

## Resonant Control of a Single-Link Flexible Manipulator

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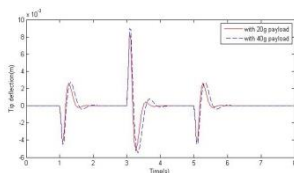
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### Graphical abstract



### Abstract

This paper presents resonant control of a single-link flexible manipulator based on the resonant modes frequencies of the system. A flexible manipulator system is a single-input multi-output (SIMO) system with motor torque as an input and hub angle and the tip deflection as outputs. The previous system which is modeled using the finite element method is considered, and the resonant modes of the system are determined. Two negative feedback controllers are used to control the system. The inner feedback control loop designed using the resonant frequencies adds damping to the system and suppress the vibration effect around the hub angle. For the outer feedback control loop, a proportional integral controller is designed to achieve a zero steady state error so that a precise tip positioning can be achieved. Simulation results are presented and discussed to show the effectiveness of the resonant control scheme.

**Keywords:** Resonant controllers; PI controller; hub angle; tip deflection; flexible manipulator

### Abstrak

Kertas ini membentangkan kawalan salunan manipulator fleksibel berdasarkan frekuensi salunan sistem. Sistem manipulator fleksibel ini adalah sistem satu-masukan multi-keluaran (MIMO) dengan daya kilas sebagai masukan manakala sudut hub dan pesongan hujung sebagai keluaran. Sistem yang telah dibangunkan menggunakan kaedah unsur terhingga telah digunakan dan mod-mod salunan sistem telah dikenal pasti. Dua sistem kawalan suabalik negatif telah digunakan. Gelung kawalan dalam direkabentuk menggunakan frekuensi salunan meningkatkan redaman sistem dan mengurangkan kesan getaran kepada sudut hub. Untuk gelung kawalan luaran, pengawal kadaran kamiran telah digunakan untuk mendapatkan ralat keadaan mantap sifar dan mencapai kedudukan hujung yang baik. Keputusan simulasi dibentangkan dan dibincangkan menunjukkan keberkesanan sistem kawalan salunan.

**Kata kunci:** Pengawal salunan; pengawal PI; sudut hub; hujung pesongan; manipulator

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### 1.0 INTRODUCTION

Control of flexible manipulators has been an interesting field of research for several years. A significant increase in the number of research for the past two decades in control of flexible manipulators has been observed, this is due to an increase in the demand for high speed robots in industries. Some advantages of flexible robot manipulators are; they can be easily driven using small sized actuator that consumed less energy, high-speed operation, low cost and lightweight. These types of robot are widely used as space robot, micro-surgery operation and nuclear plant maintenance [1]. However, on the other hand they have some disadvantages which make their control design very complicated. Due to their flexible nature, they are associated with an oscillation at the tip-end of the link that increases with inclusion of payload. Furthermore, the system produces unwanted vibrations due to elasticity of the system [2].

In order to achieved a precise deflection and suppress the effect of vibration, controller design is very essential to control the flexible robot manipulator. Main issues in designing of flexible manipulator controller are high order and non-minimum phase dynamics of the system that exist

between the tip position and the applied input torque at the hub joint of the system. Several control techniques have been applied to solve the problems of vibration and to achieve a desired tip position. These include linear state feedback control [3], adaptive control [4-5], robust control techniques based on H-infinity [6] and variable structure control [7] and intelligent control based on neural networks [8] and fuzzy logic control schemes [9]. In this area, another promising and practical controller for vibration control of flexible dynamic systems is a resonant control. An integral resonant control scheme was proposed to damp the vibration and achieve a precise tip positioning [10]. This type of control demonstrated the advantages of integral resonant controllers for their high performance and effective vibration damping when applied to flexible structures. In [11] a multivariable resonant controller was presented to control the vibration of a piezoelectric laminated cantilever beam. It has been demonstrated that the resonant controller can successfully be applied in vibration applications in such a way that all unwanted disturbances entering into the system will be rejected.

A class of resonant control scheme that can be developed to minimize structural vibration by the use of piezoelectric actuator and sensor has also been studied [12]. An effective

damping of the structure was achieved by selecting a number of resonant responses at resonant mode of the system. The controller can be tuned to select the desire number of resonant modes such that the closed loop stability is achieved. Two parallel high-Q resonant controllers were presented in [13] in which each circuit is tuned to a desired resonant frequency of the flexible structure, and this makes the controller effective at a resonant mode. Thus, this approach is only efficient and gives good result at the resonant frequency, and the system becomes ineffective at non-resonant modes. An adaptive resonant controller was proposed in [14] to attenuate vibrations effect in a cantilever structure with parameter vibration. However the controller is suitable only with structures that were exposed previously to un-modeled structure dynamic.

