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MODELING HYPERPLASTIC ELASTOMER MATERIALS USED IN TIRE COMPOUNDS: NUMERICAL AND EXPERIMENTAL STUDY

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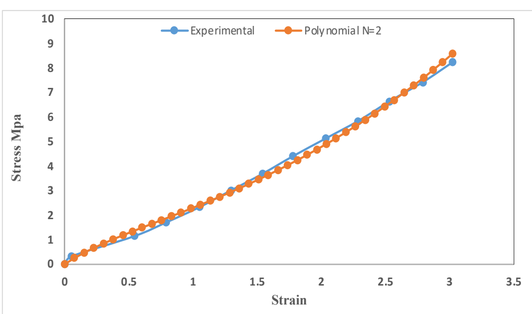
Graphical abstract



Abstract

Analyzing the rubber components of a tire, including the tread and sidewall, is crucial for assessing tire performance attributes. This evaluation enhances vehicle dynamics control and safety levels. Both finite element analysis and experimental tests are employed in this study to achieve accurate estimations. This study centers on Iraqi-manufactured tread and sidewall tires while delving into specific Dunlop components like tread and sidewalls. A highly effective methodology has been developed to ascertain material properties through experimental analysis of hyperelastic rubber models in tires. This approach is executed using ABAQUS, a widely employed commercial finite element software. Addressing rubber's intricate and diverse interactions through a straightforward yet precise phenomenological model holds immense industrial significance. Creating universally applicable design principles for these components remains a persistent challenge in modern industry. This study utilizes simulations to analyze stress-strain responses and compute material parameters for hyperelastic rubber models under tensile loading, employing computer-aided engineering (CAE) to represent stress-strain behavior, especially with varying strain amplitudes comprehensively. Focusing on elastomers, the study assesses the Ogden, Mooney-Rivlin, and reduced polynomial models by extracting coefficients from laboratory tests. It combines experimental and numerical methods to establish validated material constants. Most models yield reasonable results with acceptable deviations. The Neo-Hookean model is the simplest, fitting data up to 30% strain. The moderately complex Mooney-Rivlin and reduced polynomial models accommodate strains up to 100%. While accurate, the Ogden model exhibits higher nonlinearity and increased computational demands based on the material parameters.

Keywords: Hyperelasticity models, finite element analysis, tire rubber, material parameters

1.0 INTRODUCTION

The selection models of design materials are significant for mechanical applications since most design engineers request durable materials in service. Typically, the ultimate strength specified in the material data sheet can be used to predict failure in an isotropic material subjected to uniaxial stress. However, using materials is frequently subjected to multiaxial stress, involving orthogonal stresses; failure prediction does not simply require different and more robust techniques. A plane within a material can experience stress greater than its maximum strength, resulting in material failure, even if the orthogonal stresses are not greater than its ultimate strength. When a tensile load is applied and removed, most materials exhibit elastic behaviour, returning to their initial condition within the spectrum of load-deformation relationships. However, Hyperelastic properties are shown by rubber-like materials. The nonlinear load-deformation relationship demonstrates elastic behaviour across various deformations [1]. Hyperplastic elastomer materials with various desirable characteristics are needed in complex engineering fields. A metamaterial with multi-functionalities based on many constitutive approaches has been developed for modelling the. Polymers like rubber have become increasingly feasible in recent years with the help of additive manufacturing techniques [2].

As a function of energy density, a strain may display rubber-like behavior. Experiments on rubber-like materials with a high degree of elasticity were used to determine several efforts to replicate the stress-strain curves. However, due to the deformations, which are rubber-like in natural fabrics subjected, are too massive, and their behaviour demonstrates a considerable variation depending on the materials, it is impossible to make simpler choose an appropriate stress-energy density function experimental evidence for the stress-strain relationship [3]. This study examined the factors that influence the tensile loading of rubber. Numerical analysis entails determining what content can be published. The stress-strain behaviour of rubber is considered when comparing the performance of tires. A computer simulation approach was utilized to model and analyze the version of the virtual rubber. The use of rubber in the design of tires provides many benefits. Rubber is being tested to compare to the rubber used in traditional techniques for designing tires. Several studies have been conducted to predict rubber behavior [4,5,6]. It is known that hyperelastic materials have three invariants associated with their strain energy function. The primary ratio is used to measure these invariants. The strain energy function has only two invariants due to the rubber's near incompressibility [7]. Rubber is a substance that exhibits an extensive selection behavior. Developing a constitutive formulation is a daunting task to anticipate rubber behavior accurately. Over the years, many scientists have made substantial contributions. The Ogden Reduced Polynomial models impacted polynomial beginnings and continuum mechanics [8]. Test data must be available to make informed predictions. Finite element modelling delivers reliable estimating material parameters when tensile test data is presented in the uniaxial, biaxial, or planar form [9]. It is impossible to arrive at more precise conclusions when dealing with more complicated loads using only uniaxial test data [10]. Some models may be used to calculate the strain energy function. Other hyperelastic statistical models have been developed [11,12,13,14].

