

# Robust Helicopter Stabilization with Wind Disturbance Elimination

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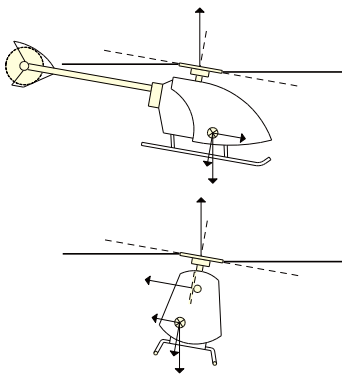
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## Graphical abstract



## Abstract

When a helicopter is required to hover with minimum deviations from a desired position without measurements of a persistent wind disturbance, a robustly stabilizing control action is vital. In this paper, the stabilization of the position and translational velocity of a nonlinear helicopter model affected by a wind disturbance is addressed. An estimate of the disturbance is introduced to be adapted using state measurements for control purposes. A nonlinear controller is then designed based on nonlinear adaptive output regulation and robust stabilization of a chain of integrators by a saturated feedback. While in the control synthesis the wind disturbance is assumed to be a sum of a fixed number of sinusoids with unknown amplitudes, frequencies and phases, a practical turbulence model is presented to the helicopter model in the control test. Simulation results show the effectiveness of the control design in the stabilization of helicopter motion and the built-in robustness of the controller in handling parameter and model uncertainties.

**Keywords:** Autonomous helicopter; robust stabilization; wind disturbance; uncertainties

## Abstrak

Apabila helikopter yang diperlukan untuk berlegar dengan penyelewengan minimum dari kedudukan yang dikehendaki tanpa ukuran gangguan angin yang berterusan, tindakan kawalan dengan kukuh stabil adalah penting. Dalam kertas ini, kestabilan kedudukan dan halaju translasi model helikopter linear terjejas oleh gangguan angin ditangani. Anggaran gangguan diperkenalkan untuk disesuaikan menggunakan ukuran negeri untuk tujuan kawalan. A pengawal linear kemudian direka berdasarkan peraturan linear output penyesuaian dan kestabilan yang mantap daripada rantaian penyatu oleh maklum balas tepu. Walaupun dalam sintesis kawalan gangguan angin dianggap sebagai satu jumlah nombor tetap sinusoids dengan amplitud yang tidak diketahui, frekuensi dan fasa, model pergolakan praktikal dikemukakan kepada model helikopter dalam ujian kawalan. Keputusan simulasi menunjukkan keberkesanan reka bentuk kawalan dalam penstabilan gerakan helikopter dan keteguhan terbina dalam pengawal dalam mengendalikan parameter model dan ketidaktentuan.

**Kata kunci:** Helikopter autonomi; penstabilan teguh; gangguan angin; ketidaktentuan

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## 1.0 INTRODUCTION

Autonomous helicopters are highly agile and have six degree of freedom maneuverability making them a favourite candidate for a wide range of practical applications including agriculture, cinematography, inspection, surveillance, search and rescue, reconnaissance, etc. For certain tasks the ability of a helicopter to follow a given state reference is crucial for a successful outcome; for instance when hovering over a ship for rescue operations or when flying close to power lines or wind turbines for inspections. In windy conditions, this becomes a significant challenge for any pilot and hence an autopilot capable of accounting effectively for the wind disturbance is a realistic alternative. In this work, the authors present a control design for longitudinal, lateral, and vertical helicopter stabilization in the presence of a wind disturbance, with intrinsic robustness property in handling parameter and model uncertainties.

Firstly, some previous works are reviewed. In Ref. 1, a feedback-feedforward proportional differential (PD) controller is developed for heave motion control. With the assistance of a

gust estimator, the controller is reported to be able to handle the influence from horizontal gusts. The effects of rotary gusts in forward and downward velocity of a helicopter are addressed in Ref. 2. It is shown that via a state feedback law, the rotary gust rejection problem is always solvable. In a work by Wang *et al.*, a multi-mode linear control strategy is designed for unmanned helicopters in the presence of model uncertainties, atmospheric disturbances and handling qualities requirements.<sup>3</sup>

In the present work, assuming that all the state variables (position, attitude and derivatives hereof) are available for measurements, a control strategy combining nonlinear adaptive output regulation and robust stabilization of a chain of integrators by a saturated feedback is carried out (see for instance, Refs. 4, 5 and 6). Guided by a control solution for vertical trajectory tracking presented in Refs. 6 and 7, the design technique presented therein is extended here for helicopter stabilization in the presence of horizontal and vertical wind disturbance. This is done by means of a robust longitudinal, lateral and vertical stabilizer capable of compensating for parameter and model uncertainties.

