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Study on Trajectory Motion and Computational Analysis of Robot Manipulator

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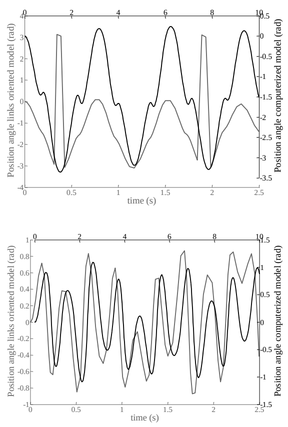
Graphical abstract



Abstract

The relation of arm and forearm for arm pitching a ball and bowling game, relation of rotating arm and the connecting rod for crankshaft and any other mechanical system will determine the performance of the system. To obtain the best performance of the system, the study of the dynamic mechanism of the system is essential. In order to study and analyze the dynamic interaction of each link for various applications, robot links manipulator is used as a simplified model for dynamic analysis. The dynamic mathematical equation of robot manipulator is derived by using the Lagrange equation. The elevation angle and angular velocity over time of link one and two is simulated using computerized algorithm. The trajectory motion of both links is illustrated and the relation between both links is determined. The results obtained are then compared with the SimMechanics model.

Keywords: Computational analysis; computational model; dynamic model; simulink model; position angle; robot manipulator; trajectory motion; velocity angle

Abstrak

Hubungan lengan atas dan lengan bawah untuk pitching bola dan permainan bowling, berhubung lengan berputar dan rod penyambung untuk aci engkol dan mana-mana sistem mekanikal yang lain akan menentukan prestasi sistem. Untuk mendapatkan prestasi yang terbaik daripada sistem itu, kajian mengenai mekanisme dinamik sistem adalah penting. Dalam usaha untuk mengkaji dan menganalisis interaksi dinamik setiap pautan untuk pelbagai aplikasi, robot pautan manipulasi digunakan sebagai model yang dipermudahkan untuk analisis dinamik. Persamaan matematik dinamik manipulasi robot diperolehi dengan menggunakan persamaan Lagrange. Sudut ketinggian dan halaju sudut dari masa ke masa pautan satu dan dua adalah simulasi menggunakan algoritma berkomputer. Gerakan trajektori kedua-dua link digambarkan dan hubungan antara kedua-dua link ditentukan. Keputusan yang diperolehi kemudiannya dibandingkan dengan model SimMechanics.

Kata kunci: Analisis pengiraan; model pengiraan; model dinamik; model Simulink; kedudukan sudut; halaju sudut; manipulasi robot; gerakan trajektori.

1.0 INTRODUCTION

Many different studies have been carried out on robot link manipulator system to improve the performance of various applications including the sports activities services and mostly in the mechanical field. In sports activities for pitching the ball in a baseball game and bowling, the optimal elevation angle of the arm and forearm are essential to achieve the best performance. Robot manipulator can imitate the arm pitching actuated at the shoulder by neglecting several degrees of freedom, 3D nature of the trajectory, spin effect through the finger and wrist.[1]

Besides, the concept of link manipulator also applied for a crankshaft mechanism mostly use for pumps, compressor, steam engine and in internal combustion engine. The crankshaft consists of connecting rod attached to an arm and a piston. It is mainly used to convert the rotational motion to the reciprocating motion or vice versa. The dynamic motion of the crankshaft can be modelled into simpler two-link manipulator where the rotating arm represented the first link and the connecting rod is the second link of two-link manipulator. The dynamic interaction between arm shaft, connecting rod and the piston (sliding part) is essential as the rotation of the shaft will determine the position of the sliding piston and vice versa.

A meal assistant service robot for helping a congenital hand disable patient or hand disable due to stroke and spinal injury is modeled into three-link manipulator system.[1] The important consideration for this simple service robot is to maintain the comfort of patient during feeding. Thus a high precision in dynamic modeling and control of each link and between spoon and mouth is an important control issue.

As well known, the dynamic modeling is an important control issue in simulation of robot motion and in design and control algorithm. Two approached of derived dynamic models have been discovered by researchers. One is the Newton-Euler formulation and the other one is the Lagrange-Euler formulation. The Euler-Lagrange formulation analyzed the system based on kinetic energy and potential energy where the links are treated as a whole. While, for the Newton-Euler formulation, the links is treated in turn. Based on previous research, there is no clear evidence describing which method is better than another. However important goal in dynamic modeling is to derive it as fast as possible and its depend on several factors including the number of links and joints in the kinematic chain, the topology of the chain whether serial or parallel and the position and orientation of the coordinate frame.

