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Mathematical Thinking in Differential Equations Among Pre-Service Teachers

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Abstract

Pre-service teachers should be equipped with mathematical thinking. Mathematical thinking is one of the most critical aims of the mathematics education has an extremely crucial role for enhancing then conceptual learning. Mathematical thinking is a process that enables students to expand the for the complexities of their ideas. This article describes mathematical thinking in differential equations among pre-service teachers. The study was carried out in the Faculty of Education, Universiti Teknologi Malaysia. A set of items was designed to test mathematical thinking level. The instrument was developed based on Mason's framework of mathematical thinking. Test items measure mathematical thinking namely, specializing, generalizing, conjecturing, and convincing. Descriptive statistics was applied for data analysis. The result indicated that students' mathematical thinking is in the low level mostly specializing. The implications of this research will lead to some recommendations and approaches to enhance mathematical thinking.

Keywords: Mathematical thinking; differential equation; pre-service teachers; Mason's framework of mathematical thinking; learning concepts

Abstrak

Guru pelatih harus dibekalkan dengan pemikiran matematik memandangkan pendidikan matematik mengiktirafnya sebagai tujuan utama untuk memantapkan pembelajaran konsep. Pemikiran matematik merupakan proses yang membenarkan pelajar mengembangkan ide yang kompleks. Artikel ini menerangkan pemikiran matematik dalam persamaan terbitan yang dibina oleh guru pelatih. Kajian dilaksanakan di Fakulti Pendidikan, Universiti Teknologi Malaysia. Item berbentuk soalan dan masalah matematik berpandukan kerangka kerja Mason dibina untuk menguji pemikiran matematik. Proses pemikiran matematik diuji dalam fasa *specializing, generalizing, conjecturing,* dan *convincing*. Analisis dilakukan secara deskriptif. Dapatan menggambarkan pemikiran matematik guru pelatih berada di peringkat rendah khususnya dalam fasa *specializing*. Implikasi kajian mengarah kepada cadangan pendekatan untuk mempertingkatkan pemikiran matematik.

Kata kunci: Pemikiran matematik; persamaan terbitan; guru pelatih; kerangka kerja pemikiran matematik Mason; pembelajaran konsep

■1.0 INTRODUCTION

The typical science or engineering student starts the university studies in mathematics with calculus and followed by differential equations in the second year. In recent years, dedicated articles that embrace changes in calculus have been written, but such changes can rarely be seen in the undergraduate level Differential Equations (DE) course. The research findings showed students' difficulties with mathematical ideas in DEs. These difficulties are based on finding closed form solutions to DEs through analytic techniques, which for long time have been the main approach in DEs classrooms, without emphasizing on mathematical thinking. The level of sophistication in mathematical thinking influences the development of appropriate instructional elements in classroom tasks and activities. Nevertheless, developing relevant questions to measure the students' level of mathematical thinking is complicated.

DE is one of the most important areas in mathematics (Firouzian, Zaleha, Roselainy, & Yudariah, 2012). Newton created DE to explain the natural phenomena (Hubbard, 1994). However, most traditional teaching methods ignore this fact and emphasize more on procedural knowledge (Engelbrecht, Bergsten, & Kågesten, 2009) rather than conceptual understanding in modeling and interpretation of DE solutions. This study, on the other hand, aimed at evaluating the teaching of DE course in undergraduate classes particularly for students majoring in mathematics education. Studies that emphasize on the effects of traditional teaching on conceptual learning in DE courses have not been carried out (Arslan, 2010). Traditional DE classrooms are carried out mainly emphasizing on symbolic manipulation, which means that instructors are interested in to obtain the correct algebraic solution for various types of DEs. Researchers suggest that teaching DEs should go beyond symbolic manipulation in order to achieve conceptual understanding. This work focused on investigating students understanding of DE in such learning environment.

1.1 Mathematical Thinking

Mathematical thinking is one of the most critical aims of the mathematics education which has an extremely crucial role for enhancing the conceptual learning. Mathematical thinking is a process that enables students to expand the complexities of their ideas. This process includes specializing, conjecturing, generalizing, and convincing (Mason, Burton, & Stacey, 1982). Combining several mental activities including exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing, and refuting characterize mathematical thinking. Watson and Mason (1998) omitted obvious activities which are common in mathematics activities such as calculating, solving, drawing and measuring. Therefore, it included only the mathematics activities which are related to thinking. Table 1 shows the main activities that are associated to mathematical thinking.

