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A Robust Stabilization using State Feedback with Feedforward

Kumeresan A. Danapalasingam[a,b*]

[a]Senior Lecturer, Department of Control & Mechatronics Engineering, Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia
[b]Associate Fellow, UTM Centre for Industrial and Applied Mathematics, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia

*Corresponding author: kumeresan@fke.utm.my

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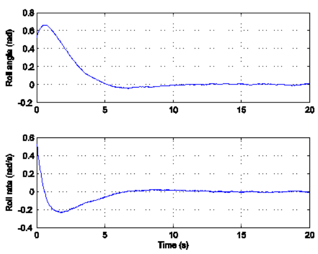
Graphical abstract



Abstract

In a general nonlinear control system a stabilizing control strategy is often possible if complete information on external inputs affecting the system is available. Assuming that measurements of persistent disturbances are available it is shown that the existence of a smooth uniform control Lyapunov function implies the existence of a stabilizing state feedback with feedforward control which is robust with respect to measurement errors and external disturbances. Conversely, using differential inclusions parameterized as nonlinear systems with state and disturbance measurement errors, it is shown that there exists a smooth uniform control Lyapunov function if there is a robustly stabilizing state feedback with feedforward. This paper demonstrates that if there exists a smooth control Lyapunov function for a general nonlinear system with persistent disturbances for which one has previously designed a feedback controller, a feedforward always exists to be augmented for stability.

Keywords: Robust stabilization; feedback; feedforward; Lyapunov function

Abstrak

Dalam sistem kawalan linear umum strategi kawalan menstabilkan sering mungkin jika maklumat lengkap mengenai input luaran yang menjejaskan sistem ini boleh didapati. Dengan mengandaikan bahawa pengukuran gangguan berterusan boleh didapati ia menunjukkan bahawa kewujudan kawalan seragam persamaan Lyapunov lancar membayangkan kewujudan suap balik keadaan stabil dengan kawalan suap depan yang teguh berkenaan dengan ralat pengukuran dan gangguan luaran. Sebaliknya, dengan menggunakan kemasukan pembezaan sebagai sistem tak linear dengan negeri dan pengukuran gangguan kesilapan, ia menunjukkan bahawa wujud kawalan seragam persamaan Lyapunov licin jika terdapat suap balik keadaan sistem dengan kukuh stabil dengan suap depan. Karya ini menunjukkan bahawa jika wujud kawalan lancar persamaan Lyapunov untuk sistem tak linear umum dengan gangguan berterusan yang mana satu sebelum ini telah direka pengawal suap balik, suap depan sentiasa wujud untuk diperluaskan untuk kestabilan.

Kata kunci: Penstabilan teguh; suap balik; suap depan; persamaan Lyapunov

1.0 INTRODUCTION

In nonlinear systems, the design of stabilizing feedback controllers guarantees stability when no persistent disturbance is present. Even though in some cases a feedback would suffice, in general a state feedback with feedforward is inevitable for stability when nonzero disturbances affect the system.[9] It could be advantageous however, if one only has to design a feedforward that can be simply augmented to an existing feedback for required stability in the presence of persistent disturbances. Some previous works on feedforward control will be reviewed here.

In Ref. 12 discrete-time feedback/feedforward controllers are developed for general nonlinear processes with stable zero dynamics. The design of the controllers is synthesized in a coupled manner where separate objectives of the feedforward and feedback controllers are realized by means of one unified control law. A feedforward only approach using artificial neural networks is reported in Ref. 6 describing a nonlinear adaptive feedforward controller for compensation of external load disturbances in the idle speed control of an automotive engine. In Ref. 7, a feedforward control is employed to handle measurable additive disturbances with linear dynamics affecting a nonlinear plant. In this paper, we study the existence of a separate robust feedforward whose control inputs can be added to those of an existing feedback to ensure stability of general nonlinear systems with persistent disturbances as one of its external inputs.