In dynamic analysis, two types of problem usually involved. One is the forward dynamics and the other is the inverse dynamics. Forward dynamic usually used in modeling and simulation as in previous research, modeled a five-fingered robot hand for a grasping task for the industrial and medical application.[2] The dynamic of this model is derived based on the Lagrangian equation. There are research on two-link manipulator presented both forward and inverse dynamic analysis.[3] In forward dynamics, the movement of mechanical system is determined based on a given force. While for inverse dynamics, the moment or can be said force required is determined to move the end effector of the link to the targeted position.

In this paper, we focus on the direct dynamic analysis derived using the Lagrangian method. Where robot manipulator is used as the simplified model of various case studies as described in the above paragraph. The trajectory motion of robot manipulator is simulated by using derived computerized algorithm and SimMechanics model. Both results will be compared.

The arrangement in this paper including the first section is the introduction, second section represents the methodology followed by result and discussion in the third section. The methodology is divided into three parts. First part comprised of a derivation of a dynamic differential model of robot manipulator. The next part is the computer integration algorithm and the third part is a physical design model using SimMechanics methods.

2.0 METHODOLOGY

2.1 Dynamics Model of Robot Manipulator

Robot links manipulator consists of two links connected by joint one and two where the center of mass of each link is located at the center of the link. For pitching ball in a baseball game and in a bowling game, link one and two represent the arm and forearm respectively. Joint one is modeled as the shoulder joint which originally have three rotational degrees of freedoms (DOF) and it is reduced to one DOF. Joint two acts as an elbow joint which also has one DOF. Thus, the total number of DOF for this model is two with two revolute joints. Figure 1 is the skeletal model of two-link manipulator related to arm mechanism in Figure 2 and Table 1 listed the related specification involved.

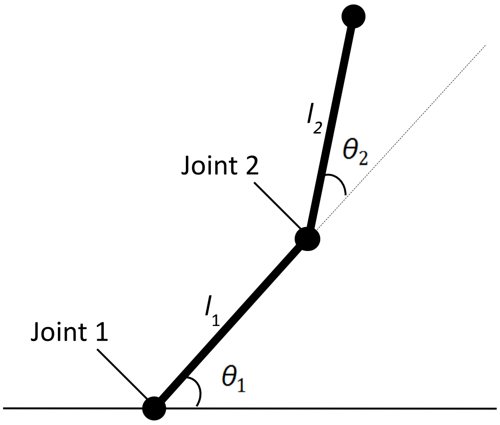


Figure 1 Robot manipulator model

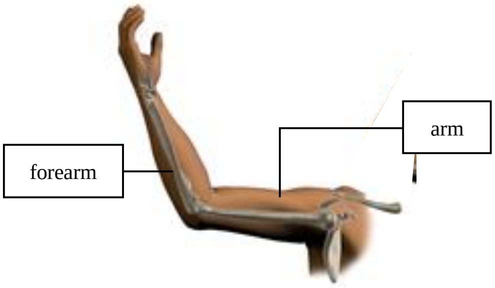


Figure 2 Arm and forearm

Table 1 Specification of two-link planar

Parameter	Definition	Unit
l_1	Length link 1	m
l_2	Length link 2	m
J_1	Joint 1	-
J_2	Joint 2	-
T_1	Apply torque 1	kgm^2/s^2
T_2	Apply torque 2	kgm^2/s^2
θ_1	Position angle of link 1	radian
θ_2	Position angle of link 2	radian
$\dot{\theta}_1$	Velocity angle of link 1	$rads^{-1}$
$\dot{\theta}_2$	Velocity angle of link 2	$rads^{-1}$
m_1	Mass of link 1	kg
m_1	Mass of link 1	kg
g	Gravitational acceleration	ms^{-2}

The dynamic mathematical model is derived via Lagrange method. The Lagrangian is defined as the difference between kinetic and potential energy of the mechanical system and Lagrangian equation is written as in (1).

