

## OPTIMAL BUFFER ALLOCATION FOR UNPACED BALANCED AND UNBALANCED MEAN PROCESSING TIME

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**Abstract.** This article discusses an optimal buffer allocation for short unpaced production line and it is assumed reliable (no machines breakdown). The main parameters that affect the line are mean processing time ( $\mu$ ) and its coefficient of variation ( $C_v$ ). Eight different mean processing time distributions were studied. Each distribution was matched with 15 different buffer allocations. Simulation method was used to estimate the line throughput rate. The results showed that the allocation of buffers affect the throughput rate. For a reliable and balanced line, the optimum buffer allocation is by equally distributing the number of buffers to each buffer slot. In the case of an extra buffer is needed after equally distribution, it is placed at the center buffer slot. Meanwhile, the best buffer allocation shape for a line with unbalanced mean (with the assumption that each station is having fixed  $C_v$  and is reliable) follows the shape of the mean processing times of that line.

**Keywords:** Unpaced production line, unbalanced mean, optimal buffer allocation, balanced line, reliable unpaced production line

**Abstrak.** Artikel ini membincangkan peruntukan penampakan optimum untuk talian pengeluaran *unpaced* yang pendek dan boleh diharap (tiada mesin rosak). Parameter utama yang mempengaruhi talian dalam kajian ini adalah min masa pemprosesan ( $\mu$ ) dan pekali variasinya ( $C_v$ ). Lapan taburan min masa pemprosesan telah dikaji. Setiap taburan dipadankan dengan 15 konfigurasi peruntukan penampakan. Kaedah simulasi digunakan bagi menganggar kadar keluaran talian untuk setiap kes. Keputusan kajian menunjukkan peruntukan penampakan tertentu mempengaruhi kadar keluaran talian. Bagi talian yang boleh diharap dan min seimbang, peruntukan penampakan yang optima adalah dengan mengagihkan bilangan penampakan secara sama rata ke setiap slot penampakan. Jika penampakan tambahan diperlukan selepas agihan dilakukan, letakkan penampakan tambahan tersebut pada slot penampakan yang di tengah. Manakala bentuk peruntukan penampakan yang baik bagi talian yang mempunyai min tidak seimbang (dengan anggapan setiap stesen mempunyai  $C_v$  tetap dan boleh diharap) ialah mengikut bentuk taburan min masa pemprosesan talian tersebut.

**Kata kunci:** Talian pengeluaran *unpaced*, min tidak seimbang, peruntukan penampakan optima, talian seimbang, talian pengeluaran *unpaced* boleh diharap

### 1.0 INTRODUCTION

There is an increasing need for management to carry out a production line analysis whilst designing the best and perfect production line that can maximize the throughput

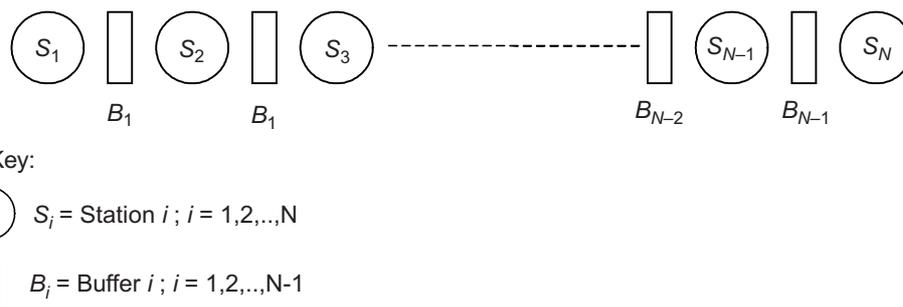
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rate, efficiency and productivity. At the same time, they also need to consider space availability, optimized manpower usage and selection of good machines and equipments to minimize (or even avoid) any downtime related to machine failure. However in a real life production line, it is impossible to get a perfect 100% balance and reliable line. This is due to many factors that caused the line to become unreliable and unbalanced. In a real life production line, the sources of unbalanced are mean processing time, coefficient of variation and a buffer allocation, which need to be considered for a reliable production line. As for an unreliable line, another two more factors need to be included, they are failure rate and repair rate.

