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ESTIMATION OF *K*-DISTRIBUTED CLUTTER BY USING CHARACTERISTIC FUNCTION METHOD

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Abstract. Detection performance of the maritime radars is often limited by the unwanted sea echo or clutter. *K*-distribution is one of the long-tailed densities which is known in the signal processing community for fitting the radar sea clutter accurately. In this paper, a novel approach for estimating the parameter of *K*-distribution is presented. The method is derived from the empirical characteristic function of the quadrature components. Simulation results show a great improvement in term of estimated bias and variance, compared with any existing non-maximum likelihood method.

Keywords: Radar system, sea clutter, *K*-distribution, parameter estimation, characteristic function

Abstrak. Prestasi pengesanan radar maritim selalunya terbatas disebabkan gema laut atau serakan yang tidak diingini. Taburan-*K* adalah salah satu ketumpatan berekor panjang, di mana ia dikenali dalam komuniti pemprosesan isyarat untuk memadan dengan tepat serakan laut. Dalam kertas kerja ini, satu pendekatan novel untuk menganggar parameter taburan-*K* dibentangkan. Kaedah ini diterbitkan dari fungsi ciri empirik komponen *quadrature*. Hasil simulasi menunjukkan pembaikan yang ketara dari segi kecenderungan dan varians teranggar, berbanding dengan mana-mana kaedah bukan kemungkinan maksimum sedia ada.

Kata kunci: Sistem radar, serakan laut, taburan-*K*, penganggaran parameter, fungsi ciri

1.0 INTRODUCTION

It has been found from practical measurements that the sea clutter for high resolution radar and low grazing angles can be well modelled by two components [1]. The first component is a spatially varying mean level that results from a bunching of scatterers associated with the sea swell structure. The second component is termed the 'speckle' component, occurs due to the multiple scatterer nature of the clutter in any range cell and has fast fluctuation [2]. Based on the two components, the overall sea clutter amplitude distribution is derived by averaging the speckle component over all possible values of the local mean level which yields the *K*-distribution [3]. The distribution is a two-parameter model which uses the scale parameter together with the shape parameter to provide a complete description of the single point statistics of a

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K-distributed process. The shape parameter is a measure of the degree of the spikiness of the data, where small values of shape parameter are associated with a spiky clutter. A recent study also has confirmed the appropriateness of *K*-distribution to model the amplitude of sea clutter [4].

The use of such statistical models to describe the clutter data necessitates the task of estimating the parameter values from data. The accuracy of the parameter estimate has direct consequences for the performance of radar detection and estimation techniques. Maximum likelihood (ML) estimation offers the optimum parameter estimate when the form of the distribution to be estimated is known. However, the *K*-distribution lacks a closed form analytical solution for the ML parameter estimate. Thus the maximum needed to be solved numerically [5,6]. Although ML based method yields asymptotically efficient estimates, the amount of computation required makes this method impractical to be implemented in real-time systems.

Over the past decades, alternative means of estimating the shape parameter have been proposed and they are generally much faster to compute. It includes method of higher order moment [7], fractional lower order moment (FLOM) [8], texture measure based method [9], gamma density approximation [7], generalised Bessel *K*-function density approximation [10] and recently neural networks based technique [11]. However, the simplest of the methods, namely the higher order moment method produces the worst results, showing largest bias and variance. The fractional lower order moment method was found to be dramatically better. In terms of accuracy and variance, both the approximation based methods are currently unsurpassed. The reason for this is that both methods are the solution of the maximum likelihood estimation formulation. Since both methods are based on density approximation, the performance degrades as the dissimilarity of the approximated density and *K*-distribution increases.

2.0 CHARACTERISTIC FUNCTION

Recently, there is growing interest in applying methods using characteristic function (CF) among the signal processing community. The interest stems from the need to apply signal models more complex than the Gaussian model. A non-Gaussian model such as alpha-stable distribution does not have a closed-form density function, except for the special case of Gaussian, Cauchy and inverse-Gaussian distribution [12]. In other cases, signals and noise do not have probability density function (PDF) in easily tractable forms but the same signals are often conveniently characterised through the characteristic function.