$$L = KE - PE \quad (1)$$

Where L is the Lagrangian, KE is the kinetic energy and PE is the potential energy. Both kinetic and potential energy for link one and link two is simplify as,

$$KE_i = \frac{1}{2} I_i \ddot{\theta}_i + \frac{1}{2} m_i {v_i}^2 \qquad i = 1, 2 \qquad (2)$$

$$PE_i = \frac{1}{2} m_1 g l_i cos\theta_i \qquad i = 1, 2 \qquad (3)$$

Assuming the center of mass of each link is located at the center of the link. Thus moment inertia of each link is,

$$I_i = \frac{1}{2} m_i {l_i}^2 \qquad i = 1, 2 \qquad (4)$$

The Lagrange equation yield the general equation of motion of the mechanical system as,

$$T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} \qquad i = 1, 2 \qquad (5)$$

Substitute the kinetic and potential energy in equation (2) and (3) into (1). Then taking the partial derivative of L given in equation (1) with respect to $\dot{\theta}_i$ then differentiate it with respect to time t. Equation (1) once again is partially differentiate with respect to θ_i and substitute into equation (5) yield an equation of motion for two-link arm as below,

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = [A]\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + [B]\begin{bmatrix} \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_2\dot{\theta}_1 \end{bmatrix} + [C]\begin{bmatrix} {\dot{\theta}_1}^2 \\ {\dot{\theta}_2}^2 \end{bmatrix} + [D] \qquad (6)$$

Where the A is the $n \times n$ generalized inertia matrix as describe in above:

$$[A] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad (7)$$

$$A_{11} = \frac{1}{3} m_1 {l_1}^2 + m_2 {l_1}^2 + \frac{1}{3} m_2 {l_2}^2 + m_2 l_1 l_2 cos\theta_2 \qquad (8)$$

$$A_{12} = \frac{1}{3} m_2 {l_2}^2 + \frac{1}{2} m_2 l_1 l_2 cos\theta_2 \qquad (9)$$

$$A_{21} = \frac{1}{3} m_2 {l_2}^2 + \frac{1}{2} m_2 l_1 l_2 cos\theta_2 \qquad (10)$$

$$A_{22} = \frac{1}{3} m_2 {l_2}^2 \qquad (11)$$

B is $n \times n$ matrix which related to centripetal acceleration and detail is as below,

$$[B] = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \qquad (12)$$

$$B_{11} = -m_2 l_1 l_2 sin\theta_2 \qquad (13)$$

$$B_{12} = B_{21} = B_{22} = 0 \qquad (14)$$

C described below is $n \times n$ matrix related to Coriolis acceleration. The Coriolis accelerations exist in this case due to the first link act as the rotating frame for the second link.

$$[C] = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \qquad (15)$$

$$C_{11} = C_{21} = 0 \qquad (16)$$

$$C_{12} = -\frac{1}{2} m_2 l_1 l_2 sin\theta_2 \qquad (17)$$

$$C_{22} = -C_{12} \qquad (18)$$

D is $n \times m$ matrix related to the gravitational acceleration,

$$[D] = \begin{bmatrix} D_{11} \\ D_{12} \end{bmatrix} \qquad (19)$$

$$D_{11} = \left(\frac{1}{2} m_1 + m_2\right) g l_1 cos\theta_1 + \frac{1}{2} m_2 g l_2 cos\theta_{12} \qquad (20)$$

$$D_{12} = \frac{1}{2} m_2 g l_2 cos\theta_{12} \qquad (21)$$

2.2 Computerize Integration of Robot Manipulator

The flow of overall process is illustrated in Figure 3. Base on the flow diagram, the derived equation of motion for robot manipulator is shown step by step in previous part. The differential equations of motion in equation (6) are highly nonlinear and cannot be solve analytically except for a simplest cases.