In this paper a Proportional Integral resonant controller is proposed to suppress vibration and oscillation to achieved precise hub angle positioning with low tip deflection of a flexible link manipulator. The controller consists of two negative feedback loops where the inner feedback-loop is designed to control the vibration by feeding back the resonant frequencies of the system to add damping to the flexible link whereas the outer feedback-loop is designed to achieve zero steady state error to obtained precise tip deflection. Simulation results show that the controller is effective and robust under various loading conditions.

2.0 MODEL DESCRIPTION

The flexible manipulator used in this study in [15] consists of a piece of thin aluminum alloy. The system parameters are; length of the flexible link  $L = 0.9$  m, Young Modulus  $E = 71 \times 10^9$  N/m<sup>2</sup>, width of the link 19.008 mm, thickness of 3.2004 mm, second moment of inertia  $I = 5.1924$  m<sup>4</sup>, and mass density per unit volume  $\rho = 2710$  kg/m<sup>3</sup>. The schematic diagram of the single link flexible manipulator system is shown in Figure 1.

In the model presented in [15] the finite element method (FEM) was used to model the system. Using FEM, the length of the manipulator link is divided into number of elements and the model order increases with an increase in the number of elements. Using a single element, a sixth order model is obtained in both state space model as given in (1), and then converted to transfer function of the hub angle and tip deflection to the input torque as described as in (2) and (3).

$$\begin{aligned} \dot{v} &= Av + Bu \\ y &= Cv \end{aligned} \tag{1}$$

where

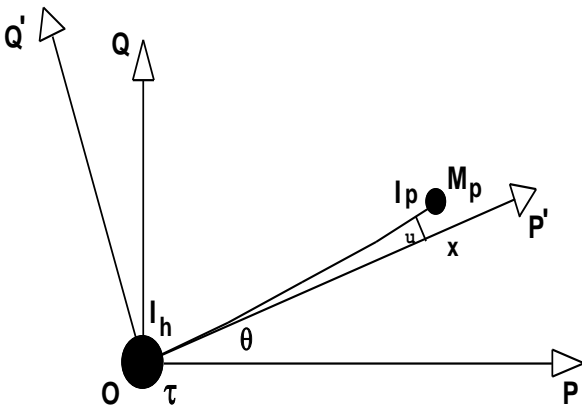


Figure 1 Schematic diagram of a flexible link manipulator

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 58209 & -27441 & 0 & -33 & -6 \\ 0 & -38548 & 16329 & 0 & -27 & 4 \\ 0 & 93918 & -58611 & 0 & 16 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1013.6 \\ -821.0 \\ 304.1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The transfer function of the hub angle to input torque can be obtained as

$$G_{hub} = \frac{1014s^4 + 4553s^3 + 4.2 \times 10^7 s^2 + 2.8 \times 10^7 s + 1.7 \times 10^{10}}{s^6 + 33.3s^5 + 9.7 \times 10^4 s^4 + 1.1 \times 10^6 s^3 + 7.2 \times 10^8 s^2} \tag{2}$$

and the transfer function of the tip deflection to input torque as

$$G_{tip} = \frac{-821s^4 - 3880s^3 - 4.3 \times 10^7 s^2}{s^6 + 33.3s^5 + 9.7 \times 10^4 s^4 + 1.1 \times 10^6 s^3 + 7.2 \times 10^8 s^2} \tag{3}$$

3.0 CONTROLLER DESIGN

In this section, design of the resonant controller is discussed. The control technique consists of two negative feedbacks. The inner loop controller is designed to add damping to the system around the hub angle to suppress the vibration effect, and the outer loop controller is designed to achieve zero steady state error in order to have an accurate tip deflection. A block diagram of the control scheme is shown in Figure 2.

3.1 Resonant Controller (Inner Loop Control)

The inner loop controller will be based on the resonant frequency at specific modes, the damping ratio  $\delta_i$  and controller gain  $\alpha_i$ . This feedback controller increases the damping effect to suppress the vibration and also guarantee unconditional stability for the closed-loop system. It is also known as collocated velocity feedback controller because it avoids closed-loop instabilities due to spillover effects. Ideally to control vibration by damping, the control should be restricted to resonant frequencies only [16].