Let X be a real-valued random variable (RV) with density function $p(x)$. The CF $\phi(t)$ of the density function $p(x)$ is by definition [13]

$$\phi(t) \equiv \int_{-\infty}^{\infty} e^{jtx} p(x)\, dx \qquad (1)$$

which is the Fourier transform of $p(x)$. The properties of the Fourier transform allow CF to behave well under shifts, scale changes and summation of RVs, thus it has been used extensively in areas such as testing for goodness of fit, testing for independence, and for parameter estimation.

The simplest estimator of the CF is the sample or empirical characteristic function (ECF) which is defined as [14]

$$\hat{\phi}(t) \equiv \frac{1}{N}\sum_{i=1}^{N} e^{jtx_i} \tag{2}$$

where $x_i; i = 1,2,\ldots,N$ represent independent and identically distributed (IID) RVs with CF $\phi(t)$. The ECF can be directly calculated from the empirical distribution. At a given t, $\hat{\phi}(t)$ is an RV and $\hat{\phi}(t)$ for $-\infty < t < \infty$ is a stochastic process [14].

3.0 ESTIMATING *K*-DISTRIBUTED QUADRATE COMPONENTS

Most estimation techniques in the past had been derived by using the envelope distribution of *K*-distribution, which is given by [2]

$$p(x) = \frac{2}{a\Gamma(\nu)}\left(\frac{x}{2a}\right)^{\nu} K_{\nu-1}\left(\frac{x}{a}\right) \quad x > 0 \tag{3}$$

where a is the scale parameter and $\nu > 0$ is the shape parameter; $\Gamma(\cdot)$ is the gamma function and K_λ is the modified Bessel function of order λ. For high resolution sea clutter, values of ν are generally observed in the region $0.1 \leq \nu \leq \infty$, where ν closed to 0.1 represents a very spiky clutter and $\nu = \infty$ represents thermal noise. As $\nu \to \infty$, the PDF in (3) tends to a Rayleigh density.

Two orthogonal components z and $z_\perp$ of *K*-distributed clutter x can be represented by a compound complex Gaussian random process as [15]

$$x(t) \equiv z(t) + z_\perp(t) = \xi(t)\{g(t) + jg_\perp(t)\} \tag{4}$$

where j denotes square root of -1, $g(t)$ and $g_\perp(t)$ are independent Gaussian processes with zero mean and unit variance. The characteristic function of the quadrature component z (or that of $z_\perp$) can be determined from [16]

$$\phi_z(t) = \int_0^{\infty} \phi_g(t\xi)p(\xi)d\xi = \left(\frac{1}{1+a^2t^2}\right)^{\nu} \tag{5}$$

where $\phi_g(t) = exp\left(-t^2/2\right)$ is the CF of a standard Gaussian distribution, and the modulating process, $\xi(t)$ has the PDF of the form

$$p(x) = \frac{2a^{\nu}\xi^{2\nu-1}}{\Gamma(\nu)} exp(-a\xi) \qquad \xi > 0, a > 0 \tag{6}$$

The inverse Fourier transform of $\phi_z(t)$ yields the univariate K-distribution PDF

$$p(z) = \frac{1}{a\pi\Gamma(\nu)}\left(\frac{|z|}{2a}\right)^{\nu-0.5} K_{\nu-0.5}\left(\frac{|z|}{a}\right) \tag{7}$$

Figure 1 shows the plots of the PDF (7) for a unit second moment and different values of ν. It is interesting to note that when $\nu = 0.5$, the PDF (7) is simply the PDF of the product of two independent Gaussian RVs. When $\nu = 1$, $p(z)$ has a Laplacian distribution. As ν approaching infinity, the PDF tends to a Gaussian distribution.

3.1 Studentized ECF Method

Since the univariate K-distributed RV has a finite second moment, Ilow *et al.* [17] have proposed a method of estimating the parameter ν by normalising the data by