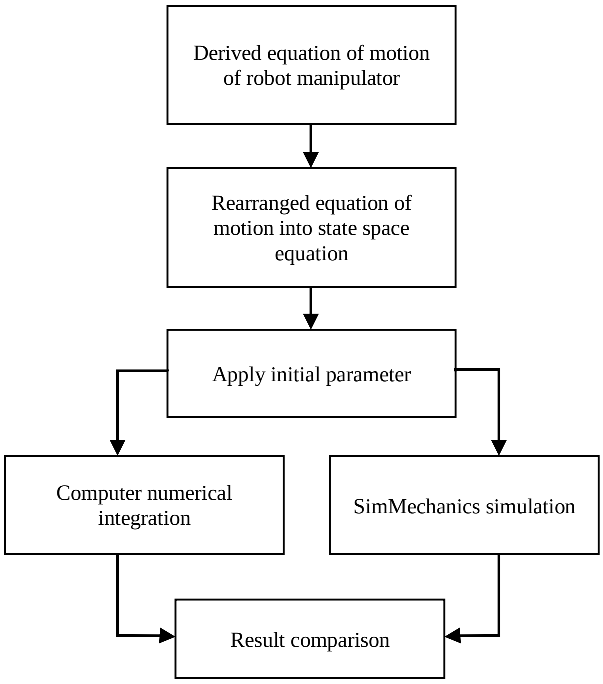


Figure 3 Overall flow process of simulation

Thus, in order to determine the mechanical movement of each link from a given value of torque, a numerical integrated is required. Therefore, MATLAB software is used to perform the numerical integration of differential equation that is rearranged into a state space form as in (22).

$$\ddot{\theta} = I^{-1}(T - h - \gamma) \qquad (22)$$

Each terms in (22) is describe as above equation

$$\ddot{\theta} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}, I = [A], T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix},$$

$$h = [B]\begin{bmatrix} \dot{\theta}_1\dot{\theta}_2 \\ \dot{\theta}_2\dot{\theta}_1 \end{bmatrix} + [C]\begin{bmatrix} \dot{\theta}_1{}^2 \\ \dot{\theta}_2{}^2 \end{bmatrix}, \gamma = [D] \quad (23)$$

Our algorithm is designed base on the state space equation (22). Type of solver used to solve this numerical equation is variable step Runge-Kutta method named ODE45 which can be apply for both first and second order differential equation.

2.3 The Dynamic Model by SimMechanics

In the previous part, the simulation of the derived mathematical dynamic model illustrated the behavior of dynamic motion in a graphical way. In this section, the simulation is represented in both graphical and 3D models of robot manipulator by using SimMechanics. SimMechanics is one of MATLAB/Simulink add-on used for modeling and simulation by applying the standard Newtonian dynamic force and torque.[4] The mechanical system in SimMechanics model is represented in block diagram which simplify and facilitate the simulation analysis without required the knowledge of tedious mathematical derivation.

Figure 4 illustrates the step involved in modeling robot manipulator in SimMechanics or can be called links oriented model. The first step is to design a 3D CAD assembly model. For this designed, SolidWorks is used. The next step is to export the CAD assembly model into the XML file. The XML file is then used to generate a SimMechanics and Simulink model. Figure 5 is the block diagram model represented every part of links manipulator. Table 2 listed out the relationship of mechanical part for Simulink model and simplify two-link manipulator model. Each revolute joint are linked to a joint sensor in which the position and velocity angle is gained. This result is compared with the derived model or can be called a computerized model present previously.