Table 1 Groups of mathematical activities associated to mathematical thinking **(**Anne Watson& John Mason 1998)

Mathematics activities					
Exemplifying	Completing	Comparing	Changing	Generalizing	Explaining
specializing	Deleting	Sorting organizing	Varying	Conjecturing	Justifying
	Correcting		Reversing		Verifying
			Altering		Convincing
					Refuting

1.2 Differential Equations

DEs are the language of nature, but there is a high inconsistency between this belief and the standard curriculum in undergraduate level courses, particularly in engineering courses. Most of the DEs cannot be solved using elementary formula (Hubbard, 1994). On the other hand, more complicated problems require both analytical and numerical strategies to be solved (Boyce, DiPrima, & Mitrea, 2001). The undergraduate curriculum in ordinary differential equations (ODEs) has commenced some crucial changes in terms of the visual and numerical aspects which require a higher level of mathematical thinking.

DEs are applied to models to explain real life problems in Economics, Physics, Biology, and other changing entities in real life. For example, changes in earth with time, velocity with distance, area of a circle with the size of its radius, path of a projectile with velocity, and bending of a beam with the applied loading. In the mathematics language, changing entities are derivative of functions (Tenenbaum, 1985) and this is one of the reasons that make mathematics important (Arslan, 2010). Thus, the most important reason to solve a DE is to understand something about an underlying process that the equation is used to model (Boyce, DiPrima, & Mitrea, 1992).

1.3 Traditional Ways of Teaching and Learning Differential Equations

There are two kinds of mathematical knowledge, namely procedural and conceptual knowledge in which the distinction between them influences the teaching of mathematics (Long, 2011). Procedural knowledge refers to mathematical definition, algorithms, and components (Hiebert, 1986) and it is divided into two parts that include format or language rules as the symbol representation system and knowledge of rules and algorithms useful in mathematical tasks. The way differential equations are taught in is so procedural/symbolic that the staff and students have no clue as to how to do anything other than teach procedures to solve specific types of equation such as separable, DEs with constant coefficients. Conceptual knowledge refers to the structure of mathematics and contains the understanding of mathematics concepts, definitions, and fact of knowledge (Long, 2011). Note also that procedural knowledge is acquired by practice (Gibson, 2008). Conceptual knowledge is gained in two ways, either by constructing the relationship between pieces of specific knowledge or by creating the relationship between current knowledge and prior knowledge. Both procedural and conceptual knowledge are crucial in teaching and learning mathematics. There are three levels of procedural knowledge with the first-order procedural knowledge level referring to skills that are automatic in nature. These skills are directed towards the completion of known goals (subjects-specific skills) like practical skills. Second-order procedural knowledge level skills are used to achieve unfamiliar goals and operate on specific procedures, for example problem solving and design. The third-order procedural knowledge skills are related to the control of other levels of procedural knowledge by ensuring the implementation is successful (Gibson, 2008). There is poverty of the teaching and the understanding of what differential equations actually are; for example, students failed to understand the solutions of DEs and their attempt to interpret the solutions of DEs were purely algebraic (Habre & Grundmeier, 2007).

All undergraduate students in science and engineering and student who are going to be a teacher must take DE as the core credit. This is especially important for pre-service teachers that will teach Calculus in high school. It is crucial for them to be equipped with mathematical thinking to focus on conceptual knowledge during their teaching without forgetting the underlying concepts. Therefore, it is important to understand the mathematical thinking level of students in mathematics education particularly in DEs, since this course includes modeling, solving, and interpreting real life problems. In addition, students confront new experiences in DEs. Therefore, it could be gathered the information about the level of mathematical thinking in this course. This study had been done based partly on previous works to explore students' conception on DE (Arslan, 2010) and difficulties faced by students in learning DEs (Rasmussen, 2001). The findings will be helpful to design and treat for next research on enhancing mathematical thinking level in DEs at undergraduate level.