Junjie Luo *et al.* (2022) developed constitutive models for additively manufactured polylactic acid (PLA). FDM-printed PLA specimens were tested under different temperatures, strain rates, stress relaxation, and expansion/contraction thermal tests [15]. Using accurate material models produced by commercial ABAQUS software, Shreyas Khandelwal *et al.* (2022) analyzed the influence of different material parameters on void growth in amorphous glassy polymers. Experimental data from the open literature have validated the numerical model [16]. Material modeling is frequently misunderstood when it comes to elastomer testing. Due to the nonlinearity and almost incompressibility of elastomers, this is a particularly challenging issue. As part of their extensive viscoelastic experiment on the digitally printed EPU, Mokarram Hossain *et al.* (2020) carried out a strain rate-dependent analysis of the mechanical properties. In the study, it was found that mechanical responses were dependent on the passage of time [17].

Although the material exhibits noticeable nonlinear viscosities that depend on the strain rate and strain, it shows notable nonlinear viscosities. An evolution law based on strain-dependent nonlinear viscosities is designed based on experimental data for the printed elastomer. It has been reported historically that several sophisticated analytical hyperelasticity models exist due to their scientific, critical importance, and technological interests in understanding rubber materials' mechanical behaviour [18,19, 20].

Using a nonlinear elastic material as a basis in engineering practice, Dastjerdi *et al.* (2023) studied hyperelastic structures using the elasticity approach to develop the governing equations. There is no linearity in any primary direction of the derived governing equations. Hence, the simulation outcomes are considered highly precise for nonlinear elastic material structures. Consequently, the derived equations are solved using a meshless solution approach known as the semi-analytical polynomial method [21]. Statistical thermodynamic and continuum mechanics models for hyperelastic materials were thoroughly studied by Yanfeng *et al.* (2006) [22]. When developing a constitutive model, researchers like Xiaofang (2005) [23] and Rugsaj and Suvanjumrat (2018) [24] used a variety of stress tests, including uniaxial, planar, and biaxial tension testing, to develop mathematical derivations and discussions. The tire industry uses numerical analysis to simulate the tire performance of the constitutive model. The finite element approach may also affect transient reactions, including dynamic transients [25,26]. Many researchers achieved experimental programs on polymers' structural properties and mechanical behaviors [27, 28,29].

Creating a novel mathematical model for semi-crystalline polymers is suggested, which are frequently used as matrix materials in various biomedical composite investigations. Several elastomers, including 3D-printed elastomers, underwent loading and unloading deformations at different rates. The findings demonstrated how particular deformation mechanisms, which can cause global or local deformations, control the mechanical response of polymers [30]. Furthermore, newly developed constitutive models generalized Anssari and Horgan's (2023) strain-invariant model for isotropic compressible elastomers [31].

On the other hand, Afshin and Andrea (2021) also used generalized Hookean strain energy to examine strain-related energy functions [32]. Multiscale constitutive models of elastomeric particulate composites based on a continuum micromechanical framework were proposed by Saadedine *et al.* (2023) [33]. In Srikanth's *et al.* (2023) model, at most, 11 parameters are used to examine the performance of filled elastomeric solids using multiple deformation modes[34]. Eduardo Vitral (2023) addressed stretch issues in neo-Hookean Mooney–Rivlin and Varga models by introducing a new stretch formulation. Different strains contrast linear and quadratic functional forms [35]. Jebur [36, 37] proposed critical sensors for the modelling of hyperelastic materials. Tam Hoai Le *et al.* (2023) used numerical simulations to estimate rubber's mechanical properties and to predict its temperature variation under cyclic compression [38].

The distribution of fibers generalized structural tensor was derived by Ciambella and Rubin 2023 [39]. Ciambella and Nardinocchi (2021) performed remodeling of an anisotropic equation compatible with a multiplication decomposition of the deformation gradient inelastic theory used for the principle of structural frame indifference [40]. According to Anna Y. Zemlyanova *et al.* (2023), the Steigmann-Ogden theory describes material surfaces in elastic matrixes is investigated. Composite materials with graphene nanoplatelets are studied using the solution, and single-layer elastic potentials are used to solve the plane strain, and a system of integral equations can solve the problem of a straight surface. A numerical solution can be obtained based on the Chebyshev polynomial series [41]. Furthermore, Anna Y. Zemlyanova *et al.* (2023) presented the numerical solution of the two-dimensional Steigmann–Ogden model of the material surface using the approximations of the boundary data that involve the series of Chebyshev's polynomials [42].

This paper uses uniaxial tensile testing to contrast many parameter-fitting methodologies based on two well-known hyperelastic constitutive models. The effectiveness of different fitting procedures is assessed using a concept known as "goodness of fit and stable fitting. "Using the constitutive rubber model with parameters obtained, the optimum appropriate approach ensures high simulation accuracy in finite element analysis. Experiments were used to verify the FEM findings and compare them to those of other hyperelastic models that had been previously published. Both micro and macro-FEM studies were carried out. Using our collected data, researchers expected to reduce construction costs and industrial development time by optimizing the foam structure. Using Abaqus test specimens, the Reduced Polynomial model's coefficients were verified. It is possible to design elastic rubber products in some ways. Few studies have tried to fully characterize the behaviour of these materials, mainly via the utilization of various loading conditions in the analysis of experimental data. Rubber on the tires Simulation has been the extent of the work so far. However, tire rubber is distinct from tire rubber in several ways. According to the findings of a literature survey, further study on rubber tire treads is needed.