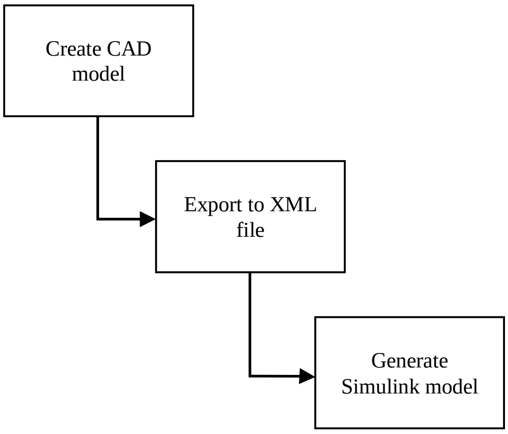


Figure 4 Flow design of links oriented model

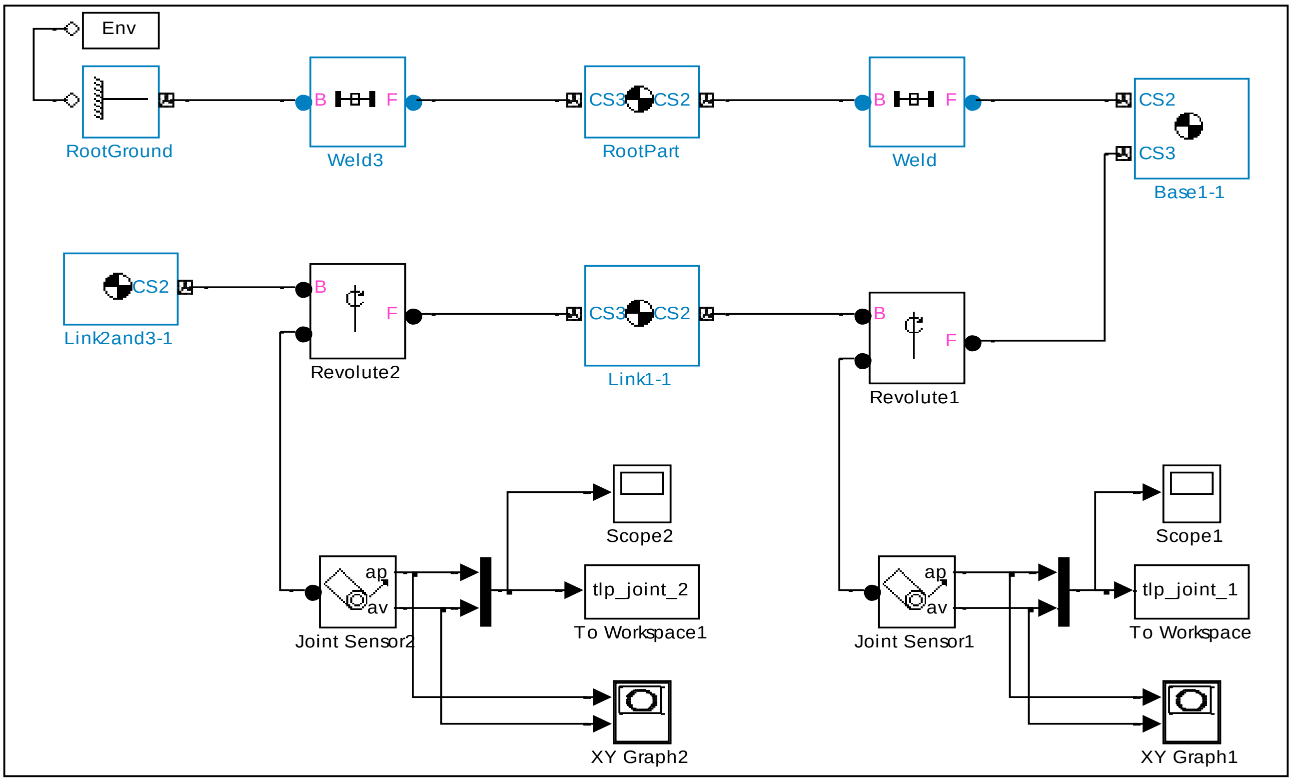


Figure 5 Block diagram of links oriented model

Table 2 Relationship of mechanical parts

Simplify model	Simulink model
Joint 1	Revolute joint 1
Link 1	Link1-1
Joint 2	Revolute joint 2
Link 2	Link2and3-1

■3.0 RESULTS AND DISCUSSION

Simulation result for computerized and link oriented model is presented in Figure 6, 7, 8 and 9. Base on the figures, the grey lines indicated the links oriented model result while the black colour is the computerized model result. The result is displayed for three complete cycles where complete cycle is from 0 to $-\pi$ (link swing clockwise direction), $-\pi$ to 0 (link swing counterclockwise direction).

For position angle of link one referred to Figure 6, after a half cycle in $-\pi$ radian range, the position angle of links oriented model undergo a sudden change compared to computerized model. This is due to link one tends to swing higher and beyond the $-\pi$ radian and reached the positive region angle as illustrated in the figure below.

From Figure 6 and 7, the position angle of link one and two, we can see that the position angle is always below zero radian, which mean in negative region except for links oriented model. It is due to the model is simulated under the gravity only as the gravitational force cause the link one to rotate in negative y axis only. Transition point happens two times in every cycle when link one reached $-\pi/2$ rad position. The transition point is phenomena happen when link two abruptly changed its sign from positive to negative or vice versa.

Figure 8 and 9 represents the comparison velocity angle of computerized and link oriented model for link one and two. From this figure, we can see that the reactions of velocity angle for both links are opposite to each other. When velocity angle of link one increased, link two will decrease in velocity and vice versa.

Overall, from the above figures, we can see that one cycle is completed in every 3 second and 0.75 second for computerized model and link oriented model respectively and repeated for the next cycle with the same pattern. Both simulations of the computerized model and links oriented model produced an identical oscillation curve but different in amplitude and frequency due to the differences in parameter model.

