Jurnal Teknologi, 49(D) Dis. 2008: 143–156

HYBRID CONTROL SCHEMES USING INPUT SHAPING AND FULL-STATE FEEDBACK FOR A FLEXIBLE ROBOT MANIPULATOR

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Abstract. This paper presents investigations into the development of hybrid control schemes with the applications of input shapers in the command shaping techniques for vibration control and input tracking of a flexible robot manipulator. A constrained planar single-link flexible manipulator is considered and the dynamic model of the system is derived using the assumed mode method. To study the effectiveness of the controllers, initially a Linear Quadratic Regulator (LQR) is developed for control of rigid body motion. This is then extended to incorporate input shaper control schemes for vibration control of the system. The input shapers are designed based on the properties of the system. Simulation results of the response of the manipulator to the shaped inputs are presented in time and frequency domains. Performances of the hybrid control schemes are examined in terms of level of input tracking capability, vibration reduction, time response specifications and robustness to parameters uncertainty. The effects of derivative order of the input shaper on the performance of the system are investigated. Finally, a comparative assessment of the hybrid control schemes to the system performance is presented and discussed. The proposed hybrid controllers are capable of reducing the system vibration while maintaining the input tracking performance of the manipulator.

Keywords: Flexible manipulator; hybrid control; input shaping; simulation; vibration control

Abstrak. Artikel ini mempersembahkan kajian berkaitan pembangunan skim kawalan hibrid melalui aplikasi pembentukan input dalam teknik arahan bentuk untuk mengawal getaran dan jejakan input bagi sebuah manipulator robot boleh lentur. Sebuah kekangan fleksibel manipulator satu-hubung telah digunakan dan model sistem dinamik telah dibentuk menggunakan kaedah anggaran mod. Untuk mengkaji keberkesanan pengawal, pada mulanya kawalan kuadratik lelurus telah dibina untuk mengawal pergerakan badan yang tegar. Ini kemudiannya dilanjutkan lagi dengan menambah skim kawalan pembentukan input untuk mengawal getaran pada sistem tersebut. Pembentukan input telah direka berdasarkan ciri-ciri sistem. Keputusan simulasi bagi sambutan manipulator tersebut telah dipersembahkan dalam domain masa dan frekuensi. Prestasi sistem kawalan hibrid telah diperiksa dari segi paras keupayaan jejakan input, pengurangan getaran, spesifikasi sambutan masa dan kelasakan terhadap parameter yang berubah-ubah. Kesan tertib pembezaan pada pembentukan input terhadap prestasi sistem telah dikaji. Akhirnya, satu penilaian perbandingan terhadap prestasi sistem kawalan hibrid telah dipersembahkan dan dibincangkan.

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Kawalan hybrid yang dicadangkan dalam projek ini berkemampuan untuk mengurangkan sistem getaran dan mengekalkan prestasi jejakan input bagi sebuah manipulator robot boleh lentur.

Kata kunci: Manipulator robot boleh lentur; kawalan hibrid; pembentukan input; simulasi; kawalan getaran

1.0 INTRODUCTION

The control strategies for flexible manipulator systems can be classified as feedforward and feedback control. Numerous feedforward control strategies have been proposed for control of vibration. These include utilisation of Fourier expansion as the forcing function to reduce peaks of the frequency spectrum at discrete points [1], derivation of a shaped torque that minimises vibration and the effect of parameter variations [2], development of computed torque based on a dynamic model of the system [3], utilisation of single and multiple-switch bang–bang control functions [4] and construction of input functions from ramped sinusoids or versine functions [5]. Moreover, feedforward control schemes with command shaping techniques have also been investigated in reducing system vibration. These include filtering techniques based on low-pass, band-stop and notch filters [6 – 9] and input shaping [10 – 11]. In filtering techniques, a filtered torque input is developed on the basis of extracting the input energy around the natural frequencies of the system. Previous experimental studies on a single-link flexible manipulator have shown that higher level of vibration reduction and robustness can be achieved with input shaping technique than with filtering techniques. However, the major drawback of the feed-forward control schemes is their limitation in coping with parameter changes and disturbances to the system [12]. Moreover, this technique requires relatively precise knowledge of the dynamics of the system.

On the other hand, feedback control techniques use measurements and estimates of the system states and changes the actuator input accordingly for control of rigid body motion and vibration suppression of the system. Several approaches utilizing closed-loop control strategies to control flexible manipulators have been reported. These include linear state feedback control [13], adaptive control [14], robust control techniques based on *H*-infinity [15], variable structure control [16] and intelligent control based on neural networks [17] and fuzzy logic control schemes [18].

This paper presents an investigation into the development of hybrid control schemes for input tracking and vibration control of a single-link flexible manipulator. A constrained planar single-link flexible manipulator is considered. Hybrid control schemes based on feedforward with full state feedback controllers are investigated. In this work, feedforward control based on input shaping with Zero-Vibration (ZV) and Zero-Vibration-Derivative-Derivative (ZVDD) shapers are considered. To demonstrate the effectiveness of the proposed control schemes, initially a LQR controller is developed for control of rigid body motion of the manipulator. This is then extended to incorporate the proposed input shapers for control of vibration of the manipulator. This paper provides a comparative assessment of the performance of hybrid control schemes with different derivative order of input shapers.