This study comprehensively explores the hyperelastic constitutive model, recognizing its critical significance in upholding tire safety. An advanced simulation model has been devised to enhance the understanding of constitutive modeling in retreaded tires to achieve this objective. Furthermore, a novel methodology is introduced for precise material property determination via experimental analysis of hyperelastic rubber models used in tires. This approach plays a paramount role in tackling the intricate and multifaceted interactions inherent to rubber, thereby delivering tangible benefits to the industrial sector.

2.0 METHODOLOGY

2.1 Parts of a Tire

Academic researchers need to know how tires work to evaluate their safety thoroughly. Studying the tire in two separate portions is necessary to comprehend tire mechanicals better. As shown in Figure 1, this research has two primary elements that will focus on the tire's material characteristics and simulations when tensile loading is applied to the material.

2.1.1 Tire Tread

The tread's principal function is to provide sufficient grip or traction for driving, stopping, and cornering. A balance must be struck between wear, traction, handling, and rolling resistance in the tread compound—the curing process results in the formation of a characteristic tread pattern in the rubber. The design strives for consistent wear across road surfaces, water channelling away from the footprint, and minimized pattern noise. While satisfying customer needs for adequate wear resistance, minimal noise, and a pleasurable ride, the tread compound and design must function well in various driving circumstances, including wet and dry weather and covered terrain.

2.1.2 Tire Sidewall

Tire sidewall rubber protects the body and plies against abrasion, impact, and fatigue due to flex. In addition, decorative features such as white or colored stripes or inscriptions adorn the sides. Sidewalls have been strengthened (shown in Figure 1). The reinforced bottom sidewalls of certain tires help improve handling and stability. A rugged rubber sidewall or additional reinforcements are often used in run-flat tire designs for low inflation pressure.

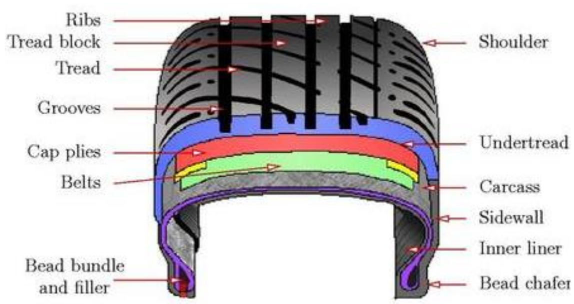


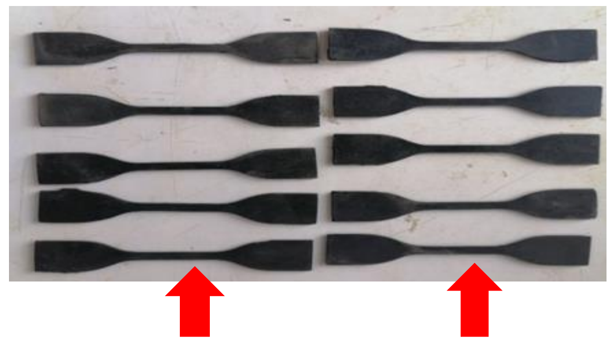
Figure 1 Tire car parts

2.2 Experimental setup

Experimental tests on hyperelastic and biomechanical model design are required to determine the input requirements [43]. Although it does not matter, a series of independent experiments determine the strain states. Individual trial outcomes are not included in this comparison as a team. Thus, the samples were used to arrive at the results for each. The test materials should be the same. There are a plethora of elastomer testing specifications in tension. Analytical criteria, on the other hand, are exploratory. The situation is vastly different regarding the most widely used test techniques. However, no national or international level has defined a practical policy test. For organizations tasked with establishing global norms on how to state and 80fulfill the input criteria for hyperelastic material, Stress tests for uniaxial were carried out.

2.2.1 Tensile Test

When defining an optimum cure state for any compound, it is possible to consider the tensile strength, elongation, or modulus. Tensile strength in rubber compounds is the maximum force applied to stretch a test sample of rubber compound until it ruptures. The rubber samples were produced according to ASTM D412 [44] (see Figure 2) through the process of calendaring, which starts by putting natural rubber [RSS] between the rolls and additive other materials mixed in a two-roll laboratory mill. The sequence is iterated multiple times once the whole material has been cured. According to Figure 3, uniaxial tensile tests were conducted using a microcomputer-controlled universal testing machine Monsanto T10 Tensometer type. Several rubber samples were used and exhibited various tensile strengths and deformations.



(a). (b)

Figure 2 Tensile test samples following ASTM standard D412, (a) RSS side of tire samples, (b) RSS tread of tire samples