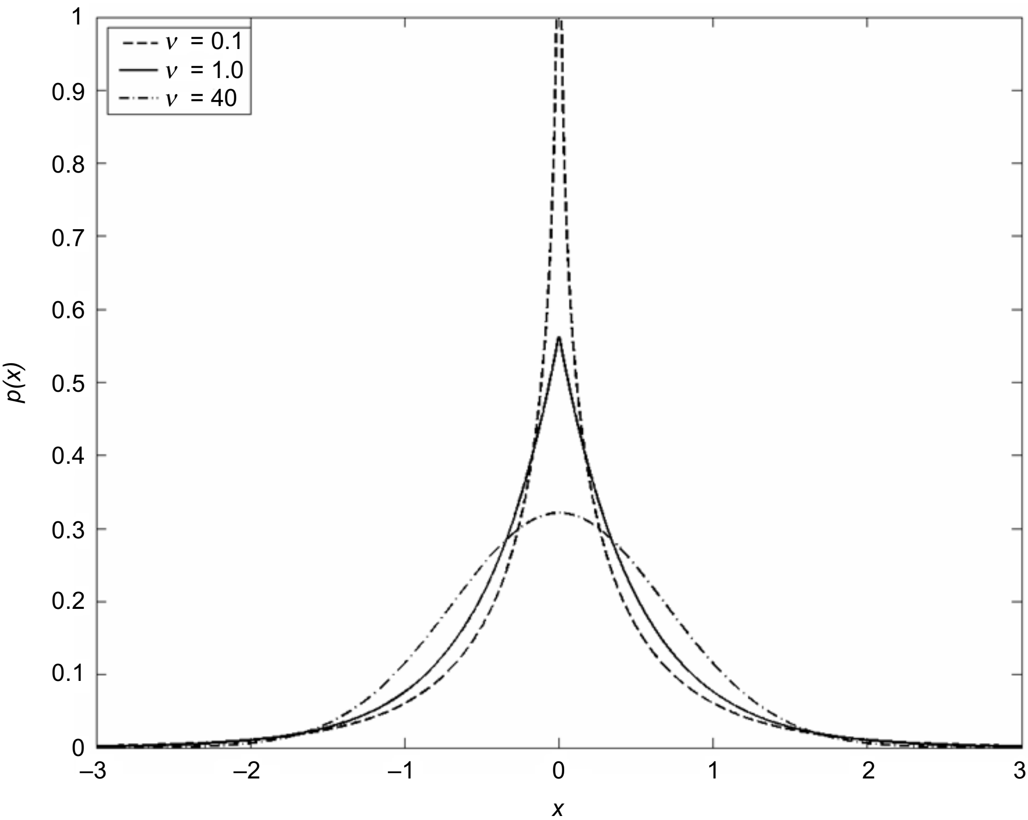

Figure 1 The probability density function of K-distributed quadrature components

the estimated standard deviation. By doing this, the ECF-based estimation methods remain invariant under scale transformations. This process is known in the statistics literature as 'ECF studentization' [18]. For the studentized data, the CF at point t is given by

$$\phi(t) = \left(\frac{1}{1 + t^2 / 2\nu} \right)^{\nu} \tag{8}$$

thus $\hat{\nu}$ can be obtained as a solution of a nonlinear equation through an iterative procedure. It has been verified in [17], theoretically and simulation, that the value of t close to 0.5 results in a good estimator performance for $\nu \in (0,1)$.

3.2 A Novel Approach

A K-distributed RV x can be transformed into its two quadrature components by

$$\begin{aligned} z &= x \cos(2\pi U) \\ z_{\perp} &= x \sin(2\pi U) \end{aligned} \tag{9}$$

where U is a uniformly distributed RV on the interval (0,1). Since the PDF of the quadrature components is an even function, the characteristic function is always real. Therefore, the empirical characteristic function can be calculated as

$$\hat{\phi}_z(t) = \frac{1}{N} \sum_{i=1}^{N} \cos(t z_i) \tag{10}$$

where z_i are the quadrature components of K-distributed RV and $\hat{\phi}_z(t)$ is computable for all real values of t. Taking the logarithm of Equation (5) and changing sign, we obtain

$$-\ln \phi_z = \nu \ln\left(1 + a^2 t^2\right) \tag{11}$$

Thus the logarithm of the CF can be seen to be a linear function, $Y = mX + c$ with

$$\begin{aligned} Y &= -\ln \phi_z(t) \\ m &= \nu \\ X &= \ln(1 + a^2 t^2) \\ \mathrm{c} &= 0 \end{aligned} \tag{12}$$

Figure 2 shows the plot of the characteristic function with its transformed axis as defined in (11), which is a straight line with gradient ν. This can be used to estimate the shape parameter for a case when scale parameter a is known a priori.