In this study, the production line is considered short unpaced or asynchronous line (up to ten stations), unbalanced mean processing time but reliable. The best allocation of buffer need to be found for this particular type of production line so that the line throughput can be maximized and the total number of work in process (WIP) on the line (both workstation and on buffer) is minimized.

Basically, the unpaced production line can be described as in Figure 1, considering the total of  $N$  stations and  $N-1$  intermediate buffer slots.



**Figure 1** General layout of unpaced production line

As in Figure 1, material is not pushed by demand but it is in a pull mode. General assumption is that the raw material is always available before station-1,  $S_1$  (first station never starved) and the completed assembled material can always be placed after station- $N$ ,  $S_N$  (last station never blocked). Therefore, there are infinite buffer slots before  $S_1$  and after  $S_N$  but limited buffer slots between stations.

Every single station will have its own mean processing time ( $\mu$ ) and coefficient of variation ( $C_v$ ). In this study, unbalanced line is taken into consideration where the  $\mu$  will be random variables and  $C_v$  will be fixed.

## 2.0 BALANCED AND UNBALANCED PRODUCTION LINE

Production lines may be divided into two groups: synchronous and asynchronous lines. In synchronous lines, the movement of jobs is coordinated (all jobs move to

the next workstation simultaneously). So the number of jobs in the system remains constant, and there is no need to put buffers between stations.

In asynchronous lines, the movement of jobs is not coordinated. The operator or machine starts to process the job as soon as one becomes available. Upon process completion, the job immediately moves to the next workstation, as long as there is space for it. Thus, operator or machine can become starved (no job available) or blocked (no space to put a completed job). Asynchronous lines are almost always unpaced (In the paced line, the time allowed for an operator or machine to work on the job is limited). The number of jobs in the system may fluctuate (considerably) and buffers are needed to prevent starvation and blocking (i.e. loss of capacity) as much as possible.

Buffer allocation is an important, yet intriguingly difficult issue in physical layout and location planning for production systems. Optimum buffer allocation can optimize system performances such as minimizing work in process, cycle time and blocking probability and maximizing throughput. In theory, the buffer allocation problem is by itself a difficult NP-hard combinatorial optimization problem [1]. It is made even more difficult by the fact that the objective function is not obtainable in closed form for interrelating the integer decision variables (i.e. buffer sizes) and the performance measures of the system.

There have been substantial efforts devoted to design buffer coordination in the production system. Among the early researchers include studies conducted by Conway *et al.* [2], Altiook and Perros [3], Hillier *et al.* [4] and Papadoulos and Vidalis [5]. In solving this problem, different approaches have been used. The approaches broadly can be categorized into analytical methods and simulation. An example of more current research is by Hemachandra and Eedupuganti [6]. They developed an analytical models of open assembly system that consist of two assembly lines and a single joining operation. In the study, they investigated parameters such as service time and arrival rates that affect the configuration of buffer. Recently, an artificial intelligence model had been used to solve this problem, especially for large solution space and when fast decision making is needed. Altiparmak *et al.* [7] demonstrated a simulation model based on neural network approach. They investigated the buffer allocation on asynchronous assembly system subjected to failure. Despite of their promising results, the neural network has a drawback. It is an empirical and data driven model, which means the approach depends heavily on data.

Generally, researches have focused their study based on whether the line is balanced or unbalanced and whether it is reliable or unreliable. A balanced line is a line which have mean processing times that are equally distributed. Whereas, a line is called unbalanced if the mean processing times at each station are not equal. If there is no workstation failure then the line can be called a reliable line, otherwise, the line is unreliable.

There are 3 types of unbalanced line as explained by Papadopoulos and Vidalis [8];

- (i) Mean processing time unbalanced –  $\mu_i \neq \mu_{(i+1)}$  for at least one pair of  $i$  and  $(i+1)$  but  $Cv_i = Cv_{(i+1)}$  for all  $i$ .
- (ii) Coefficient of variation unbalanced –  $Cv_i \neq Cv_{(i+1)}$  for at least one pair of  $i$  and  $(i+1)$  but  $\mu_i = \mu_{(i+1)}$  for all  $i$ .
- (iii) Fully unbalanced –  $\mu_i \neq \mu_{(i+1)}$  and  $Cv_i \neq Cv_{(i+1)}$  for at least one pair of  $i$  and  $(i+1)$ .

