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ANALYTICAL AND NUMERICAL INVESTIGATION OF FREE VIBRATION CHARACTERISTICS IN LIGHTWEIGHT ALUMINIUM FOAM PANELS

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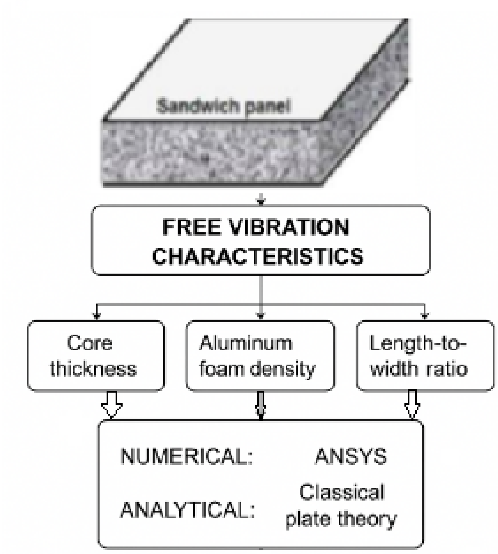
Graphical abstract



Abstract

This study investigates the vibration behavior of aluminum foam composite panels widely used in lightweight aerospace and automotive structures. Accurate prediction of their natural frequencies remains challenging due to the influence of core thickness, density, and geometric aspect ratio. To address this, an analytical model based on conventional Kirchhoff plate theory combined with the Donghui Xu and Ju-Heng Ju equations was developed to estimate the effective Young's modulus and natural frequencies. The analytical results were validated using numerical simulations conducted in ANSYS. Quantitative comparisons showed close agreement between analytical and numerical results. For a core density of 400 kg/m³ and thickness of 5 mm, the first natural frequency (m=1, n=1) was 275.64 Hz analytically and 273.12 Hz numerically (0.87% error). Increasing thickness to 10 mm at the same density yielded 456.35 Hz analytically versus 447.75 Hz numerically (1.89% error). At a density of 500 kg/m³ and thickness of 5 mm, frequencies were 267.97 Hz and 264.91 Hz, respectively (1.14% error). Overall error percentages ranged from 0.32% to 2.42%. Results indicate that increasing core thickness enhances stiffness and raises natural frequencies, whereas increasing density or aspect ratio (a/b) reduces frequency values. The low error range confirms the reliability of the proposed analytical model for engineering design applications.

Keywords: Lightweight sandwich panels, classical plate theory, free vibration analysis, aluminum foam core, FEA of sandwich panels

1.0 INTRODUCTION

Early construction of heavily loaded structures relied on dense, impermeable materials such as steel, subjected to uniform stress and boundary conditions. Over time, composite materials, particularly sandwich structures, emerged as lighter and more efficient alternatives. These structures are widely used, making them essential in manufacturing composite products [1–4]. Metallic foams, particularly aluminum foams, are cellular materials characterized by a solid metal matrix interspersed with interconnected voids, resulting in a unique combination of low density, high strength, and excellent energy absorption capabilities. These properties make metallic foams ideal for a wide range of engineering applications, especially in the aerospace, automotive, and construction industries. Their high strength-to-weight ratio, combined with superior impact resistance, makes them particularly suitable for structural components that need to absorb energy or withstand heavy loads, such as crash-resistant components, armored panels, and load-bearing structures [5–7].

Aluminum foam, a type of metallic foam, is highly regarded for its exceptional porosity and outstanding physical and mechanical properties. Engineers favor aluminum foam structures for their ability to absorb impact energy, provide thermal insulation, and reduce noise. In the construction industry, aluminum foam stands out for its ability to combine lightweight properties with structural strength and design efficiency. Its excellent strength-to-weight ratio and ease of manufacturing make it a preferred material for creating durable yet lightweight structures, contributing to improved performance and sustainability in modern engineering [8, 9]. Currently, aluminium foam-based sandwich panels are extensively used in the aerospace and automotive industries, where reducing weight is crucial without compromising strength or performance. These panels provide superior energy absorption and an enhanced strength-to-weight ratio compared to conventional solid panels, making them an ideal choice for applications that demand both durability and efficiency [9–13]. Recent progress in sandwich panel technology, especially with metal foam as a core material, has greatly improved its mechanical and dynamic properties. Fiber reinforcement and integration of nanomaterials have played an important role in increasing structural performance under both static and dynamic conditions. These innovations have led to strong, more durable ,and very effective materials, making them quickly valuable for high protest engineering applications [14–17]. The inclusion of functionally classified material (FGM) in both fixed and porous structures has been an important development, which enhances the balance between load and resistance and war and total structural efficiency [18, 19]. Functionally classified material (FGM) enables a gradual infection of physical properties of the thickness of the panel,and optimizes thermal stability, mech-anical strength and vibration resistance. This spontaneity in the composition increases the panel's ability to meet dynamic stresses and environmental absorptions and downs, which causes FGM -to be particularly valued in applications that require high performance, durability and structural efficiency [20, 21]. The density of the core plays a crucial role in defining the mechanical properties of sandwich panels, directly influencing their bending strength and energy absorption. Higher core densities improve bending strength but may compromise energy absorption. This balance makes aluminium foam sandwich panels (AFSPs) particularly well-suited for applications that demand both strength and impact resistance, such as automotive battery enclosures, where fire resistance and sound absorption are also critical factors [15]. Steel-aluminum foam-steel sandwich panels have also been shown to be strong and stiff enough for lightweight structural systems, demonstrating the versatility and strength of these composite materials [22]. Extensive research has applied numerical and analytical modeling techniques to study the free vibration behavior of sandwich panels, especially those with metal foam cores. These studies have analyzed how material properties, geometric configurations, and boundary conditions affect vibration performance. Recent advancements, particularly in finite element analysis (FEA), have enabled a more detailed examination of the dynamic response of functionally graded porous panels, offering valuable insights into their mechanical performance and structural efficiency [21]. Amir *et al*. studied functionally graded porous panels, considering geometric nonlinearity in their vibration analysis [23]. Emad *et al.* proposed a new analytical model for the free vibration analysis of a simply supported rectangular functionally graded sandwich plate,which considers porosity-dependent core properties [24]. Muthna *et al*. An analytical model was suggested for cylindrical panels made of porous functionally classified materials (FGM), focusing on how porosity levels and thickness of the content affect their free vibration properties. Their study provides valuable insight into structural behaviour of such panels and exposes the effect of material distribution on vibration performance and mechanical stability [20]. The study of vibration behavior is important as it enhances and supports how to dampen vibration, which is the basis for improving dynamic performance. Many researchers have addressed this topic, including the study by Jebur *et al.* and Ali *et al.* [25, 26]. Using different damping on vibrations can reduce the impact of vibrations and enhance the dynamic stability of the system. Which leads to improving the vibration characteristics and dynamic response. Thus, this research supports the importance of using numerical and analytical models to improve the design of dynamic systems such as sandwich panels.

A numerical model using the homogenization method has also been proposed, facilitating efficient vibration analysis through finite element analysis and validating these models against conventional three-dimensional models [27]. Studies of Djamaluddin [28] have shown that final analysis can improve the natural frequencies of the sandwich foam core structures, and support experimental comparison. Wang *et al*. Different hole distribution and their effects on basic physical properties including young module [29]. Additionally, Badarloo and Salehipour developed closed-loop analytical solutions to analyze the vibration behaviour of curved sandwich panels with composite nano face sheets. Meanwhile, Rahmani *et al*. investigated syntactic foams using higher-order sandwich panel theory,offering valuable insights into the Eigenmodes of simply supported beams [30].