■2.0 METHODOLOGY

This study tries to know students' mathematical thinking level particularly pre-service teachers. The data was gathered via questionnaires that consisted of a set of items constructed based on Questions and Prompt (Watson & Mason, 1998). The samples were chosen from the Faculty of Education at Universiti Teknologi Malaysia (UTM) during semester II 2011/2012. The samples were identified through purposive sampling as the researchers wanted to know the level of mathematical thinking in DE among pre-service teachers who had just taken the course. The students were selected in February and they had taken DE course in the previous December. The set of items was distributed to 52 pre-service teachers to be answered in an hour and 10 minutes, but only 51 questionnaires were gathered.

■3.0 INSTRUMENTATION

The ways that the data is gathered (i.e. instrumentation) are strategic for designing the research (Cohen, Manion, Morrison, & Morrison, 2007). Mathematical thinking power can be invoked through exemplifying, completing, correcting, sorting, changing, reversing, specializing, generalizing, deleting, comparing, organizing, varying, altering, conjecturing, convincing, and justifying. The mathematical structures are referred as definitions, facts, theorems and properties; examples and counter–examples; techniques and instructions; conjectures and problems; representation and notation; explanations, justifications, proof and reasoning; and links, relationships and connections. The structures of mathematics and mental mathematics activities in terms of mathematical thinking were combined to produce a grid as shown in Figure 1 (Watson & Mason, 1998). This grid was adopted to ask questions that would depict the students' mathematical thinking level; for examples, the question *Which of the following equations are differential equations?* is related to specializing level of mathematical thinking and the question *What is the definition of differential equation?* is related to explaining level of mathematical thinking.

	Exemplifying, specializing	*Completing, deleting and correcting*	*Comparing, sorting and organizing*	*Changing, varying, reversing and altering*	*Generalizing, conjecturing*	*Explaining, justifying, verifying convincing, refuting*
Definitions	**Show me an example of a**	**What must be added without affecting the equation?**	**What is the same and what different?**	**Change …so that it describes a…**	**Describe all … in one word, one sentence?**	**What is the definition of a DE?**
Facts, theorems and properties	**Tell me something that must be true if…**	**To be… complete and delete or correct properties as required?**	**What is the same and what is the different …?**	**What… do you get if you change …to…?**	**Is it always, sometimes, never, …**	**How can be sure that the solution given is true?**
Examples, counter-examples	**Which features of …make it an example of …**	**What (additional) properties must a … have so that it is an example of a**	**Sort the following according to the method used to solve**	**Change one aspect of the example so that…**	**Of what is … an example?**	**Tell me what is wrong with…**
Techniques and instruction	**Tell me how to solve….**	**Complete the solution, Delete unnecessary steps**	**Is … a technique for….?**	**Show that every member of the family of functions ... is a solution of…**	**What can change, and what must still stay the same?**	**How can we be sure that …?** **Is it always true that…?**
Conjectures and problems	**Tell me what your current conjecture is.**	**What other information is needed in order to answer the question..?**	**What is the same and what is different about…?**	**How would your conjecture change if you changed…to…?**	**Describe all… in one word , one sentence, one diagram**	**Why do you want to solve a differential equation?**
Representation and notations	**Show me a way to (write, depict, graph, use calculator)**	**Complete the missing parts in the …**	**Is … a useful notation for…?**	**Write an equivalent first ODE.**	**Of what is…. An example?**	**Justify why….has slope fields as shown in the diagram?**
Explanations, *Justification, proofs and reasoning*	**How would you explain(justify)**	**Provide and insert missing steps?** **Correct the following steps?**	**What is the same and what is different about…?**	**Explain, justify, prove if we know …but do not know …**	**Describe possible solutions for the slope field.**	**Could you give a general rule to solve differential equations graphically?**
Links, relationships and connections	**Give an example of relationship between … and ….**	**…and … are the same in that both are … but different in that …**	**Make a connected chain from…**	**What if….?**	**Of what is…. An example?**	**Explain connection between graphical and analytical solutions of a differential equation.**

