

Multivariable PID Using Singularly Perturbed System

Mashitah Che Razali^a, Norhaliza Abdul Wahab^{a*}, Sharatul Izah Samsudin^b

^aFaculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia

^bFaculty of Electronics and Computer Engineering, Universiti Teknikal Malaysia Melaka, 76100 UTeM Durian Tunggal, Melaka, Malaysia

*Corresponding author: aliza@fke.utm.my

Article history

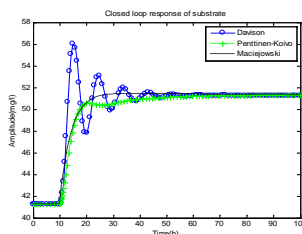
Received :5 August 2013

Received in revised form :

28 November 2013

Accepted :15 January 2014

Graphical abstract



Abstract

The paper investigates the possibilities of using the singularly perturbation method in a multivariable proportional-integral-derivative (MPID) controller design. The MPID methods of Davison, Penttinen-Koivo and Maciejowski are implemented and the effective of each method is tested on wastewater treatment plant (WWTP). Basically, this work involves modeling and control. In the modeling part, the original full order system of the WWTP was decomposed to a singularly perturbed system. Approximated slow and fast models of the system were realized based on eigenvalue of the identified system. The estimated models are then used for controller design. Mostly, the conventional MPID considered static inverse matrix, but this singularly perturbed MPID considers dynamic matrix inverse. The stability of the singularly perturbed system is established by using Bode analysis, whereby the bode plot of the model system is compared to the original system. The simulation results showed that the singularly perturbed method can be applied into MPID. The three methods of MPID have been compared and the Maciejowski shows the best closed loop performance.

Keywords: Singularly perturbed system; slow and fast model; multivariable PID

Abstrak

Penyelidikan ini mengkaji beberapa kemungkinan penggunaan kaedah bersendirian resah pada pengawal pembolehubah *proportional-integral-derivative* (MPID). Beberapa kaedah MPID iaitu Davison, Penttinen-Koivo dan Maciejowski telah dilaksanakan dan keberkesanan bagi setiap kaedah telah diuji pada loji rawatan sisa kumbahan (WWTP). Pada dasarnya terdapat dua peringkat utama yang terlibat di dalam kerja-kerja ini iaitu bahagian pemodelan dan kawalan. Pada bahagian pemodelan, sistem asal WWTP telah diuraikan kepada sistem bersendirian resah. Pada bahagian ini, model perlahan dan cepat diperolehi dengan berdasarkan nilai eigen sistem yang telah dikenal pasti. Kemudian, model yang diperolehi telah digunakan untuk mereka bentuk pengawal pembolehubah. Daripada matriks songsang statik yang telah dipertimbangkan dalam konvensional MPID, kes ini telah menggunakan MPID bersendirian resah berdasarkan matriks songsang dinamik. Kestabilan sistem bersendirian resah telah ditentukan dengan menggunakan analisis Bode, di mana *bode plot* bagi sistem tersebut dibandingkan dengan sistem asal. Keputusan simulasi menunjukkan bahawa kaedah bersendirian resah boleh digunakan ke dalam MPID. Tiga kaedah MPID telah dibandingkan dan kaedah Maciejowski menunjukkan prestasi gelung tertutup yang terbaik.

Kata kunci: Sistem bersendirian resah; model perlahan dan cepat; MPID

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1.0 INTRODUCTION

1.1 Singularly Perturbed System

Singularly perturbation technique has attracted the attention of many researchers from various fields of studies over the past four decades [1]. It is a common tool for system modeling, system analysis and controller design. In control design, the most important is how to mathematically describe the dynamic system to be controlled and the level of modeling itself. Singularly perturbation technique can decompose and simplify the higher order of the original system into the slow and fast model [1–3]. By applying this technique, singularly perturbed system is obtained. Therefore, singularly perturbed system is a system that contains a reduced order system which is known as a slow and fast model. Singularly perturbed system has a multi-

scale characteristic [4], [5] and ill-conditioned dynamics [6] like wastewater treatment plant. In control system design, a reduced order system has the capability to make the synthesis, analysis and design of the controller simpler [7].