Recent innovations in aluminum foam core sandwich panels (AFSP) with reinforced blankets have made them highly effective in aerospace applications. These panels offer structural robustness and safeguard against micrometeoroids and orbital debris [31]. Additionally, ongoing research is focused on optimizing the performance of advanced composite materials for structural applications [32]. The continued development of these materials is crucial for improving the dynamic behavior of sandwich panels under various loading conditions, making them even more efficient for demanding applications in the aerospace, automotive, and construction sectors.

The integration of aluminum foam in sandwich panels has changed structural material design, which provides an optimal combination of light efficiency, energy absorption ,and mechanical strength. Posts in material science, calculation modeling ,and hybrid core design are continuously improved and processed to these panels for high performance applications. Their adaptability and better structural properties make them inevitable in modern engineering, and expect to unlock even more capacity with ongoing research and expand the boundaries of their applications in the future.

Although several prior studies [33–36] have extensively investigated the vibration characteristics of foam-based sandwich structures through analytical and numerical approaches, this study offers a distinctive contribution by focusing specifically on aluminum foam composite panels, which have received relatively limited attention in this context. Unlike previous works that often isolated individual parameters, such as thickness or density, this research presents an integrated model that captures the combined influence of foam density, core thickness, and panel aspect ratio (a/b) on the natural frequencies of the system.

What sets this work apart is the dual-methodology approach: it combines Classical Plate Theory (CPT) with Finite Element Analysis (FEA) using ANSYS, enabling both theoretical formulation and numerical validation of the results. This hybrid method enhances prediction accuracy and provides a robust analytical framework for early-stage design and vibration analysis of lightweight structures.

Furthermore, while studies like those by Sadeq and Mohsen (2023) [20] and Guangdong Sun *et al.* (2023) [21] investigated the effects of material properties such as density and thickness, they did not sufficiently consider the aspect ratio as a determining factor in free vibration behavior. This study addresses that gap by demonstrating how changes in panel geometry significantly influence dynamic responses, contributing to a more comprehensive understanding of aluminum foam sandwich panels.

By doing so, the study not only validates the analytical results against numerical findings with minimal error margins (as low as 0.23%) but also establishes a reliable methodology for enhancing the dynamic performance of structural components in aerospace and automotive applications.

2.0 METHODOLOGY

2.1 Mechanical Properties Structure

The foam's density, modulus of elasticity, and tensile strength are assumed to be uniform throughout the material. The assumption of the aluminium foam core as a homogeneous medium facilitates analytical and numerical modelling, particularly when the microstructural details of the foam are not critical [32, 37]. This approximation emphasizes the macroscopic mechanical properties of the material, rendering it especially appropriate for applications where stiffness and density are paramount [38].

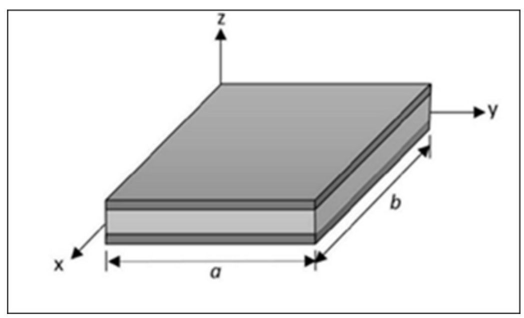


Figure 1 Thin rectangular sandwich panel with aluminium metal foam core and aluminium sheets

The foam's density, modulus of elasticity, and tensile strength are assumed to be uniform throughout. Treating the aluminum foam core as homogeneous simplifies analysis. Analytical and numerical models use this method to simplify calculations, especially when foam cellular details are not important [32, 37]. The homogeneous approximation emphasizes the material's macroscopic mechanical properties, making it suitable for stiffness and density-critical applications [34]. Where ρ_f, ρ_s. The density of the foam and the solid material and E_f, E_s. The Young's modulus of the foam and the solid material.

The Dongwei Shu and Joo-Heng Goh modules was used to calculate the aluminum foam's Young's modulus from its density and solid material properties. Equation 1 calculates closed-cell foam Young's modulus from density [39].

$$E = C_1\phi^2 E_s\left(\frac{\rho}{\rho_s}\right)^2 C_2(1-\phi)E_s\left(\frac{\rho}{\rho_s}\right)+ \quad (1)$$

Where Table 1 shows the material properties of aluminum and the constants used in the Gibson-Ashby equation. Table 2 shows the relationship between foam density and modulus of elasticity calculated using Equation 1. Where C_1empirical constant for the contribution of the solid material for closed-cell foam, C_2 empirical constant representing the cell wall bending and other effects. for closed-cell foam [37] and ϕ Fraction of solid material located in the cell edges.

Table 1 Material properties for aluminium and constants which [34]

Constant	value
C_1 for closed	0.69
C_2 for closed	1
ϕ	0.68

Table 2 Density and elastic modulus of aluminum foam

Density of foam kg/m³	ρ foam/ρ solid	Elastic modulus GPa
400	0.148	3.76
500	0.185	4.85
600	0.222	6.05
700	0.259	7.22
800	0.296	8.49
1000	0.370	11.21

2.2 Analytical Solution

The governing differential equations for the vibrational behavior of sandwich structures were derived. This method corresponds to Kirchhoff's Classical Plate Theory, which applies to thin plates. The following assumptions apply to the linear elastic small deflection theory for thin plates [37]. The plate thickness (h) is minimal in comparison to its lateral dimensions.

The plate material is presumed to be elastic, homogeneous, and isotropic with $E_s = 70$ GPa and $\rho_s = 2730\ \mathrm{kg/m^3}$. At the outset, the plate is level. The mid-plane deflection is minimal, resulting in a slight slope of the deflected surface, rendering the square of the slope insignificant relative to unity. Lines that are initially perpendicular to the mid-plane remain linear and perpendicular throughout deformation, with their length remaining constant. Vertical shear strains (γ_{xz} and γ_{yz}) and the normal stress component (ε_{zz}) are deemed negligible. The normal stress across the thickness (σ_{zz}) is minimal relative to other stress components and may be disregarded in stress-strain relationships. Given the moderate displacements of the plate, it is presumed that the central surface remains unstressed post-bending.

$$D\nabla^2\nabla^2 w(x,y,t) = p(x,y,t) - \rho h\frac{\partial^2 w}{\partial t^2}(x,y,t). \quad (2)$$

$$(3) \quad \frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 m_{yy}}{\partial y^2} + \rho h\frac{\partial^2 \omega}{\partial t^2} = p_z$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \rho h\frac{\partial^2 \omega}{\partial t^2} = 0 \quad (4)$$

Where Mx, My, and Mxy are the bending and twisting moments per unit length of the plate [40], Kirchhoff's hypothesis states that the strain displacement for plates can be expressed as the transverse displacement of the plate's mid-surface. This displacement is caused by the stress that an elastic body experiences when subjected to an applied load.

