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A NOVEL ANALYTICAL SOLUTION OF MERKEL EQUATION FOR COUNTERFLOW COOLING TOWERS

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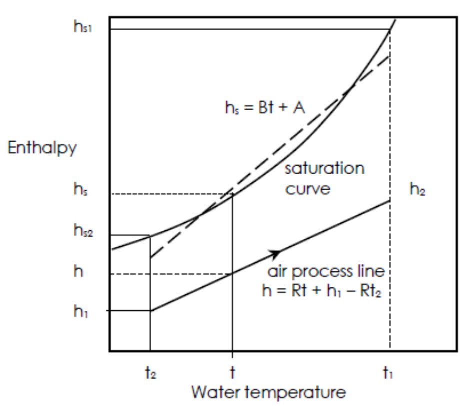
Graphical abstract



Abstract

Accurate performance analysis of direct-contact air-water counterflow cooling towers is important in practice for energy conservation and cost savings purposes. In general, accurate analysis can often be made using Merkel's theory. However, a simple and accurate analytical solution of the Merkel equation is not yet available. In this paper, a novel and accurate analytical solution is presented, obtained from direct integration of the Merkel equation. The new method gives excellent agreement when compared with the use of standard Merkel's method when predicting the outlet water temperatures with maximum root-mean-square error of 0.2% for a practical range of operating conditions. The new method also predicts the outlet water temperatures for four experimental cooling towers with maximum root-mean-square error of 0.8%. A limited validation shows that the method is also applicable to performance analysis of an actual crossflow cooling tower. The new method is a valuable addition to the existing methods of solving the Merkel equation.

Keywords: Cooling towers, enthalpy potential, NTU, Merkel equation, evaporative cooling

Abstrak

Analisis perlakuan yang jitu terhadap menara penyejukan sentuhan-terus udara-air aliran berlawan adalah penting di lapangan untuk tujuan penjimatan tenaga dan kos. Secara umum, analisis yang jitu boleh dibuat menggunakan teori Merkel. Namun, penyelesaian beranalisis yang mudah dan jitu terhadap persamaan Merkel belum lagi ditemui. Kertas kerja ini membentangkan satu penyelesaian beranalisis yang novel dan jitu, diperolehi menerusi pengamiran secara lansung persamaan Merkel. Kaedah baru ini memberikan hasil yang baik apabila dibandingkan dengan kaedah Merkel yang piawai untuk meramal suhu air yang keluar daripada menara pendingin dengan punca ralat punca-min-kuasa-dua 0.2% untuk julat operasi yang praktikal. Kaedah baru ini juga meramalkan suhu air yang keluar empat menara pendingin ujikaji dengan ralat punca-min-kuasa-dua maksimum sebanyak 0.8%. Satu kajian yang terhad menunjukkan kaedah ini boleh digunakan untuk analisis perlakuan menara pendingin aliran bersilang yang sebenar. Kaedah baru ini merupakan satu penambahan bernilai kepada kaedah sedia ada untuk penyelesaian persamaan Merkel.

Kata kunci: Menara pendingin, keupayaan entalpi, NTU, persamaan Merkel, penyejukan tersejat

1.0 INTRODUCTION

Direct-contact air-water counterflow cooling towers are widely used in the air conditioning and other industries where a large amount of heat needs to be rejected to the environment. The cooling towers not only reject a large amount of heat efficiently, but at the same time conserve water usage by recycling the process water [1]. The thermal performance of these towers has been the subject of great interest in the past, and the present, where mathematical models of varying degrees of complexity have been proposed and scrutinized. Among the numerous models proposed, Merkel's theory [2] for cooling tower analysis is arguably the oldest, simplest, widely used, and yet reasonably accurate model that is available. The solutions to Merkel equation have been obtained using numerical, graphical, and simplified analytical methods.