2.0 THE FLEXIBLE MANIPULATOR SYSTEM

The single-link flexible manipulator system considered in this work is shown in Figure 1, where X_0OY_0 and XOY represent the stationary and moving coordinates frames respectively, while τ represents the applied torque at the hub. E, I, ρ, A, I_h and m_p represent the Young modulus, area moment of inertia, mass density per unit volume, cross-sectional area, hub inertia and payload mass of the manipulator, respectively. In this study, an aluminium type flexible manipulator of dimensions $900 \times 19.008 \times 3.2004$ mm³, $E = 71 \times 10^9$ N/m², $I = 5.1924 \times 10^{11}$ m^4, $\rho = 2710$ kg/m^3 $mp = 0$ kg and $Ih = 5.8598 \times 10^{-4}$ kgm^2 is considered.

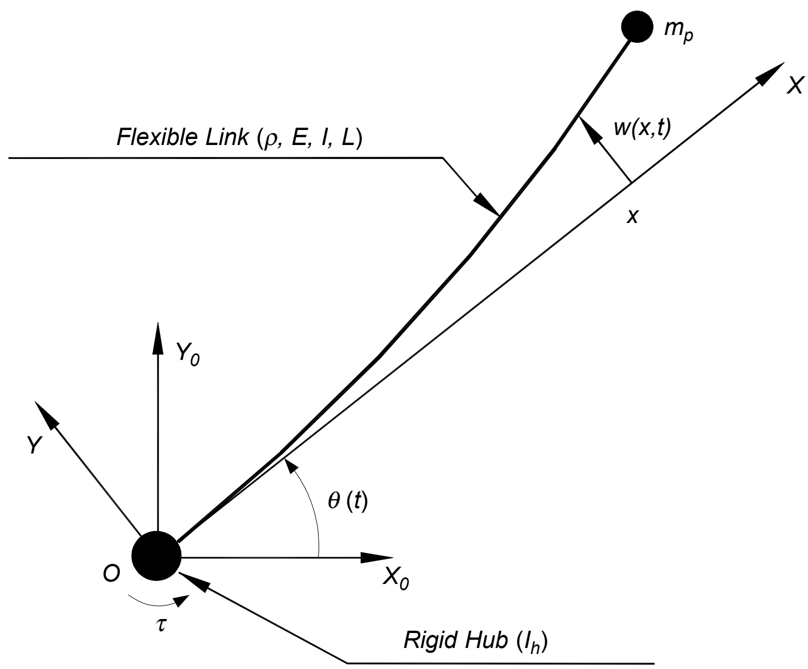


Figure 1 Description of the flexible manipulator system

3.0 MODELLING OF THE FLEXIBLE MANIPULATOR

This section provides a brief description on the modelling of the flexible robot manipulator system, as a basis of a simulation environment for development and assessment of the input shaping control techniques.

The assume mode method with two modal trajectory is considered in characterising the dynamic behaviour of the manipulator incorporating structural damping and hub inertia. Further details of the description and derivation of the dynamic model of the system can be found in Subudhi *et al.* [19]. The dynamic model has also been validated with experimental exercises where a close agreement between both theoretical and experimental results has been achieved in Martin *et al.* [20].

Considering revolute joints and motion of the manipulator on a two-dimensional plane, the kinetic energy of the system can thus be formulated as

$$T = \frac{1}{2}(I_H + I_b)\dot{\theta}^2 + \frac{1}{2}\rho\int_0^L \left(\dot{v}^2 + 2\dot{v}x\dot{\theta}\right)dx \tag{1}$$

where I_b is the beam rotation inertia about the origin O_0 as if it were rigid. The potential energy of the beam can be formulated as

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx \tag{2}$$

This expression states the internal energy due to the elastic deformation of the link as it bends. The potential energy due to gravity is not accounted for since only motion in the plane perpendicular to the gravitational field is considered.

Next, to obtain a closed-form dynamic model of the manipulator, the energy expressions in (1) and (2) are used to formulate the Lagrangian $L = T - U$. Assembling the mass and stiffness matrices and utilising the Euler-Lagrange equation of motion, the dynamic equation of motion of the flexible manipulator system can be obtained as

$$M\,\ddot{Q}(t) + D\,\dot{Q}(t) + KQ(t) = F(t) \tag{3}$$

where *M, D* and *K* are global mass, damping and stiffness matrices of the manipulator respectively. The damping matrix is obtained by assuming the manipulator exhibit the characteristic of Rayleigh damping. *F*(*t*) is a vector of external forces and *Q*(*t*) is a modal displacement vector given as

$$Q(t) = \begin{bmatrix}\theta & q_1 & q_2 & \cdots & q_n\end{bmatrix}^T = \begin{bmatrix}\theta & q^T\end{bmatrix}^T \tag{4}$$

$$F(t) = \begin{bmatrix}\tau & 0 & 0 & \cdots & 0\end{bmatrix}^T \tag{5}$$

Here, q_n is the modal amplitude of the *i*th clamped-free mode considered in the assumed modes method procedure and *n* represents the total number of assumed modes. The model of the uncontrolled system can be represented in a state-space form as