The transverse displacements that occur as a result of the stress-strain relationships allow for the description of stresses [37]. A representation of the normal and shear stresses that are acting on the plate is denoted by the symbols σ_x, σ_y, and σ_{xy}, respectively. E_{xx}, E_{yy} E_{xy} , G_{xy} , v_{xy} and v_{yx} are the mechanical properties of plate materials sections that are measured in the x and y directions correspondingly. Moreover, the deflection of the plate in the z-direction is denoted by the symbol w(x, y, t)

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial u}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial u}{\partial x}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial w_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial w_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}\\ \frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x\,\partial y}\end{Bmatrix} \quad (5)$$

$$\left.\begin{aligned}\sigma_x &= -z\left(\frac{E_{xx}}{1-v_{xy}v_{yx}}\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + \frac{E_{yy}\vartheta_{xy}}{1-v_{xy}v_{yx}}\frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\\ \sigma_y &= -z\left(\frac{E_{xx}\vartheta_{yx}}{1-v_{xy}v_{yx}}\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + \frac{E_{yy}}{1-v_{xy}v_{yx}}\frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\\ \sigma_{xy} &= -2ZG_{xy}\frac{\partial w_{(x,y,t)}}{\partial x\partial y}^2\end{aligned}\right\} \quad (6)$$

$$\begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix} = \int_{-z/2}^{z/2}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} \quad (7)$$

$$\rho h = \int_{-h/2}^{h/2} \rho \ \partial z. \tag{8}$$

In homogeneous materials, the mechanical properties are represented as follows:
$E_{xx} = E_{yy} = E$ & $G_{xy} = G = \frac{E}{2(1+v)}$ Also $v_{xy} = v$,
Thus, stress relationships become:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E}{1-v^2}\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \\ \frac{E}{1-v^2}\left(v\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\ \frac{E}{(1+v)}\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}. \tag{9}$$

Sandwich plates have three layers: an upper face with a thickness of h_{up}, a lower face with h_{lp}, and a thick core (metal foam) in between with h_f. this configuration provides structural strength and stiffness while minimizing weight.

The stress relationships for each layer of the sandwich panel can be expressed in terms of the properties of each individual layer. The properties for upper face equal to the properties of lower face is made of homogenous aluminum in this study.

$E_{xy} = E_{yx} = E_{up} = E_{lp} = E_{AL}$, $v_{xy} = v_{yx} = v_{up} = v_{lp} = v_{AL}$, $\rho = \rho_{upAL} = \rho_{lpAL} = \rho_{AL}$, $h_{up} = h_{lp} = h_{AL}$ =thick of sheet face

$$G_{xy} = G_{upAL} = G_{lpAL} = G_{AL} = \frac{E_{AL}}{2(1+v_{AL})}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{AL}}{1-v_{AL}}\left(\frac{\partial^2 w}{\partial x^2} + v_{AL}\frac{\partial^2 w}{\partial y^2}\right) \\ \frac{E_{AL}}{1-v_{AL}^2}\left(v_{AL}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\ \frac{E_{AL}}{(1+v_{AL})}\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}. \tag{10}$$

The moment in the sandwich panel:

$$\left.\begin{aligned} M_x^{total} &= M_x \text{upper layer} + M_x \text{foam} + M_x \text{lower layer} \\ M_y^{total} &= M_y \text{upper layer} + M_y \text{foam} + M_y \text{lower layer} \\ M_{xy}^{total} &= M_{xy} \text{upper layer} + M_{xy} \text{foam} + M_{xy} \text{lower layer} \end{aligned}\right\} \tag{11}$$

$$\rho = (\rho h \text{ upper layer} + \rho h \text{ foam} + \rho h \text{ lower layer})/h^{total} \tag{12}$$

Sub equation 7 and 9 in equation 11 to find the total moments in the sandwich panel

$$M_x^{total} = \frac{-E_{AL}}{1-v_{AL}^2}\left(\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + v_{AL}\frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\int_{-h_f/2-h_{AL}}^{-h_f/2} z^2\, dz + \frac{-E_f}{1-v_f^2}\left(\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + v_f\frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\int_{-h_f/2}^{h_f/2} z^2\, dz + \frac{-E_{AL}}{1-v_{AL}^2}\left(\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + v_{AL}\frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\int_{\frac{h_f}{2}}^{\frac{h_f}{2}+h_{AL}} z^2\, dz. \tag{13}$$

$$M_y^{total} = \frac{-E_{AL}}{1-v_{AL}^2}\left(v_{AL}\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + \frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\int_{-h_f/2-h_{AL}}^{-h_f/2} z^2\, dz + \frac{-E_f}{1-v_f^2}\left(v_f\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + \frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\int_{-h_f/2}^{h_f/2} z^2\, dz + \frac{-E_{AL}}{1-v_{AL}^2}\left(v_{AL}\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 + \frac{\partial w_{(x,y,t)}}{\partial y^2}^2\right)\int_{\frac{h_f}{2}}^{\frac{h_f}{2}+h_{AL}} z^2\, dz. \tag{14}$$

$$M_{xy}^{total} = \frac{-E_{AL}}{1+v_{AL}}\frac{\partial w_{(x,y,t)}}{\partial x \partial y}^2 \int_{-h_f/2-h_{AL}}^{-h_f/2} Z^2\, dz + \frac{-E_f}{1+v_f}\frac{\partial w_{(x,y,t)}}{\partial x \partial y}^2 \int_{-h_f/2}^{-h_f/2} Z^2\, dz + \frac{-E_{AL}}{1+v_{AL}}\frac{\partial w_{(x,y,t)}}{\partial x \partial y}^2 \int_{\frac{h_f}{2}}^{\frac{h_f}{2}+h_{AL}} Z^2\, dz. \tag{15}$$

To simplify the calculation process, let

$$A_f = \frac{E_f h_f^3}{12(1-v_f^2)}. \tag{16}$$

$$A_{AL} = \frac{E_{AL}}{3(1-v_{AL}^2)}\left(h_{AL}^3 + \frac{3h_{AL}^2 h_f}{2} + \frac{3h_f^2 h_{AL}}{4}\right) \tag{17}$$

Then the total moment simplified to

$$M_x^{total} = -(2A_{AL} + A_f)\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 - (2A_{AL}v_{AL} + A_f v_f)\frac{\partial w_{(x,y,t)}}{\partial y^2}^2 \tag{18}$$

$$M_y^{total} = -(2A_{AL}v_{AL} + A_f v_f)\frac{\partial w_{(x,y,t)}}{\partial x^2}^2 - (2A_{AL} + A_f)\frac{\partial w_{(x,y,t)}}{\partial y^2}^2. \tag{19}$$

$$M_{xy}^{total} = -(2A_{AL}(1-v_{AL}) + (1-v_f))\frac{\partial w_{(x,y,t)}}{\partial x \partial y}^2 \tag{20}$$

Sub equation 18, 19 and 20 in equation of motion equation 4

$$-(2A_{AL} + A_f)\frac{\partial^4 w_{(x,y,t)}}{\partial x^4} - (2A_{AL}v_{AL} + A_f v_f)\frac{\partial^4 w_{(x,y,t)}}{\partial x^2 \partial y^2} + -2(2A_{AL}(1-v_{AL}) + A_f(1-v_f))\frac{\partial^4 w_{(x,y,t)}}{\partial x^2 \partial y^2} + -(2A_{AL}v_{AL} + A_f v_f)\frac{\partial^4 w_{(x,y,t)}}{\partial x^2 \partial y^2} - (2A_{AL} + A_f)\frac{\partial^4 w_{(x,y,t)}}{\partial y^4} - (2\rho_{AL}h_{AL} + \rho h_f)\frac{\partial w_{(x,y,t)}}{\partial t^2}^2 = 0. \tag{21}$$

Equation 21 can be solved using the method of separation of variables by assuming a deflection function as specified in Equation 22 [40].