Merkel's theory is described in the ASHRAE handbook [1] for analyzing the performance of direct-contact air-water counterflow cooling towers, with an in-depth review of the theory, its promise and shortcoming by Baker and Shryock [3]. The theory was employed by Simpson and Sherwood [4] in their experimental study on the performance of small-scale mechanical draft cooling towers, when calculating the empirical coefficients of total heat transfer. Assuming linearized saturated air enthalpy variation with water temperature, the effectiveness-NTU method has been proposed as an alternative to the number of transfer units (NTU) analysis based on Merkel's theory [5,6]. Braun *et al.* [6] have found that Merkel's model produced results that were close to those obtained from effectiveness-NTU and detailed models. Kloppers and Kroger [7] stated that Merkel and effectiveness-NTU methods predict practically identical tower outlet water temperatures, which is expected since both methods are based on the same simplifying assumptions [8,9].

The application of Merkel's theory narrows down to calculating the dimensionless number, known as the NTU. When the NTU is calculated for a set of hypothetical conditions, it is known as the required coefficient, and it represents the degree of difficulty [1]. On the contrary, the NTU calculated for a set of test results is known as the available coefficient [1].

The NTU calculation invoking Merkel's theory is usually made using numerical integration methods, mainly due to the nonlinear relation between saturated air enthalpy and the water temperature. A four-point Tchebycheff integration formula is often used to produce results with an acceptable degree of accuracy and is adopted by the Cooling Technology Institute for its testing protocol [10]. Singh and Das [11] have also used the simple four-point integration method to calculate the NTU in their multi-objective optimization study on the performance of an induced draft cooling tower. Benton *et al*. [12] have used the four-point Tchebycheff integration method and found that cooling tower performance using Merkel's theory compared favorably with some of the well-known empirical and analytical methods. As an alternative, the numerical integration method of Stoecker and Jones [13] requires dividing the tower into a finite number of control volumes, with uniform water temperature differential across each control volume. The added advantage of this approach is that it is possible to calculate the state points of air through the tower. A similar approach was taken by Tomas *et al.* [14] in their experimental work to evaluate the performance of new alternative cooling tower fill materials. With additional work, Sutherland [15] has solved the governing differential equations using a fourth order Runge-Kutta method, and performed the integration using a four-point Gaussian Quadrature technique. On the other hand, Mansour and Hasab [16] have employed Simpson's rule to evaluate the NTU to verify their new effectiveness-NTU correlation model. In a related work, Picardo and Variyar [17] have used MATLAB's integration routine when solving Merkel's equation. Merkel's theory has also been adopted by Costelloe and Finn [18], Serna-Gonzalez *et al.* [19], Pontes *et al.* [20], Mohiuddin and Kant [21], Navaro *et al*. [22], and Ruiz *et al*. [23] to calculate the number of transfer units. The related integration in Merkel's NTU calculation can also be made with the aid of Stevens diagram, as described by Threlkeld [24]. These and many other works based on Merkel's theory are a testimony to the widespread acceptance of the theory, and given its simplicity and acceptable accuracy, it has become the de facto theory of choice.

When the saturated air enthalpy is expressed as a second-order polynomial function of water temperature, Clouse [25] has solved Merkel equation analytically to arrive at a direct closed-form solution for the counterflow cooling tower NTU. However, the reverse process of calculating the unknown outlet water temperature requires a trial-and-error iterative type of calculation, that is, there is no direct solution.

In this study, a novel and accurate analytical solution has been presented for estimating the outlet water temperature of direct-contact air-water counterflow cooling towers. The simple solution obtained from direct integration of the governing Merkel integral equation has a huge advantage over the traditional numerical integration methods currently being used.

2.0 METHODOLOGY

Merkel's theory for the analysis direct-contact air-water counterflow cooling towers is based on the following assumptions [1,3,7].

1. Water loss by evaporation is negligible.
2. The Lewis number is unity.
3. The specific heats for air and water are constant.
4. Thermal resistance in the water film is neglected.

5. The rate of heat transfer from the water to the air is proportional to the difference between the saturated air enthalpy at the water temperature and the free airstream unsaturated air enthalpy.
6. The cooling tower is adiabatic.
7. Steady flow conditions prevail.