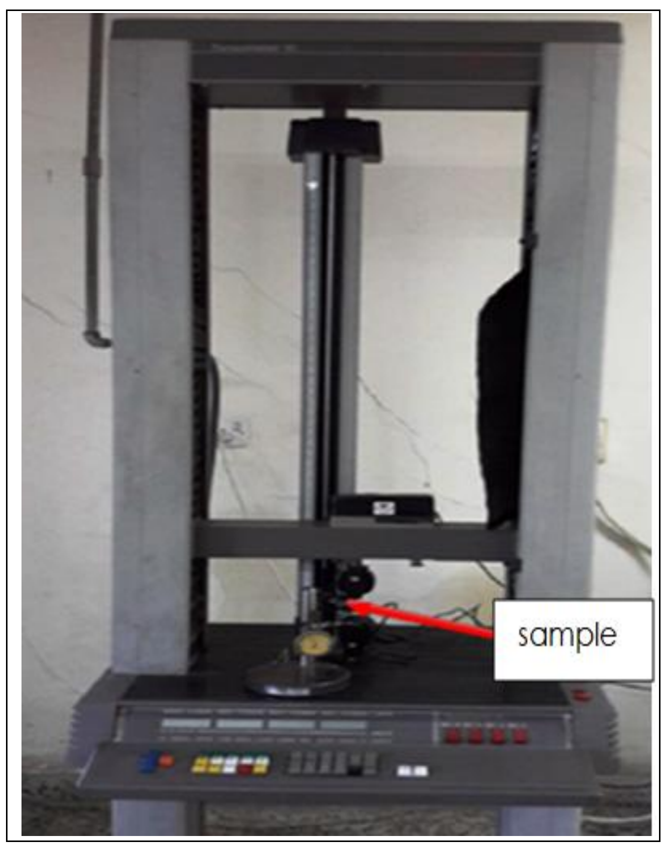


Figure 3 Tensile test setup

2.3 Finite Element Method Simulation

Composite Finite Element Method (FEM) can help model complex three-dimensional composites due to its accuracy and simplicity for material analysis [45,46,47].

The numerical model is valuable for predicting rubber materials' overall trends and behaviors under various conditions .While there is generally good agreement between numerical predictions and experimental measurements, some discrepancies are expected due to the inherent complexities of real-world testing.

In this research, ABAQUS, a finite element analysis software, used the Ogden, Mooney-Rivlin, and Reduced polynomial models to simulate finite element simulations on rubber. The tensile stress loading is done in ABAQUS finite element software as described below:

2.3.1 Single Element Simulation

The coefficients of the hyperelastic model can be measured using a single 3D continuum reduced integration, hybrid C3D8RH element with unit dimension. The single element can be subjected to uniaxial tensile loading to make predictions. It can directly compare the coefficients to the experimental data used to develop them. Three distinct loading methods are outlined in a phase-by-phase style. In step 3, which follows step 1, tensile stress is applied. The final stage is to provide planar tension.

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Three distinct loading methods are outlined in a phase-by-phase style. In step 3, which follows step 1, tensile stress is applied. The final stage is to provide planar tension. Once steps 2 and 4 are completed, reloading can reuse the element. Three hyperelastic constitutive models are used:

1- Ogden model (For N=1 to 6)

$$\mathcal{W} = \sum_{n=1}^{\infty} \frac{\mu_n}{\alpha_n}\left(\beta_1^{\alpha_n} + \beta_2^{\alpha_n} + \beta_3^{\alpha_n} - 3\right) \quad (1)$$

2- Mooney-Rivlin formulation

$$\mathcal{W} = \mathcal{C}_{10}(\partial_1 - 3) + \mathcal{C}_{01}(\partial_2 - 3) + \frac{1}{\mathcal{D}_1}(\mathrm{J_{el}} - 1) \quad (2)$$

3- Reduced polynomial models (For N=3)

$$\mathcal{W} = \sum_{i=1}^{n} \mathcal{C}_{10}\binom{n}{k}.x^k a^{n-k} = \begin{pmatrix} \sum_{i=1}^{N} \mathcal{C}_{10}\left(\bar{\partial}_1 - 3\right)^i + \\ \sum_{k=1}^{N} \frac{1}{\mathcal{D}_k}(\mathrm{J} - 1)^{2k} \end{pmatrix} \quad (3)$$

Equation 3 represents a general form of the Reduced polynomial models; as an individual case, N=1 represents a Neo-Hookean model, while N=2 represents a Mooney-Rivlin model.

3.0 RESULTS AND DISCUSSION

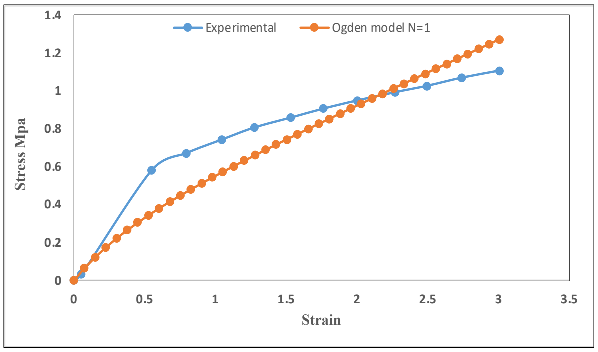
3.1 Experimental and FEM Simulation

The testing findings aim to specify and fulfill the input specifications of mathematical material models in structural, nonlinear finite element analysis software. The Abaqus includes the ability to fit hyperelastic curves to diverse material models. Abaqus is used to analyze stress-strain data from a single-axis test. Abaqus generates a strain energy equation for each stress and strain data set. Researchers can compare material models using Abaqus' hyperelastic curve fitting tool.