$$\begin{aligned} \dot{x} &= A_x + B_u \\ y &= C_x \end{aligned} \tag{6}$$

with the vector $x = \begin{bmatrix} \theta & \dot{\theta} & q_1 & q_2 & \dot{q}_1 & \dot{q}_2 \end{bmatrix}^T$ and the matrices A and B are given by

$$\begin{aligned} \mathbf{A} &= \left[\begin{array}{c|c} 0_{3\times3} & I_{3\times3} \\ \hline -M^{-1}K & -M^{-1}D \end{array}\right], \quad && \mathbf{B} = \left[\begin{array}{c} 0_{3\times1} \\ \hline M^{-1} \end{array}\right] \\ \mathbf{C} &= \left[\begin{array}{c|c} I_{1\times3} & 0_{1\times3} \end{array}\right], && \mathbf{D} = [0] \end{aligned} \tag{7}$$

4.0 LINEAR QUADRATIC REGULATOR (LQR) CONTROL SCHEME

A more common approach in the control of manipulator systems involves the utilization linear quadratic regulator (LQR) design [21]. In order to design the LQR controller a linear state-space model of the flexible manipulator was obtained by linearising the equations of motion of the system. For a linear time invariant (LTI) system

$$\dot{x} = Ax + Bu, \tag{8}$$

The technique involves choosing a control law $u = \psi(x)$ which stabilizes the origin (i.e., regulates x to zero) while minimizing the quadratic cost function

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt \tag{9}$$

where $Q = Q^T \geq 0$ and $R = R^T \geq 0$. The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function.

The matrices Q and R are called the state and control penalty matrices, respectively. If the components of Q are chosen large relative to those of R, then deviations of x from zero will be penalized heavily relative to deviations of u from zero. On the other hand, if the components of R are large relative to those of Q, then control effort will be more costly and the state will not converge to zero as quickly.

A famous and somewhat surprising result due to Kalman is that the control law which minimizes J always takes the form $u = \psi(x) = -Kx$. The optimal regulator for a LTI system with respect to the quadratic cost function above is always a linear control law. With this observation in mind, the closed-loop system takes the form

$$\dot{x} = (A - BK)x \tag{10}$$

and the cost function J takes the form

$$J = \int_0^\infty x(t)^T Qx(t) + (-Kx(t))^T R(-Kx(t))dt \tag{11}$$
$$= \int_0^\infty x(t)^T (Q + K^T RK)x(t)dt$$

Assuming that the closed-loop system is internally stable, which is a fundamental requirement for any feedback controller, allows the computation value of the cost function for a given control gain matrix K.

5.0 INPUT SHAPING CONTROL SCHEME

The design objectives of input shaping are to determine the amplitude and time locations of the impulses in order to reduce the detrimental effects of system flexibility. These parameters are obtained from the natural frequencies and damping ratios of the system. The input shaping process is illustrated in Figure 2. The corresponding design relations for achieving a zero residual single-mode vibration of a system and to ensure that the shaped command input produces the same rigid body motion as the unshaped command yields a two-impulse sequence with parameter as

$$t_1 = 0,\ t_2 = \frac{\pi}{\omega_d},\ A_1 = \frac{1}{1+K},\ A_2 = \frac{K}{1+K} \tag{12}$$

where

$$K = e^{-\zeta\pi / \sqrt{1-\zeta^2}},\ \omega_d = \omega_n\sqrt{1-\zeta^2}$$

with ω_n and ζ representing the natural frequency and damping ratio, and t_j and A_j are the time location and amplitude of impulse j, respectively. The robustness of the input shaper to errors in natural frequencies of the system can be increased by solving the derivatives of the system vibration equation. This yields a four-impulse sequence with parameter as

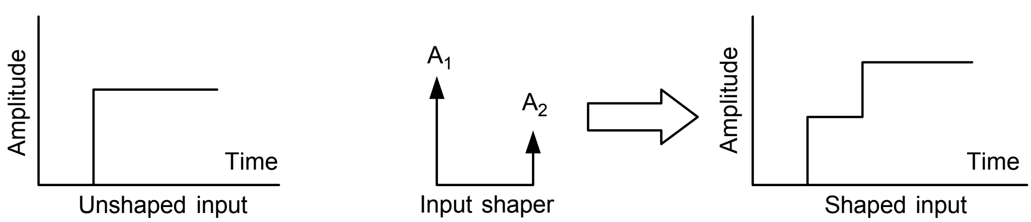


Figure 2 Illustration of input shaping technique

$$t_1 = 0,\ t_2 = \frac{\pi}{\omega_d},\ t_3 = \frac{2\pi}{\omega_d},\ t_4 = \frac{3\pi}{\omega_d}$$

$$A_1 = \frac{1}{1+3K+3K^2+K^3},\quad A_2 = \frac{3K}{1+3K+3K^2+K^3}$$
$$A_3 = \frac{3K^2}{1+3K+3K^2+K^3},\quad A_4 = \frac{K^3}{1+3K+3K^2+K^3} \tag{13}$$

where K as is equation (12).

6.0 IMPLEMENTATION AND RESULT

In this investigation, hybrid control schemes for tracking capability and vibration suppression of the flexible manipulator are examined. Initially, a Linear Quadratic Regulator (LQR) is designed. This is then extended to incorporate input shaping scheme for control of vibration of the system.