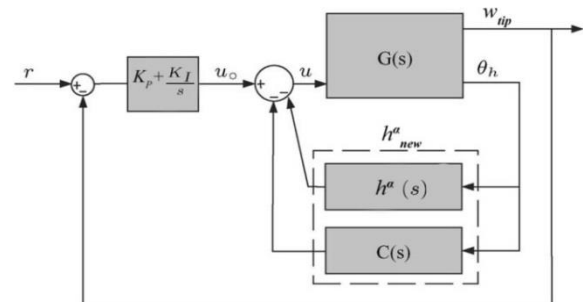


Figure 2 Block diagram of a resonant controller

The general model structure of a resonant controller is in the form of approximation of differentiator around resonant frequencies of the system. The resonant controller model form is given as

$$h_i^\alpha = \sum_{i=1}^N \frac{\alpha_i s^2}{s^2 + 2\delta_i \omega_i s + \omega_i^2} \tag{4}$$

where  $N$  is the number of modes need to be controlled,  $w_i$  is the resonant frequency and  $\alpha_i$  is a constant parameter ranging

from 0 to 150. In this study the resonant controller is designed based on one element and without payload. As reported in [15], using experiment the first two resonant modes,  $\omega_1$  and  $\omega_2$  are obtained as 75.3 rad/s and 221.2 rad/s respectively. Similarly through experiments, the damping ratios,  $\delta_1$  and  $\delta_2$  are obtained as 0.007 and 0.015 respectively. For two modes resonant frequencies,  $N = 2$ ,

$$h_i^\alpha = h_1 + h_2 \quad (5)$$

In this design  $\alpha_1$  and  $\alpha_2$  are chosen to be 120. For  $i = 1, 2$  Equation (5) can be obtained as

$$h_i^\alpha = \frac{120s^2}{s^2+1.056s+5689.5} + \frac{120s^2}{s^2+6.6s+48400} \quad (6)$$

In order to improve system stability and increase the response speed, a phase lead can be used to shift the pole to the left half s-plane. To accomplish this, a first order lead compensator is designed using the root locus method. The lead compensator can be described as

$$G_c(s) = K \frac{s+z}{s+p} \quad (7)$$

where  $K$  is the compensator gain,  $z$  and  $p$  are the zero and pole of the compensator respectively. In this work,  $K$ ,  $p$  and  $z$  are deduced as 70, 75 and 4.92 respectively. The lead compensator can be obtained as

$$G_c(s) = 70 \frac{s+4.92}{s+75} \quad (8)$$

Hence the new controller  $h_{new}$  which is a combination of resonant controller  $h_i^\alpha$  and the phase lead compensator  $G_c(s)$  can be obtained as

$$h_{new} = \frac{120s^2}{s^2+1.056s+5689.5} + \frac{120s^2}{s^2+6.6s+48400} + 70 \frac{s+4.92}{s+75} \quad (9)$$

The closed-loop transfer function of the hub angle with the resonant controller is given by

$$G_{hub}^{closed-loop} = \frac{G_{hub}(s)}{1+h_{new}G_{hub}(s)} \quad (10)$$

Figures 3 and 4 show Bode plots of the open loop system and closed-loop system with resonant controller. It can be observed that the resonant controller adds damping to the system and increases both phase and gain margins of the system.

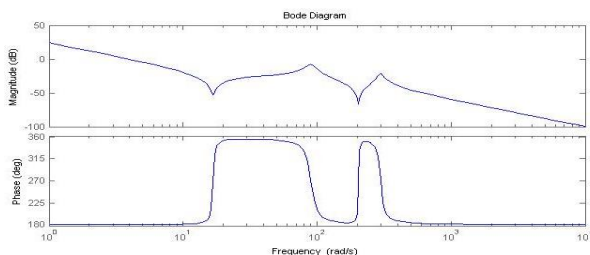


Figure 3 Bode plot of the open loop system

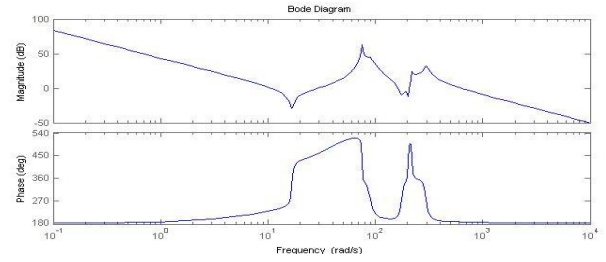


Figure 4 Bode plot of the closed-loop system with controller

### 3.2 Outerloop Controller (PI Controller)

A Proportional Integral (PI) controller is designed to achieve zero steady state error so that a precise tip deflection of the flexible link can be achieved. The PI controller can be described as