According to Figures 3 to 7, there are hyperelastic material models. Abaqus includes the ability to fit hyperelastic curves to diverse material models. Abaqus relies on defined stress–nominal strain data when conducting uniaxial tests. The hyperelastic models were deemed the most accurate as they were able to accurately fit experimental data points at both moderate and significant strain levels. Mooney-Rivlin parameters and Ogden models were used for all deformation modes in Abaqus. Models like Ogden (N= from 1 to 6) were the most accurate since they may fit experimental findings at small and big strain values when forecasting rubber formulation behaviour. Abaqus estimated all deformation modes using the Ogden and the Mooney-Rivlin coefficients. The materials parameters were predicted using uniaxial tensile test data, and these parameters are listed in Tables 1 and 2 with different fitting errors.

The investigation of experimental stress-strain diagrams and the numerical modelling of the sidewall [Side wall] rubber using natural rubber [RSS] can be used to manufacture tires for heave applications because of high failure stress Dunlop company specifications. It is important to note that the sidewall layers of a tire are subjected to stresses found within the tire from the pressure inside, compression, and buckling from the car's weight. As a result, Young's modulus is significantly high (1.98 MPa at a strain rate of 300% at the rupture point, and the failure stress is 22.684 Mpa).

For the outer layer of the tire (Tread) using the natural rubber [RSS], the bending loading is dominant due to car weight and the tensile stresses due to the air load in the tire. The main job of this layer is to absorb the high-impact loading, especially for the people transporting cars. The requirements of the material should have high stiffness and a little bit small Young's modulus (2.29 MPa at 300% strain rate and failure stress 21.218 Mpa. Figures 4, 5, 6, and 7 demonstrate how material parameters affect the distribution of stresses on the material surface. A small value of normalized strain tends to produce sharper plots.



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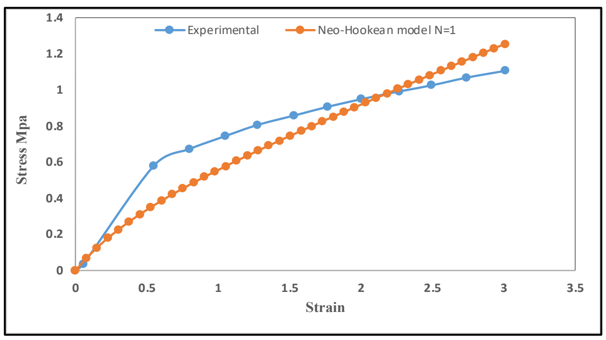

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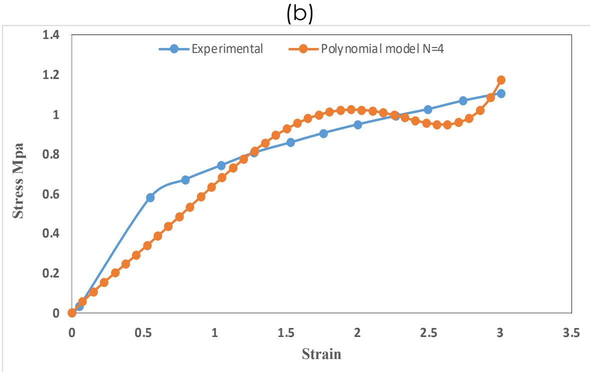
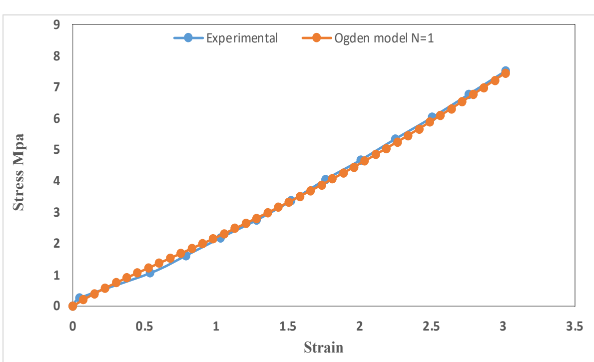


(c)

Figure 4 Uniaxial tensile test results for RSS between the experimental and the (a) Ogden model N=1, (b) Neo-Hookean N=1, (c) Polynomial N=4



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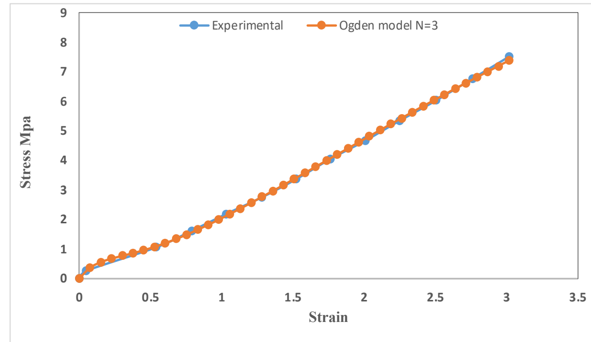

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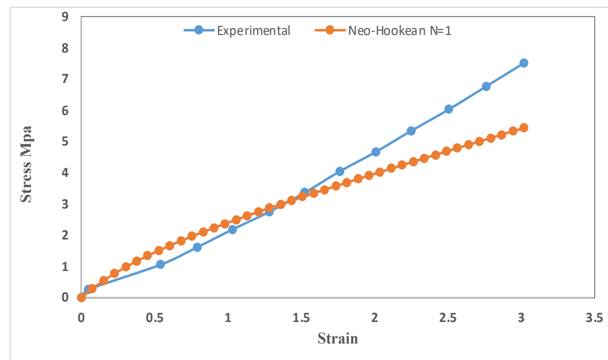