This paper focuses on a production line with condition (i) as mentioned above. Mean processing time is the most difficult variable to fix constant in an actual production line due to the complexity of assembly process. The unbalanced mean effect will become higher if the assembly process is more complex.

The management normally will decide the desired throughput rate and total buffer capacity. Then, the production line layout is designed based on that requirement. By a given total buffer capacity, an optimal buffer allocation capacity need to be found. This is the main objective of this paper.

### 3.0 PRODUCTION LINE MODEL AND METHODOLOGY

The type of production line used in this study is unpaced short line with number of stations,  $N \leq 10$  stations. This type of short line is normally known as cell line concept where there is no conveyor belt used. Each  $\mu$  (for each station) is independent and not paced by a conveyor or specific given time. Many electronics companies with small and medium size products such as computer peripheral and personal audio are practicing this type of line for their assembly processes.

A few assumptions were made while developing the production model for this study. These assumptions are:

- (i) All variables ( $\mu$  and  $C_v$ ) are assumed independent random variables, where  $C_v = \frac{\sigma}{\mu}$ .
- (ii) All stations are reliable, meaning that there are zero (0) station failure and repair rate.
- (iii) The line is subjected to fully balanced and unbalanced for  $\mu$ .

Unbalanced line normally happen in actual production line due to the difficulty of getting a constant mean processing time for all stations. There may be a slight difference between one station to another in terms of  $\mu$  and/or  $C_v$ . In this study, 8 possible  $\mu$  distribution patterns (Case 1 – Case 8 in Figure A1 Appendix A) for the unbalanced line are taken into consideration. For each case, the optimum buffer allocation is determined in order to achieve the highest throughput for a given total buffer,  $B$ . The total buffer size given is based on a minimum allowable buffer sets (usually finalized by management which minimizes WIP).

Secondly, it is assumed that there is a possibility of achieving almost reliable line target by having the best machines, multi-skilled and highly motivated operators and a good support engineering groups who can predict and plan effectively through statistical tools. However, the 100% reliable line is still impossible to be achieved.

In this study, the throughput rate is estimated by using simulation technique. All possibilities of buffer allocation have been investigated and from the results, the optimum buffer allocation for the related  $\mu$  distribution patterns was decided based on the maximum throughput rate.

Parameters used in the simulation are:

- Number of stations,  $N = N \leq 10$ ;
- Number of buffer slots,  $BUF = \text{number of stations}-1$ ;
- Maximum loops,  $NRUNS : 50$  runs
- Accuracy = 0.1 which means that accuracy is simulated to get more than 90% accuracy. This is to simplify the software simulation iteration process and to avoid system hang up problem.
- Minimum constant processing time for station- $i$  is fixed to 0.
- Mean processing time for station- $i$ ,  $\mu$  is based on 8 possible  $\mu$  distribution patterns (Case 1 – Case 8 in Figure A1 Appendix A). For the simulation purposes,  $\mu$  of a line need to be converted into ratio which total up to the number of stations:

$$\sum_{i=1}^N \bar{\mu}_{i(ratio)} = N; \text{ where } \mu_{i(ratio)} = \left( \frac{\mu_i}{\mu_{total}} \right) \times N \quad (1)$$

where,  $\mu_{total}$  = total mean process time of the whole production line.

- $C_v$  of processing time at station  $i$  is fixed to 0.1.

The following section will discuss the results for a line consisting of 8 stations with total number of buffer of 14 pieces (average of 2 buffers per slot  $\times$  7 slots).

## 4.0 RESULTS AND DISCUSSIONS

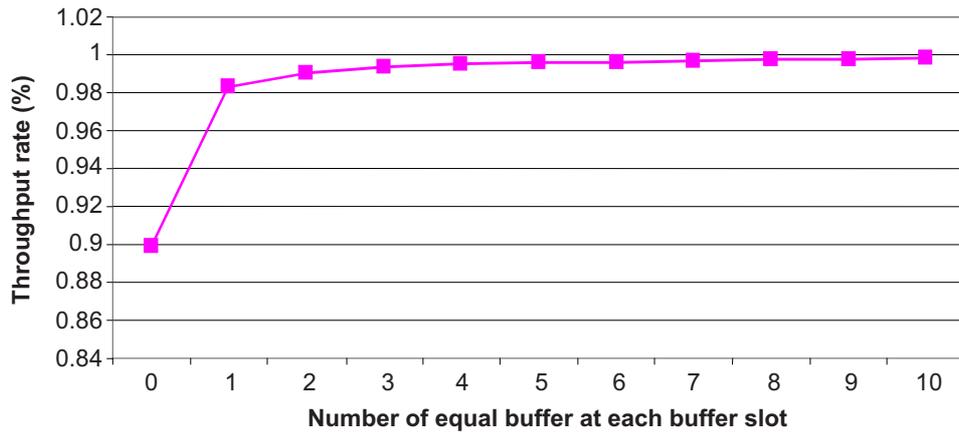
This section will discuss two main simulation results; fully balanced line and unbalanced line.