$$G_{PI} = K_p + \frac{K_I}{s}$$

where  $K_p$  and  $K_I$  are proportional and integral gains respectively.

In this study the PI controller is design using Ziegler-Nichols tuning technique in MATLAB. The appropriate value of  $K_p$  and  $K_I$  were obtained as 4.576 and 0.00565 respectively. By substituting these values in equation (13) yield the PI controller as

$$G_{PI} = 4.576 + \frac{0.00565}{s} \quad (11)$$

Therefore, the closed-loop transfer function of the tip deflection with the PI controller is given as

$$G_{tip}^{closed-loop} = \frac{G_{tip}(s)}{1+K_{PI}G_{tip}(s)} \quad (12)$$

Figure 5 shows the root locus of the closed-loop transfer function of the tip deflection with resonant and PI controllers. The root locus shows that all the poles of the system are within the stability region. Hence with the controller the system is stable.

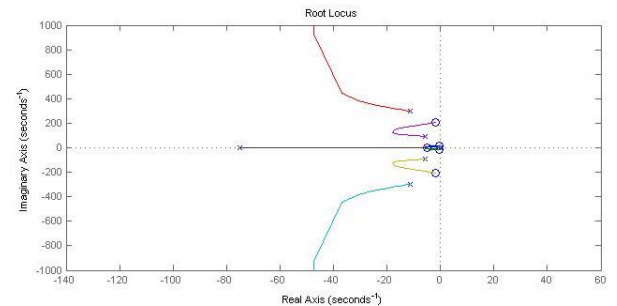


Figure 5 Root locus of the tip deflection with the controllers

## 4.0 SIMULATION RESULTS

In this section, simulation results obtained by implementing the proposed controllers are presented. As shown in Figures 3 and 4, phase and gain margin of the system increase with the resonant controller. Figures 6 and 7 show open loop response of the hub angle and tip deflection without payload with a bang-bang input. It is noted that the manipulator accelerates and finally stops at approximately 85 degree. However,

significant vibration is noted during motion. Moreover, tip deflection response in Figure 7 reveals significant vibration at the tip.

The closed-loop system with resonant controller as in Equations (9) and (11) is then simulated. Figures 8 and 9 show the hub angle and tip deflection responses of the system using the controllers. It is observed that the hub angle moves to the desired location with zero steady state error. Moreover, the tip deflection is significantly reduced observed to be approximately regulated at zero deflection with acceptable maximum deflection of 0.007 m. The proposed control scheme has successfully added damping around the hub angle and efficiently suppressed the vibration resulting in a precise tip positioning.