The tracking performance of the Linear Quadratic Regulator (LQR) applied to the flexible manipulator systems was investigated by firstly setting the value of vector K and $\bar{N}$ which determines the feedback control law and for elimination of steady state error capability respectively. Using the LQR function in the Matlab, both vector K and $\bar{N}$ were set as

$$K = \begin{bmatrix} 3.1623 & 4.1006 & 42.7052 & 0.4956 & 0.6449 & 6.7017 \end{bmatrix}$$
$$\bar{N} = \begin{bmatrix} 3.1623 \end{bmatrix}$$

The natural frequencies were obtained by exciting the flexible manipulator with an unshaped unit step reference input under LQR controller. The input shapers were designed for pre-processing the unit step reference input and applied to the system in a closed-loop configuration, as shown in Figure 3.

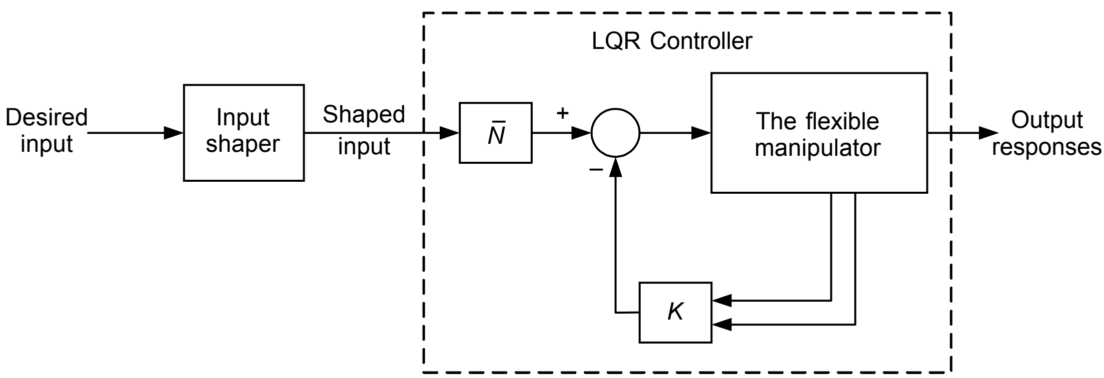
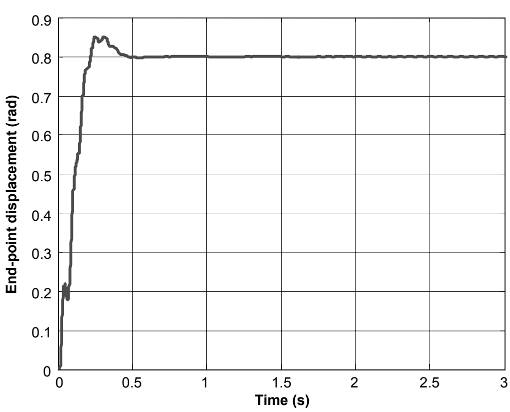


Figure 3 Block diagram of hybrid control schemes configuration

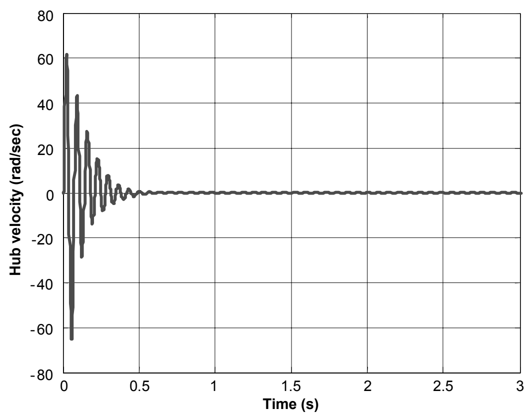
6.1 Linear Quadratic Regulator Control

In this work, the input is applied at the hub of the flexible manipulator. The unit step command is required in order for the manipulator to follow a trajectory at 0.8 radian. The first two modes of vibration of the system are considered, as these dominate the dynamic of the system.

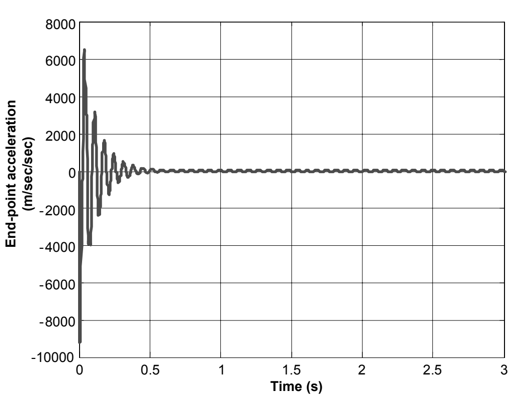
Figure 4 shows the responses of the flexible manipulator system to the unshaped unit step reference input in time-domain and frequency domain (spectral density). These results were considered as the system response to the unshaped input under tracking capability and will be used to evaluate the performance of the input shaping techniques. The steady-state end-point trajectory of 0.8 radian for the flexible manipulator system was achieved within the rise and settling times and overshoot of