In this work, by adding a feedforward term and restricting the persistent disturbance to be a Lipschitz function, the work in Ref. 9 is extended using similar approach therein to accommodate our purposes. While only a feedback is considered in the main reference Ref. 9, here we employ a feedback with feedforward control and a stricter smooth uniform control Lyapunov function for robust stability. This paper is organized as follows: Section 2.0 contains the problem statement and some definitions. The main theorem of this paper, Theorem 2.1 as well as the converse Lyapunov theorem from Ref. 3, Theorem 2.2 are also stated here. Subsequently, a simulation example is given in Section 3.0. The paper is concluded in Section 4.0.

■2.0 MAIN RESULTS

This work concerns the development of a feedforward control strategy for general nonlinear control systems of the type

$$\dot{x} = f(x, u, d), \quad x \in \mathbb{R}^n, u \in \mathbb{U}, d \in \mathbb{D}, \tag{1}$$

where $\mathbb{U}$ is a compact subset of $\mathbb{R}^c$, persistent disturbance $d = d(\cdot)$ is a Lipschitz function taking values in some compact set $\mathbb{D} \in \mathbb{R}^w$ containing 0 and $f: \mathbb{R}^n \times \mathbb{U} \times \mathbb{D} \to \mathbb{R}^n$ is a continuous function. Given an existing stabilizing feedback $k: \mathbb{R}^n \to \mathbb{U}$ designed for (1) with $d = 0$, the feedforward stabilization problem is that of finding a feedforward control $l: \mathbb{R}^n \times \mathbb{R}^w \to \mathbb{U}$ with $l(x, 0) = 0$ such that the origin in $\mathbb{R}^n$ is asymptotically stable with respect to the trajectories of the closed-loop system

$$\dot{x} = f(x, k(x) + l(x, d), d). \tag{2}$$

The remainder of this section provides a series of essential definitions and theorems.

A function $V: \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is said to be *positive* (definite) if $V(0) = 0$ and $V(x) > 0$ for all $x \neq 0$, and *proper* if the sublevel set $\{x: V(x) \leq a\}$ is compact for all $a > 0$.

Definition 2.1: A smooth function $V: \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is defined as a *smooth uniform control Lyapunov function* for system (1) if V is positive, proper and satisfies the following *infinitesimal decrease* condition: There exists a continuous positive function $W: \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that, for any bounded set $\mathbb{X} \subset \mathbb{R}^n$,

$$\min_{u \in \mathbb{U}} \langle \nabla V(x), f(x, u, d) \rangle \leq -W(x), \quad \forall x \in \mathbb{X}, x \neq 0, \forall d \in \mathbb{D}, \tag{3}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathbb{R}^n$ (cf. (14) in Ref. 9).

It follows from the infinitesimal decrease condition (3) that there always exists a state feedback with feedforward $m: \mathbb{R}^n \times \mathbb{R}^w \to \mathbb{U}$ which satisfies

$$\langle \nabla V(x), f(x, m(x, d), d) \rangle \leq -W(x), \quad \forall x \in \mathbb{X}, x \neq 0, \forall d \in \mathbb{D}. \tag{4}$$

Here, we define the state feedback with feedforward as

$$m(x, d) := k(x) + l(x, d). \tag{5}$$

Such a control m will be in general discontinuous.[4,8] It will be shown that a feedback k and a feedforward l satisfying (5) and (4) will drive the state of the system (2) to the origin in $\mathbb{R}^n$ and this stabilizing state feedback with feedforward m is robust with respect to state measurement errors $e_x(\cdot)$, disturbance measurement errors $e_d(\cdot)$ and external disturbances $w(\cdot)$ in the perturbed system

$$\dot{x} = f\big(x, m\big(x + e_x(t), d + e_d(t)\big), d(t)\big) + w(t). \tag{6}$$

As described in Definition 2.2, robustness in this context refers to the insensitivity of m in handling measurement errors and additive external disturbances to drive all states to an arbitrary neighborhood of the origin for fast enough sampling and small enough measurement errors and external disturbances.