### 4.1 Balanced Line

In this experiment, balanced line is referred to the production line that consists of stations with equal  $\mu$ . Even though in a real production line, this type of line is impossible to achieve, a study on this is useful to get basic findings for further study in unbalanced line.

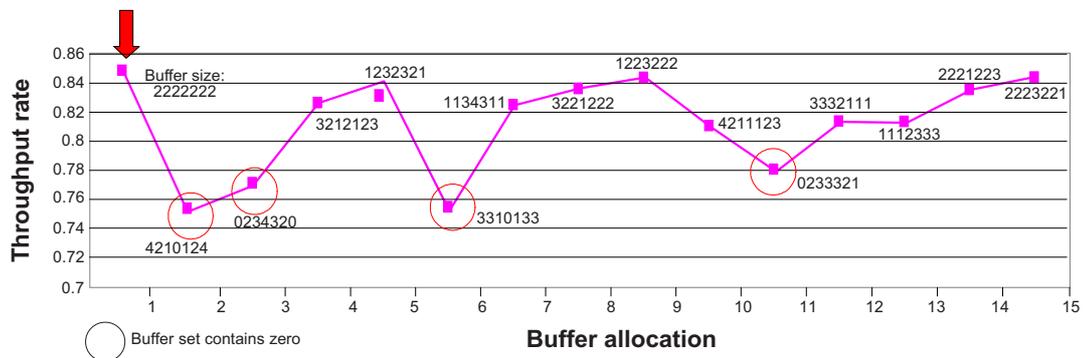
### 4.1.1 The Effects Increment in Number of Buffers

In this simulation experiment, the general effect of equal increment number of buffers to throughput rate is being investigated. Figure 2 shows that the throughput increases when the equal buffer in each buffer slot is being added from 0 to 10 pieces.



**Figure 2** Throughput rate trend for number of equal buffers incremental

The throughput rate increases significantly after a buffer is added (from 0 to 1) and slightly increases after each increment. Further experiments showed that if there is a zero buffer in any buffer slot, throughput rate will always be at a minimum side. This is shown in Figure 3. As in Figure 3, a point pointed by the arrow, a set of buffer allocation is shown as 2222222. It means that a uniform number of buffer has been distributed among the buffer slots (each slot consists of 2 pieces of buffer). The circled points showed that the set contains zero buffer, resulted in minimum throughput rate.



**Figure 3** Affect of zero buffer on the throughput rate

Based on these experiments, two conclusions can be made. Those conclusions are:

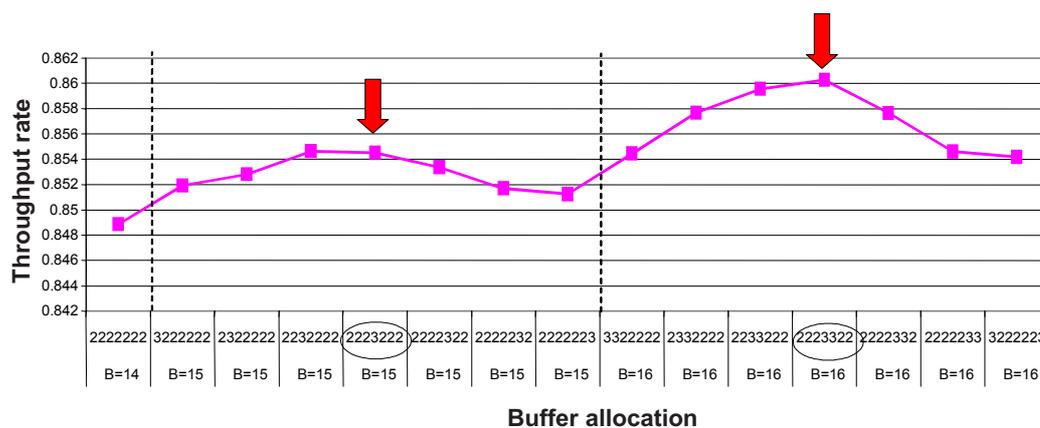
- (i) Zero buffer need to be avoided since it will minimize the throughput rate. Therefore, no zero buffer if possible.
- (ii) Throughput rate starts to saturate when buffer size increases from one onwards. Therefore, optimum buffer allocation needs to be determined in this range of buffers.