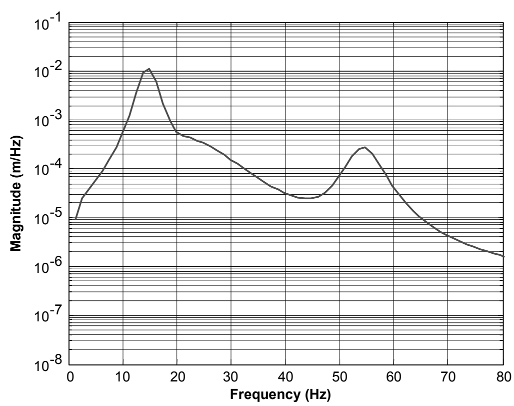
(a) End-point displacement



(b) Hub-velocity



(c) End-point acceleration



(d) Spectral density of residual motion

Figure 4 Response of the flexible manipulator to the unshaped unit step torque input under LQR controller

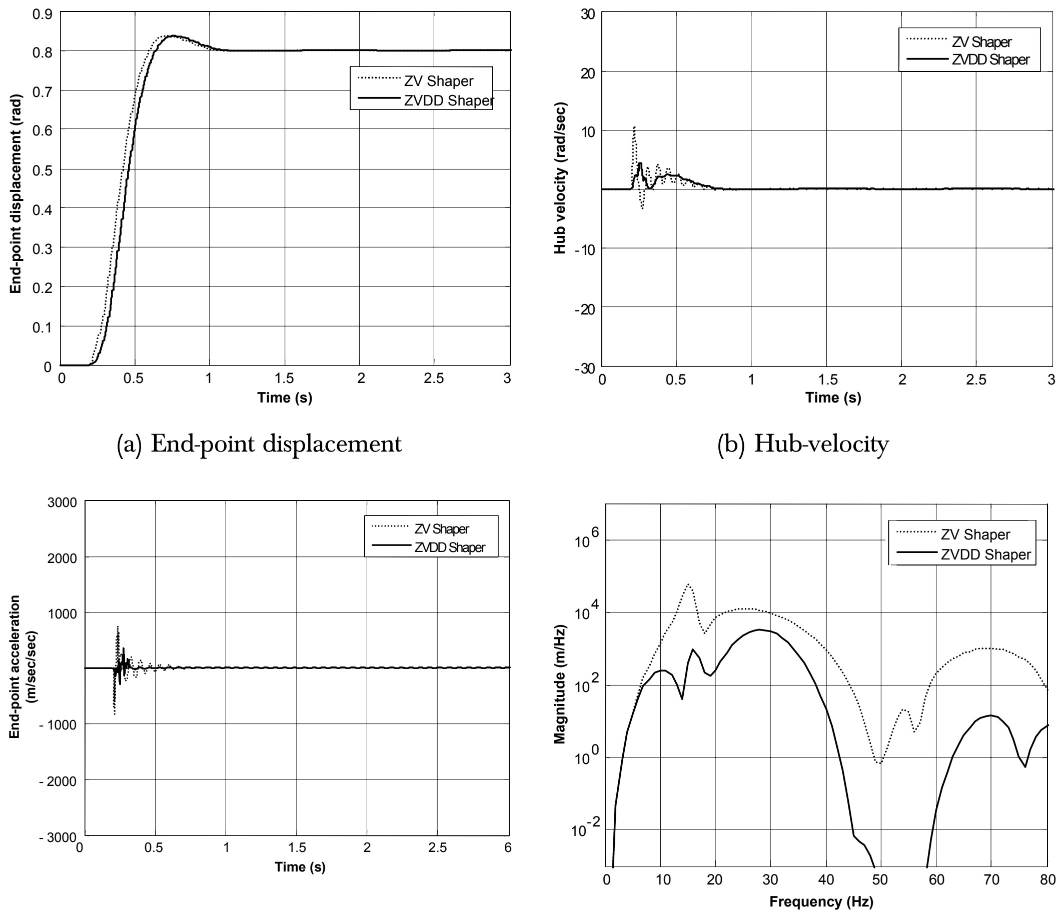
0.269 s, 0.678 s and 5.11 % respectively. Resonance frequencies of the system were obtained by transforming the time-domain representation of the system into frequency domain using power spectral density. The vibration frequencies of the flexible manipulator system were obtained as 16 and 56 Hz for the first two modes of vibration.

6.2 Hybrid Control

In the case of hybrid control schemes, a ZV (two-impulse sequence) and ZVDD (four-impulse sequence) shapers were designed for two modes utilising the properties of the system. With the exact natural frequencies of 16 and 56 Hz, the time locations and amplitudes of the impulses were obtained by solving equations (12) and (13). For evaluation of robustness, input shapers with error in natural frequencies were also evaluated. With the 30% error in natural frequency, the system vibrations were considered at 20.8 and 72.8 Hz for the two modes of vibration. Similarly, the amplitudes and time locations of the input shapers with 30% erroneous natural frequencies for both the ZV and ZVDD shapers were calculated. For digital implementation of the input shapers, locations of the impulses were selected at the nearest sampling time of simulation.