In [8], the combination of a singularly perturbed system with the optimal control theory and two-stage design of linear feedback control has been discussed. Similar research approach with a combination of optimal control theory with the singularly perturbed system have been presented in [9] and [10]. The papers study the coexistence of slow and fast dynamics in an integrated process network. A controller design framework consisting of properly coordinated controllers in slow and fast models has been investigated by [11]. Decentralized model predictive control for different dynamic system is presented. In [12], singularly perturbed system has been modeled using a real Schur form method. The paper shows that any two-time scale

system can be modeled in the singularly perturbed form via a transformation into an order of real Schur form, followed by balancing.

In [13], reduced order dynamical modeling of reaction systems by using a singularly perturbation technique has been proposed. A systematic method was used for transforming the natural mass-balance model of reaction systems with slow and fast reactions into the two-time scale standard form of singularly perturbation technique. In [14] stability analysis of singularly perturbed systems via vector Lyapunov methods has been proposed. The design of a composite feedback control and the stability analysis of the closed loop full order system were applied with singularly perturbation technique. In conjunction, [15] apply an interpolation method to control tuning parameter, ϵ of singularly perturbed systems. It can build globally stable and efficient controllers. State estimation of two-time scale multiple models with application to wastewater treatment plant have been studied in [16]. The proposed method was applied for nonlinear systems with two-time scales.

From the reviews, it was found that the significance of the singularly perturbed system is the possibility to deal with the reduced order model instead of the original system and possibility to study the important properties such as robustness in the presence of un-modeled dynamics [14]. Singularly perturbed system also able to alleviate the difficulties of both dimensionality and stiffness problem [17]. Most of the control systems are dynamic, where the decomposition into stages is dictated by different time scales: slow and fast. Singularly perturbed system basically has two different parts of eigenvalue [18] represented for slow and fast dynamic. It was able to naturally describe the dynamics characteristic of many practical systems such as wastewater treatment plant, chemical industry, power systems, boiler system and robotic field [19]. Here, focused on the analysis of singularly perturbed system of wastewater treatment plant was given.

To the best of our knowledge, there are two methods to obtain the singularly perturbed system: analytical [4], [14], [20], [21] and linear analysis [22]. Singularly perturbed system obtained based on linear analysis is straighter forward than analytical analysis and has been applied in this work. Analytical analysis will cause great difficulty in terms of solving partial differential equation. Using linear analysis, data is obtained from subspace system identification and the state variables were identified from literature study on the related system. More details about the methods to obtain a singularly perturbed system can be obtained from the book of [23]. Based on linear analysis, the number of slow and fast poles needs to be identified based on pole location. Then, the new state space for the n number of slow poles and m number of fast poles are obtained using equations (1) until (6).

$$A_{slow} = A_{11} - A_{12}A_{22}^{-1}A_{21} \quad (1)$$

$$B_{slow} = B_1 - A_{12}A_{22}^{-1}B_2 \quad (2)$$

$$C_{slow} = C_1 - C_2A_{22}^{-1}A_{21} \quad (3)$$

$$A_{fast} = A_{22} \quad (4)$$

$$B_{fast} = B_2 \quad (5)$$

$$C_{fast} = C_2 \quad (6)$$

After that, state space of a singularly perturbed system was obtained using equations (7) to (12).

$$A_{spm} = \begin{bmatrix} A_{slow} & Z_{12} \\ Z_{21} & A_{fast} \end{bmatrix} \quad (7)$$

$$B_{spm} = \begin{bmatrix} B_{slow} \\ B_{fast} \end{bmatrix} \quad (8)$$

$$C_{spm} = [C_{slow} \quad C_{fast}] \quad (9)$$

$$D_{spm} = D \quad (10)$$

where

$$Z_{12} = \text{zeros}(m, 1) \quad (11)$$

$$Z_{21} = \text{zeros}(1, m) \quad (12)$$

1.2 Multivariable Control Strategies

Nowadays, several approaches have been proposed to control single input single output (SISO) system [24] such as predictive model, Zeigler Nichol, Cohen Coon, pole placement and gain-phase margin method. However, there is still lack of control design techniques for multiple input multiple output (MIMO) systems. Due to the many practical industry are based on MIMO systems, multivariable control design needs to be enhanced. Most of the practical industry used PID controller in control applications [25]. This is due to a simple structure and easiness to implement [7], [26], [27]. Others than that, PID controllers are able to give good stability and high reliability [28].