With reference to the second conclusion, by increasing the number of buffers on the line, production cost indirectly will increase due to the space required to place the buffer i.e. WIP. Therefore, the total buffer size ( $B$ ) and buffer quantity in each slot ( $b_i$ ) need to be minimized. This can be done by finding the optimum buffer allocation of the production line.

#### 4.1.2 Buffer Allocation for Unsymmetrical Pattern

As shown in Figure 3, when the buffers are equally distributed among the buffer slot, the throughput rate is high. But, the throughput is affected when the allocation is unsymmetrical. This is shown in Figure 4.

When there is a case of unsymmetrical allocation of buffer after equally distributed, simulation results showed that this extra buffer needs to be allocated at the center buffer slot of the line. Refer to Figure 4 for details. We introduce an extra factor,  $E$  which referred to extra of buffer after equally distributed to each buffer slot. For example, for this 8 stations line, if  $B = 15$  then  $E = 1$  ( $15 - (7 \times 2)$ ). The theory is to allocate this  $E$  buffer at center buffer slot of the line, in this case, the fourth buffer slot. If for an example,  $B = 16$ , then  $E = 2$  ( $16 - (7 \times 2)$ ). So, the 2 pieces extra



**Figure 4** Unsymmetrical buffer allocation

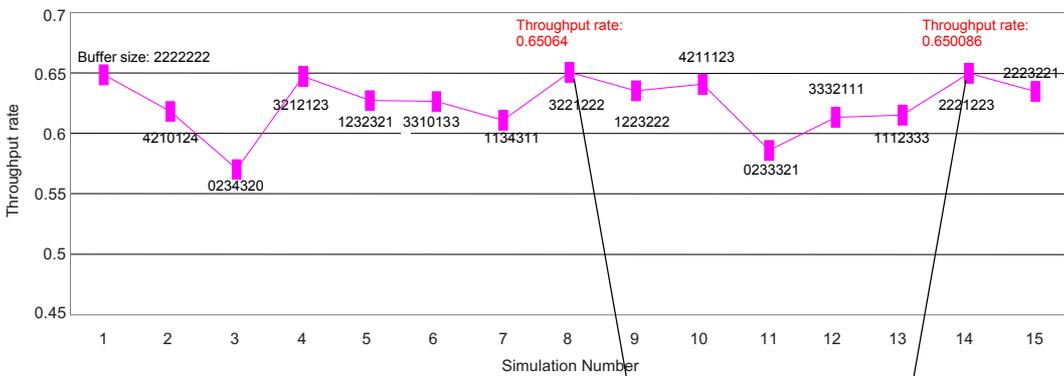
buffers are to be allocated at slot 4 and 5. This result is supported by earlier findings that center buffer slot have the highest effect on variables ( $\mu$ ,  $C_v$  and buffer) from its downstream and upstream stations [8]. Figure 4 shows that there is a repeating trend for the throughput curve when the extra buffers allocated at center buffer slots of the line and the same applies to  $B = 15$  and 16.

The results can be concluded as:

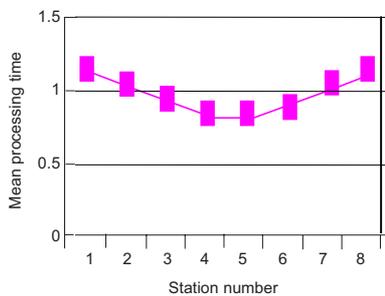
- The optimum buffer allocation is by equally distributing the number of buffer to each buffer slot. In the case of an extra buffer after the distribution, it should be placed at center buffer slot/s.

