Jurnal Teknologi

Full paper

Review of the Role of Analytical Modelling Methods in Riverbank Filtration System

Shaymaa Mustafa[a], Arifah Bahar[a,b], Zainal Abdul Aziz[a,b*], Saim Suratman[c]

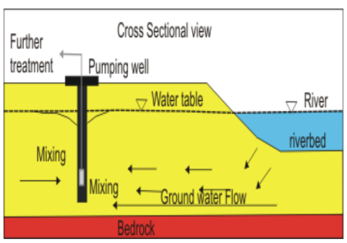
[a]*UTM Centre for Industrial & Applied Mathematics, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia*
[b]*Department of Mathematical Sciences, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia*
[c]*National Hydraulics Research Institute Malaysia (NAHRIM),Lot 5377, Jalan Putra Permai, 43300 Seri Kembangan, Malaysia*

*Corresponding author: zainalabdaziz@gmail.

Article history

Received :6 April 2014
Received in revised form :
4 September 2014
Accepted :15 November 2014

Graphical abstract



Abstract

Riverbank filtration (RBF) technology is applied in several countries around the world as one of the main sources of drinking water supply both from quantitative and qualitative point of view. Consequently, several analytical modelling methods, mostly based on the transformation techniques, are developed in literature to describe different processes which occur in RBF system. An extensive overview of these analytical methods, their uses and limitations are discussed. The review disclosed that most analytical models usually are concerned in evaluating stream depletion rate rather than contaminants transport especially the transportation of pesticides and pathogens. Laplace and Fourier methods are more popular methods used by researchers to solve the system of partial differential equation that developed to simulate the RBF problem.

Keywords: Riverbank filtration; analytical modelling; transformation techniques; contaminant transport; pumping well; groundwater

Abstrak

Teknologi penapisan tebing sungai (RBF) digunakan di beberapa negara seluruh dunia sebagai salah satu sumber utama bekalan air minuman, kedua-duanya daripada pendekatan kuantitatif dan kualitatif. Hasilnya, beberapa kaedah pemodelan matematik yang kebanyakannya berdasarkan kepada kaedah transformasi telah dibangun dalam kajian untuk menerangkan proses berbeza yang berlaku dalam sistem RBF. Gambaran yang luas berkaitan kaedah analitik, kegunaan dan kekurangan telah dibincangkan. Kajian ini mendedahkan bahawa kebanyakan model analitik lebih menekankan kadar penyusutan aliran sungai berbanding pengangkutan pencemar terutamanya pengangkutan racun perosak dan patogen. Kaedah Laplace dan Fourier adalah kaedah yang lebih terkenal digunakan oleh penyelidik untuk menyelesaikan sistem persamaan pembezaan separa yang dibangunkan untuk mensimulasi masalah RBF itu.

Kata kunci: Penapisan tebing sungai; pemodelan analitikal; kaedah transformasi; pengangkutan pencemar; perigi mengepam; air bawah tanah

■1.0 INTRODUCTION

Riverbank filtration (RBF) is a natural technique for surface water treatment, based on the natural removal of pollutants from water during its transfer to the aquifer (Figure 1). This technology is applied in USA and several European countries. The first RBF system was constructed in Glasgow, Scotland in 1810 [1, 2]. Approximately RBF system produces 16% of the drinking water supplies in Germany, 50% in the Slovak Republic, 45% in Hungary, and 5% in the Netherlands [3]. Recently, RBF technique had been applied for the first time in Malaysia in Pilot project conducted in Jenderam Hilir, located in Langat Basin, Selangor, Malaysia [4].Therefore, it becomes essential to develop mathematical models for describing different processes which occur in RBF system.

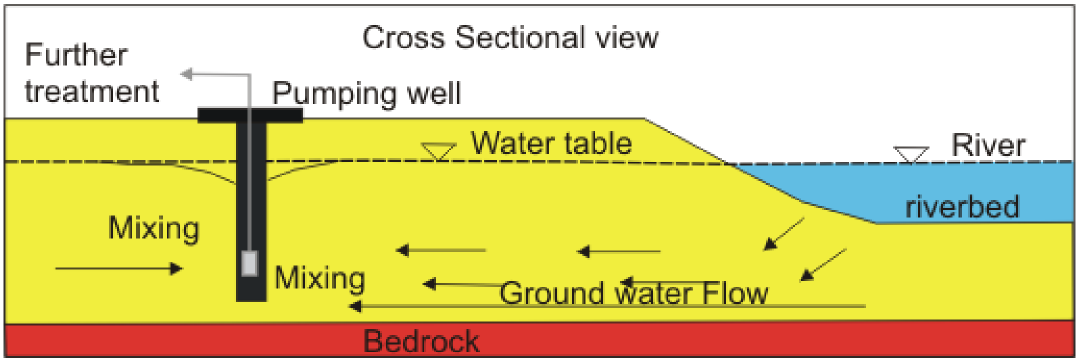


Figure 1 Illustrates a simple river bank filtration system (modified from [5])