Figure 1 shows the schematic diagram of a counterflow direct-contact air-water cooling tower. Ambient air enters at the bottom of the tower with mass flow rate of G kg/s, and enthalpy h_1 kJ/kg. Hot water enters at the top of the tower with mass flow rate of *L* kg/s, temperature t_1 °C, and leaves at a lower temperature t_2 °C. A graphical description of the enthalpy transfer between saturated air at the water temperature, and the unsaturated free airstream is shown in Figure 2. The *NTU* for the cooling tower, employing Merkel's theory is given by [1,3,7]

$$NTU = c_w \int_{t_2}^{t_1} \frac{dt}{h_s - h} \qquad (1)$$

where h_s is the saturated air enthalpy and h is the unsaturated free air stream enthalpy. Equation 1 is known as the Merkel equation, in integral form [9]. The saturated air enthalpy is a non-linear function of water temperature, making analytical evaluation of the integral challenging. Clouse [25] utilized a second-order polynomial function for the saturated air enthalpy and obtained closed form analytical solutions for the *NTU*. However, when the outlet water temperature, t_2 is the only unknown, there is no simple solution to the analytical equations, except for the case where the discriminant of the denominator in the integral of Equation 1 is equal to zero. As a result, in this paper, a new and accurate linearized saturated air enthalpy model is proposed to overcome this limitation. In the new model, the saturated air enthalpy is assumed to vary linearly with water temperature, as depicted in Figure 2.

$$h_s = Bt + A \qquad (2)$$

The gradient *B* is calculated from,

$$B = \frac{h_{s1} - h_{s2}}{t_1 - t_2} \qquad (3)$$

where h_{s1} and h_{s2} are the actual saturated air enthalpy at t_1 and t_2, respectively. The intercept, A is modeled after Maclaine-cross and Banks [26], who used it for the variation of saturated air humidity ratio with water temperature,

$$A = [2(h_{s2} + h_{sm}) - h_{s1}]/3 - Bt_2 \qquad (4)$$

where h_{sm} is the actual saturated air enthalpy at the mean water temperature, $0.5(t_1 + t_2)$. Equation 2 would be an exact least square fit if the actual saturated air enthalpy curve were a parabola [26].

The saturated air enthalpy, h_s J/kg, at water temperature, t °C, is calculated using the standard psychrometric equations [27]

$$h_s = c_a t + w_s(c_v t + h_{fg0}) \qquad (5)$$

$$w_s = 0.622 P_s / (P_{atm} - P_s) \qquad (6)$$

$$\ln P_s = -5800.2206T^{-1} + 1.3914993 - 0.04860239T + 0.41764768\text{x}10^{-4}T^2 - 0.14452093\text{x}10^{-7}T^3 + 6.5459673 \ln(T) \qquad (7)$$

where c_a = 1006 J/kg °C, is the specific heat of dry air; w_s is the saturated air humidity ratio; c_v = 1880 J/kg °C, is the specific heat of water vapor; h_{fg0} = 2501000 J/kg, is latent heat of vaporization of liquid water at 0°C; P_{atm} = 101325 Pa, is the atmospheric pressure; P_s is the partial pressure of saturated water vapor; and $T = t + 273.15$ K, is the water temperature. Alternatively, the simpler polynomial equation of Stoecker and Jones [13] can used to obtain h_s.

It can be shown that the equation for the air process line, for the free airstream enthalpy is given by [25]

$$h = Rt + h_1 - Rt_2 \qquad (8)$$

where h_1 is inlet air enthalpy, $R = c_w L/G$ is the gradient of the air process line, and c_w is the specific heat of liquid water. Equation 1 can then be written as follows,

$$NTU = c_w \int_{t_2}^{t_1} \frac{dt}{(B-R)t + A - h_1 + Rt_2} \qquad (9)$$

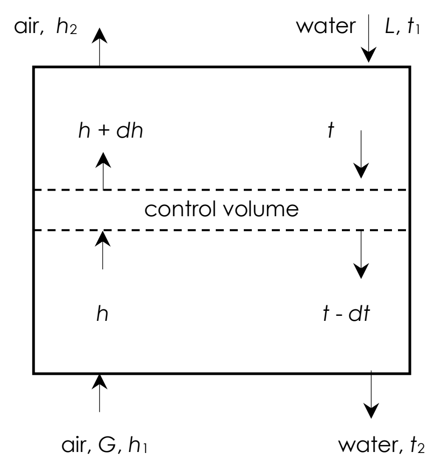


Figure 1 Schematic diagram of counterflow cooling tower

Equation 9 is integrated analytically to give the *NTU* when the gradient of the air process line, *R* and the gradient of the linearized saturated air enthalpy line, *B* are not equal,

$$NTU = \frac{c_w}{(B-R)} ln\left[\frac{(B-R)t_1+Rt_2+A-h_1}{Bt_2+A-h_1}\right] \quad (10)$$

and, when the outlet water temperature is the only unknown, Equation 10 is rearranged to give,

$$t_2 = \frac{(B-R)t_1 + (A-h_1)(1-e^{\phi})}{Be^{\phi}-R} \quad (11)$$

where $\phi = (B-R)NTU/c_w$.