Previous study, [29] used Davison, Penttinen-Koivo, Maciejowski and a new method proposed by them as a control method. The Davison method which only uses integral action was able to provide good decoupling characteristics at low frequencies. Whereas, Penttinen-Koivo method which uses both integral and proportional action was able to provide good decoupling characteristics at both low and high frequencies. By using Maciejowski method, the plant is diagonalized at a particular bandwidth frequency where it can minimize an interaction around the system. All of these methods require plant step-tests and the determination of plant frequency response at the single frequency. In [30] the control method for PI multivariable controller has been studied. They used open-loop, closed-loop and open-closed loop test to validate the controller structure. They also used Davison and Penttinen-Koivo method to control the applied system.

However, multivariable control techniques that are already available are model-based which is not practical for many applications due to the different configuration and input characteristic. Previous studies only determine the output feedback control at a steady state. In order to improve the effectiveness and credibility of multivariable control technique, the application of multivariable PID control technique to a singularly perturbed system of wastewater treatment plant at a dynamic state was discussed. The methods involved are Davison, Penttinen-Koivo and Maciejowski method.

2.0 WASTEWATER TREATMENT PLANT

Figure 1 show an activated sludge process which fundamentally consists of two components: aerated reactor and settler. In the aerated reactor, microorganisms survive and grow. Air is introduced into the system to create an aerobic environment that meets the needs of the biological community, and to keep the activated sludge properly mixed. The microorganisms uses dissolved oxygen (C) to oxidize the organic material in the wastewater and gain energy through consuming the substrate.

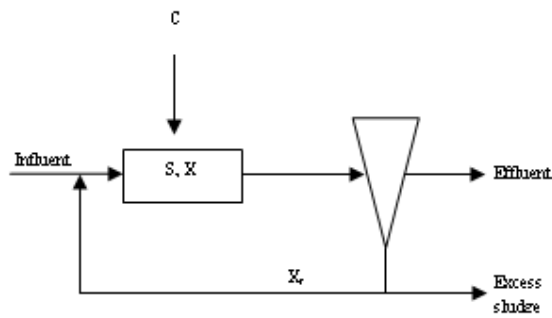


Figure 1 Activated sludge process

Equations (13) until (16) show the nonlinear differential equations which describe the behavior of the activated sludge reactor system.

$$\dot{X}(t) = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad (13)$$

$$\dot{S}(t) = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \quad (14)$$

$$\dot{C}(t) = -\frac{K_o\mu(t)X(t)}{Y} - D(t)(1+r)C(t) + K_{La}(C_s - C(t)) + D(t)C_{in} \quad (15)$$

$$\dot{X}_r(t) = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \quad (16)$$

where

$X(t)$ = Biomass, $S(t)$ = Substrate, $X_r(t)$ = Recycled biomass, $C(t)$ = Dissolved oxygen, $D(t)$ = Dilution rate, $\mu(t)$ = Specific growth rate, S_{in} = Substrate concentrations of influent steams, C_{in} = Dissolved oxygen concentrations of influent steams, C_s = Constant represents maximum dissolved oxygen, Y = Rate of microorganism growth, K_o = Model constant, K_{La} = Constant represents oxygen transfer rate coefficient, r = Ratio of recycled to the influent flow rate, β = Ratio of waste flow to the influent flow rate.