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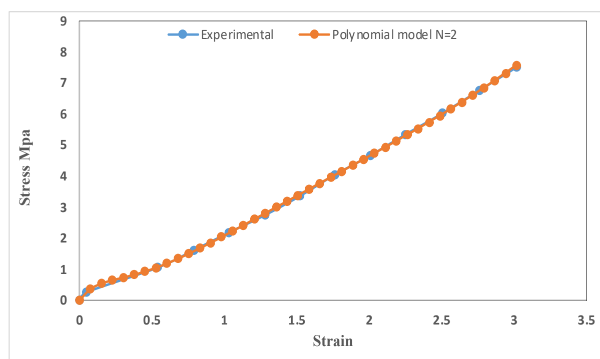

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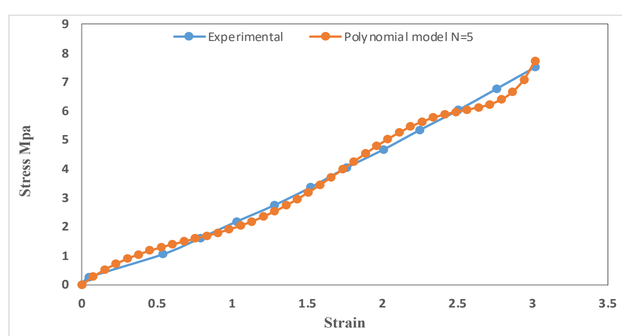
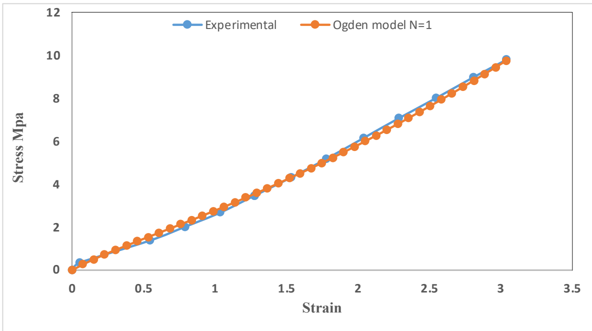


(e)

Figure 5 Material stress-strain diagram comparison for the Natural rubber RSS between the experimental and the: (a) Ogden model N=1, (b) N=3, (c) Neo-Hookean N=1, (d) Polynomial N=2, (e) N=5, in the outer layer (TREAD)



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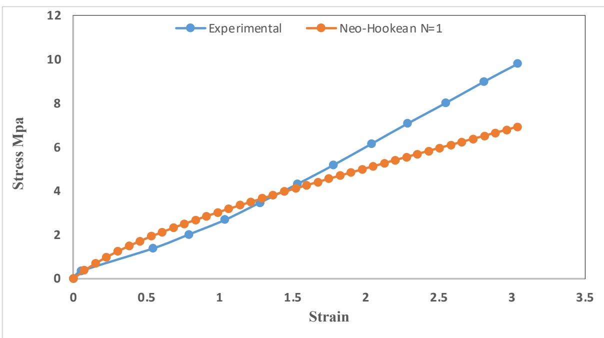

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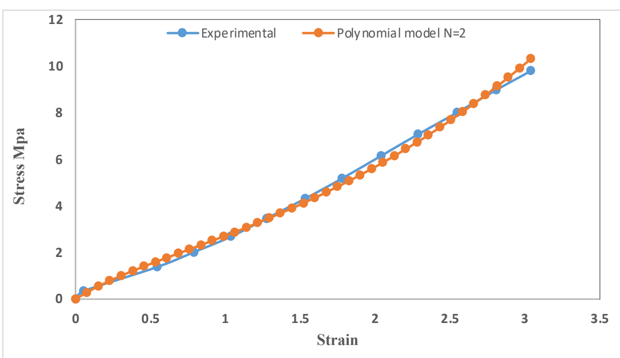

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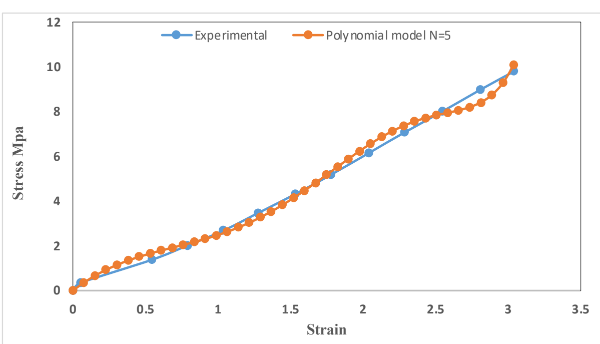
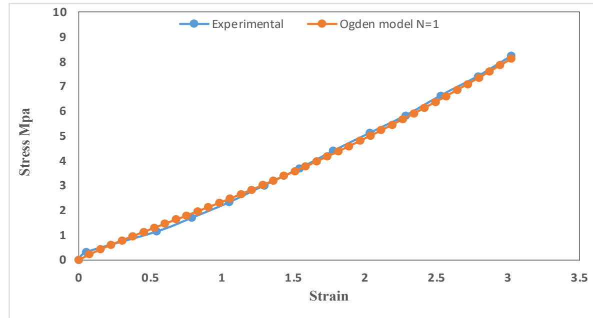