Next, the state trajectory of a system with a discontinuous control is defined similarly to Ref. 4. Let $\pi = \{t_i\}_{\geq 0}$ be any partition of $[0, +\infty]$ with

$$0 = t_0 < t_1 < \cdots$$

and $\lim_{i \to \infty} t_i = +\infty$. The π-trajectory of the perturbed system (6) starting from x_0, under the action of a possibly discontinuous state feedback with feedforward m and in the presence of disturbance $d(\cdot)$, state measurement errors $e_x(\cdot)$, disturbance measurement errors $e_d(\cdot)$ and external disturbances $w(\cdot)$, is defined recursively on the intervals $[t_i, t_{i+1}], i = 0,1, \dots,$ as the solution of the differential equation

$$\dot{x}(t) = f\big(x(t), u_i, d(t)\big) + w(t), \quad a.a. t \in [t_i, t_{i+1}], \tag{7}$$

where $u_i = m\big(x(t_i) + e_x(t_i), d(t_i) + e_d(t_i)\big)$, $x(0) = x_0$. To be noted, $x(\cdot)$ may fail to exist on one of the intervals $[t_i, t_{i+1}]$ when there exists a $T < +\infty$ such that the $x(\cdot)$ only exists on $(0, T]$ and $\lim_{t \uparrow T} |x(t)| = +\infty$ where $|\cdot|$ denotes the Euclidean norm. Such an $x(\cdot)$ is called a *blown-up* trajectory.

Definition 2.2: The state feedback with feedforward m is *robustly s-stabilizing* (sampling stabilizing) if for any $0 < r < R$ there exists positive $T = T(r, R)$, $\delta = \delta(r, R)$, $\eta = \eta(r, R)$ and $M(R)$ such that for any state measurement error $e_x(\cdot)$, disturbance measurement errors $e_d(\cdot)$ (arbitrary bounded functions $e_x: [0, +\infty) \to \mathbb{R}^n$ and $e_d: [0, +\infty) \to \mathbb{R}^w$) and external disturbances $w(\cdot)$ (measurable essentially bounded function $w: [0, +\infty) \to \mathbb{R}^n$) for which

$$|e_x(t)| \leq \eta, |e_d(t)| \leq \eta, \quad \forall t \geq 0, \quad \|w(\cdot)\|_\infty \leq \eta, \tag{8}$$

and any partition π with $diam := sup_{i \geq 0}(t_{i+1} - t_i) \leq \delta$, every π-trajectory with $x(0) \leq R$ does not blow-up and satisfies the following relations:

1. Uniform attractivity

$$|x(t)| \leq r, \quad \forall t \geq T \tag{9}$$

2. Bounded overshoot

$$|x(t)| \leq M(R), \quad \forall t \geq 0 \tag{10}$$

3. Lyapunov stability

$$\lim_{R \downarrow 0} M(R) = 0 \tag{11}$$

The following is the main theorem of this paper.

Theorem 2.1: The control system (1) admits a smooth uniform control Lyapunov function if and only if there exists a robustly s-stabilizing state feedback with feedforward m.

In the proof of the sufficiency part of Theorem 2.1, it can be shown that if there exists a stabilizing state feedback with feedforward m that is robust with respect to state and disturbance measurement errors and external disturbances for the control system (1), then the differential inclusion

$$\dot{x} \in G(x) \tag{12}$$

with multivalued function

$$G(x) := \bigcap_{\varepsilon>0} \overline{co} \bigcup_{d\in\mathbb{D}} f(x, m(x+\varepsilon B, d+\varepsilon B), d) \tag{13}$$

is strongly asymptotically stable, where B is a closed unit ball and $\overline{co}\, S$ the closure of the convex hull of a set S. One can easily show that the multifunction (13) satisfies Hypothesis (H) which is given as follows:

H(1) The multifunction G is upper semicontinuous, i.e. for any $x \in \mathbb{R}^n$ and any $\varepsilon > 0$ there is a $\delta > 0$ such that,

$$G(x') \subset G(x) + \varepsilon B, \quad \forall x' \in x + \delta B.$$

H(2) $G(x)$ is a compact convex subset of $\mathbb{R}^n$ for each $x \in \mathbb{R}^n$.