### 4.2 Buffer Allocation for Unbalanced Line

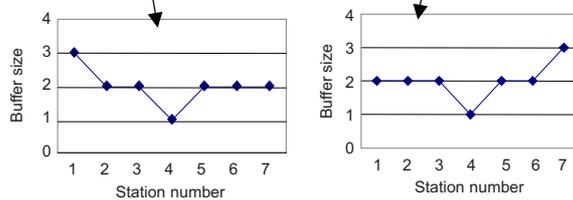
In this simulation, the study considered 8 possible  $\mu$  distribution pattern (i.e.  $\mu$  vs station number as shown in Figure A1 Appendix A). For each pattern, 15 possible buffer distribution shapes (Figure A2 Appendix A) have been matched in order to find which distribution gives the best throughput rate. The simulation showed that the best buffer allocation shapes (that give the best throughput rate) followed the  $\mu$  distribution patterns. For example, Figure 5 shows the result for  $\mu$ -unbalanced that



(a) Throughput rate for different buffer allocation for Case 3



(b) Unbalanced Mean processing time ( $\mu$ ) shape

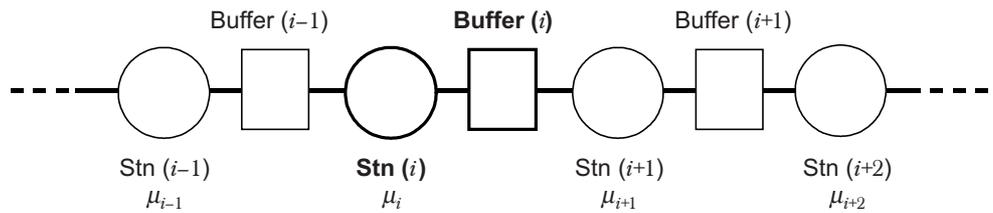


(c) Case H and Case P: Buffer allocation shape

**Figure 5** Selected result for unbalanced processing time

followed the  $\mu$  distribution pattern as in Case 3 (Figure A1 Appendix A). The results showed that buffer allocations Case H and Case P (Figure A2 Appendix A) gave the highest throughput rate of the line. The shape of both curves (Case H and Case P) were similar to the shape in Case 3 (in this case it is a bowl shape).

This finding can be explained technically as in Figure 6. Consider any station- $i$  and buffer- $i$  at any location within the production line.



Note: Station processing time,  $\mu$

**Figure 6** Stations and buffers in a production line

If  $\mu_{i-1} > \mu_i > \mu_{i+1} \Rightarrow$  there is a higher possibility for station  $i$  to be starved. So, buffer  $i$ ,  $b_i$  need to be set to higher capacity (relatively to the next buffer  $(i+1)$ ,  $b_{i+1}$ ), to avoid station  $i$  from starving.

If  $\mu_{i-1} < \mu_i < \mu_{i+1} \Rightarrow$  there is a higher possibility for station  $i$  to be blocked. So, buffer  $i$ ,  $b_i$  need to be set to higher capacity (relatively to previous buffer  $(i-1)$ ,  $b_{i-1}$ ), to avoid station  $i$  from blocking.

The simulation results for all possible combination are shown in Figure A3 Appendix A. It showed that in order to get the best throughput rate, the shape of buffer allocation followed the shape of mean processing time of the line. For example, for the mean processing times that followed the shape in Case 4, there were five possible patterns of buffer allocation which maximize the throughput rate. The patterns were Case G, Case Q, Case J, Case L and Case C and the throughput rate were 0.629, 0.627, 0.626, 0.628 and 0.625 respectively. All the buffer allocation patterns were similar to the shape of mean processing time of the line. From these result, it can be concluded that:

- For any  $\mu$ -unbalanced line, the best buffer allocation shape is to follow the shape of  $\mu$  distribution of the line.

## 5.0 CONCLUSION

This study covers a case of buffer allocation for short production line that consist of eight stations, balanced and unbalanced mean processing time, reliable and with fixed coefficient of variation. It used a simulation technique to measure a throughput rate of the line. This throughput rate was affected by the allocation of buffers between

stations. In order to minimize the space of buffers and maximize the throughput rate, the best possible buffer distribution should be allocated. In balanced line, zero buffer should be avoided and buffers should be equally distributed throughout the line. On the other hand, in unbalanced line, the distribution of buffers followed the shape of mean processing time distribution. These findings are a part of a research for determining optimum buffer allocation for the unpaced line and its concept can be applied as a guideline to a management in designing the production line.