In order to obtain a linear model from the nonlinear model identified in the identification step, operating conditions are identified. The operating conditions for this plant are

Biomass, $X(0) = 217.7896$
 Substrate, $S(0) = 41.2348$
 Dissolved oxygen, $C(0) = 6.1146$
 Recycled biomass, $X_r(0) = 435.5791$

Equations (17) and (18) show the state space model which represented a singularly perturbed system of wastewater treatment plant at the operating point with two considered control outputs which are substrate, S and dissolved oxygen, C . This state space model is obtained according to the equations (1) to (12).

$$\delta\dot{Z} = \begin{bmatrix} 0 & 0.1234 & 0.2897 & 0 \\ -0.05077 & -0.3219 & -0.4457 & 0 \\ -0.02538 & -0.09493 & -1.975 & 0 \\ 0 & 0 & 0 & -0.066 \end{bmatrix} \begin{bmatrix} X \\ S \\ C \\ X_r \end{bmatrix} + \begin{bmatrix} -87.12 & 0 \\ 134 & 0 \\ -9.083 & 0.06994 \\ 8e-005 & 0 \end{bmatrix} \begin{bmatrix} D \\ W \end{bmatrix} \quad (17)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} S \\ C \end{bmatrix} \quad (18)$$

State space equations in (17) and (18) can be represented in transfer function as shown in equation (19).

$$G(s) = \begin{bmatrix} \frac{134s^2+273.1s+8.869}{s^3+2.297s^2+0.6071s+0.01195} & \frac{-0.03117s-0.001029}{s^3+2.297s^2+0.6071s+0.01195} \\ \frac{-9.083s^2-13.43s-0.1847}{s^3+2.297s^2+0.6071s+0.01195} & \frac{0.06994s^2+0.02251s+0.0004382}{s^3+2.297s^2+0.6071s+0.01195} \end{bmatrix} \quad (19)$$

The eigenvalues of the system are

$$Poles = [-1.9957, -0.0214, -0.2798, -0.0660] \quad (20)$$

Based on a physical viewpoint the fast dynamics correspond to the dissolved oxygen and the slow dynamic correspond to the substrate. Figure 2 shows the magnitude plot for both original and singularly perturbed system. As can be seen the dynamics of the singularly perturbed system is much identical to the original system. This indicated that, singularly perturbation technique did not disturb the stability of the system instead reduced the order of the system.

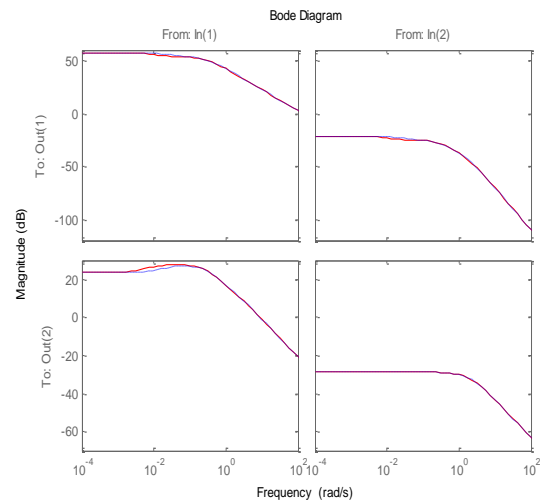


Figure 2 Magnitude bode plot of original system (red) and singularly perturbed system (blue)

3.0 MULTIVARIABLE PID CONTROL TUNING

Controller is implemented in most of process to regulate and deal with the system parameters. Additionally, handle with uncertainties in parameters. Controller implementation affects the performance of the system. It is really important to analyse the stability of the controlled system. Applying singularly perturbation technique to the control design cause the analysis and design of the systems become easier due to the two-time scale properties. The control of a singularly perturbed system comprised of two stages. Firstly, the singularly perturbed system is decomposed into slow and fast model and then designed the composite controller [3]. In nonlinear singularly perturbed system, many approaches have been developed. The approaches used different conditions on the properties of the used functions, different assumptions, different theorem and different lemma [3], [4], [20], [21], [31], [32]. Due to the ability to control multivariable system, Davison, Penttinen-Koivo and Maciejowski method are implemented accordingly.