$$w(x,y,t) = w(x,y)\cdot w(t) \tag{22}$$

Examine a rectangular plate with dimensions length a and width b, where all four edges are simply supported, as illustrated in Figure 4. To assess the deflection characteristics of the plate in relation to the x and y axes, while adhering to the boundary conditions w=0 and M=0 along all four edges, the deflection equation of the plate, dependent on x and y, is presented in Equation 23 [41].

$$\left.\begin{aligned}
&\left(\frac{\partial^2 W_2}{\partial x^2}+v\frac{\partial^2 W_2}{\partial y^2}\right)\Big|_{(a,y)} = 0 \text{ and } W(a,y)=0\\
&\left(\frac{\partial^2 W_2}{\partial x^2}+v\frac{\partial^2 W_2}{\partial y^2}\right)\Big|_{(0,y)} = 0 \text{ and } W(0,y)=0\\
&\left(\frac{\partial^2 W_2}{\partial y^2}+v\frac{\partial^2 W_2}{\partial x^2}\right)\Big|_{(x,b)} = 0 \text{ and } W(x,b)=0\\
&\left(\frac{\partial^2 W_2}{\partial y^2}+v\frac{\partial^2 W_2}{\partial x^2}\right)\Big|_{(x,0)} = 0 \text{ and } W(x,0)=0
\end{aligned}\right\} \tag{23}$$

When, applying the four boundary conditions, the displacement equation can be written as follows[42] where a, b represent the width of sandwich and Length of sandwich

$$w(x,y) = \sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi y}{b}(m,n=1,2\,...). \tag{24}$$

By deriving the Equation 23 with respect to the second derivative in the x-direction, y-direction, and the xy-direction. Then, the fourth-order Equation 24 sub in Equation 21.

$$\begin{bmatrix}-(2A_{AL}+A_f)\left(\frac{m\pi}{a}\right)^4-(2A_{AL}v_{AL}+A_fv_f)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2-\\ 2\left(2A_{AL}(1-v_{AL})+A_f(1-v_f)\right)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\\ -(2A_{AL}v_{AL}+A_fv_f)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2-(2A_{AL}+A_f)\left(\frac{n\pi}{b}\right)^4\end{bmatrix}w(t)-(2\rho_{AL}h_{AL}+\rho h_f)\frac{d^2w(t)}{dt^2}=0 \tag{25}$$

By comparison, the Equation 25 with Equation 26 can be easily identified as having a simple second-order form, yielding the result ω_n^2 [38]

$$\frac{d^2w(t)}{dt^2}+\omega_n^2w(t)=0 \tag{26}$$

Then,

$$\omega_n^2=\left[-(2A_{AL}+A_f)\left(\frac{m\pi}{a}\right)^4-(2A_{AL}v_{AL}+A_fv_f)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2-2\left(2A_{AL}(1-v_{AL})+A_f(1-v_f)\right)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2-(2A_{AL}v_{AL}+A_fv_f)\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2-(2A_{AL}+A_f)\left(\frac{n\pi}{b}\right)^4\right]/-(2\rho_{AL}h_{AL}+\rho h_f) \tag{27}$$

$$w_n=\sqrt{\omega_n^2} \tag{28}$$

In the theoretical analysis, MATLAB was used to calculate the natural frequency ω_n for each value of n, m =(1,1),(1,2),(2,1),(2,2),(3,1) and (1,3) for the sandwich and plotted the mode shape of free vibration.

2.3 Investigation of Sandwich Panels with Metal Foam Core for Analysis

Numerical techniques such as finite elements (FEM) are powerful and widely used to analyze the free vibration behavior of complex structures, including sandwich plates. The versatility of the method allows accurate modeling of heterogeneous materials, geometric complexity, and various boundary and loading conditions associated with such structures. Numerous studies [43, 44] have shown that FEM can capture the dynamic responses of sandwich plates with different core and face configurations. It is possible to investigate how key parameters such as the thickness of the core, the material properties of the face, and the support conditions affect the natural frequencies and modes based on the effect of these parameters on the model. FEM remains the preferred approach for researchers and engineers who wish to gain reliable insights into sandwich structure vibrational performance [45, 46] to a detailed finite element model was developed using ANSYS 2021 R1 to analyze the free vibration characteristics of aluminum foam sandwich panels [47] as shown in Figure 2. ANSYS Design Modeler provided a comprehensive environment for parametric geometry creation, automated meshing, and structural simulation. The model was constructed in 3D using the SOLID186 element, a 20-node higher-order solid element capable of representing large deformations, plasticity, creep, and incompressibility effects. Each node possesses three degrees of freedom (translations in x, y, and z directions). A structured mesh was generated automatically, resulting in 73370 elements and 323324 nodes, as shown in the mesh visualization. A mesh sensitivity study was conducted to ensure that the results were not significantly influenced by mesh size, confirming the stability and reliability of the numerical model. Simply supported boundary conditions were applied along all edges of the panel, consistent with the analytical assumptions. These were implemented by constraining out-of-plane displacements while allowing in-plane movement.

The mechanical properties of the materials were assumed to be homogeneous and isotropic. The aluminum face sheets were assigned standard mechanical properties explained in Table 1, while the Young's modulus of the aluminum foam core was calculated based on the Donghui Xu and Ju-Heng Ju equations model, using the relative density of the foam explained in Table 2. These values were manually defined in ANSYS to align with the analytical model.

To simulate perfect adhesion between the face sheets and the foam core, Add Frozen contact constraints were used in ANSYS, effectively eliminating relative displacement at the interfaces. Following model setup, modal analysis was performed to extract the natural frequencies and corresponding mode shapes under various conditions, including changes in core density, thickness, and aspect ratio (a/b). This numerical approach provided a robust basis for comparison with the analytical results and enabled a more accurate assessment of the sandwich panel's dynamic behavior.

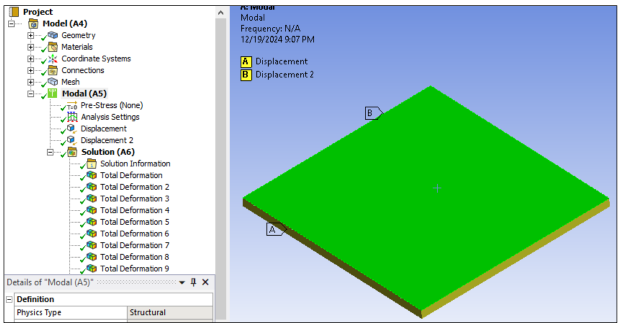


Figure 2 Sandwich Panel Modal Analysis in ANSYS with simply supported Boundary Conditions

3.0 RESULTS AND DISCUSSION

In this study, a new mathematical model was proposed to analyze the free vibration characteristics of sandwich panels with metallic foam cores, as described in Equation 26. The investigation focused on the influence of various foam core properties, theoretically determined using the Gibson-Ashby model outlined in Equation 1. Natural frequencies corresponding to different metallic foam cores were evaluated under varying foam densities. To validate the analytical solution, numerical simulations were performed using ANSYS 2021 R1 software. The results were presented in tabular form and visualized through multiple curves generated using MATLAB programming. Because numerical models take into consideration more realistic deformations and complicated stress distributions, which theoretical models might miss, the numerical natural frequencies are typically found to be lower than the theoretical values [48, 49]. This is clearly shown in Figure 3 and Table 3 and Table 4.