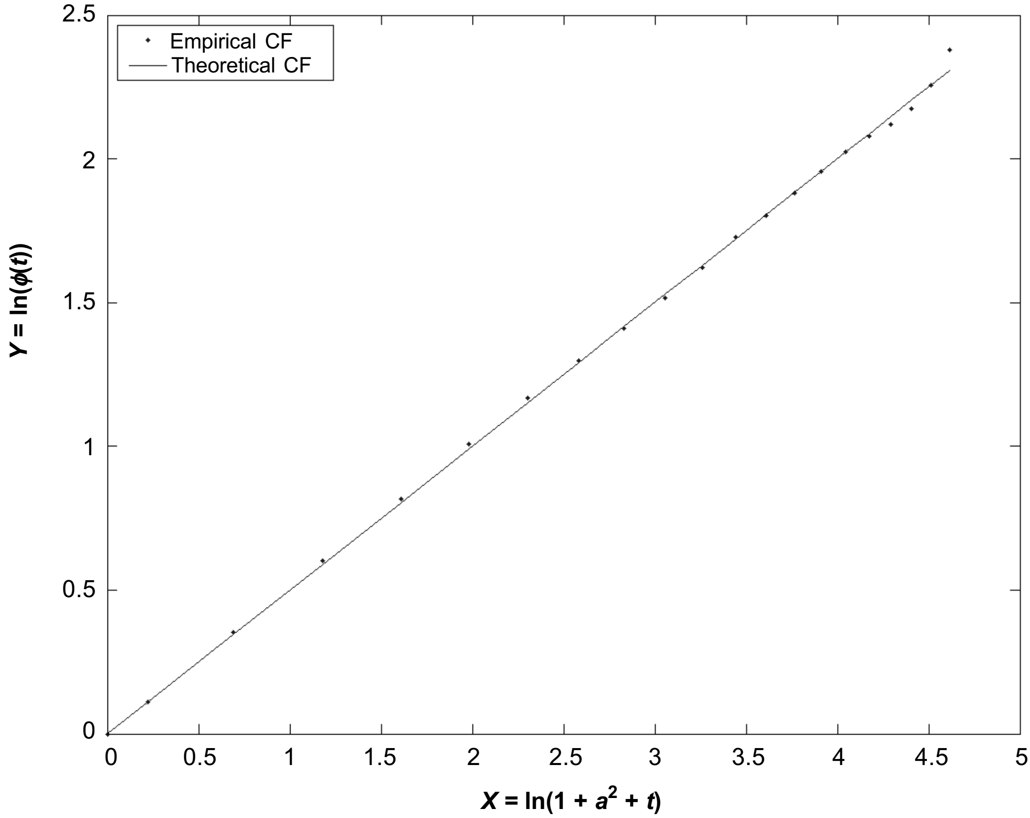

Figure 2 The logarithm of characteristic function of *K*-distributed quadrature components

When only one value of t is selected, the shape parameter can be estimated as

$$\hat{v} = -\frac{\text{In}\hat{\phi}(t)}{\ln\left(1+\text{a}^2 t^2\right)} \tag{13}$$

Alternatively, t can be set more than one and the gradient of the slope can be estimated by using a least square estimator of the regression parameters. Suppose that the responses Y_i, corresponding to input values X_i, $i = 1,2,...,N$ are to be observed and used to estimate the gradient, the shape parameter v can be estimated by

$$\hat{v} = \frac{\sum_{i=1}^{N} X\left(Y - \bar{Y}\right)}{\sum_{i=1}^{N} X\left(X - \bar{X}\right)} \tag{14}$$

for non-zero t, where X and Y are as defined in (12).

4.0 SIMULATION RESULTS AND DISCUSSION

For computational simplicity and to avoid fluctuation at the tail of the characteristic function (in case of a small number of samples), it is proposed that the value t should be in the range $0 \leq t \leq 2$. Throughout the simulation, a is considered to be known ($a = 1$). K-distributed data was generated for 15 different values of the shape parameter as 0.1 to 1.5 in increment of 0.1. This range is of great importance in the radar imaging applications [10]. The number of samples were chosen to be $N = 256$ and 512. The estimation was performed over 1000 independent trials, where in each case the averages were obtained.