(d)

Figure 6 Comparison of the stress-strain diagram for a material prepared according to DUNLOP tire company in the outer layer (TREAD) between the experimental and the: (a) Ogden model, (b) Neo-Hookean N=1, (c) Polynomial N=2, (d) Polynomial N=5



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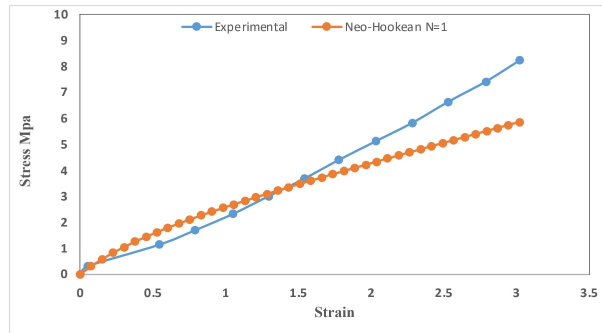

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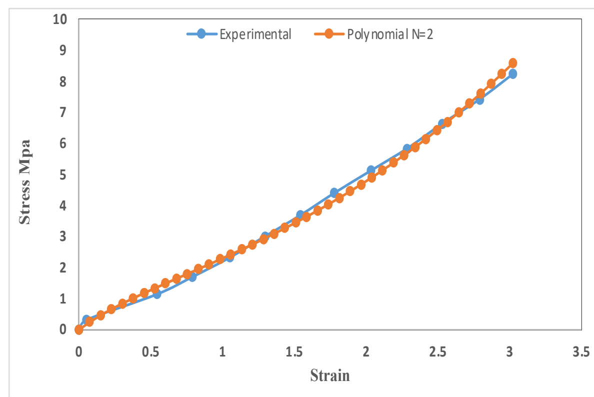

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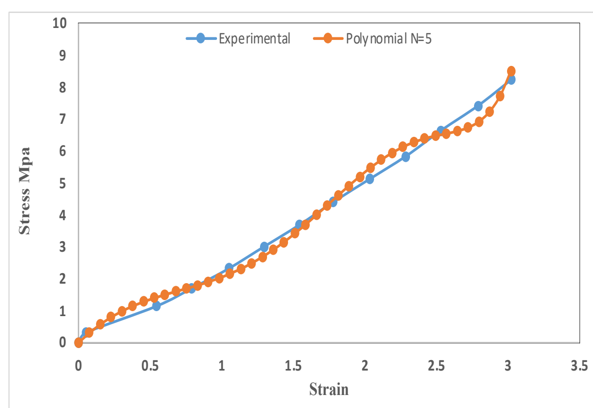

(d)

Figure 7 Material stress-strain diagram comparison prepared according to DUNLOP tire company in the sidewall layers between the experimental and the: (a) Ogden model, (b) Neo-Hookean N=1, (c) Polynomial N=2, (d) Polynomial N=5

Hyperelastic material like rubber can sustain finite deformations with no volume change and can only move in an isochoric direction. Many applications of continuum and computational mechanics rely on this assumption and idealization. Roughly speaking, stability problems in the hyperelastic models remain big challenges at small and large strains. However, the investigation of experimental stress-strain diagrams and the numerical modelling of the sidewall [Side wall] rubber using natural rubber [RSS] can be used to manufacture tires for heave applications because of high failure stress Dunlop company specifications. Using Abaqus ' hyperelastic material curve fitting capability, researchers can compare different hyperelastic material models with test data. The sidewall layers are subjected to tensile stresses due to air pressure inside the tire, compression, and buckling due to car weight. Therefore, Young's modulus is considerably high (1.98 MPa at a 300% strain rate, and failure stress is 22.684 MPa). For the outer layer of the tire (Tread) using the natural rubber [RSS], the bending loading is dominant due to car weight and the tensile stresses due to the air load in the tire. The main job of this layer is to absorb the high-impact loading, especially for the people transporting cars.

Material requirements for this project include increased stiffness and a slight decrease in Young's modulus (2.29 MPa at 300% strain rate and failure stress of 21.218 MPa). The simulation results show that the nonlinear coefficients in the Ogden constitutive model do not make fitting the experimental data easy; thus, the nonlinear least squares method must be used.

Table 1 Estimation of Ogden hyperelastic constitutive model parameters with different fitting errors

Model No 1:	**Parameters**							
	μ1	**α1**	**μ2**	**α2**	**μ3**	**α3**	**Status**	**Fitting discrepancy %**
n=1	0.31	2.03	-	-	-	-	Stable	
n=2	-	-	-	-	-	-	Unstable	
n=3	-	-	-	-	-	-	Unstable	**56**
n=4	-	-	-	-	-	-	Unstable	
n=5	-	-	-	-	-	-	Unstable	
n=6	-	-	-	-	-	-	Unstable	
n=1	1.04	2.69		-	-	-	Stable	
n=2	-	-	-	-	-	-	Unstable	
n=3	-	-	-	-	-	-	Unstable	**31.22**
n=4	-	-	-	-	-	-	Unstable	
n=5	-	-	-	-	-	-	Unstable	
n=6	-	-	-	-	-	-	Unstable	
n=1	0.98	2.66		-	-	-	Stable	
n=2							Unstable	
n=3	-22.99	-3.32	-0.98	2.78	26.21	3.90	Stable	**26.9**
n=4	-	-	-	-	-	-	Unstable	
n=5	-	-	-	-	-	-	Unstable	
n=6	-	-	-	-	-	-	Unstable	
n=1	1.22	2.70	-	-	-	-	26.13	
n=2	-	-	-	-	-	-	Unstable	
n=3	-	-	-	-	-	-	Unstable	**26.13**
n=4	-	-	-	-	-	-	Unstable	
n=5	-	-	-	-	-	-	Unstable	
n=6	-	-	-	-	-	-	Unstable	