Tables 3 and 4 indicate that the metal foam directly affects the natural frequency, as Saeed, Badshah and *et al.* [50] showed that the density of the material plays a crucial role in the dynamics of systems with different thicknesses. The effect of density on the natural frequency becomes more pronounced with increasing thickness of the metal core, as shown in the table. Higher-density systems have more mass, which leads to a decrease in the natural frequency.

Theoretical analysis of laminates containing foam cores shows that the relationship between total mass (represented by density and thickness) and natural frequency is related to a balance between stiffness (which depends on the area, Poisson's ratio, and Young's modulus) and equivalent mass (determined by density and thickness). As the density increases, the natural frequency increases due to increased stiffness. However, increasing porosity leads to a decrease in frequency due to decreased stiffness and increased mass, which has been shown in previous studies [51, 52, 53].

This decrease in frequency with increasing density can be explained by the relationship between mass and stiffness. Gibson and Ashby [32] demonstrated the relationship between density and stiffness in cellular materials such as metal foams, where higher density tends to reduce the natural frequency of the structures. This is consistent with the results of the current study that natural frequencies decrease with increasing density, as pointed out by Zahra *et al.* [34] who analyzed the effect of density on structural properties and found that materials with higher density reduce natural frequencies due to their increased mass.

Furthermore, Gao *et al.* [54] supported these results through their theoretical and numerical investigations, where they confirmed the inverse relationship between density and natural frequencies. The study of Ellitore [55] confirmed that low-density panels have higher natural frequencies due to their lower mass. These results are consistent with the current study, enhancing the scientific understanding of the relationship between metal foam density and natural frequency.

Although increasing density reduces the natural frequency, it also increases Young's modulus, which is a major factor affecting structural stiffness. The natural frequency of a structure is the result of the interaction between mass and stiffness, where Young's modulus is a measure of stiffness. The higher the Young's modulus, the more resistant the material is to deformation under external loads. The results indicate that when Young's modulus increases, the natural frequency of a sandwich structure decreases at certain values of thickness and a/b ratio, as shown in Figure 4to Figure 5. An increase in stiffness leads to higher natural frequencies, provided that the mass remains constant, according to recent research results. For example, a study on composite materials reinforced with shape memory alloy showed that increasing stiffness leads to higher natural frequency [37].

On the other hand, the effect of thickness on the natural frequency of metal foams in composite structures is clear, as shown in the third and fourth tables for each value of M.N. This study deals with the effect of thickness on the natural frequency using numerical and analytical models and shows that increasing the thickness leads to an increase in the equivalent mass, which reduces the natural frequency. Where we notice that the analytical natural frequency increases from 275.64 to 456.35 and the numerical frequency increases from 273.12 to 447.75 by increasing the thickness of the aluminum foam from 5 to 10 mm at a density of 400. This resulted in an increase in the analytical natural frequency by 65.6%, while the numerical frequency increased by 63.9%. Many studies support the current understanding of the effect of thickness on the natural frequency of metal foams, which helps in improving the design of composite structures. Where Thi *et al.* found that variations in thickness can alter the mass distribution and stiffness of the foam, affecting its vibrational characteristics [56]. Mouthanna *et al.* and Pham-Tan *et al.* found that thicker plates tend to have lower natural frequencies due to increased mass, while thinner plates can resonate at higher frequencies [57, 58].

When a/b is the ratio of width to length (the plate), increasing this ratio from 1 to 1.5 has a significant effect on the natural frequency of the system. When a/b = 1, the plate is in a dimensionally balanced shape, resulting in an even distribution of mass and stiffness. However, when a/b increases to 1.5, the plate becomes longer in one direction, resulting in more mass in that direction, which increases the flexibility of the system and reduces its stiffness. This change in dimensions reduces the natural frequency because the system becomes more susceptible to motion in the longer direction, and thus the frequency decreases. Increasing the dimensions in a given direction increases the equivalent mass in that direction, which reduces the ability to oscillate rapidly and lowers the natural frequency of the system.

Based on these results, the effect of increasing dimensions on the natural frequency in systems containing metal foams can be understood, as the change in the geometric shape (such as a/b) has a significant effect on the dynamic properties of the system. Where the frequency of the analytical sandwich decreases when a/b changes from 1 to 1.5 from 275.64, 267.97, 261.13, 254.87, 249.24, 239.35 to 199.07, 193.53, 188.59, 184.08, 180.01, 172.87 at density of 400, 500, 600, 700, 800, and 1000, respectively, with metal foam thickness 5 mm, the natural frequency decreased from 456.35, 438.20, 423.04, 409.77, 398.46,, 379.78 To 329.58, 316.48, 305.53, 295.95, 287.78, 274.29 with 10mm metal foam thickness. As shown in Table 3 and Table 4.

Table 4 shows the natural frequencies calculated both numerically and analytically. The analytical natural frequencies were derived using Equation (27). It was observed that there is a good agreement between the theoretical and numerical results for all values. There was great agreement of the fundamental natural frequency with discrepancies ranging from 0.23% to 2.42 % depending on the case, as shown in Table 2 for the foam density of 400 kg/m³ with a thickness of foam equal to 10 mm when n,m = 1. This indicates the good accuracy and reliability of both methods in predicting the natural frequencies of the system. The close match between these approaches validates the model and assumptions used in the theoretical analysis.

Numerical and theoretical results differ because analytical models simplify assumptions and neglect nonlinear deformations, structural imperfections, and complex dynamic effects. These assumptions can make theoretical results less accurate than numerical simulations. However, numerical models like FEA account for more material and structural effects, such as stress and deformation distribution across the structure and nonlinear effects [46].

There is a strong correlation between the refinement of the mesh and the accuracy of numerical simulations. When using finite element analysis (FEA), the structure is broken down into smaller components known as the mesh. The more finely the mesh is, the more realistic and detailed the results are. Finer meshes, on the other hand, call for a greater number of computational resources, which in turn increases the amount of time required for simulations.

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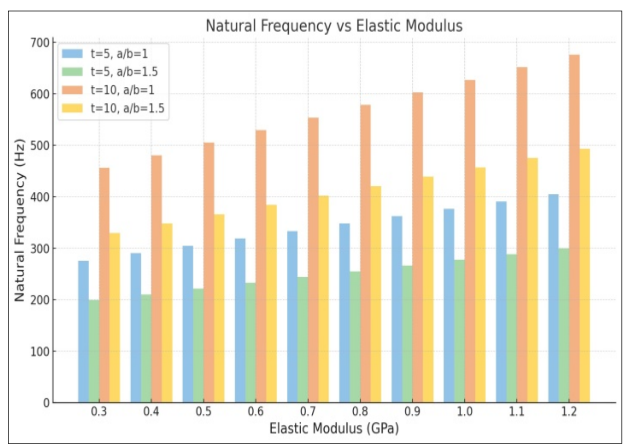

Figure 3 The relationship between natural frequency and modulus of elasticity of aluminum foam sandwich panels

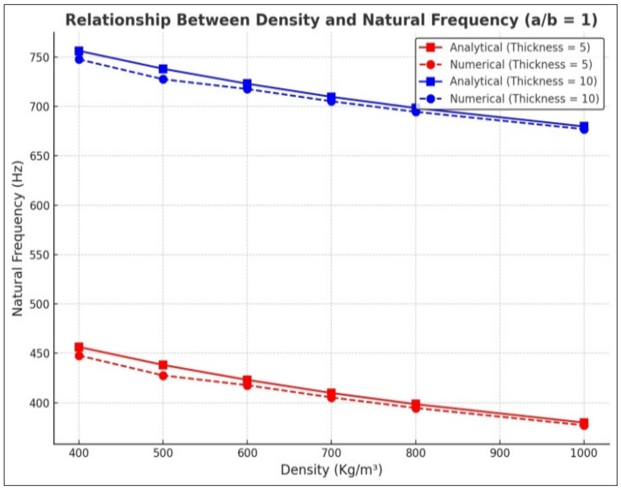

Figure 4 The relationship between density and natural frequency of aluminum foam sandwich panels