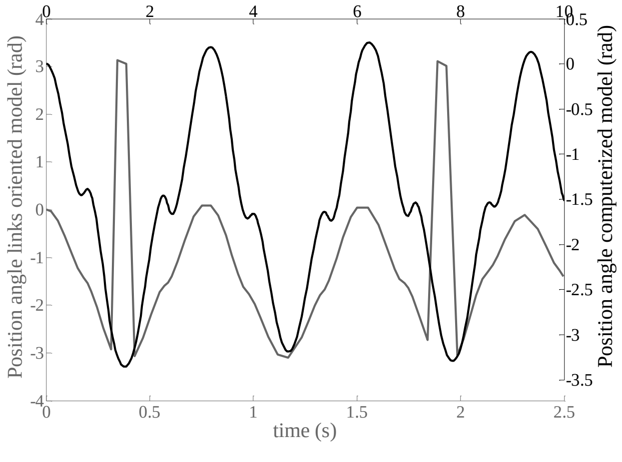


Figure 6 Comparison of link one position angle for computerized and links oriented model

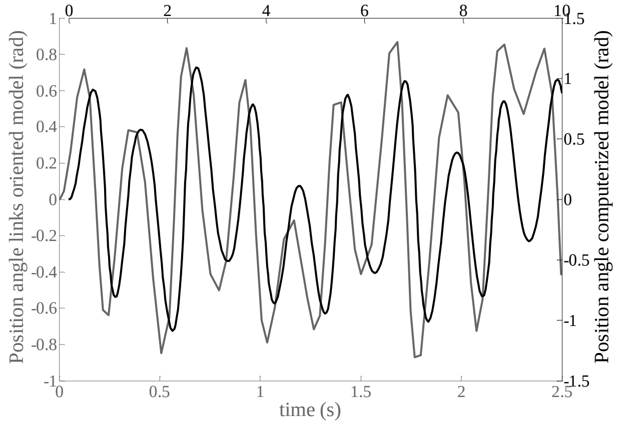


Figure 7 Comparison of link two position angle for computerized and links oriented model

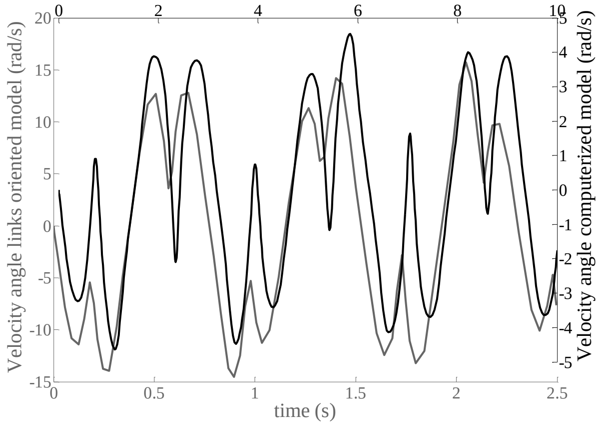


Figure 8 Comparison of link one velocity angle for computerized and links oriented model

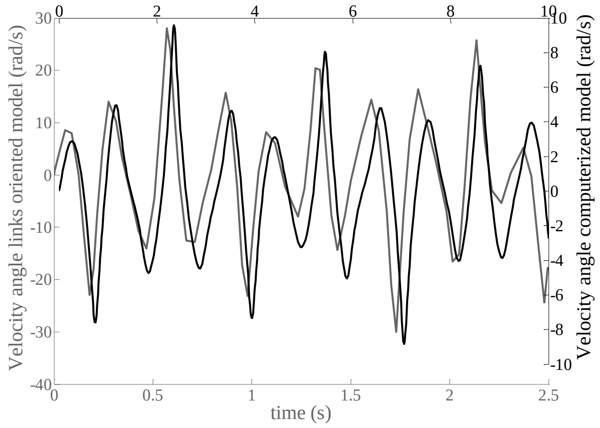


Figure 9 Comparison of link two velocity angle for computerized and links oriented model

The XY plot in Figure 10, 11, 12 and 13 is the trajectory motion of link one and two for computerized and link oriented model. As in the figure below, X is the position angle and Y is the velocity angle with the initial value zero and marked as '●'. The trajectory motion of link one for computerized and links oriented model as in Figures 10 and 11 start to swing from the starting point marked '●' in clockwise direction. When the link one angle reaches $-\pi/2$ radian, the velocity angle gradually reduced to zero. Then, the speed once again will reduce to zero during completing a half cycle at angle $-\pi$ radian. During completing one cycle, link one will swing back to the initial position in a counterclockwise direction and this overall process is repeated.