When the gradients of the air process line and the linearized saturation line are equal, $R = B$, integration of Equation 9 gives

$$NTU = \frac{c_w(t_1-t_2)}{Rt_2+A-h_1} \quad (12)$$

and, when the outlet water temperature is the only unknown, Equation 12 is rearranged to give,

$$t_2 = \frac{\frac{c_w}{NTU}t_1 - A + h_1}{\frac{c_w}{NTU} + R} \quad (13)$$

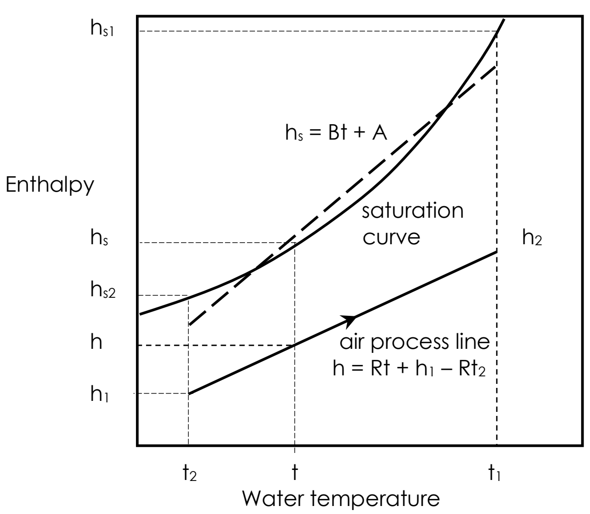


Figure 2 Merkel analysis on enthalpy-temperature diagram

Equation 10 through Equation 13 are the new explicit solutions to Equation 1, the Merkel equation. Most of the calculations will use Equations 10 and 11, and very rarely use Equations 12 and 13, for which R is exactly equal to B.

The *NTU* calculation is a simple process. When *L/G*, t_1, t_2 and h_1 are known, such as from experimental data, calculate A and B using Equation 3 and Equation 4. Either Equation 10 or Equation 12 is then used to calculate the *NTU*. The actual saturated air enthalpy is calculated using Equations 5 through 7.

In the opposite case, when *NTU*, *L/G*, t_1 and h_1 are known, the outlet water temperature prediction is carried out in two steps. Firstly, an estimate of the outlet water temperature, t_2 is made. The saturation wet bulb temperature, t_{wbs} which corresponds to the inlet air enthalpy, h_1 should serve as the estimate. Equations 5 through 7 are solved by iteration to obtain t_{wbs} when h_s is replaced with h_1 and t is replaced with t_{wbs}. Next, calculate A and B, and the outlet water temperature is then calculated using Equation 11 or Equation 13. In the second step, repeat the first step but using the new estimate of the outlet water temperature in place of t_{wbs}. The last calculated value of t_2 is then the desired solution. An attempt to perform more than two iterations would cause the RMSE and the single-point errors to have a slight increase in magnitude. A converged solution is really the result of an attempt to conform the actual saturated air enthalpy function to the linearized equation when it should be the other way around. In other words, a linearized equation can never replace a non-linear saturated air enthalpy function but can only come close to replicating it.