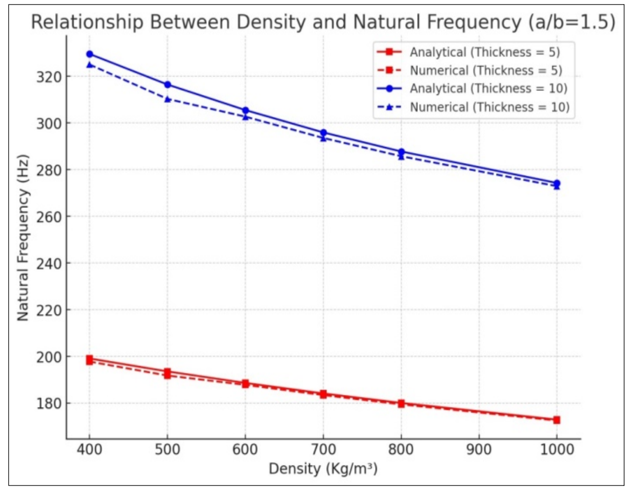

Figure 5 The relationship between density and natural frequency of aluminium foam sandwich panels

Table 3 Numerical and analytical natural frequencies of sandwich panel with the metal foam core at a=400 mm

Metal Foam Density kg/m3	Elasic Moduls GPa	Thick of metal foam	a/b	Natural frequency HZ	m=1 n=1	m=1 n=2	m=1 n=3	m=2 n=1	m=2 n=2	m=3 n=1
400	3.76	5	1	analytical	275.64	689.09	1378.18	689.09	1102.54	1378.18
				numerical	273.12	673.73	1319.1	673.73	1064.	1319.1
			1.5	analytical	199.07	382.83	689.09	612.52	796.28	1301.62
				numerical	197.75	377.98	775.8	600.32	673.7	1063.9
		10	1	analytical	456.35	1140.86	2281.73	1140.86	1825.38	2281.73
				numerical	447.75	1089.5	2090.2	1089.5	1699.2	2090.2
			1.5	analytical	329.58	633.81	1140.86	973.17	1318.33	2154.96
				numerical	325.05	617.39	1250.4	617.39	1089.5	1699.
500	4.85	5	1	analytical	267.97	669.92	1339.84	669.92	1071.88	1339.84
				numerical	264.91	654.1	1282.5	654.1	1033.9	1282.5
			1.5	analytical	193.53	372.18	669.92	595.49	774.13	1265.41
				numerical	191.77	366.71	753.38	582.73	654.08	1033.8
		10	1	analytical	438.20	1095.50	2191.01	1095.50	1752.81	2191.01
				numerical	427.59	1042.3	2004.8	1042.3	1628.2	2004.8
			1.5	analytical	316.48	608.61	1095.50	973.78	1265.92	2069.29
				numerical	310.31	589.88	1196.7	930.73	1042.3	1628.1
600	6.05	5	1	analytical	261.13	652.83	1305.65	652.83	1044.52	1305.65
				numerical	259.58	643.27	1268.5	643.27	1020.4	1268.5
			1.5	analytical	188.59	362.68	652.83	580.29	754.38	1233.12
				numerical	187.78	359.68	741.63	572.73	643.28	1020.3
		10	1	analytical	423.04	1057.60	2115.19	1057.60	1692.15	2115.19
				numerical	417.68	1025.2	1991.9	1025.2	1611.5	1991.9
			1.5	analytical	305.53	587.55	1057.60	940.09	1222.11	1997.68
				numerical	302.71	577.29	1179.1	914.33	1025.2	1611.4
700	7.22	5	1	analytical	254.87	637.18	1274.37	637.18	1019.49	1274.37
				numerical	253.57	629.18	1243.2	629.18	999.23	1243.2
			1.5	analytical	184.08	353.99	637.18	566.39	736.30	1203.57
				numerical	183.39	351.48	725.62	560.06	629.2	999.2
		10	1	analytical	409.77	1024.43	2048.87	1024.43	1639.09	2048.87
				numerical	405.27	997.06	1944.3	997.06	1570.8	1944.3
			1.5	analytical	295.95	569.13	1024.43	910.61	1183.79	1935.04
				numerical	293.58	560.49	1147.4	888.9	997.09	1570.7
800	8.49	5	1	analytical	249.24	623.09	1246.19	623.09	996.95	1246.19
				numerical	248.13	616.28	1219.6	616.28	979.66	1219.6
			1.5	analytical	180.01	346.16	623.09	553.86	720.02	1176.96
				numerical	179.42	344.03	710.92	548.48	616.3	979.65
		10	1	analytical	398.46	996.15	1992.30	996.15	1593.84	1992.30

Metal Foam Density kg/m3	Elasic Moduls GPa	Thick of metal foam	a/b	Natural frequency HZ	m=1 n=1	m=1 n=2	m=1 n=3	m=2 n=1	m=2 n=2	m=3 n=1
				numerical	394.6	972.62	1902.	972.62	1534.9	1902.
			1.5	analytical	287.78	553.42	996.15	885.47	1151.11	1881.62
				numerical	285.76	546.04	972.66	866.83	1119.9	1535.1
1000	11.21	5	1	analytical	239.35	598.38	1196.77	598.38	957.42	1196.77
				numerical	238.59	593.38	1176.9	593.38	944.5	1176.9
			1.5	analytical	172.87	332.44	598.38	531.90	691.47	1130.28
				numerical	172.48	330.92	684.76	527.98	593.41	944.52
		10	1	analytical	379.78	949.46	1898.92	949.46	1519.14	1898.92
				numerical	377.06	931.81	1829.6	931.81	1474.2	1829.6
			1.5	analytical	274.29	527.48	949.46	843.97	1097.16	1793.43
				numerical	272.91	522.1	1073.6	830.08	931.86	1474.2

Table 4 Discrepancy between numerical and analytical natural frequencies for metal foam sandwich panels at a= 400 mm

Metal foam density kg/m3	Thick of metal foam	a/b	analytical Natural frequency HZ	Numerical Natural frequency HZ	Discrepancies percent %
400	5	1	275.64	273.12	0.91
		1.5	199.07	197.75	0.66
	10	1	456.35	447.75	1.88
		1.5	329.58	325.05	1.37
500	5	1	267.97	264.91	1.14
		1.5	193.53	191.77	0.91
	10	1	438.20	427.59	2.42
		1.5	316.48	310.31	1.95
600	5	1	261.13	259.58	0.59
		1.5	188.59	187.78	0.43
	10	1	423.04	417.68	1.27
		1.5	305.53	302.71	0.92
700	5	1	254.87	253.57	0.51
		1.5	184.08	183.39	0.37
	10	1	409.77	405.27	1.1
		1.5	295.95	293.58	0.8
800	5	1	249.24	248.13	0.45
		1.5	180.01	179.42	0.33
	10	1	398.46	394.6	0.97
		1.5	287.78	285.76	0.71
1000	5	1	239.35	238.59	0.32
		1.5	172.87	172.48	0.23
	10	1	379.78	377.06	0.72
		1.5	274.29	272.91	0.5

Figures illustrate the normalized mode shapes derived from the free vibration analysis. The displayed amplitudes are not due to external excitation but are intrinsic to the structure's vibration modes. Consequently, the study exclusively examines free vibration characteristics, and the findings align with the objectives of modal analysis. This analysis was carried out with the assistance of both analytical models and numerical simulations. The natural vibration behavior of the composite panel is depicted by the graphs that were derived from numerical analysis (which was generated using ANSYS software) and theoretical calculations (which were computed using MATLAB). The simulation of the behavior of the composite panel when subjected to free vibrations is the primary objective of these graphs. The deformation patterns and displacements that take place on the surface of the panel at each natural frequency are broken down into these visual representations, which provide insight into the phenomenon. Engineering professionals can identify vibration modes (mode shapes) and the natural frequencies that correspond to them in the system with the assistance of the graphs.