In the first simulation, comparison is made between method as in Equation (13) and (14). The value of t was chosen to be 0.1, 0.5, 1 for (13) and $t = 0.1{:}0.1{:}1$ for (14). Figure 3 and 4 shows the estimated bias and variance of the estimates, respectively, for $N = 512$. It can be seen that as the value of t increases, the variance of the estimates is decreased. However, the estimated bias increases with respect to t. This is the normal property of the empirical characteristic function. Therefore, as a trade-off between estimated bias and variance, we proposed the use of linear regression as in Equation (14). It can be seen that this method possesses the optimum variance and keep the bias to its minimum.

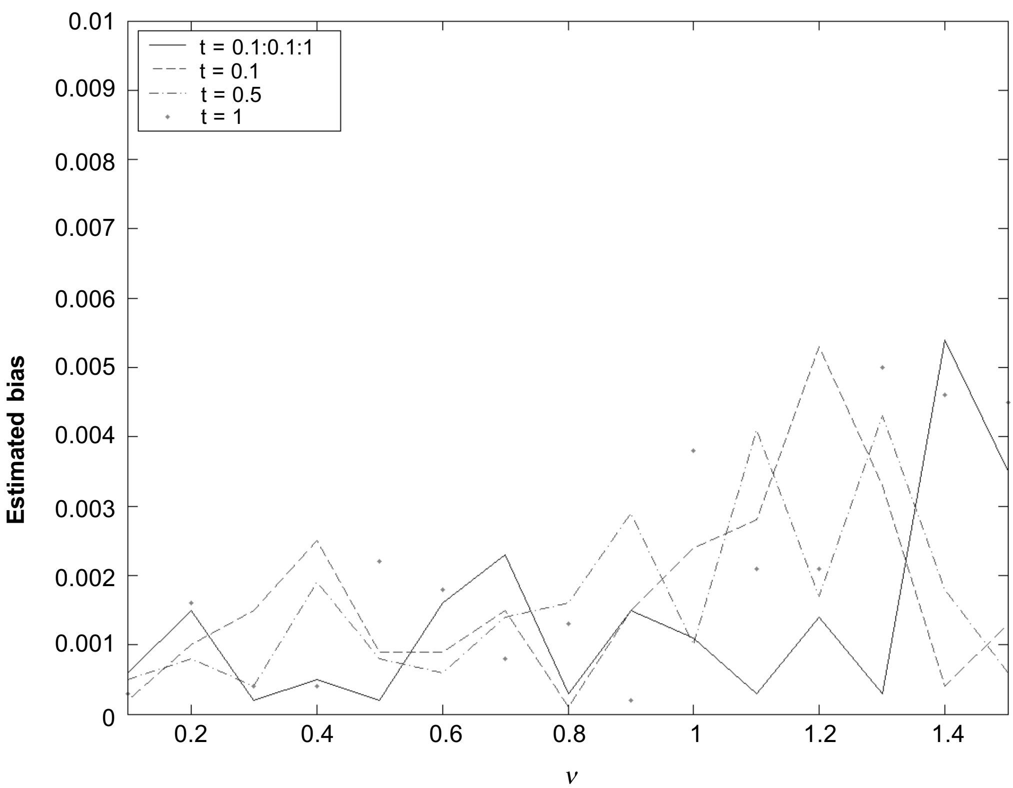

Figure 3 Comparison of estimated bias for the proposed method

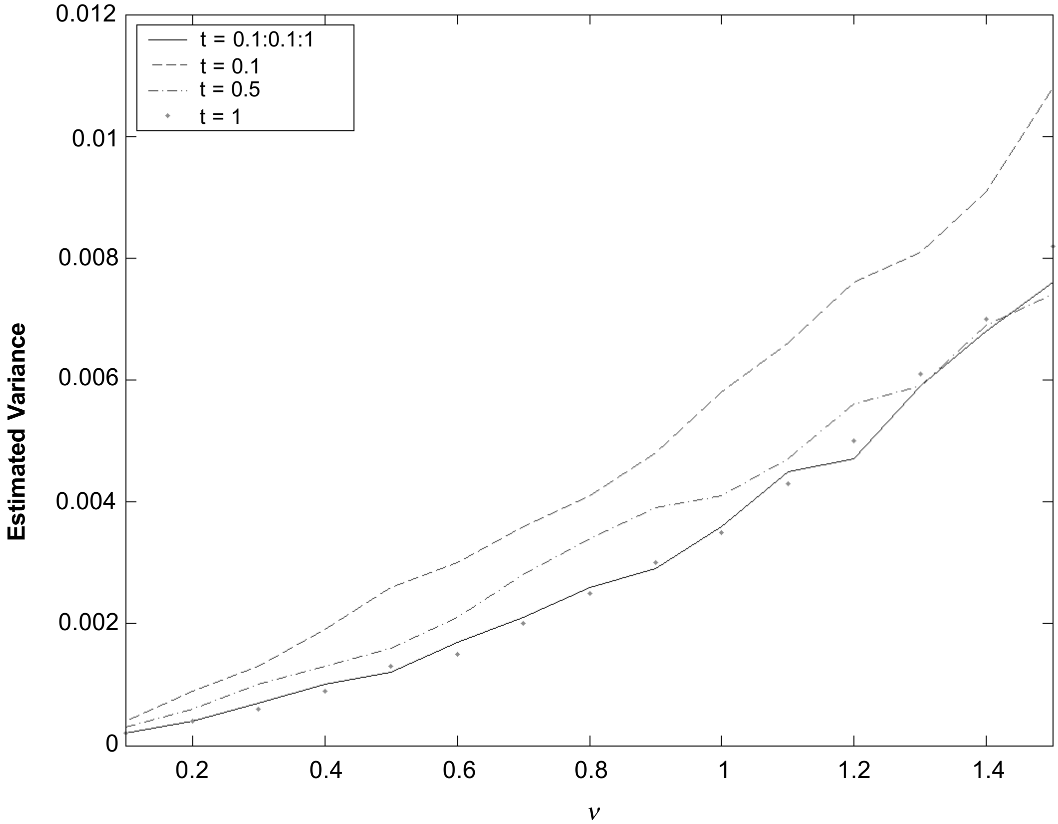

Figure 4 Comparison of variance of estimates for the proposed method

In the next simulation, a comparison is made between the proposed method and previously published methods. Fractional lower order moment (FLOM) method was chosen since it is among the fastest to compute [8], while Raghavan's gamma density approximation method produces the best performance in terms of its variance [7]. The performance of the studentized ECF based estimator is too poor to compare. Figure 5 and 6 show the estimated bias of the estimators for N = 256 and 512 respectively. It can be seen that the proposed method recorded the smallest bias compare with both Raghavan's and FLOM method. The estimated bias decreases as the number of sample increases. The maximum bias recorded by using the proposed method was 0.0088 (N = 256; ν =1.3, $\hat{\nu}$ = 1.3088), where FLOM method estimates was 1.3787 and Raghavan's method estimate it as 1.3622. From Figure 5 and 6, we can easily see that the proposed method based on ECF is unbiased even for small sample sizes.

The plotted values of the variance of the estimates for N = 256 and 512 are shown in Figure 7 and 8, respectively. For $\nu < 0.5$, the performance of the Raghavan's and FLOM method is slightly better than that of the proposed method by a margin of less than 0.015. However, as ν increases, the proposed method outperforms both