In Section 2.0, a mathematical model governing a model helicopter and a problem statement are given. This is followed by Section 3.0, which covers the vertical, longitudinal and lateral stabilization. After the presentation of simulation results in Section 4.0, conclusion and future works are discussed in Section 5.0.

## 2.0 PRELIMINARIES

In this section, a mathematical model of a helicopter is described after which a problem statement is given.

### 2.1 Helicopter Model

The position and attitude of a helicopter in the three-dimensional Euclidean space  $\mathbb{E}^3$  is described by the relative position and orientation respectively between a body-fixed coordinate frame  $\mathcal{F}_b$  and an inertial coordinate frame  $\mathcal{F}_i$ . A coordinate frame  $\mathcal{F}$  is denoted by the set  $\mathcal{F} = \{O, i, j, k\}$ , where  $i, j$  and  $k$  are right-handed, mutually orthogonal unit vectors with a common origin  $O$  in  $\mathbb{E}^3$ .

The motion of the center of mass of a helicopter is expressed in an inertial coordinate frame  $\mathcal{F}_i = \{O_i, i_i, j_i, k_i\}$  (whose axes are henceforth denoted as  $x, y$  and  $z$  axis) as

$$M\ddot{p}^i = \mathcal{R}f^b \quad (1)$$

where  $M$  is the mass and  $p^i = [x \ y \ z]^T \in \mathbb{R}^3$  is the position of the center of mass of the helicopter with respect to  $O_i$ . The rotation matrix  $\mathcal{R} := \mathcal{R}(\mathbf{q})$  is parametrized in terms of unit quaternions  $\mathbf{q} = (q_0, \mathbf{q}) \in S^4$  where  $q_0$  and  $\mathbf{q} = [q_1 \ q_2 \ q_3]^T$  denote the scalar and the vector parts of the quaternion respectively, as given by

$$\mathcal{R} = \begin{bmatrix} 1 - 2q_2^2 - 2q_3^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\ 2q_1q_2 + 2q_0q_3 & 1 - 2q_1^2 - 2q_3^2 & 2q_2q_3 - 2q_0q_1 \\ 2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & 1 - 2q_1^2 - 2q_2^2 \end{bmatrix}$$

The overall force  $f^b$  that acts on the helicopter in a body-fixed coordinate frame  $\mathcal{F}_b = \{O_b, i_b, j_b, k_b\}$  is given by

$$f^b = \begin{bmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{bmatrix} + \mathcal{R}^T \begin{bmatrix} 0 \\ 0 \\ Mg \end{bmatrix} + \mathcal{R}^T \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} \quad (2)$$

where  $g$  is the gravitational acceleration and,  $d_x, d_y$  and  $d_z$  are wind disturbances that affect the helicopter motion in  $x, y$  and  $z$  axis respectively. Moreover  $X_M = -T_M \sin a$ ,  $Y_M = T_M \sin b$ ,  $Z_M = -T_M \cos a \cos b$  and  $Y_T = -T_T$ , where  $T_M, T_T, a$  and  $b$  are main rotor thrust, tail rotor thrust, longitudinal main rotor tip path plane tilt angle and lateral main rotor tip path plane tilt angle respectively (see Figure 1).

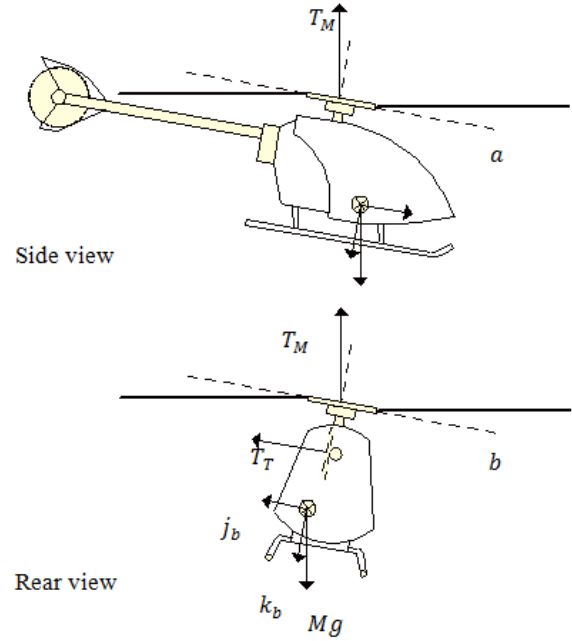


Figure 1 Side and rear view of the helicopter

For convenience of subsequent analysis, it is assumed that  $a$  and  $b$  are small such that

$$\sin a \approx a, \sin b \approx b, \cos a \approx 1, \cos b \approx 1. \quad (3)$$