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APPENDIX A

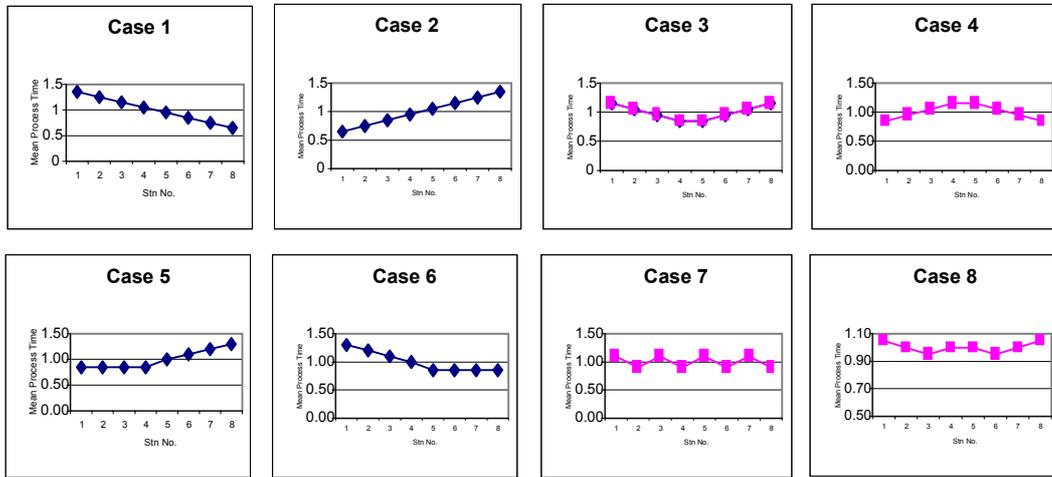


Figure A1 The distribution pattern of mean processing time ( $\mu$ ) for each workstation