The trajectory motion of link two in Figure 12 and 13 for computerized and links oriented model is affected by the position angle of link one. Link two will have a minimum velocity angle when link one angle is in 0 and $-\pi$ radian where at this position, link two experienced the highest potential energy but the lowest kinetic energy by link one. The maximum velocity is reached when link one is in $-\pi/2$ radian position which means link two experienced the highest kinetic energy by link one.

Overall, the trajectory motion for both computerized and links oriented model formed a similar pattern with slightly different in scale due to different parameter model.

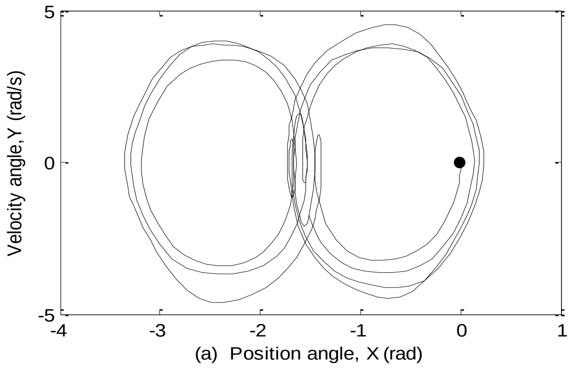


Figure 10 XY plot of link one for computerized model

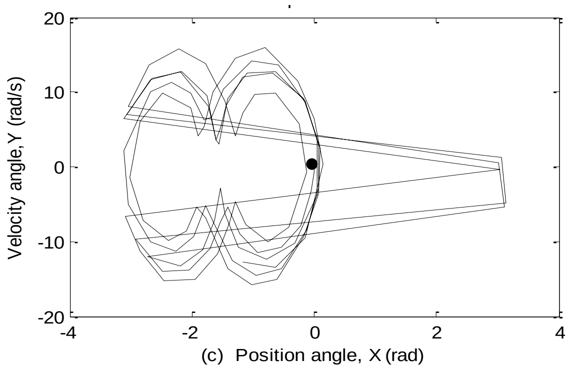


Figure 11 XY plot of link one for links oriented model

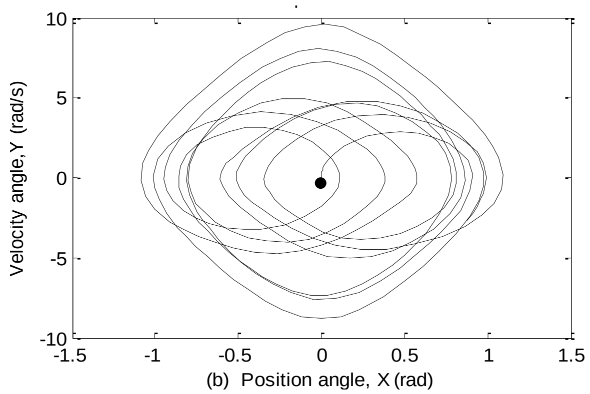


Figure 12 XY plot of link two for computerized model

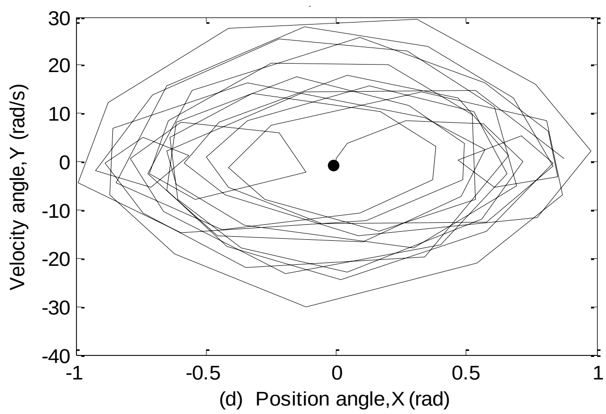


Figure 13 XY plot of link two for links oriented model

■4.0 CONCLUSION

The direct dynamic mechanical system of robot manipulator is derived based on Lagrangian method. We performed the identical simulation result between derived and SimMechanic model. From the result, both methods satisfied the principle of two-link manipulator model. Thus, SimMechanic is convenient for complicated system which the deriving the mathematical system will be cumbersome and difficult. For further research work, we plan to model and analyze a controller system to the link manipulator model.

Acknowledgement

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