Figure 1 Questions & prompt for mathematical thinking (Adapted from Anne Watson& John Mason 1998)

The instrument for this study consisted of three questions which were Q1, Q2, and Q3. Q1 had several parts that were numbered alphabetically from A to M such as Q1-A, Q1-B, so on. Each part in Q1 had some items numbered numerically; for example Q1-A-1 means item 1 from part A in Q1. Question 2 had three parts including Q2-F, Q2-S, and Q2-I. Question 2 was drawn to identify formulating (F), solving (S), and interpreting (I) of real life problem. These were referred alphabetically where Q2-F referred to formulating; Q2-S referred to solving of DEs; and Q2-I referred to interpretation of solution. Question 3 only had one part, which was Q3-1.

4.0 RESULT

Students' answers to the items of the questionnaire divided into three parts including *Right answer*, *Wrong answer*, and *No answer*. Descriptive statistics was used to data analysis such as frequency and percentage. Figure 2 shows the proportion of answers to DEs problem into three types of answers including *Right answer, Wrong answer,* and *No answer*. The horizontal axis shows the questions in the instrument and the vertical axis presents the percentage of different answers to the corresponding questions. As can be seen, from Q1-A-1to Q1-B-6 were answered by a majority of participants, which were related to *exemplifying* and *specializing* of mathematical thinking level. Q2 and Q3 were not answered too much that were related to higher level of mathematical thinking such as *generalizing*, *conjecturing*, and *convincing*.

Table 2 presents the frequencies and percentages of three types of answers given by the participants in this study. A great majority of the respondents answered Q1-A-1 correctly (98.03 %). 34 students responded incorrectly to Q1-A-3 (66.66 %). Approximately, all students answered all items in Q1. Q1-B-2 was answered correctly by a majority of students (94.11 %). However, 64 respondents (68.62 %) answered Q1-B-1 wrongly. Similar situation was observed for Q1-B-3 where around 34 of them provided the wrong answer. Many respondents did not respond to items of Q1-C, for example 38 students (74.50 %) did not answer to Q1-C-2. Nevertheless, 9.80 % of the respondents answered Q1-C-2 correctly. 76.47 % of the respondents provided the right answer to Q1-D-1, but 18 (35.29%) participants replied wrongly to Q1-D-2. Interestingly, equal number of respondents responded to item Q1-D-3 correctly and incorrectly (41.17%)

The percentage of *No answer* items increased from Q1-E-1 to Q3-I. As can be seen in Table 2, 39 respondents (76.47 %) did not answer Q1-E-2 and 23.53 % of the respondents answered wrongly. There was no correct answer provided by any respondent for this question as well. There were some similarities between Q1-F-1 and the items in Q1-E-2 where majority of the respondents chose not to provide any answer; 33 respondents (64.70 %) did not answer Q1-F-1. Although 21.57 % of the respondents did answer but provided the wrong answer, there were still 13.72 % of them who answered correctly. A great number of respondents (68.63 %) left item Q1-G-1 unanswered as well and hardly any respondent replied correctly (5.88 %). The rest of the respondents responded to this item wrongly (25.49 %). For item Q1-I-1, the percentages of wrong and right answer were approximately the same. However, it was not answered by 11 respondents (21.57%). The number of respondents who did not answer (56.86 %) Q1-J-1 was greater than those who answered this item correctly (7.84 %).

Items Q1-K-1, Q1-L-1, Q1-L-2, and Q1-M-1 were not answered by a great majority of respondents with the percentages reported as 72.54 %, 92.15 %, 94.11 %, and 90.19 % respectively. Hardly any of these items were responded correctly (3.92 %). However, the percentages of the right answers were higher than the wrong answers. A considerable numbers of respondents also answered Q2-F correctly (72.54 %) and only 1.96 % of the respondents answered wrongly. The number of questions unanswered increased from Q2 to Q3 (from 50.98 % to 88.23 %). No respondent answered Q1-E-2, Q2-I, and Q3-1 correctly. The number of wrong answers provided for items Q1-E-2 and Q2-I were the same (21.57 %).