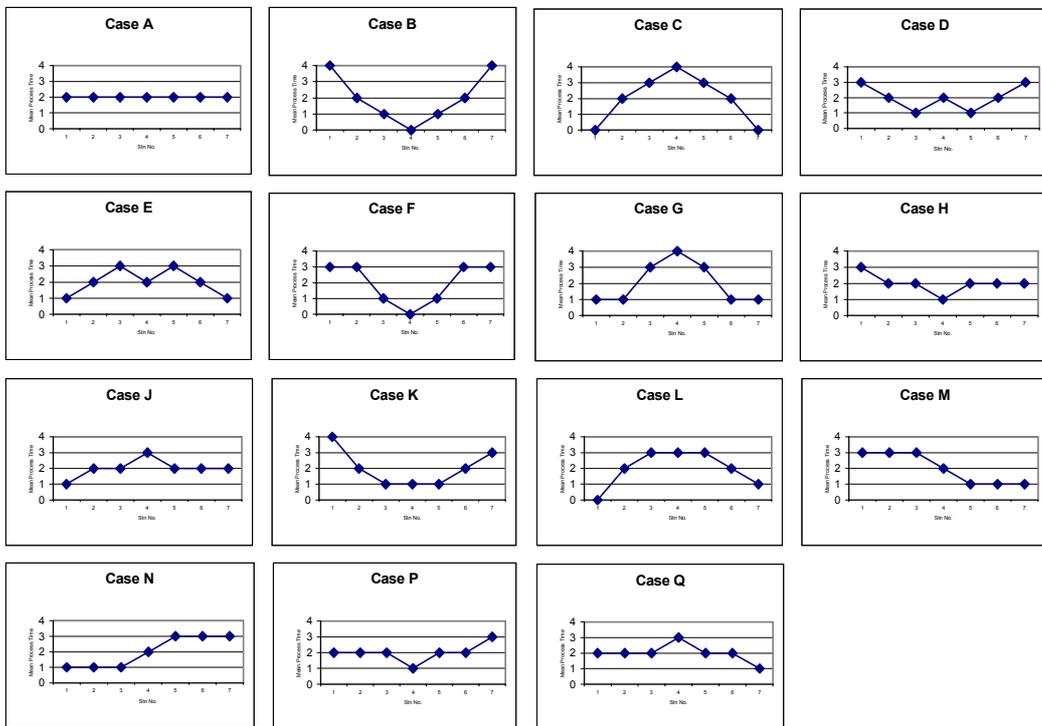
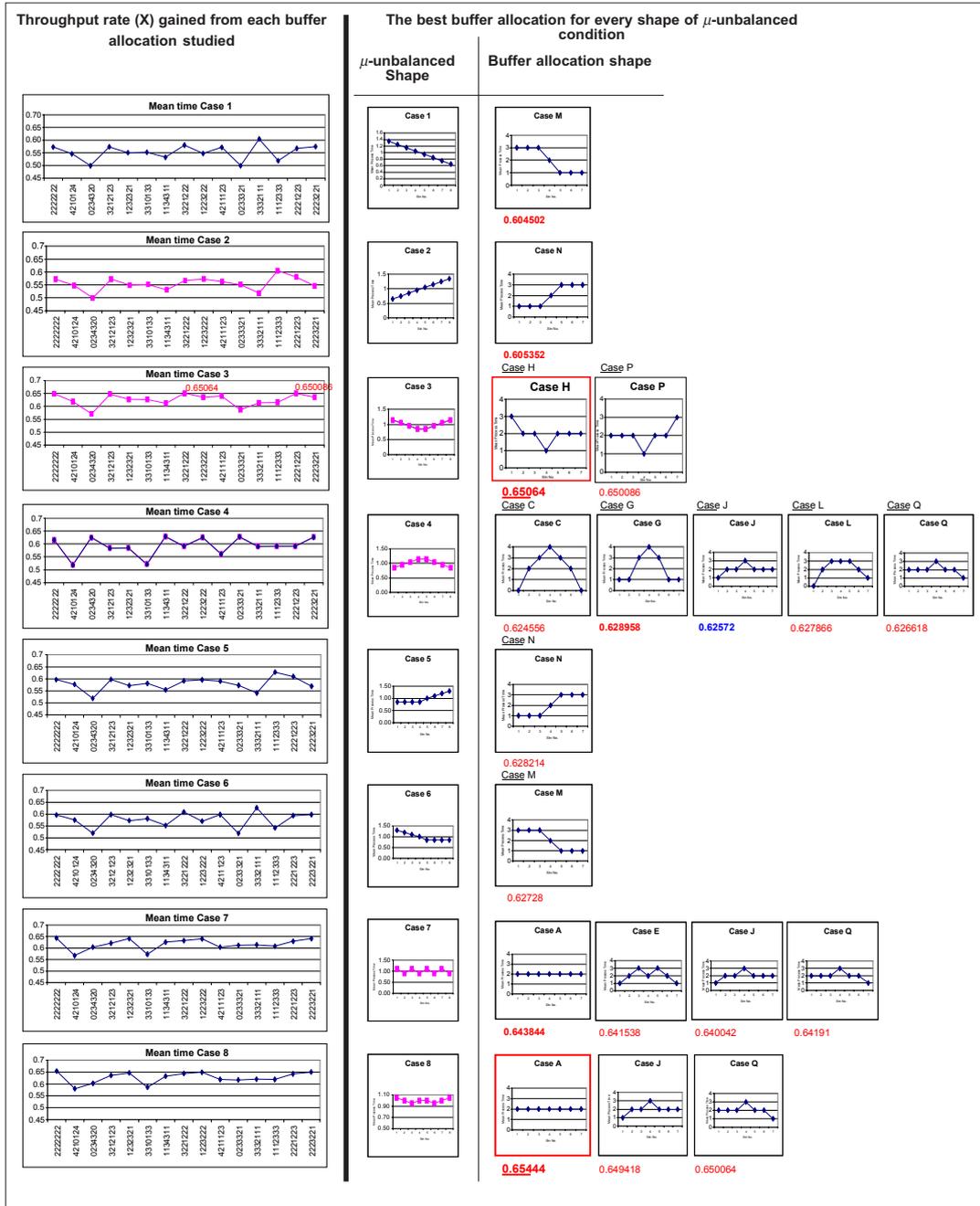


Figure A2 Buffer allocation shape



**Figure A3** Simulation results for different mean process time pattern and its buffer allocation