3.0 RESULTS AND DISCUSSION

3.1 Comparison with Standard Merkel's Method

The simple analytical method in Section 2.0 was validated against the standard Merkel's method for predicting the outlet water temperatures for *NTU* between 0.5 and 2.5 in an increment of 0.25, and *L/G* between 0.5 and 3 in an increment of 0.125, for a total of 210 data points. The atmospheric pressure was 101.325 kPa, and the inlet water temperatures were 30°C, 35°C, and 40°C. The inlet air enthalpy was 60 kJ/kg, 75 kJ/kg, and 90 kJ/kg. In the standard Merkel calculations, the tower was divided into vertical control volumes, where each control volume had a 0.05°C water temperature differential. The numerical integration of Equation 1 was carried out using Simpson's one-third rule. When the *NTU* was specified, the standard Merkel calculations involved a systematic trial-and-error method to predict the outlet water temperature, where convergence was achieved when the predicted outlet water temperatures gave *NTU* values with errors of magnitudes lower than 0.001% of the known *NTU*.

Figure 3 show the errors in the outlet water temperature predictions by the analytical method, when compared to the solutions of the standard Merkel's method for the extreme case at h_1 = 60 kJ/kg, and t_1 = 40°C. In can be seen that the errors are small with root mean square error (RMSE) of 0.20%, and the largest single-point error is 1.35%, at the lowest *L/G* of 0.5 and largest *NTU* of 2.5, where the cooling range, (t_1-t_2) is the largest at 16.1°C. The RMSE was calculated from the 210 single-point errors for each pair of *NTU* and *L/G* combination as described in the preceding paragraph. In general, the error increases as the cooling range shown in Figure 4 increases. A large cooling range means that Equation 2 becomes less effective at replicating the actual saturation curve due to the increased curvature.

Table 1 summarizes the errors for several h_1, and t_1 combination. All things equal, an increase in the inlet air enthalpy reduces both the RMSE and the maximum single-point errors. However, an increase in the inlet water temperature increases both the RMSE, and the maximum single-point errors. The increased errors are due in part to an increase in the cooling range, $t_1 - t_2$.

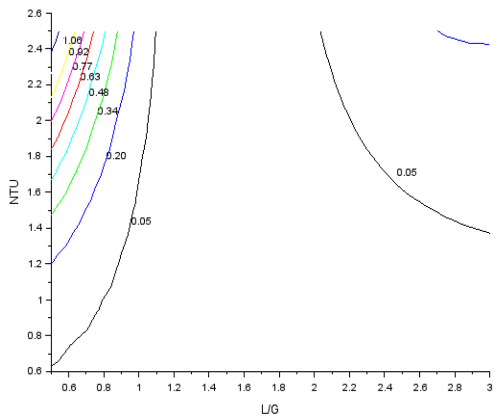


Figure 3 Contours of percentage errors in outlet water temperatures by analytical method when compared to standard Merkel solutions (t_1 = 40 °C, h_1 = 60 kJ/kg)

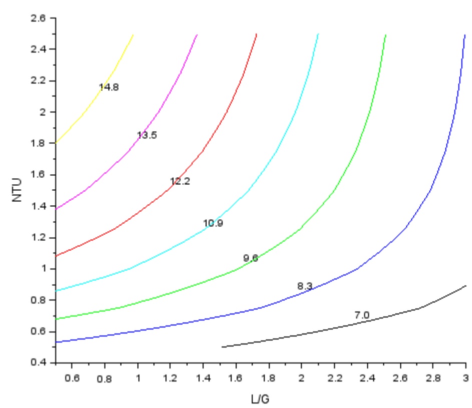


Figure 4 Contours of cooling range (°C) estimates by analytical method (t_1 = 40 °C, h_1 = 60 kJ/kg)

Table 1 Errors in predicted outlet water temperatures by analytical method at different inlet water temperatures, and inlet air enthalpies ($0.5 \leq L / G \leq 3$, $0.5 \leq NTU \leq 2.5$)

h_1 (kJ/kg)	t_1 (°C)	RMSE (%)	Max. error (%)
60	40	0.20	1.35
(t_{wbs} = 20.5°C)	35	0.08	0.52
	30	0.03	0.17
75	40	0.17	1.07
(t_{wbs} = 24.6°C)	35	0.06	0.37
	30	0.03	0.10
90	40	0.13	0.78
(t_{wbs} = 28.0°C)	35	0.04	0.23
	30	0.03	0.06