The percentages of answers related to each level of mathematical thinking of the participants are presented in Table 3. The majority of students answered the questions, which they were engaging in exemplifying and specializing level of mathematical thinking (66.94%). However, hardly any of students answered the questions related to generalizing and conjecturing level of mathematical thinking (5.88%).

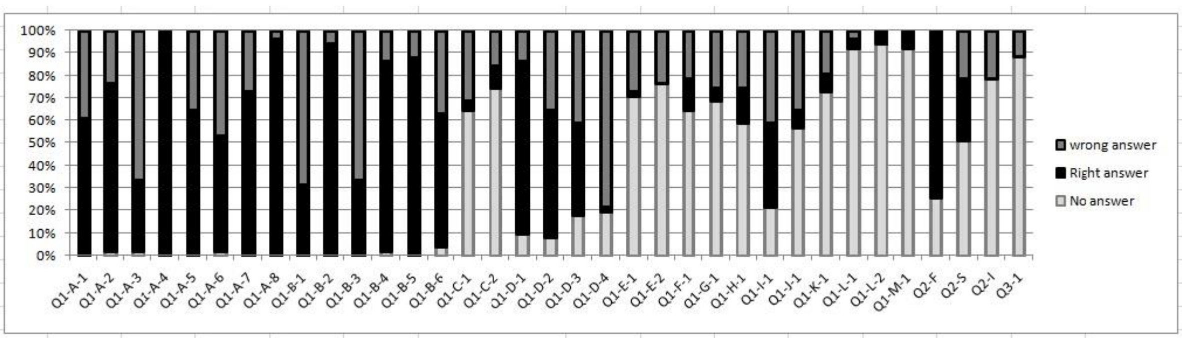


Figure 2 Differential equations proportions answered by participants

Table 2 Percentage of answers to DE questions provided by pre-service teachers

Question	NO answer	Right answer	Wrong answer
Q1-A-1	0 (0%)	31(60.79%)	20(39.21%)
Q1-A-2	1(1.96%)	38(74.50%)	12(23.52%)
Q1-A-3	1(1.96%)	16(31.37%)	34(66.66%)
Q1-A-4	0(0%)	50(98.03%)	1(1.96%)
Q1-A-5	0(0%)	33(64.70%)	18(35.29%)
Q1-A-6	1(1.96%)	26(50.98%)	24(47.05%)
Q1-A-7	0 (0%)	37(72.54%)	14(27.45%)
Q1-A-8	0 (0%)	49(96.07%)	2(3.92%)
Q1-B-1	0 (0%)	16(31.37%)	35(68.62%)
Q1-B-2	0 (0%)	48(94.11%)	3(5.88%)
Q1-B-3	0(0%)	17(33.33%)	34(66.66%)
Q1-B-4	1(1.96%)	43(84.31%)	7(13.72%)
Q1-B-5	0(0%)	44(86.27%)	6(11.76%)
Q1-B-6	2(3.92%)	30(58.82%)	19(37.25)
Q1-C-1	33(64.70%)	2(3.92%)	16(31.37%)
Q1-C-2	38(74.50%)	5(9.80%)	8(15.68%)
Q1-D-1	5(9.80%)	39(76.47%)	7(13.72%)
Q1-D-2	4(7.84%)	29(56.86%)	18(35.29%)
Q1-D-3	9(17.64%)	21(41.17%)	21(41.17%)
Q1-D-4	10(19.61%)	1(1.96%)	40(78.43%)
Q1-E-1	36(70.59%)	1(1.96%)	14(27.45%)
Q1-E-2	39(76.47%)	0(0%)	12(23.53%)
Q1-F-1	33(64.70%)	7(13.72%)	11(21.57%)
Q1-G-1	35(68.63%)	3(5.88%)	13(25.49%)
Q1-H-1	30(58.82%)	8(15.68%)	13(25.49%)
Q1-I-1	11(21.57%)	19(37.25%)	21(41.18%)
Q1-J-1	29(56.86%)	4(7.84%)	18(35.29%)
Q1-K-1	37(72.54%)	4(7.84%)	10(19.60%)
Q1-L-1	47(92.15%)	2(3.92%)	2(3.92%)
Q1-L-2	48(94.11%)	2(3.92%)	1(1.96%)
Q1-M-1	46(90.19%)	3(5.88%)	1(1.96%)
Q2-F	13(25.49%)	37(72.54%)	1(1.96%)
Q2-S	26(50.98%)	14(27.45%)	11(21.57%)
Q2-I	40(78.43%)	0(0%)	11(21.57%)
Q3-1	45(88.23%)	0(0%)	6(11.76%)