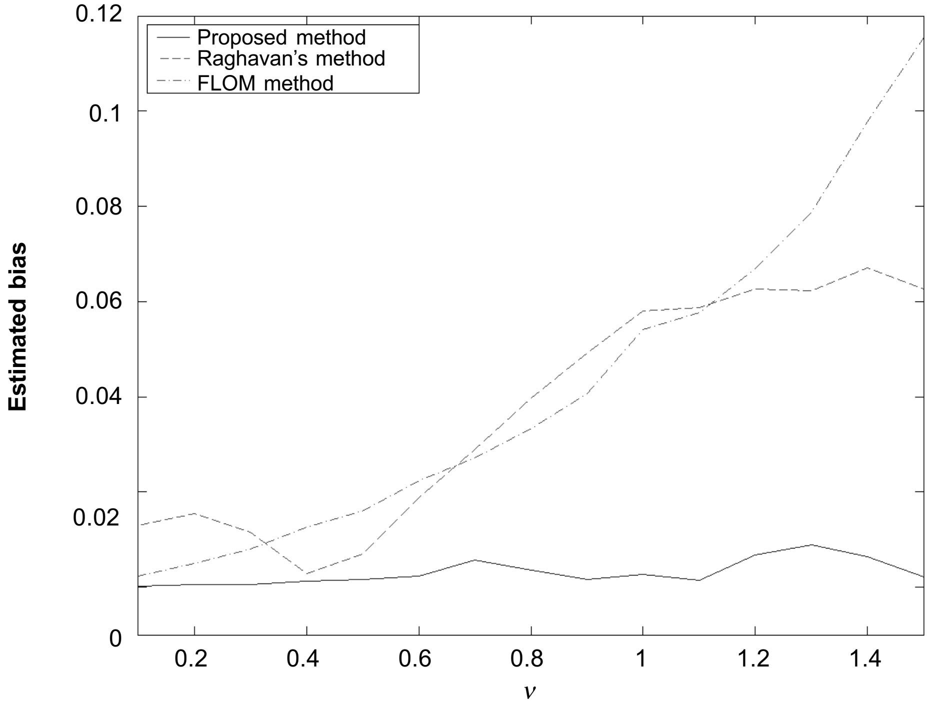

Figure 5 Estimated bias of estimates of ν for N=256

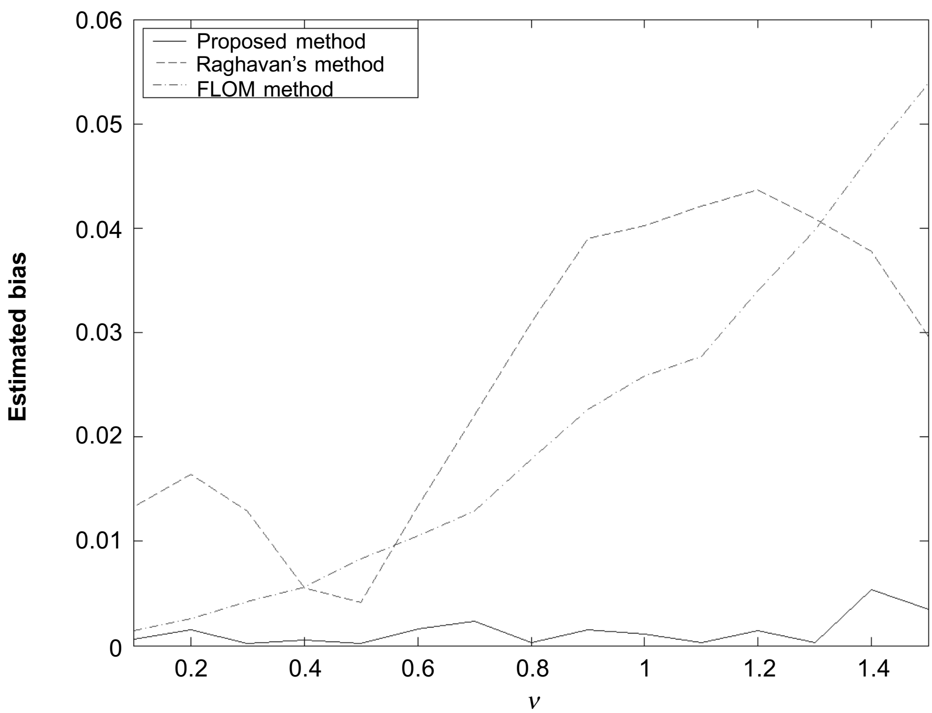

Figure 6 Estimated bias of estimates of ν for N=512

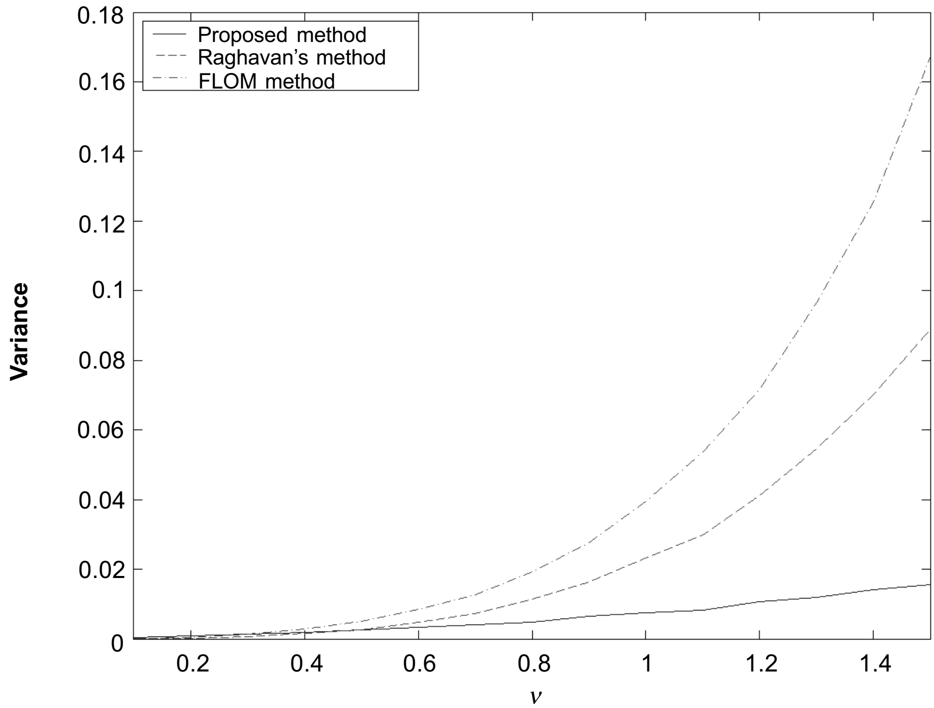

Figure 7 Variance of the estimators for N=256

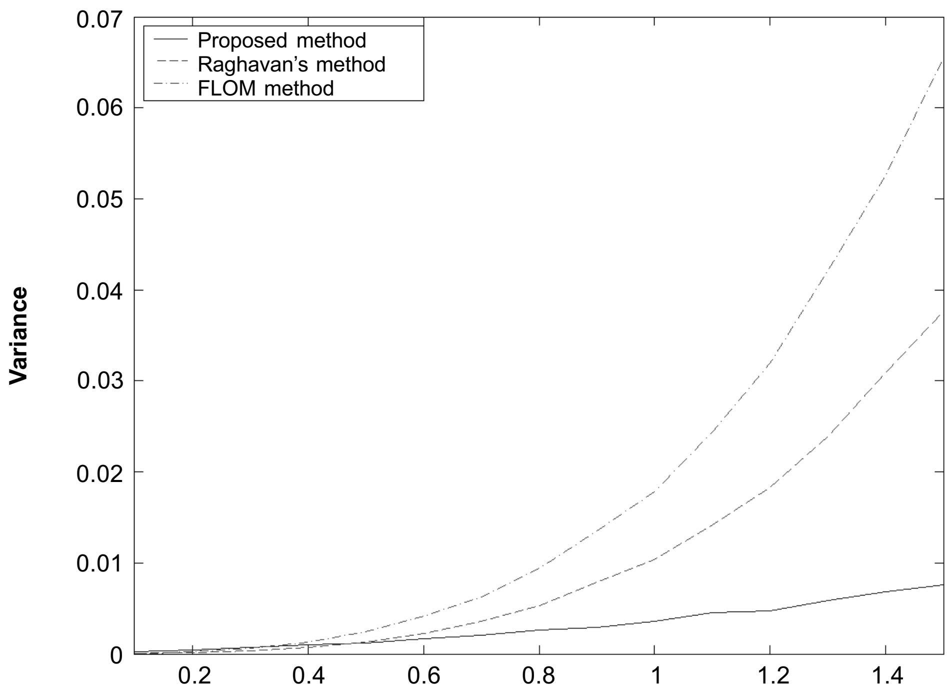

Figure 8 Variance of the estimators for N=512

methods by a large margin. It can be seen that the proposed method can offer an estimate of accuracy less than 0.04, as measured by the variance, for ν in this range. The accuracy of the estimator increases as the sample size increased. For $\nu = 1.5$ and $N = 512$, $\hat{\nu}$ could be estimated by FLOM method as 1.755, while the proposed method only deviates by 0.0872.

5.0 CONCLUSIONS

It has been shown that the shape parameter of *K*-distribution can be estimated via empirical characteristic function. The simulation results show that the method is unbiased even for a small size sample. Although the variance of the proposed method is larger for small ν, the difference is insignificant. As ν increases, the performance of the proposed method is much better compared to the other two methods. In addition, the proposed method is computationally tractable and does not require solving non-linear equations or complicated functions.

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