The single-point errors can be reduced by dividing the cooling tower into two or more sections. As an example, divide the cooling range into two equal intervals, with the mid-point water temperature of $t_x = 0.5(t_1 + t_2)$. The partial NTU_1 is then calculated for the first section between t_x and t_1. The *NTU* for the second section between t_2 and t_x is then obtained by subtracting NTU_1 from the whole tower *NTU*, and the outlet water temperature is then calculated for the second section. The calculations are made in two iterations. The initial estimate of the outlet water temperature is obtained from the analysis for the one-section whole cooling tower. This technique gives the best results for *L/G* lower than about 1.5. This is not a problem since for *L/G* greater than 1.5, the maximum errors in the outlet water temperature predictions are always lower than 0.2% for the one-section whole tower analysis. By using the technique, the maximum error of 1.35% in Figure 3, and Table 1 is reduced to 0.21%. This is possible because when the tower is divided into two sections, the linearized saturated air enthalpy equation in each section becomes closer to the actual saturated air enthalpy due to reduced curvature of the saturation curve in each section.

The overall excellent agreement with the standard Merkel's method when predicting the outlet water temperatures proves the reliability and accuracy of the new analytical method. It must be realized that in the comparison study, the analytical method used Merkel's *NTU* (since comparisons were made to the more accurate Merkel's outlet water temperatures) to predict the outlet water temperatures using Equation 11 or Equation 13 and has performed remarkably well. In Section 3.2 both empirical *NTU* calculations, and prediction of the outlet water temperatures were made using the analytical equations described in Section 2.0, exclusively.

3.2 Empirical Validation

The new analytical method in Section 2.0 was validated using the experimental data of Simpson and Sherwood [4]. Firstly, the empirical number of transfer units were calculated using the explicit equations described in Section 2.0, and using least square regression analysis, the coefficient c and exponent *n* were obtained for the following correlation [1].

$$NTU = c\left[\frac{L}{G}\right]^n \tag{14}$$

Secondly, using the same experimental data, Equation 14 was used to calculate the empirical *NTU*, and the outlet water temperatures were predicted using Equation 11 or Equation 13.

The outlet water temperature prediction was a two-step process as explained at the end of Section 2.0. Table 2 shows the empirical correlation coefficients and exponents for the data of Simpson and Sherwood [4]. The coefficient of determination, r^2 was quite low for tower R1, where data scatter was quiet large. However, the resulting empirical *NTU* correlation was sufficiently accurate as it enabled accurate prediction of the outlet water temperatures to be made.

Table 2 Empirical correlation coefficients

Tower	c	n	r^2
R1	1.501	-0.390	0.40
R2	1.225	-0.688	0.93
M1	0.873	-0.866	0.96
M2	1.016	-0.682	0.90

Figure 5 shows the predicted outlet water temperatures for the four experimental cooling towers by the new analytical method. The RMSE of the predicted outlet water temperatures for cooling towers R1, R2, M1, and M2 are 0.78%, 0.58%, 0.79%, and 0.58%, respectively. As a comparison, when using the standard Merkel's method, the RMSE are 0.79%, 0.66%, 0.83%, and 0.57%, respectively, for towers R1, R2, M1, and M2. The close agreements between both methods, and the low RMSE values confirm the validity of the new method as a practical tool in performance analysis of direct-contact air-water counterflow cooling towers. The magnitude of the errors is remarkably small, considering that for the experimental data, all energy balances checked within 15%, and the majority checked within 8% [4].

Merkel's theory assumes that the water droplets have a uniform temperature. In reality the interior of the water droplet has a higher temperature than that of the surface. This should not be a great concern since empirical *NTU* values include the influence of the actual internal heat conduction in the water droplet [13]. The empirical *NTU* also includes the effects of cooling in the spray chamber above the filling, and in the open space below the filling. In other words, it reflects the performance of the whole tower assembly [1]. In short, the empirical *NTU* tacitly includes all the heat and mass transfer phenomena that occur in cooling towers. As a result, the new and simple analytical method is most useful for performance analysis of actual counterflow cooling towers where experimental data are reduced to an empirical correlation of Equation 14, and the outlet water temperatures can be predicted using Equation 11 or Equation 13, when there are changes in the inlet or flow conditions.

The simple analytical equations can easily be incorporated into, for example, chiller-tower optimization algorithms for accurate analysis at off-design operating conditions. When chiller loads are low, it is possible to operate the cooling tower fan at reduced speed, such that the combined fan and chiller energy usage is minimum. Such optimization study has been conducted by Yu and Chan [28].