3.1 Davison Technique

Control and tuning method proposed by Davison for MIMO system only used integral action [29], [30]. The controller expression is represented by equation (21).

$$\underline{u}(s) = k_i \frac{1}{s} \underline{e}(s) \quad (21)$$

where k_i as the integral feedback gain which is defined as in equation (22).

$$k_i = \varepsilon G(0)^{-1} \quad (22)$$

ε is the tuning parameter and $\underline{e}(s)$ is the controller error. Equations (21) and (22) are used to determine the static control. To meet the practical needs, equation (22) are modified as represented by equation (23).

$$k_i = \varepsilon G(s)^{-1} \quad (23)$$

To obtain the optimum control system, tuning parameter, ε can be tuned simultaneously in a range of $0 < \varepsilon \leq 1$. For dynamic control, the best tuning parameter, ε obtained was 0.85. After the process of minimal realization, thus will give the control matrix as equation (24).

$$k_i = \begin{bmatrix} \frac{0.006343s^2 + 0.00204s + 0.00004}{s^2 + 0.03301s} & \frac{0.002827}{s} \\ \frac{0.82379s^2 + 1.2183s + 0.01675}{s^2 + 0.03301s} & \frac{12.1533s + 24.2579}{s} \end{bmatrix} \quad (24)$$

$$(K_p + K_i)_{11} = \frac{0.00209s^5 + 0.0053s^4 + 0.002426s^3 + 0.00035s^2 + 1.165e - 5s + 1.117e - 7}{s^5 + 2.33s^4 + 0.6827s^3 + 0.03198s^2 + 0.0003944s} \quad (28)$$

$$(K_p + K_i)_{12} = \frac{0.0006652s^4 + 0.00155s^3 + 0.0004543s^2 + 2.128e - 5s + 2.624e - 7}{s^5 + 2.33s^4 + 0.6827s^3 + 0.03198s^2 + 2.128e - 5s + 2.624e - 7} \quad (29)$$

$$(K_p + K_i)_{21} = \frac{0.2714s^5 + 0.9126s^4 + 0.8329s^3 + 0.1878s^2 + 0.005848s + 4.71e - 5}{s^5 + 2.33s^4 + 0.6827s^3 + 0.03198s^2 + 0.0003944s} \quad (30)$$

$$(K_p + K_i)_{22} = \frac{4.004s^5 + 15.06s^4 + 16.09s^3 + 4.044s^2 + 0.185s + 0.002262}{s^5 + 2.33s^4 + 0.6827s^3 + 0.03198s^2 + 0.0003944s} \quad (31)$$

3.3 Maciejowski Technique

Maciejowski method combined integral, proportional and derivative action. The controller expression is represented by equation (32)

$$K = \left(K_p + K_i \frac{1}{s} + K_d s \right) \quad (32)$$

where K_i , K_p and K_d are the integral, proportional and derivative gain which are defined as equations (33) until (35). ρ , ε and δ are scalar tuning parameters.

$$K_p = \rho G^{-1}(j\omega_b) \quad (33)$$

$$K_i = \varepsilon G^{-1}(j\omega_b) \quad (34)$$

$$K_d = \delta G^{-1}(j\omega_b) \quad (35)$$

For Maciejowski method, the tuning was done around bandwidth frequency, ω_b . For wastewater treatment plants with implementation of singularly perturbation technique, the bandwidth frequency, ω_b is around $0\text{Hz} < \omega_b < 1130\text{Hz}$. The

3.2 Penttinen-Koivo Technique

Method proposed by Penttinen-Koivo is slightly advanced than Davison method[30]. This method used integral and proportional action to control the system. The controller expression is represented in equation (25)

$$\underline{u}(s) = \left(k_p + k_i \frac{1}{s} \right) \underline{e}(s) \quad (25)$$

where k_i and k_p are integral feedback and proportional gain which represented as

$$k_i = \varepsilon G(0)^{-1} \quad (26)$$

$$k_p = (CB)^{-1}\rho \quad (27)$$

ε and ρ are the tuning parameters and $\underline{e}(s)$ is the controller error. Equation (26) which is similar to equation (22) underwent the same modification and is represented as equation (23). By using Penttinen-Koivo method with some modification, the tuning parameter ε and ρ is set to 0.20 and 0.08 respectively. The resulting control matrix is shown in equations (28) until (31).

