaching to Learners' Mathematical Future*. Paper presented at the 2009 Curtis Centre Mathematics and Teaching Conference, University of California at Los Angeles.

Ball, D. L., Hill, H.C. & Bass, H. 2005. Knowing Mathematics for Teaching: Who Knows Mathematics Well Enough to Teach Third Grade and How Can We Decide? *American Educator*. 29(1): 14–17, 20–22, 43–46.

Ball, D. L., Thames, M. H. & Phelps, G. 2008. Content Knowledge for Teaching: What Makes it Special? *Journal of Teacher Education.* 59(5): 389–407. doi:101177/0022487108324554.

Bills, L., Dreyfuss, T., Mason, J., Tsamir, P., Watson, A. & Zaslavsky, O. 2006. Exemplification in Mathematics Education. In J. Novotná, H. Moraová, M. Krátká & N. Stehliková (Eds.). *Proceedings of 30th Conference of the International Group for the Psychology of Mathematics Education*: Prague: Psychology of Mathematics Education. 1: 126–154.

CDC. 2006. *Integrated Curriculum for Secondary Schools: Curriculum Specifications Additional Mathematics Form 5*. Putrajaya: Ministry of Education Malaysia.

Hiebert, J., Gallimore, R. & Stigler, J. W. 2002. A Knowledge Base for the Teaching Profession: What Would It Look Like and How Can We Get One? *Educational Researcher.* 31(5): 3–15.

Hill, H. C., Rowan, B., & Ball, D. L. 2005. Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal*. 42(2): 371–406.

Kennedy, M. M. 2002. Knowledge and Teaching. *Teachers and Teaching: Theory and Practice.* 8(3): 354–370. doi: 10-1080/135406002100000495.

KPM. 2006. *Terma Rujukan Konsep Guru Cemerlang*. Kuala Lumpur: Kementerian Pelajaran Malaysia.

Leinhardt, G. 2001. Instructional Explanations: A Commonplace for Teaching and Location for Contrast. In V. Richardson (Ed.). *Handbook of Research on Mathematics Teaching*. 4th ed. Washington DC: American Educational Research Association. 333–357.

Pass, F. G. W. C. & Merrienboer, J. J. G. V. 1994. Variability of Worked Examples and Transfer of Geometrical Problem-Solving Skills: A Cognitive-load Approach. *Journal of Educational Psychology*. 86(1): 122–133.

Quilci, J. L. & Mayer, R. E. 1996. Role of Examples in How Students Learn to Categorize Statistics Word Problems. *Journal of Educational Psychology*. 88(1): 144–161.

Reed, S. & Bolstad, C. A. 1991. Use of Examples and Procedures in Problem Solvings. *Journal of Experimental Psychology: Learning, Memory and Cognition*. 17(4): 753–766.

Rowland, T. 2008. The Purpose, Design and Use of Examples in the Teaching of Elementary Mathematics. *Educational Studies of Mathematics*. 6: 149–163. doi:10.1007/s10649-008-9148y.

Shulman, L. S. 1986. Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*. 15(2): 4–14.

Trafton, J. G. & Reiser, B. J. 1993. The Contribution of Studying Examples and Solving Problems to Skills Acquisition. In M. Polson (Ed.). *Proceedings of the Fifteenth Annual Conference of the Cognitive Science Society* Hillsdale, NJ: Erlbaum. 1017–1022.

Watson, A. & Mason, J. 2002. Extending Example Spaces as a Learning/Teaching Strategy In Mathematics. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the Psychology of Mathematics Education*. Norwich: University of East Angelia. 2:377–385.

Watson, A. & Mason, J. 2005. *Mathematics as a Constructive Activity: Learners Generating Examples.* Mahwah: Lawrence Erlbaum Associates.

Zaslavsky, O. & Zodik, I. 2007. Mathematics Teachers' Choices of Examples that Potentially Support or Impede Learning. *Research in Mathematics Education*. 9(1): 143–155. doi:10.1080/14794800008520176.

Zaslavsky, O., Harel, G. & Manaster, A. 2006. A Teacher's Treatment of Examples as Reflection of Her Knowledge-Base. In J. Novotná, H. Moraová, M. Krátká & N. Stehliková (Eds.). *Proceedings of 30th Conference of the International Group for the Psychology of Mathematics Education*: Prague: Psychology of Mathematics Education. 1: 457–464.

Zodik, I. & Zaslavsky, O. 2008. Characteristics of the Teachers' Choice of Examples in and for the Mathematics Classroom. *Educational Studies in Mathematics*. 69: 165–182. doi:10.1007/s10649-008-9140-6.