To simplify force equation (2), it is assumed that  $X_M = 0$  and  $Y_M + Y_T = 0$  (see e.g. Ref. 8). Thus, a simplified force equation is obtained,

$$f^b = \begin{bmatrix} 0 \\ 0 \\ -T_M \end{bmatrix} + \mathcal{R}^T \begin{bmatrix} 0 \\ 0 \\ Mg \end{bmatrix} + \mathcal{R}^T \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} \quad (4)$$

Subsequently the following equations of translational motion can be derived,

$$\ddot{x} = \frac{-(2q_1q_3 + 2q_0q_2)T_M}{M} + \frac{d_x}{M} \quad (5)$$

$$\ddot{y} = \frac{-(2q_2q_3 - 2q_0q_1)T_M}{M} + \frac{d_y}{M} \quad (6)$$

$$\ddot{z} = \frac{-(1 - 2q_1^2 - 2q_2^2)T_M}{M} + g + \frac{d_z}{M} \quad (7)$$

Now for the angular dynamics,

$$J\dot{\omega}^b = -S(\omega^b)J\omega^b + \tau^b, \dot{\mathbf{q}} = \frac{1}{2} \begin{bmatrix} -q^T \\ q_0I + S(\mathbf{q}) \end{bmatrix} \omega^b \quad (8)$$

where  $\omega^b \in \mathbb{R}^3$  represents the angular velocity in  $\mathcal{F}_b$  and  $J$  is the inertia matrix. In a generic notation, given any vector  $v \in \mathbb{R}^3$ ,

$$S(v) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$$

The external torques  $\tau^b$  expressed in  $\mathcal{F}_b$  are given by the following equation,

$$\tau^b = \begin{bmatrix} \tau_{f_1} \\ \tau_{f_2} \\ \tau_{f_3} \end{bmatrix} + \begin{bmatrix} R_M \\ M_M + M_T \\ N_M \end{bmatrix} \quad (9)$$

where  $\tau_{f1}$ ,  $\tau_{f2}$  and  $\tau_{f3}$  are moments generated by the main and tail rotors.  $R_M = c_b^M b - Q_M \sin a$ ,  $M_M = c_a^M a - Q_M \sin b$ ,  $N_M = -Q_M \cos a \cos b$  and  $M_T = -Q_T$  are the torques of aerodynamic forces, where  $Q_M = c_M^Q T_M^{1.5} + D_M^Q$ ,  $Q_T = c_T^Q T_T^{1.5} + D_T^Q$  and  $c_b^M$ ,  $c_a^M$ ,  $c_M^Q$ ,  $c_T^Q$ ,  $D_M^Q$ , and  $D_T^Q$  are physical parameters.<sup>8</sup> With approximations  $T_M^{1.5} \approx T_M$ ,  $T_T^{1.5} \approx T_T$  and (3), a compact torque equation is obtained,

$$\tau^b = A(T_M)\mathbf{v} + B(T_M), \mathbf{v} := [a \ b \ T_T]^T \quad (10)$$

where  $A(T_M)$  and  $B(T_M)$  are a matrix and a vector whose entries are functions of  $T_M$  with dependence on the helicopter parameters.