MATLAB-generated graphs compare theoretical equation-calculated natural frequencies to ANSYS-calculated ones. This comparison validates theoretical models against numerical data and ensures accuracy. By showing the relationship between density and natural frequency, Python plots help optimize panel design based on material properties like density and Young's modulus. In order to detect critical resonances, which could result in the panel's mechanical failure or permanent deformation, it is essential to identify the vibration modes that are present [47]. Engineers enhance the structural design of aluminum foam components by analyzing vibration modes. This process helps increase the components' resistance to harmful vibrations. By comparing numerical and analytical results, engineers can identify vibration modes and detect critical resonances that might lead to panel failure or permanent deformation. Through this analysis, structural designs are refined to minimize vibrations, ultimately extending the lifespan of aluminum foam components. Optimization of panel density and stiffness for aerospace and automotive applications is achievable through free vibration analysis. Ensuring the accuracy of theoretical models and verifying their alignment with real-world behavior is essential. When analytical and numerical results closely match, engineers can confidently use analytical models for more efficient calculations, saving both time and effort. This process of validation helps ensure that theoretical approaches are reliable for use in design and analysis in the future. Although numerical and analytical models produced encouraging outcomes, it is essential to keep in mind that the model is founded on the Classical Plate Theory, which presupposes a particular thickness and material homogeneity. In studies that involve metallic foams, this methodology is generally accepted to simplify the process overall.

The findings of this research align closely with numerous prior studies examining the influence of density, foam thickness, and aspect ratio on the natural frequencies of aluminum foam sandwich panels. Previous studies, including those by Gibson and Ashby [37] demonstrated that the natural frequency of metal foam systems is considerably influenced by density, with natural frequencies rising as density increases due to augmented stiffness, a finding corroborated by this research. The findings of this research align with those derived from numerical models in Wang *et al.* [29] which demonstrated that an increase in density results in a reduction of natural frequency due to the augmented mass, thereby diminishing stiffness. Our findings corroborate the assertions made by Amir *et al.* [24] that natural frequency rises with augmented foam thickness, attributable to the enhanced stiffness conferred by thicker foam. Our research demonstrated that augmenting the foam thickness from 5 mm to 10 mm results in a substantial elevation in the natural frequency, corroborating findings from prior studies employing finite element simulation. Research, including that of Mouthanna *et al.* [20] Length-to-width ratio (A/B) significantly affects the natural frequency. In particular, an increase in this ratio in 1 to 1.5 results in natural frequency loss is responsible for extended flexibility in the system with a longer axis and reduces the frequencies. Our current research confirmed this effect and revealed remarkable reduction in frequencies with an increased relationship, and the conclusions documented in before -literature.

During the thickness of the foam, the effect of increased foam density on the natural frequency ignored, as these studies focus only on thickness without explaining density variations. This research examined the mutual relationship between density, thickness and aspect conditions, which improved profit accuracy and provided a greater intensive understanding of the effect of these variables. This comparison with pre-literature suggests that current research results are consistent with the trends prevailing in the field, it shows that integration of analytical and numerical models acts as an effective means of examining natural frequencies in these complex systems.

Notwithstanding the significant findings of this study, certain limitations must be acknowledged. The study is based on classical plate theory, which posits that the thickness is small relative to the panel dimensions. The metal foam was presumed to be homogeneous, which may not be precise in applications, necessitating more intricate models that consider heterogeneous material distribution or significant deformations in foam panels. The numerical analysis in the study utilized ANSYS software, which employs a specific element type, potentially constraining the accuracy of results in instances involving panels with atypical dimensions or properties not encompassed by the model. The study concentrated on the density, thickness, and aspect ratio of the aluminum foam panels, neglecting the influence of the foam's cellular structure, which can impact the vibrational characteristics of the panels.

Finally, the study faces some limitations, most notably its reliance on classical plate theory (CPT), which neglects transverse shear effects, and its consideration of metallic foam as a homogeneous material, which does not accurately reflect the true structure of foams. Furthermore, the study did not include actual experimental verification. Therefore, it is proposed in the future to use more accurate analytical models, such as FSDT, and to study the effect of porosity irregularity, along with the possibility of enhancing properties by adding nanomaterials or designing complex internal structures within the foam core. These approaches will contribute to improving the accuracy of the models and expanding their engineering applications.