Definition 2.3: The differential inclusion (12) is strongly asymptotically stable if it has no blown-up solutions and

1. (Attractivity) for any solution $x(\cdot)$

$$\lim_{t\to\infty} x(t) = 0. \tag{14}$$

2. (Strong Lyapunov stability) for any $\varepsilon > 0$, there is $\delta > 0$ such that every solution of (12) with $x(0) < \delta$ satisfies

$$|x(t)| \le \varepsilon, \quad \forall t \ge 0. \tag{15}$$

Strong asymptotic stability of differential inclusion (12) implies that there are no solutions $x(\cdot)$ of (12) exhibiting finite time blow-up and for any positive $r < R$ there exist $T = T(r, R)$ and $M(R)$ such that any solution with $x(0) < R$ satisfies (9) and (10), and (11) holds.[3]

Definition 2.4: The smooth function $V: \mathbb{R}^n \to \mathbb{R}_{\ge 0}$ is said to be a smooth strong Lyapunov function for the differential inclusion (12) if it is positive, proper and satisfies the following infinitesimal decrease condition:

$$\min_{z\in G(x)} \langle \nabla V(x), z\rangle \le -W(x), \tag{16}$$

where W is a positive continuous function. The following theorem is proved in Ref. 3.

Theorem 2.2: Under Hypothesis (H), the multifunction G is strongly asymptotically stable if and only if there exists a smooth strong Lyapunov function for G.

It is worth mentioning here that the multifunction (13) can be shown to satisfy Hypothesis (H).

3.0 SIMULATION RESULTS

We will now show the existence of a robustly s-stabilizing state feedback with feedforward for the control of wing rock motion of an aircraft.[11] From Theorem 2.1, we know that this is an implication of the existence of a smooth uniform control Lyapunov function for the system in question. The following are the equations governing a wing rock motion with disturbance and neglecting actuator dynamics, see e.g. Ref. 14.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + \Delta(x_1, x_2) + d \end{aligned} \tag{17}$$

where $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ represent the roll angle ϕ and roll rate p respectively, $u \in \mathbb{R}$ is the control input, $d \in \mathbb{R}$ is the persistent disturbance and $\Delta(x_1, x_2) := b_0 + b_1 x_1 + b_2 x_2 + b_3|x_1|x_2 + b_4|x_2|x_2 + b_5 x_1^3$ with $b_0 = 0$, $b_1 = -0.01859521$, $b_2 = -0.015162375$, $b_3 = -0.6245153$, $b_4 = 0.00954708$ and $b_5 = 0.02145291$. Note that we have assumed $u \in \mathbb{R}$ for simplicity so that given $u := -x_1 - x_2 - \Delta(x_1, x_2) - d$, the function $V(x_1, x_2) := {}^1\!/_2\, x_1^2 + {}^1\!/_2\, x_2^2$ satisfies

$$\min_{u\in\mathbb{R}} \langle \nabla V(x_1, x_2), f(x_1, x_2, u, d)\rangle = \min_{u\in\mathbb{R}} [x_1 x_2 + x_2 u + x_2 \Delta(x_1, x_2) + x_2 d] = -x_2^2$$

and is therefore a smooth uniform control Lyapunov function for (17). Using the model reference adaptive controller from Ref. 1 as a feedback $k(x)$ and a feedforward $l(d) := -d$, we will demonstrate that they form a robustly s-stabilizing state feedback with feedforward $m(x, d) := k(x) + l(d)$ as assured by Theorem 2.1. In the simulation we assume that all states and persistent disturbance $d(t) = \sin(t)$ can be measured. Additionally we set the state measurement errors, disturbance measurement errors and external disturbances to be uniformly distributed random numbers, i.e., $e_x(\cdot), e_d(\cdot), w(\cdot) \in [-0.1, 0.1]$ and employ a uniform partition π of [0,20] with $t_{i+1} - t_i = 0.02, i = 1,2, \ldots$.

The objective of the control is to suppress the wing rock motion $\phi = p = 0$. In Figure 1, in the absence of disturbance, it could be seen that the state feedback is robustly s-stabilizing in the face of state measurement errors $e_x(\cdot)$ and external disturbances $w(\cdot)$. This capability is diminished however, when disturbance is fed to the system as shown in Figure 2. The validity of Theorem 2.1 is proven in Figure 3 when the combination of the existing state feedback $k(x)$ and the feedforward $l(d)$ stabilizes the motion and is robust with respect to state measurement errors $e_x(\cdot)$, disturbance measurement errors $e_d(\cdot)$ and external disturbances $w(\cdot)$. Thus, in this example we have shown that if there exists a smooth uniform control Lyapunov function and a previously designed robustly s-stabilizing feedback in the absence of disturbance, one could find a feedforward so that the state feedback with feedforward is robustly s-stabilizing for nonzero disturbances.