simulation gives the best result during $\omega_b = 0.05\text{Hz}$ with $\varepsilon = 0.312$, $\rho = 1$ and $\delta = 0$ respectively. The simulation gives control matrix in complex value. That value was converted to real data and represented as equation (36)

$$K_p + K_i + K_d = \begin{bmatrix} 0.002493 & 0.00433 \\ 1.518 & 37.62 \end{bmatrix} \quad (36)$$

4.0 RESULT AND DISCUSSION

To obtain the results, simulation using Matlab/Simulink was conducted for several tuning parameters. Simulation is based on wastewater treatment plant that is already going through singularly perturbation technique. This technique simplifies the original system while maintaining the system stability, hence reducing the complexity during the analysis and design of the control system. Simulation was done until the best performance was obtained. The simulation was carried out in two stages. First stage is during substrate change, and the second stage is during dissolved oxygen change. For each change, the step input was injected at $t = 10\text{h}$. The substrate is set with respect to the step change from 41.2348 mg/l to 51.2348 mg/l and

dissolved oxygen change from 6.1146 mg/l to 4.1146 mg/l, respectively.

Figure 3 shows the closed loop response and the process interaction of substrate and dissolved oxygen outputs. Figure 3a and Figure 3c are the response during substrate change whereas Figure 3b and Figure 3d are the response during dissolved oxygen change. Based on Figure 3a and 3d, it shows that the measure output: substrate and dissolved oxygen are able to reach the desired point even though variations exist at the set point. It shows that all singularly perturbed MPID controller designs are able to keep the concentration of the substrate and dissolved oxygen close to the desired value. However, due to the control characteristic which only applies integral gain, control action based on Davison method provides a response with the highest percentage of overshoot (%OS). By using Penttinen-Koivo method, the output response shows better improvement. The presence of both integral and proportional gain is able to minimize the %OS and offer a better settling time (T_s). However, proportional gain needs to be tuned wisely. High value of proportional gain can cause the system to become unstable, while small value of proportional gain may reduce the sensitivity of the controller. Compared to the Davison and Penttinen-Koivo method, Maciejowski method gives the best performance with small %OS and faster T_s .

Figure 3b and 3c shows the interaction responses. The response obtained has proven that substrate and dissolved

oxygen are coupled since the step changes in the substrate disturb the dissolved oxygen correspondingly and vice versa. If there is no process interaction, dissolved oxygen should not be affected when the substrate is changed. Fortunately, process interactions were reduced for each controller design where the Maciejowski method provides less interaction.

Figure 4 shows the closed loop responses of inputs: air flow rate and dilution rate where Figure 4a and Figure 4c are the response during substrate change whereas Figure 4b and Figure 4d are the response during dissolved oxygen change.

From the Figure 3 and 4, it clearly shows that control action using Davison method is able to reach the desired point but resulting in high %OS. It gives high oscillation which is not required in control design. The tuning parameter is selected based on the best closed loop response. By using Penttinen-Koivo method, the control action was improved. However, compared to Davison and Penttinen-Koivo method, Maciejowski method gives the best performance. It is proven that frequency analysis can improve closed loop response of the system compared to the uncompensated system. It could reduce the interaction around the system. An important feature in Maciejowski method is the selection of frequency. Frequency must be selected properly because indecorously choosing a frequency can lead to instability.