The mathematical model of the helicopter given by (5) – (8) is subject to parameter uncertainties. Taking  $\mu$  as a vector of all uncertain parameters with a nominal value  $\mu_0$ , the additive uncertainty is given by  $\mu_\Delta = \mu - \mu_0 \in \mathcal{P}$ , where  $\mathcal{P}$  is a given compact set. Correspondingly,  $M = M_0 + M_\Delta$ ,  $J = J_0 + J_\Delta$ ,  $A(T_M) = A_0(T_M) + A_\Delta(T_M)$  and  $B(T_M) = B_0(T_M) + B_\Delta(T_M)$ . Now defining a state vector  $\mathbf{x} = [p^i \ \dot{p}^i \ \omega^b \ \mathbf{q}]^T$  and taking the vector  $\mathbf{u} = [T_M \ \mathbf{v}]^T$  as the control input, the helicopter model acted by the wind disturbance  $\mathbf{d} = [d_x \ d_y \ d_z]^T$  is summarized as

$$\dot{\mathbf{x}} = F(\mathbf{x}, \mathbf{u}, \mathbf{d}) \quad (11)$$

where  $F$  is a vector of functions (1) and (8) considering the full force and torque equations (2) and (9) respectively.

## 2.2 Problem Statement

The primary goal of Section 3.0 is to design a controller that is able to stabilize the helicopter affected by the disturbance  $\mathbf{d}$ . The disturbance that affects the acceleration of the helicopter can be written as a linear combination of  $N$  (possibly  $\infty$ ) sinusoidal functions of time modeled in the following form,

$$d_j = \sum_{i=1}^N A_{ji} \cos(\Omega_i t + \varphi_{ji}) \quad (12)$$

with unknown amplitudes  $A_{ji}$ , phases  $\varphi_{ji}$  and frequencies  $\Omega_i$ , for  $j = x, y, z$ . Note that a great variety of disturbances can be modeled in this way, i.e., as a trigonometric polynomial or as a Fourier series. In our setup, considering computational limitations in computer control it is assumed that  $\mathbf{d}$  can be approximated with a small  $N$  but we emphasize that theoretically this can be carried out for arbitrary  $N$ . In Section 4.0 the sufficiency of a controller with only a small  $N$  in suppressing the effects of a disturbance given by the Von Karman turbulence model is demonstrated. It can be shown that the disturbance (12) is generated by the following linear time-invariant exosystem,

$$\begin{aligned} \dot{w}_j &= S(\varrho)w_j, \\ d_j &= RS^2(\varrho)w_j, j = x, y, z \end{aligned} \quad (13)$$

where  $w_j \in \mathbb{R}^{2N}$ ,  $\varrho = [\Omega_1 \ \Omega_2 \ \dots \ \Omega_N]^T$ ,  $S(\varrho) = \text{diag}(H(\Omega_1), \dots, H(\Omega_N))$  with

$$H(\Omega_i) = \begin{bmatrix} 0 & \Omega_i \\ -\Omega_i & 0 \end{bmatrix}, i = 1, \dots, N$$

and  $R = [1 \ 0 \ 1 \ 0 \ \dots \ 1 \ 0]$  a  $1 \times 2N$  matrix (see Ref. 6). We remark that the initial condition  $w_j(0)$  of the exosystem represents the amplitudes  $A_{ji}$  and phases  $\varphi_{ji}$  of the disturbance. As the control design proceeds in the next section, it will be

clear how the representation of the disturbance in such a form can be advantageous in the development of an internal model for stabilizing control input generation.

We will now concisely state the overall objective of the control design. Given the helicopter system (11), develop a robust controller of the form

$$\begin{aligned} \dot{\xi} &= \phi(\xi, p^i, \dot{p}^i), \\ \mathbf{u} &= \theta(\xi, p^i, \dot{p}^i, q, \omega^b) \end{aligned} \quad (14)$$

such that the helicopter asymptotically tracks the reference position and quaternion (vector part),  $[x^{ref}(t) \ y^{ref}(t) \ z^{ref}(t)]^T = [0 \ 0 \ 0]^T$  and  $q^{ref}(t) = [q_{d1}(t) \ q_{d2}(t) \ 0]^T$  respectively, in the presence of parameter uncertainties involving the helicopter geometry, the aerodynamic forces and the disturbance. In the above formulation,  $\xi$  is the state of the controller and,  $q_{d1}(t)$  and  $q_{d2}(t)$  are functions of a disturbance estimate. Note that  $q^{ref}(t)$  is the horizontal axis of rotation to be tracked to counter the horizontal disturbances  $d_x, d_y$ .

## 3.0 CONTROLLER DESIGN

A controller of the form (14) is designed in two stages. Firstly, a control law is constructed for  $\mathbf{u}$  to achieve vertical (motion in  $z$  direction) stabilization. Next, it can be shown that the stabilizing  $\mathbf{v}$  can be tuned separately (without jeopardizing the vertical stabilization) to render the horizontal ( $x$  and  $y$  direction) motion stable.