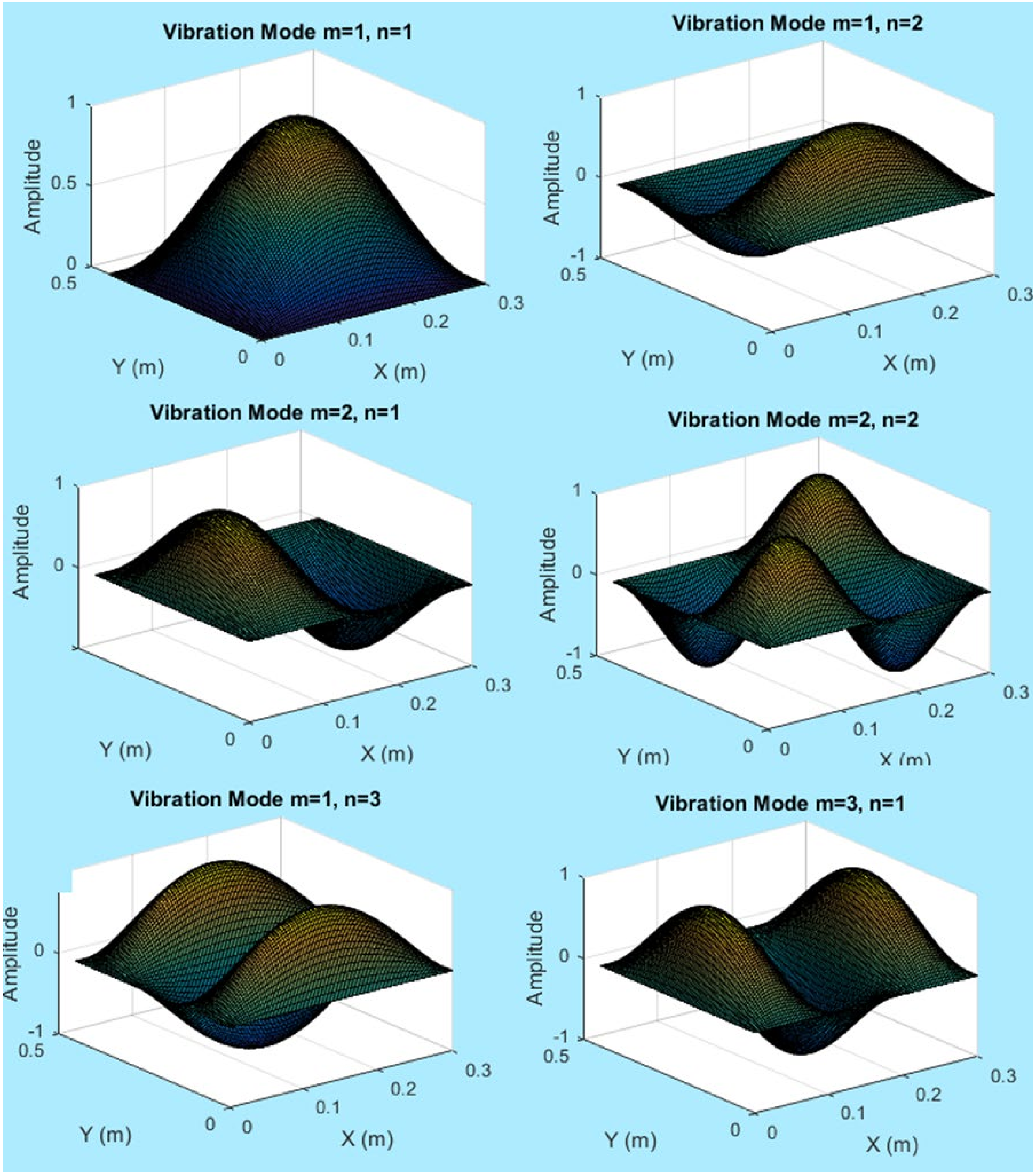

Figure 6 Influence of varying mode numbers (n, m) of the analytical natural frequencies of aluminum foam sandwich panel in MATLAB

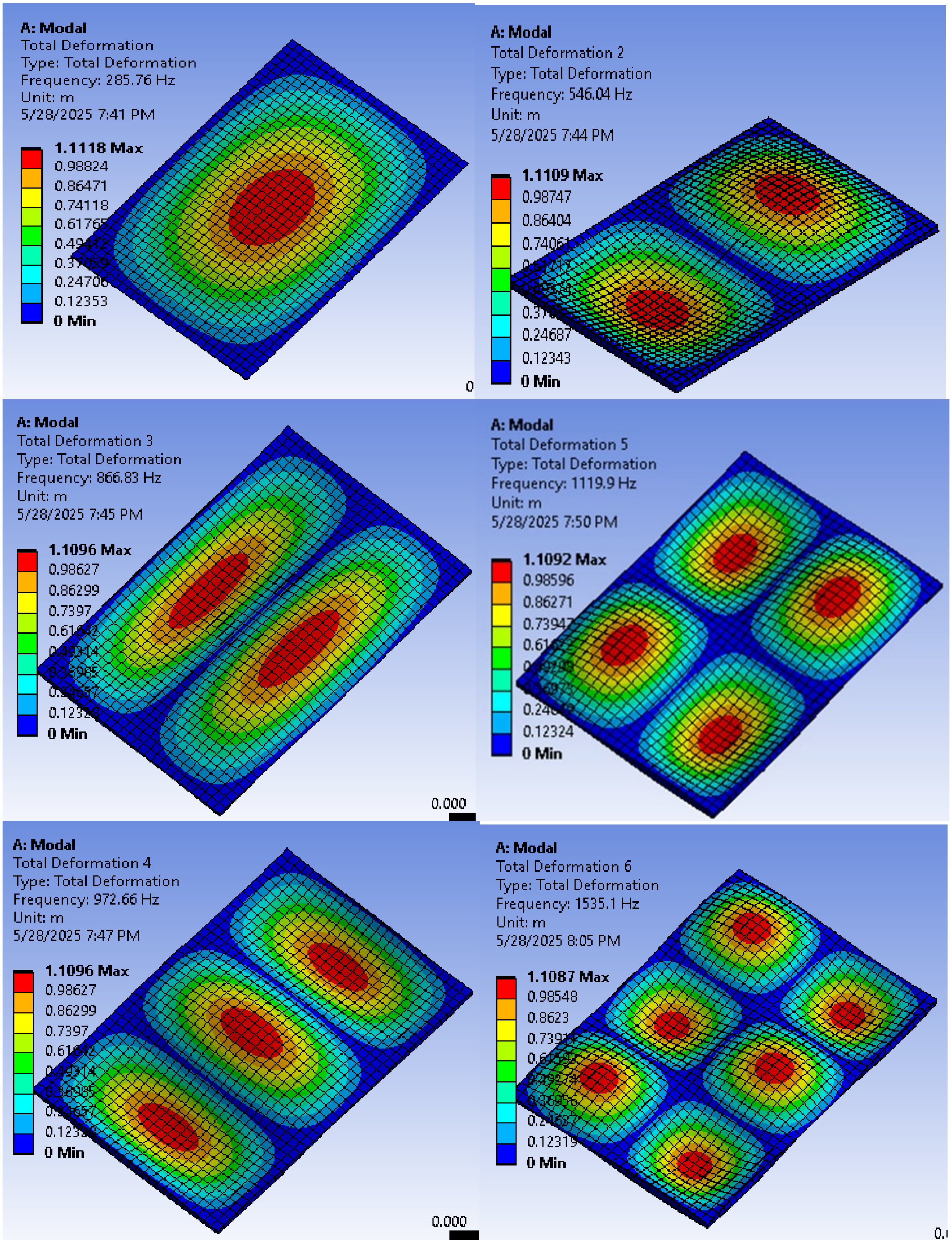

Figure 7 Influence of varying mode numbers (n, m) of the analytical natural frequencies of aluminum foam sandwich panel in Ansys (ρ foam=800 $\mathbf{kg/m^3}$, a/b=1.5, Thick of metal foam=10 mm)

4.0 CONCLUSIONS

This study developed and validated a mathematical model based on Kirchhoff's Classical Plate Theory combined with the Donghui Xu and Ju-Heng Ju formulations to investigate the free vibration characteristics of aluminum foam sandwich panels. The results demonstrated that core density significantly influences the dynamic response of the panels. Increasing foam density leads to a reduction in natural frequencies due to the associated increase in mass, despite the simultaneous enhancement in stiffness. For instance, the first natural frequency decreased from 275.64 Hz to 239.35 Hz when the density increased from 400 to 1000 kg/m³ at a thickness of 5 mm and an aspect ratio (a/b) of 1. Furthermore, the analytical model exhibited high predictive accuracy when compared with finite element simulations conducted using ANSYS, with error percentages ranging between 0.23% and 4.35%, confirming the reliability of the proposed formulation. The findings also highlight the importance of design optimization, where achieving a balanced combination of foam density, core thickness, and aspect ratio can enhance structural performance in weight-sensitive applications such as aerospace and automotive engineering, particularly in terms of stiffness and vibration control.

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Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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Appendix

Symbol	Description
$w(x, y, t)$	Transverse displacement
ρ	Density
h	Thickness
M_x, M_y	Bending moments in x and y directions
M_{xy}	Twisting moment
$\sigma_x, \sigma_y, \sigma_{xy}$	Normal and shear stresses
E_{xx}, E_{yy}	Young's modulus in the x and y directions
G_{xy}	Shear modulus
v_{xy}, v_{yx}	Poisson's ratio
$p(x, y, t)$	External loading
ω_n	Natural frequency
A_f, A_{AL}	Constants related to foam and aluminum materials
z	Displacement in the z-direction
m, n	Mode numbers
$\sigma_x, \sigma_y, \sigma_{xy}$	Normal and shear stresses