The system responses of the flexible manipulator to the shaped unit step input with exact natural frequencies using LQR control with ZV and ZVDD shapers are shown in Figure 5. It can be noted that the vibration of the end-point displacement, hub velocity and end-point acceleration responses were significantly reduced. Table 1 summarises the levels of vibration reduction of the system responses at the first two modes in comparison to the LQR control. Higher levels of vibration reduction were obtained using LQR control with ZVDD shaper as compared to the case with ZV shaper. However, with ZVDD shaper, the system response is slower. Hence, it is evidenced that the speed of the system response reduces with the increase in number of impulse sequence. The corresponding rise time, settling time and overshoot of the end-point trajectory response using LQR control with ZV and ZVDD shapers with exact natural frequencies are depicted in Table 1. It is noted that a slower end-point trajectory response with less overshoot, as compared to the LQR control, was achieved.

To examine the robustness of the shapers, the shapers with 30% error in vibration frequencies were designed and implemented to the flexible manipulator system. Figure 6 shows the response of the manipulator to the shaped input using ZV and ZVDD shapers with erroneous natural frequencies. The vibrations of the system were considerable reduced as compared to the system with LQR control. However, the level of vibration reduction is slightly less than the case with exact natural frequencies. Table 1 summarises the levels of vibration reduction with erroneous natural frequencies in comparison to the LQR control. The time response specifications of the end-point trajectory with error in natural frequencies are



(a) End-point displacement

(b) Hub-velocity

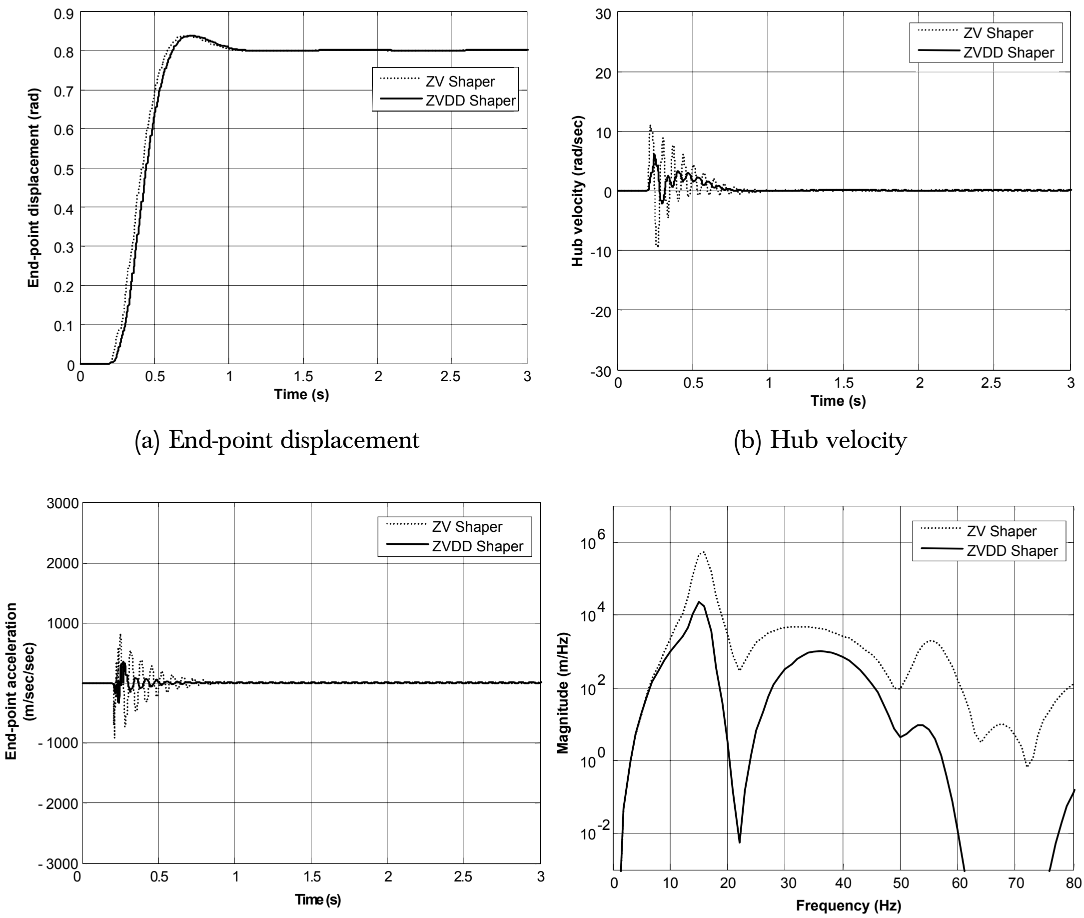
(c) End-point acceleration

(d) Spectral density of residual motion

Figure 5 Response of the flexible manipulator with exact natural frequencies

Table 1 Level of vibration reduction of the end-point acceleration and specifications of end-point trajectory response for hybrid control schemes

Frequency	Types of shaper	Attenuation (dB) of vibration end-point acceleration		Specifications of end-point trajectory response		
		Mode 1	Mode 2	Rise time (s)	Settling time (s)	Overshoot (%)
Exact	ZV	37.39	78.85	0.257	0.703	4.78
	ZVDD	69.81	202.9	0.262	0.745	4.64
Error	ZV	14.86	27.31	0.260	0.696	4.89
	ZVDD	44.38	81.27	0.259	0.730	4.71



(a) End-point displacement

(b) Hub velocity

(c) End-point acceleration

(d) Spectral density of residual motion

Figure 6 Response of the flexible manipulator with 30% error in natural frequencies

summarised in Table 1. It is noted that the response is slightly faster for the shaped input with error in natural frequencies than the case with exact frequencies. However, the overshoot of the response is slightly higher than the case with exact frequencies. Significant vibration reduction was achieved for the overall response of the system to the shaped input with 30% error in natural frequencies, and hence proved the robustness of the input shapers.