The designed controller that makes a stable hover possible can now be summarized as follows,

### 1. Vertical dynamics stabilizer

$$\dot{\xi}_z = (F + G\hat{\Psi})\xi_z + k_2 G(\dot{z} + k_1 z) + FG M_0 \dot{z} \quad (15)$$

$$\begin{aligned} \dot{\hat{\Psi}}_2 &= \gamma \xi_{z2}^T (\dot{z} + k_1 z) - \text{tas}_d(\hat{\Psi}_2) \\ T_M &= \frac{g M_0 + \hat{\Psi} \xi_z + k_2 (\dot{z} + k_1 z)}{1 - \text{sat}_c(2q_1^2 + 2q_2^2)} \end{aligned} \quad (16)$$

### 2. Longitudinal and lateral dynamics stabilizer

$$\begin{aligned} \dot{\xi} &= \hat{F}\xi + g_{st} \\ \hat{d} &= \hat{P}\xi \\ q_d &= -K_d \tilde{D}_0(t) \hat{d} \\ \omega_d &= Q_d \dot{q}_d \\ q_r &= q^* + q_d \\ \mathbf{v} &= A_0^{-1}(T_M)(K_P(q_r - q) - K_P K_D(\omega^b - \omega_d) - B_0(T_M)) \end{aligned} \quad (17)$$

where

$$\begin{aligned} \xi &= \begin{bmatrix} \xi_y \\ \xi_x \end{bmatrix}, \xi_i = \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \end{bmatrix} \in \mathbb{R} \times \mathbb{R}^{2N}, i = x, y \\ \hat{F} &= \begin{bmatrix} F + G\hat{\Psi} & 0 \\ 0 & F + G\hat{\Psi} \end{bmatrix}, \hat{P} = \begin{bmatrix} \hat{\Psi} & 0 \\ 0 & \hat{\Psi} \end{bmatrix} \\ g_{st} &= \begin{bmatrix} k_4(\dot{y} + k_3 y)G \\ k_4(\dot{x} + k_3 x)G \end{bmatrix}, k_3, k_4 > 0 \end{aligned}$$

Thus, a controller of the form (14) is developed where

$$\xi = \begin{bmatrix} \xi \\ \xi_z \\ \hat{\Psi}_2^T \end{bmatrix}$$

To conclude the control design, consider the controller above and choose the design parameters accordingly. Then for any initial conditions  $w(0) \in \mathcal{W}$ ,  $\eta(0) \in \mathcal{Z}$ ,  $(x(0), \dot{x}(0), y(0), \dot{y}(0)) \in \mathbb{R}^4$ ,  $q_0(0) > 0$ ,  $(q(0), \omega(0)) \in \mathcal{Q} \times \Omega$  where  $\mathcal{Z}$  is an arbitrary compact set,  $(x(t), \dot{x}(t), y(t), \dot{y}(t))$  converges to a neighborhood of the origin which can be rendered arbitrarily small by choosing  $K_P$  and  $K_D$  sufficiently large and small respectively. In addition,  $\lim_{t \rightarrow \infty} \|z(t), \dot{z}(t)\| = 0$ .

4.0 SIMULATION RESULTS

Hover flight of an autonomous helicopter equipped with the proposed autopilot and influenced by a wind disturbance is simulated.

The simulation results presented here are based on a model of a small autonomous helicopter from Ref. 12. To test the robustness property of the controller, parameter uncertainties are taken up to 30% of the nominal values. Even though the controller is designed based on simplified force and torque equations as described by (4) and (10) respectively, the helicopter model assumes full torque, (9) and full force equations. The wind disturbance shown in Figure 2 is presented to the helicopter as a persistently acting external force generated by the Von Karman wind turbulence model. To further challenge the controller, only 5-dimensional internal models (15) and (17) are used for each axis. Position of the helicopter in the face of the wind disturbance when disturbance measurements are available without parameter uncertainties are shown in Figure 3. When wind disturbance measurements are not available, helicopter position in the presence of parameter uncertainties without ( $\gamma = 0$ ) and with ( $\gamma = 1$ ) disturbance adaptation are given in Figures 4 and 5 respectively.