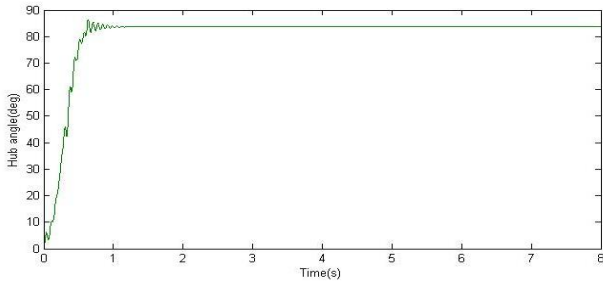


Figure 6 Open loop response of hub angle

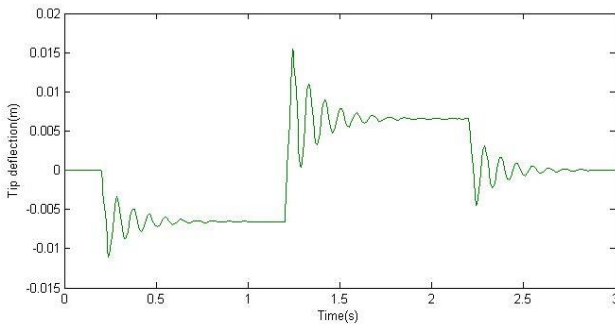


Figure 7 Open loop response of tip deflection

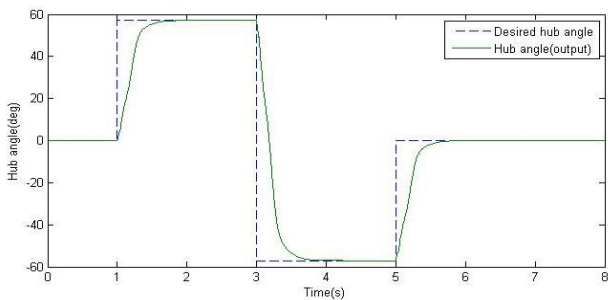


Figure 8 Hub angle response with the controller

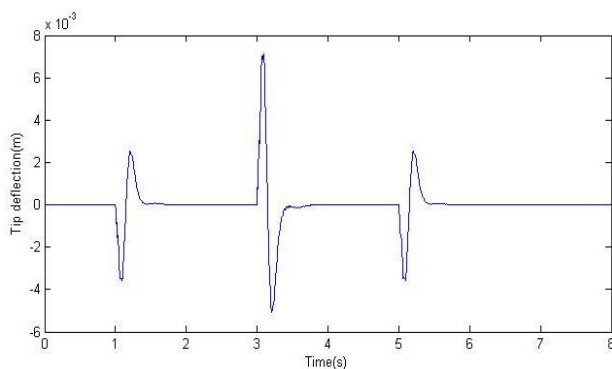


Figure 9 Tip deflection response with the controller

### 4.1 Robustness

To examine robustness of the proposed controller, the manipulator is tested with payloads of 20 g and 40 g. Figures 10 and 11 shows the hub angle and tip deflection responses respectively for the system with payloads of 20 g and 40 g. It is noted that the desired angle of 58 degrees can be achieved with the controller in both loading conditions. In both cases, zero steady-state error is achieved. However, overshoots of 2% and 5% are observed for the system with 20 g and 40 g respectively. This is as expected as the same PI parameters are used. The tip deflection response with both 20 g and 40 g shows a similar pattern of deflection as the system without payload. No significant change is noted indicating robustness of the resonant control in reducing vibration of the flexible manipulator.

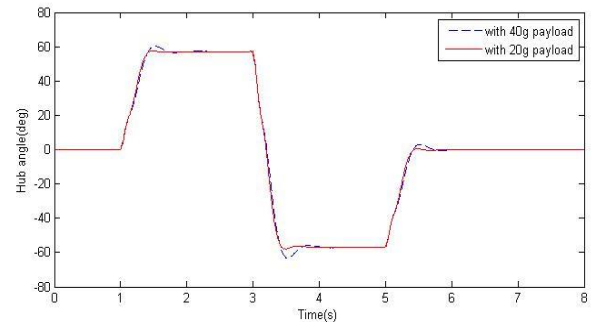


Figure 10 Hub angle response of the system with 20 g and 40 g payload

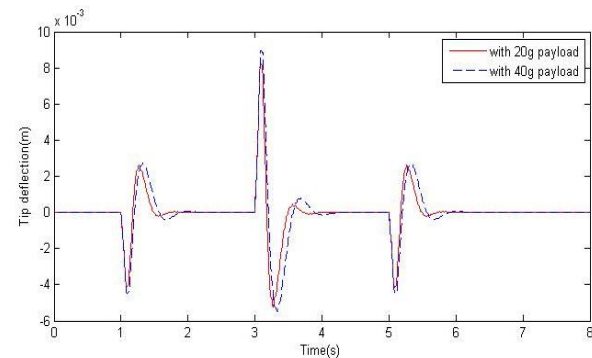


Figure 11 Tip deflection response of the system with 20 g and 40 g payload

### 5.0 CONCLUSION

This paper has presented a resonant control of a single link flexible manipulator. Resonant and PI controllers were successfully designed to add damping and suppressed vibration to achieve an accurate tip deflection. A lead compensator was designed to shift the poles to left side of the s-plane for stability. Simulation results have shown that the resonant controller have significantly reduced tip deflection of the system. Moreover, the hub angle response successfully achieved the desired angle. Examining the system with payloads of 20 g and 40 g shows that the proposed control scheme is robust to payload variations. Almost similar hub angle and tip deflection responses have been obtained.

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