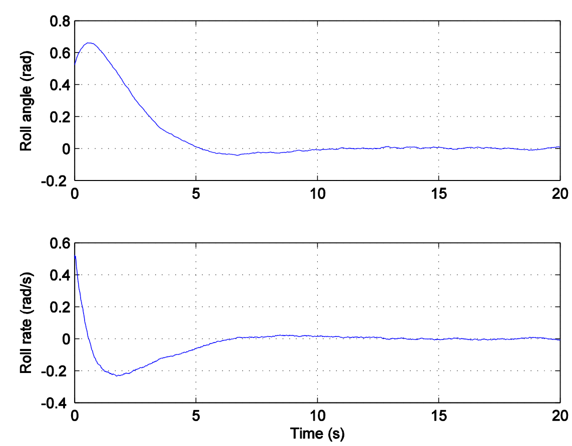


Figure 1 Roll angle and roll rate using state feedback only with $d = 0$

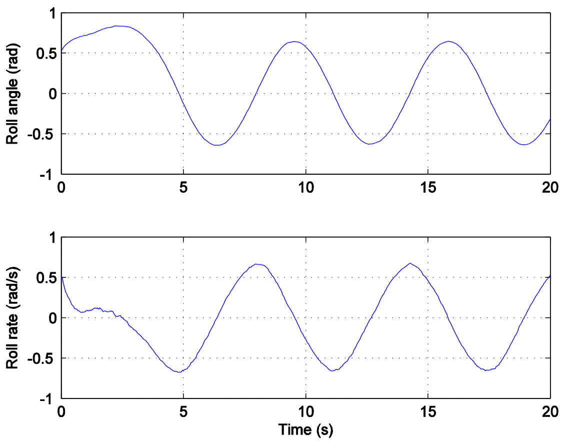


Figure 2 Roll angle and roll rate using state feedback only when $d \neq 0$

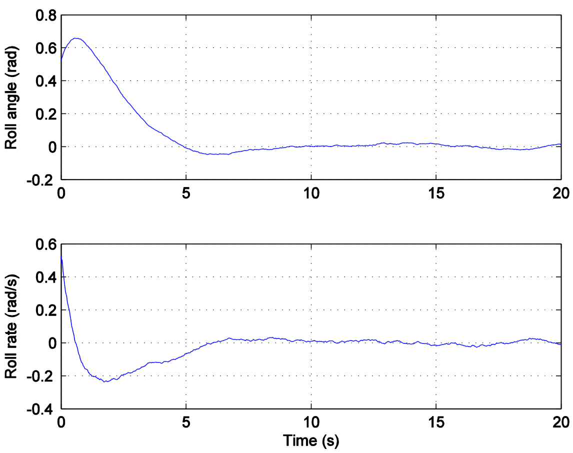


Figure 3 Roll angle and roll rate using state feedback with feedforward when $d \neq 0$

■4.0 CONCLUSION

In this theoretical work, we have proven that given a smooth uniform control Lyapunov function, there always exists a robustly s-stabilizing state feedback with feedforward m which could be implemented as a combination of a feedback k and a feedforward l, that is robust with respect to state and disturbance measurement errors and external disturbances. To prove that the reverse is also true, general nonlinear control systems with state and disturbance measurement errors are represented by parameterized differential inclusions. If there exists a robustly s-stabilizing state feedback with feedforward m, it is shown that the differential inclusion is strongly asymptotically stable. Since strong asymptotic stability implies the attraction of all of the solutions to an arbitrary neighborhood of the origin, a smooth control Lyapunov function is proven to exist. With the establishment of the present theoretical foundation, the authors expect to produce a practical implementation of the feedforward control for disturbance rejection as a future work.

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