Without disturbance adaptation, while the controller fails to stabilize the  $x$  and  $y$  positions,  $z$  does converge fairly close to zero as could be seen in Figure 4. Apparently,  $T_M$  is still capable of acting as a vertical stabilizer to a certain degree although the disturbance adaptation is turned off due to the presence of other terms in (16). The importance of information on the disturbance to the longitudinal/lateral stabilizer is demonstrated in Figure 5. Now that the disturbance adaptation is turned on,  $z$  converges to zero and,  $x$  and  $y$  converge to a small neighbourhood of the origin. In Figure 3, even with disturbance measurements and perfect knowledge of helicopter parameters, a poor  $z$  position control is obtained. The slight offset in  $z$  position is a consequence of the absence of integral action in the vertical control.

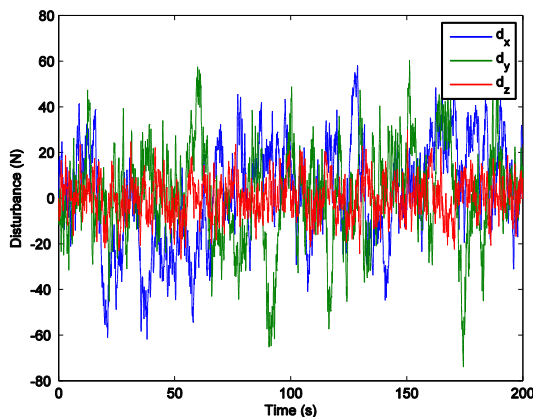


Figure 2 Wind disturbance

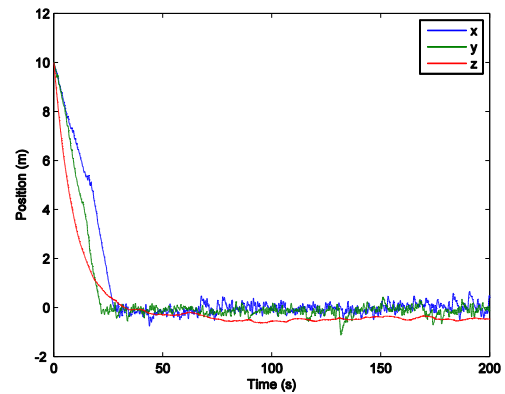


Figure 3 Position when disturbance measurements are available without parameter uncertainties

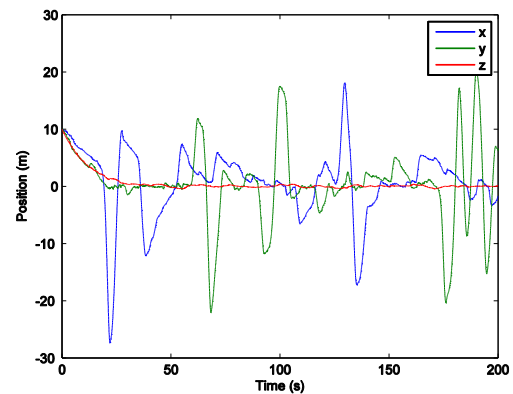


Figure 4 Position when disturbance adaptation is turned off

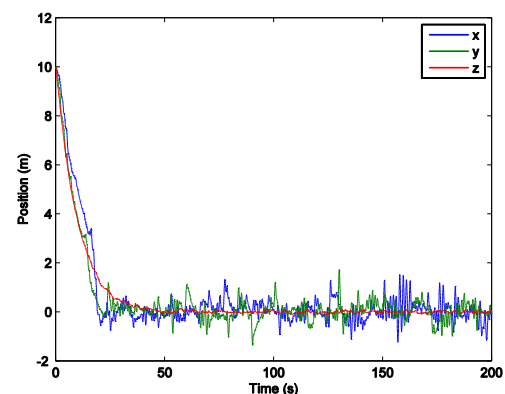


Figure 5 Position when disturbance adaptation is turned on

5.0 CONCLUSION

A robust controller for helicopter stabilization to reject wind disturbance is presented. The wind disturbance affecting the helicopter is assumed to be a function of time of a fixed structure with unknown parameters. By designing an internal model that estimates the disturbance, a control design is carried out for longitudinal, lateral and vertical dynamics stabilization. Despite the presence of helicopter parameter and model uncertainties, simulation results clearly demonstrates the effectiveness of the control technique. As future works, indoor and outdoor flights are to be carried out to test the feasibility of the proposed controller. That gives an immediate challenge caused by the presence of servo dynamics and limitations on wind disturbance that could be handled.

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