Table 2 Estimating Reduced Polynomial hyperelastic constitutive model parameters with different fitting errors

Model No 1, (N number):	**Parameters**								
	D1	**C10**	**C01**	**D2**	**C20**	**C11**	**C02**	**Status**	**Fitting discrepancy %**
* 1	0	0.15	0	-	-	-	-	Stable	
2	-	-	-	-	-	-	-	Unstable	
3	-	-	-	-	-	-	-	Unstable	**56**
4	-	-	-	-	-	-	-	Unstable	
5	-	-	-	-	-	-	-	Unstable	
6	-	-	-	-	-	-	-	Unstable	
* 1	0	0.68	-	-	-	-	-	Stable	
2	0	0.55	0	-	0.0163	0	0	Stable	**23.68**
3	-	-	-	-	-	-	-	Unstable	

Model No 1, (N number):	D1	C10	C01	D2	C20	C11	C02	Status	Fitting discrepancy %
				Parameters					
4	-	-	-	-	-	-	-	Unstable	
5	-	-	-	-	-	-	-	Unstable	
6	-	-	-	-	-	-	-	Unstable	
* 1	0	0.86	0	-	-	-	-	Stable	**66.09**
2	0	0.69	0		0.0219	0	0	Stable	
3								Unstable	
4								Unstable	
5								Unstable	
6								Unstable	
4	D1	C10	C01	D2	C20	C11	C02		**30.54**
	0	0.132	0	0	1.91e-2	0	0		
	D3	C30	C21	C12	C03	D4	C40		
	0	-2.23e-3	0	0	0	0	-7.29e-5		
5	D1	C10	C01	D2	C20	C11	C02		**15.11**
	0	0.84	0	0	-9.60E-2	0	0		
	D3	C30	C21	C12	C03	D3	C30		
	0	2.63e-2	0	0	0	0	2.63e-2		
	D4	C40	C31	C22	C13	C04	D5		
	0	-2.15E-3	0	0	0	0	0		
	C50	C41	C32	C23	C14	C05	C50		
	5.91e-5	0	0	0		0	5.91e-5		
5	D1	C10	C01	D2	C20	C11	C02		**17.34**
	0	0.66	0	0	-7.74e-2	0	0		
	D3	C30	C21	C12	C03	D4	C40		
	0	-2.11e-2	0	0	0	0	-1.75e-3		
	C31	C22	C13	C04	D5	C50	C41		
	0	0	0	0	0	4.85e-5	0		

* As a special case for N=1 the model represents a Neo-Hookean model

4.0 CONCLUSIONS

This study investigates various hyperelastic models, including the Reduced Polynomial (Mooney-Rivlin and Neo-Hookean models) and Ogden constitutive models, widely used in engineering for rubber component design. Using ABAQUS finite element software, we conduct regression analysis based on uniaxial tensile tests with rubber samples commonly found in tire manufacturing. Results from parameter fitting optimization indicate satisfactory model performance compared to experimental data. Particularly, in characterizing rubber compression deformation, the Ogden model outperforms the Reduced Polynomial model in uniaxial tension analysis. The study enhances our understanding of hyperelastic rubber models, emphasizing the Ogden model's effectiveness as it approaches its function's end. Overall, this research contributes to the practical application of hyperelastic models in engineering, aided by ABAQUS software for accurate parameter fitting.

In the context of hyperelastic materials, it's crucial to calibrate finite element (FE) models using data from multiple tensile tests. Recognizing a material's performance within its inherent constraints holds paramount significance, as these constraints are generally inherent to most material models.

According to the findings, it is concluded that the Neo-Hookean model is the simplest, fitting data up to 30% strain. There is no limit to strains of up to 100% for the Mooney-Rivlin and reduced polynomial models, which are both moderately complex. The use of the Ogden model leads to a greater level of nonlinearity and computational demands. The remarkable alignment between experimental and numerical results indicates the sound manufacturing of the tensile test samples, where the fitting discrepancy between the two techniques was found to be not more than 15.11 %. This paper provides extensive findings of significant value to researchers and manufacturers. It streamlines the process of evaluating mechanical properties in rubber products, saving both time and effort. The insights presented herein are poised to pique the interest of future researchers in this field.

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Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

List of symbols

Symbol	Description
$\mathcal{W}$	Strain energy density
β_i (i=1,2,3.)	Stretch ratio
μ_i	Material constant related to the initial shear modulus.
α_i	Empirically calculated material constants
D, C	Dimensionless material property
N	Order of the polynomial
n	Number of data points
β	Extensions of the deformation
$\bar{\partial}_1$	Principal strain invariants
J	Incompressibility constraint

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