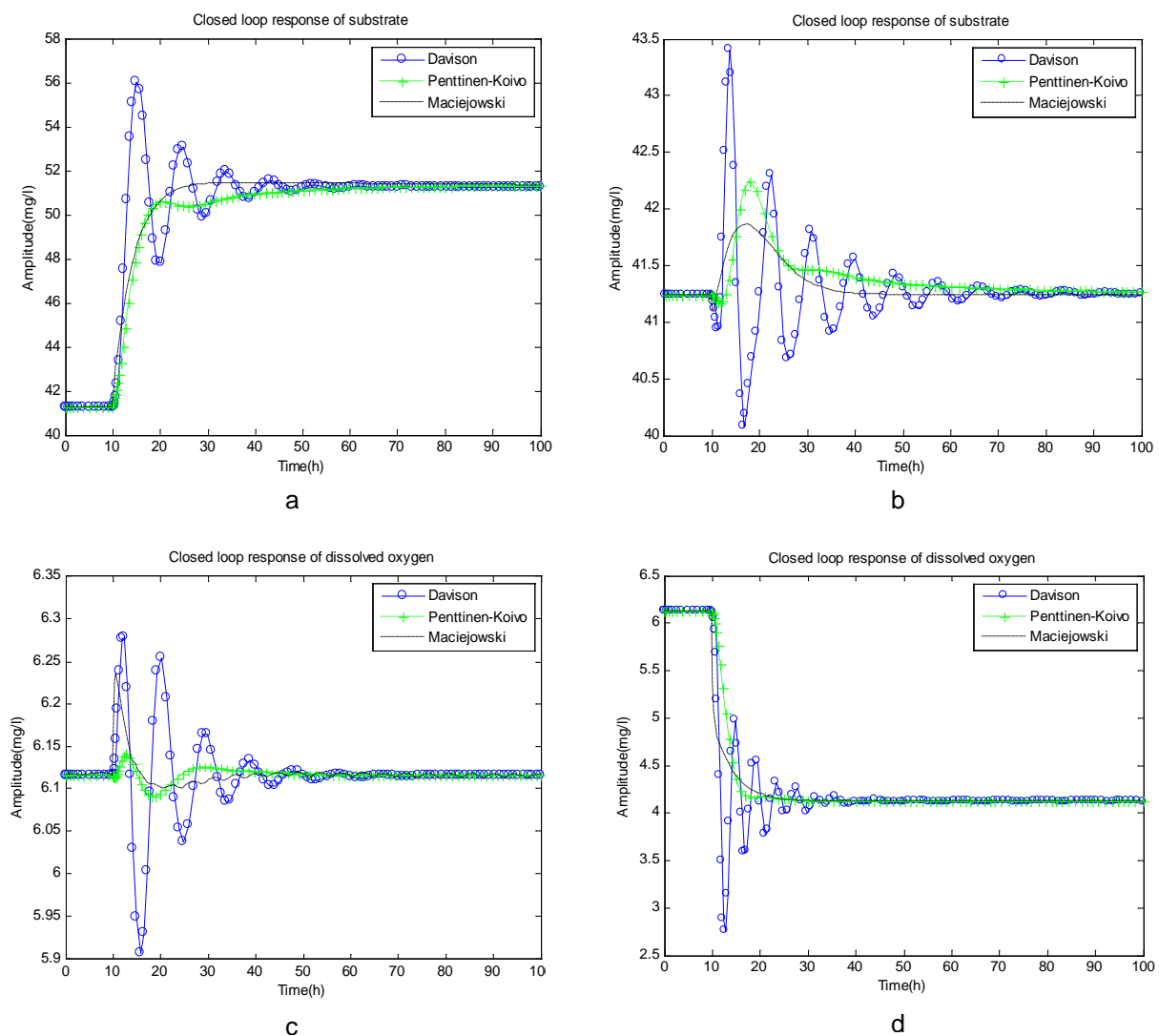


Figure 3 Closed loop responses of substrate and dissolved oxygen

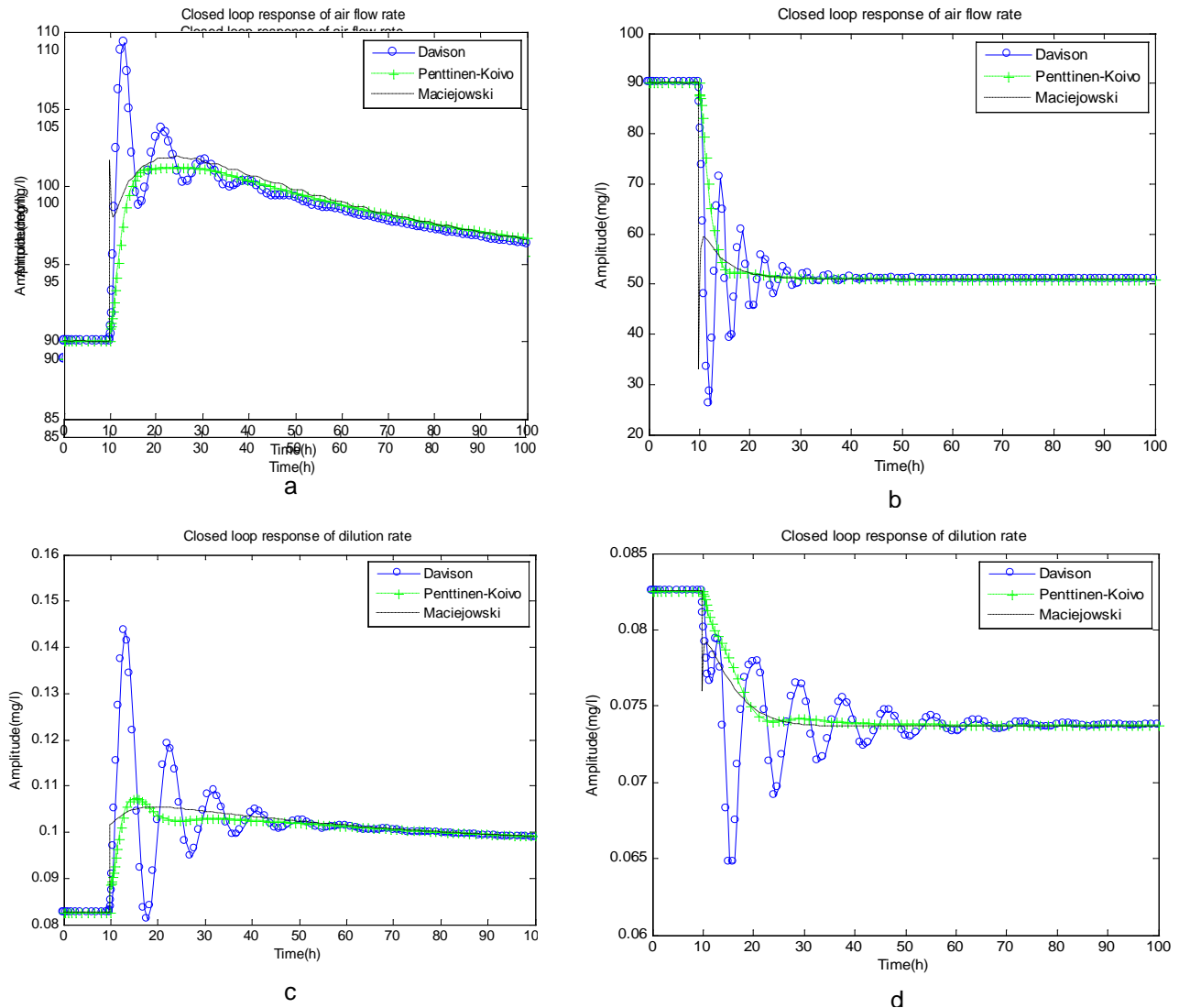


Figure 4 Closed loop responses of air flow rate and dilution rate

5.0 CONCLUSION

Analysis of a singularly perturbed system of wastewater treatment plant using multivariable PID control design was presented. Modification of previous methods enables the proposed approach to obtain dynamic control. Simulation results have shown that the proposed method is able to control dynamic system where the outputs are able to track the set point change and produced less interaction. Based on Davison, Penttinen-Koivo and Maciejowski method, a suitable choice of parameters affects the ability to control the dynamic system gradually.

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