Table 3 The percentage of mathematical thinking level of participants

Mathematical Thinking Level	Exemplifying Specializing	Completing Deleting Correcting	Comparing Sorting Organizing	Changing Varying Reversing Altering	Generalizing Conjecturing	Explaining Justifying Verifying Convincing Refuting
Percentage	34.14 (66.94%)	3.5 (6.86%)	22.5 (43.27%)	2.66 (5.23%)	3 (5.88%)	6 (11.76%)

5.0 DISCUSSION AND CONCLUSION

This study was carried out to identify the level of mathematical thinking in DEs among pre-service teachers. The findings showed that most students' mathematical thinking level is low, particularly in specializing. This has been highlighted in all answered items concerning specializing; for example items in Q1 (Q1-A-4). However, the findings were rather disappointing because the questions aimed to recognize modeling and interpreting abilities were not answered by the participants or the answers were wrong. This might be caused by two reasons: the students forgot the procedure of solving DEs or they lacked the knowledge about modeling and interpretation of solutions. The first issue can be overcome by changing the teaching and learning strategies in terms of retention in DEs. Kwon (2005) found that using inquiry-oriented teaching strategies has a positive effect on retention in DE classroom. However, one of the most significant findings emerged from this study is that students do not have enough knowledge to solve DE problems both procedurally and conceptually. The findings confirmed the research finding by Rasmussen (2001) who further classified students' difficulties in learning DE into two parts: firstly, the solution dilemma which refers to the interpretation of solutions and secondly, the lack of intuitions and images which refers to graphical and numerical items in DEs. Similar problems appeared in this study; for instance, hardly any participant answered item Q1-M-1 that required them to find the solution curves based on the given graph. In addition, no one answered Q3-1 correctly.

Furthermore, Arslan (2010) concluded that the learning type of most students in learning DEs is procedural where the students fail to develop conceptual learning accordingly. Apparently, based on the statistics in Table 2, the frequency of conceptual items were quite low such as Q2-F, Q2-I, and Q3-1(0 %). The other major finding was that almost all students did not respond to Q1-G-1 and Q2-I. This was because they did not emphasize on graphical solution and tend to calculate the symbolic solution only. The present study confirmed previous findings and contributed additional evidence which showed that the students should be engaged in thinking in mathematics. This will enable better development of appropriate teaching and learning tools to manipulate mathematics concepts symbolically and graphically. An implication of these findings is that the elements of DEs instruction should be taken into account including the curriculum content, the role of teacher, the assessment, and the teaching environment.

References

Arslan, S. 2010. Traditional Instruction of Differential Equations and Conceptual Learning. *Teaching Mathematics and its Applications*. 29(2): 94–107.

Cohen, L., Manion, L., Morrison, K., & Morrison, K. R. B. 2007. *Research Methods in Education*. Psychology Press.

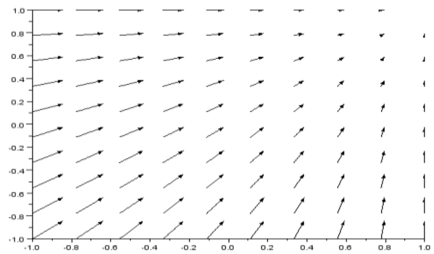
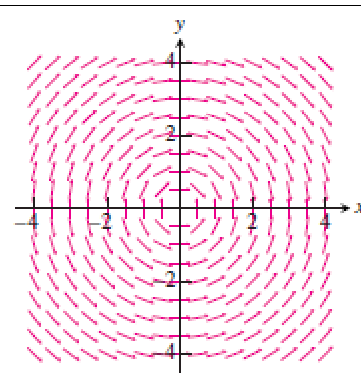
Kwon, O. N., Rasmussen, C., & Allen, K. 2005. Students' Retention of Mathematical Knowledge and Skills in Differential Equations. *School Science and Mathematics*. 105(5): 227–239.