6.3 Comparative Performance Assessment

By comparing the results presented in Table 1, it is noted that the higher performance in the reduction of vibration of the system is achieved using LQR control with ZVDD shaper. This is observed and compared to the LQR control with ZV shaper at the first two modes of vibration. For comparative assessment, the levels of vibration reduction of the end-point acceleration using LQR control with both ZV and ZVDD shapers are shown with the bar graphs in Figure 7. The result shows that, highest level of vibration reduction is achieved in hybrid control schemes using the ZVDD shaper, followed by the ZV shaper for both modes of vibration. Therefore, it can be concluded that the LQR control with ZVDD shapers provide better performance in vibration reduction as compared to the LQR control with ZV shapers in overall.

Comparisons of the specifications of the end-point trajectory responses of hybrid control schemes using both ZV and ZVDD shapers are summarised in Figure 8 for the rise times and settling times. It is noted that the differences in rise times of the

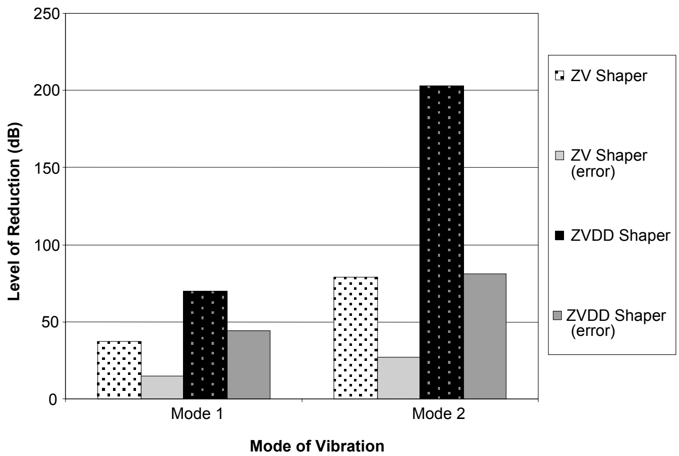


Figure 7 Level of vibration reduction with exact and erroneous natural frequencies using ZV and ZVDD shapers

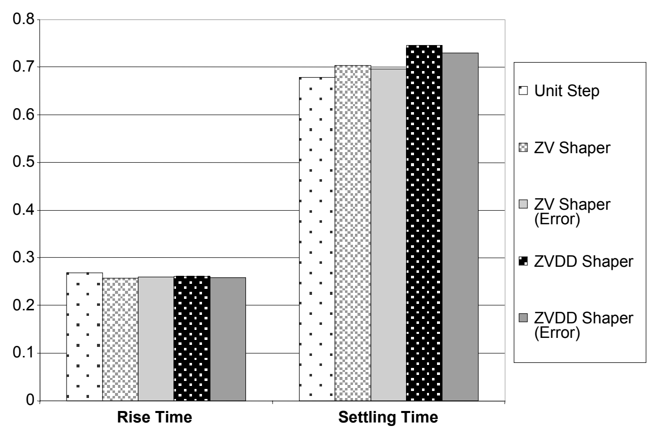


Figure 8 Rise and settling times of the end-point trajectory with exact and erroneous natural frequencies using ZV and ZVDD shapers

end-point trajectory response for the LQR control with ZV and ZVDD shapers are negligibly small. However, the settling time of the end-point trajectory response using the LQR control with ZV shaper is faster than the case using the ZVDD shaper. It shows that, by incorporating more number of impulses in hybrid control schemes resulted in a slower response.

Comparison of the results shown in Table 1 for the shaping techniques with error in natural frequencies reveals that the higher robustness to parameter uncertainty is achieved with the LQR control with ZVDD shaper. For both case of the ZV and ZVDD shapers, errors in natural frequencies can successfully be handled. This is revealed by comparing the magnitude of vibration of the system in Figure 7. Comparisons of the end-point trajectory response using LQR control with ZV and ZVDD shapers with erroneous natural frequencies are summarised in Figure 8. The results show a similar pattern as the case with exact natural frequencies. The system response with ZVDD shaper provides slightly slower responses than the ZV shaper.