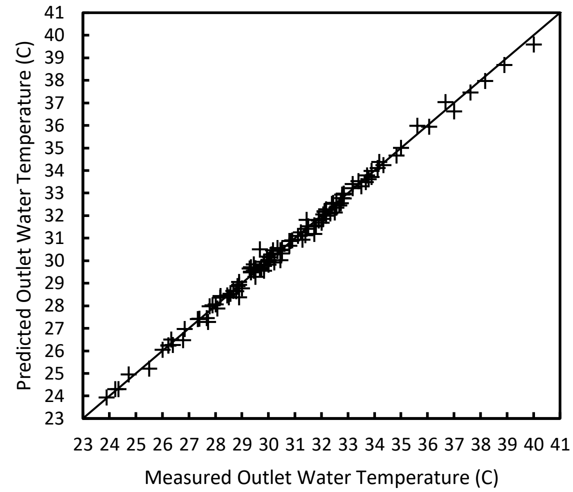


Figure 5 Analytically predicted versus measured outlet water temperatures for counterflow towers R1, R2, M1, and M2 of Simpson and Sherwood [4]

The new analytical method was also used to predict the performance of an actual crossflow cooling tower. Figure 6 shows the analytically predicted outlet water temperatures for the crossflow tower of Baker and Shryock [3], with large cell, and the RMSE was 0.74%. The coefficient *c* was 0.849, and the exponent *n* was -0.537, with r^2 of 0.95 when the experimental data was fitted to Equation 14. The close agreements between the predicted and measured results give an indication that the analytical method can be applied to performance analysis of actual crossflow cooling towers. However, more results are needed to establish the efficacy of the new method for actual crossflow cooling towers.

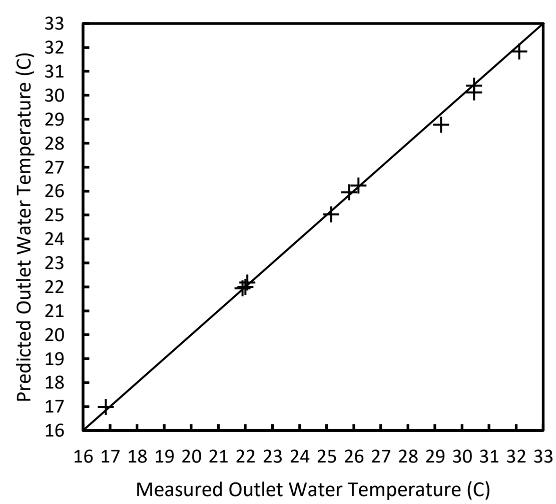


Figure 6 Analytically predicted versus measured outlet water temperatures for crossflow tower of Baker and Shryock with large cell [3]

4.0 CONCLUSION

A new and novel analytical method for direct calculation of counterflow direct-contact air-water cooling tower *NTU*, based on Merkel's theory has been developed. It is assumed that the saturated air enthalpy can be linearized with the water temperature, over the inlet and outlet water temperature interval. The same *NTU* equation can also be used to simply predict the unknown outlet water temperature in a simple two-step process. This is highly desirable because it is always the main problem to obtain the outlet water temperature when solving the Merkel equation. Most of the existing methods require a trial-and-error, and lengthy iterative method of solution. In a comparison study, the new method predicted the outlet water temperatures with a maximum RMSE of 0.2% when compared to the predictions by the standard Merkel's method for t_1 between 30 and 40 °C, *L/G* between 0.5 and 3, *NTU* between 0.5 and 2.5, and h_1 between 60 and 90 kJ/kg. The new method also predicted the outlet water temperatures of four experimental counterflow cooling towers, with a maximum RMSE of 0.8%. As a result, the new method has great potential for practical application. This is especially true when an empirical *NTU* correlation is available, which will lead to quick and accurate analysis of counterflow cooling towers as an important heat exchanger in water-cooled refrigeration, air-conditioning, and many important industrial process systems. A limited validation study shows that the new method can also be used for performance analysis of actual crossflow cooling towers. In conclusion, the new method is a valuable contribution to the heat exchanger community.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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