Rasmussen, C. L. 2001. New Directions in Differential Equations: A Framework for Interpreting Students' Understandings and Difficulties. *The Journal of Mathematical Behavior*. 20(1): 55–87.

Watson, A., & Mason, J. 1998. *Questions and Prompts for Mathematical Thinking*: Association of Teachers of Mathematics.

Appendix

No	Question
Q1 **Procedurally Oriented Items**	**A. Which of the following equations are differential equations?** **1.** $y' + ky = -t \quad k \in \mathbb{Z}$ ☐ **2.** $y = y^2 - t$ ☐ **3.** $y'' = ay'^2$ ☐ **4.** $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 7y = 0$ ☐ **5.** $(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$ ☐ **6.** $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ ☐ **7.** $dx = t\sqrt{x}\,dt$ ☐ **8.** $\frac{dy}{dx} = \frac{x+y}{x-y+2}$ ☐
	B. Which of the following equations are linear differential equations and which are non linear differential equations? **1.** $y''' + 5y' + 9 = 0$ .... **2.** $\frac{dy}{dx} = -5y$ … … … … … … … … … .. **3.** $\frac{dy}{dx} = -5y(1-y)$ … … … … … … …. **4.** $y''y = e^x$ … … … … … … … … … … **5.** $y'^2 - y = \sin x$ … … … … … … … … …. **6.** $y'' - y' - 3y = 3x\sin x$ … … … … … … … … … ….
	~~C.~~ What must be added without affecting the equations? **1.** $\frac{dy}{dx} = 2x + y \Rightarrow z' = \cdots$ **2.** $\frac{dy}{dx} = \frac{y^5+x^5}{x^4y^4} \Rightarrow y' - \cdots y = \cdots$
	D. Sort the following according to the method used to solve. **1.** $\frac{dy}{dx} = (1+e^{-x})(y^2-1)$ .... **2.** $2(x+2y)dx + (y-x)dy = 0 \quad , \quad y(1) = 0$ .... **3.** $x(1-\sin y)dy = (\cos x - \cos y - y)dx$ .... **4.** $\begin{cases} \frac{dy}{dx} = xy^3 + x^2 \\ y(0) = 0 \end{cases}$ ....
	E. Write an equivalent first-order differential equation and initial condition for *y*. **1.** $x = \ln y' + \sin y'$ … … … … … … … … … … … … … … … … .. **2.** $xy' = y + \sqrt{x^2 + y^2}$ … … … … … … … … … … … … … … ….. **F. Show that every member of the family of functions $y = \frac{c}{x} + 2$ is a solution of the first-order differential equation $\frac{dy}{dx} = \frac{1}{x}(2-y)$.** ....

	G. Describe possible solutions for the slope field: 
	H. How can you be sure that the solution given above is true? ………………………………………………………………………………………………… **I. What is the definition of differential equation?** ………………………………………………………………………………………………… **J. Why do we need to include $\frac{dy}{dx}$ in the definition of a differential equation?** ………………………………………………………………………………………………….. **K. Could you give a general rule to solve differential equations graphically?** ………………………………………………………………………………………………… **L. Consider differential equation $y' = f(x, y)$** Is it always true that …? **1. In analytical view, $y_1(x)$ is a solution for the differential equation if $y'_1(x) = f(x, y_1(x))$. …………………** **2. In geometric view, $y_1(x)$ is a solution for the differential equation if the slope of $y_1(x)$ is equal to the slope of direction field $f(x, y_1(x))$. …………………** **M. What are the similarities and differences between analytical view and geometrical view to solve a differential equation?** …………………………………………………………………………………………………..
Q2 Modeling Items	**A. When a cake is removed from an oven, its temperature is measured at 300ºF. Three minutes later its temperature is 200ºF. How long will it take for the cake to cool off to a room temperature of 70ºF?**
Formulating the differential equation:	
Solving the differential equation	
Interpretation the solution	
Q3 Graphical items	**A. Justify why $y' = -\frac{x}{y}$ has slope fields as shown in the diagram?** (a)