7.0 CONCLUSION

The development of hybrid control schemes for input tracking and vibration suppression of a flexible manipulator has been presented. The hybrid control schemes have been developed based on LQR with feedforward control using the ZV and ZVDD input shapers technique. The proposed control schemes have been implemented and investigated within the simulation environment of a single-link flexible manipulator. The performances of the control schemes have been evaluated in terms of level of input tracking capability, vibration reduction, time response specifications and robustness. Acceptable input tracking capability and vibration suppression have been achieved with both control strategies. A comparison of the results has demonstrated that the LQR control with input shaping using ZVDD shapers provide higher level of vibration reduction as compared to the cases using ZV shapers. In term of speed of the responses, ZVDD shapers results in a slower tracking response with less overshoot. It has also demonstrated that input shaping technique is very robust to error in natural frequencies especially with higher number of impulses. It is noted that the proposed hybrid controllers are capable of reducing the system vibration while maintaining the input tracking performance of the manipulator.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge Universiti Teknologi Malaysia (UTM) and the Ministry of Science, Technology, and Innovation (MOSTI) for the research funding under eScience Research Fund (03-01-06-SF0026, Vote 79031).

REFERENCES

[1] Aspinwall, D.M., 1980. Acceleration Profiles for Minimising Residual Response. *Transactions of ASME: Journal of Dynamic Systems, Measurement and Control.* 102(1): 3-6.

[2] Swigert, J. C. 1980. Shaped Torque Techniques. *Journal of Guidance and Control*. 3(5): 460-467.

[3] Moulin, H. and E. Bayo. 1991. On the Accuracy of End-point Trajectory Tracking for Flexible Arms by Non-causal Inverse Dynamic Solution. *Transactions of ASME: Journal of Dynamic Systems, Measurement and Control*. 113: 320-324.

[4] Onsay, T. and A. Akay. 1991. Vibration Reduction of a Flexible Arm by Time Optimal Open-loop Control. *Journal of Sound and Vibration*. 147(2): 283-300.

[5] Meckl, P. H. and W. P. Seering. 1990. Experimental Evaluation of Shaped Inputs to Reduce Vibration of a Cartesian Robot. *Transactions of ASME: Journal of Dynamic Systems, Measurement and Control*. 112(6): 159-165.

[6] Singhose, W. E., N. C. Singer and W. P. Seering. 1995. Comparison of Command Shaping Methods for Reducing Residual Vibration. *Proceedings of European Control Conference.* Rome: 1126-1131.

[7] Tokhi, M. O. and H. Poerwanto. 1996. Control of Vibration of Flexible Manipulators Using Filtered Command Inputs. *Proceedings of International Congress on Sound and Vibration*. St. Petersburg, Russia: 1019-1026.

[8] Pao, L.Y. 2000. Strategies for Shaping Commands in the Control of Flexible Structures. *Proceedings of Japan-USA-Vietnam Workshop on Research and Education in Systems*, *Computation and Control Engineering*. Vietnam: 309-318.

[9] Tokhi, M. O. and A. K. M. Azad. 1996. Control of Flexible Manipulator Systems. *Proceedings of IMechE-I: Journal of Systems and Control Engineering.* 210: 283-292.

[10] Singer, N. C. and W. P. Seering. 1990. Preshaping Command Inputs to Reduce System Vibration. *Transactions of ASME: Journal of Dynamic Systems, Measurement and Control.* 112(1): 76-82.

[11] Mohamed, Z. and M. O. Tokhi. 2002. Vibration Control of a Single-link Flexible Manipulator Using Command Shaping Techniques. *Proceedings of IMechE-I: Journal of Systems and Control Engineering.* 216(2): 191-210.

[12] Khorrami, F., Jain, S. and Tzes, A., 1994. Experiments on Rigid Body-based Controllers with Input Preshaping for a Two-link Flexible Manipulator. *IEEE Transactions on Robotics and Automation.* 10(1): 55-65.

[13] Cannon, R. H. and E. Schmitz. 1984. Initial Experiment on the End-point Control of a Flexible One-link Robot. *Int. J. Robotics Res*. 3(3): 62-75.

[14] Hasting, G. G. and W. J. Book. 1990. A Linear Dynamic Model for Flexible Robot Manipulators. *IEEE Control Systems Mag*. 10: 29-33.

[15] Feliu, V., K. S. Rattan and H. B. Brown. 1987. Adaptive Control of a Single-link Flexible Manipulator. *IEEE Control Systems Mag*. 7: 61-64.

[16] Moser. 1993. A. N. Designing Controllers for Flexible Structures with *H*-infinity/*m*-synthesis. *IEEE Control Systems Mag.* 13(2): 79-89.

[17] Gutierrez, L. B., P. L. Lewis and J. A. Lowe. 1998. Implementation of a Neural Network Tracking Controller for a Single Flexible Link: Comparison with PD and PID Controllers. *IEEE Trans. Ind Electronics.* 45(3): 307-318.

[18] Moudgal, V. G., K. M. Passino and S. Yurkovich. 1994. Rule Based Control for a Flexible-link Robot. *IEEE Trans. Control Systems Technol*. 2(4): 392-405.

[19] Subudhi, B. and A. S. Morris. 2002. Dynamic Modelling, Simulation and Control of a Manipulator with Flexible Links and Joints. *Robotics and Autonomous Systems*. 41: 257-270.

[20] Martins, J. M., Z. Mohamed, M. O. Tokhi, J. Sá da Costa and M. A. Botto. 2003. Approaches for Dynamic Modelling of Flexible Manipulator Systems. *IEE Proceedings-Control Theory and Application.* 150(4): 401-411.

[21] Ogata, K. 1997. *Modern Control Engineering*. Upper Saddle River, NJ: Prentice-Hall International.