One of the main advantages of RBF systems is the improvement in water quality in comparison to direct use of river water for drinking water [6]. RBF systems effectively reduce the concentrations of particulates (suspended solids, turbidity), pathogens, Gardaí, dissolved organic carbon, and many (but not all) organic and inorganic compounds. This technique does not require high degree of technical process so it is less expensive and more efficient. Moreover, it does not has any environmental impact [7]. Depending on the degree of filtering and contaminant concentration in produced water, additional treatment may be provided for the pumped water prior to distribution. As a minimum, RBF acts as a pre-treatment step in drinking water production and, in some instances, can serve as the final treatment just before disinfection. More details concerning the technology of bank filtration can be found in Ray [1]. Figure 2 illustrates the factors influencing RBF system.

Mathematical modelling of RBF had been studied since 1941 [8-20]. Both numerical and analytical approaches are used to solve several problems arisen in RBF system. However, analytical solutions are more useful in quick estimation of concentrations, simplicity of input parameters and free from numerical errors. Also these models are used to verify numerical solutions. The aim of this paper is to summarize the main analytical models that are developed to simulate different problems in RBF systems. This review is divided into four sections. Section1: gives a brief background of RBF technology. Section 2 summarizes the governing equation, initial and boundary conditions and methods of solutions commonly used in RBF systems. Section 3 yields various interesting subjects related to analytical modelling in RBF systems. Finally, Section 4 discusses on some conclusions and recommended future work.

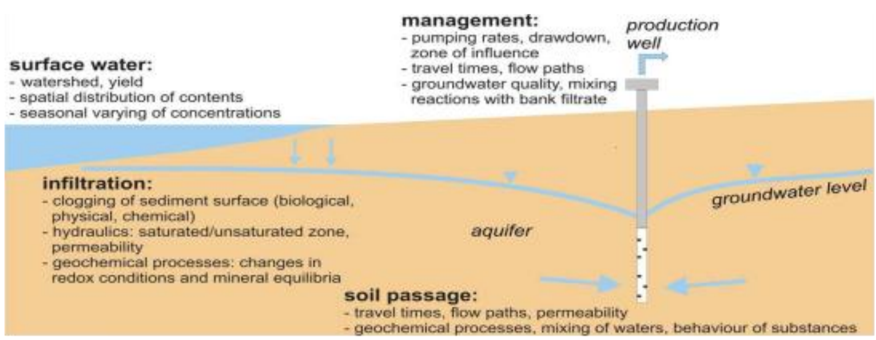


Figure 2 Illustrates the factors affecting RBF systems (from [21])

■2.0 FORMULATION OF THE ANALYTICAL METHODS

RBF process can be governed by a partial differential equation (PDE) or a system of PDEs with suitable initial and boundary conditions. In this section, the main governing equations, initial and boundary conditions and analytical solutions of these governing equations are reviewed.

2.1 Basic Principle and Governing Equations

Two types of equations are used in RBF system. The contaminant transport equation describes the transfer of pollutants from river to well and depends on the change of concentration of compounds with time, and the groundwater flow equation describes the transient flow problems in which the hydraulic head changes with time. Table 1 summarizes the main governing equations and process in RBF technique.

Table 1 Summary of basic governing equations of RBF process

Description of process	Governing Equations \ Main Law	Parameters
Flow: There are four equations usually used to describe water flow in RBF system: 1) Darcy Law: related hydraulic gradient $\frac{\partial h}{\partial s}$ to discharge velocity *q*. 2) A 3D groundwater flow equation in homogeneous and anisotropic aquifer. 3) The flow equation that uses to assess stream depletion by pumping test from the well. 4) For streambeds with small horizontal and vertical dimensions.	1) Darcy Law: $q = -K\frac{\partial h}{\partial s} = -KI$ [22]. 2) $\frac{\partial}{\partial x}K_x\frac{\partial h}{\partial x} + \frac{\partial}{\partial y}K_y\frac{\partial h}{\partial y} + \frac{\partial}{\partial z}K_z\frac{\partial h}{\partial z} = S\frac{\partial h}{\partial t} + R$ [23] 3) $T(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}) = S\frac{\partial h}{\partial t} + w$ where $w = Q\delta(x-l)\delta(y) - \lambda(H-h)\delta(x) - \frac{K_r}{b}(H-h)$ [12, 24, 25]. 4) $\lambda = \frac{K_r w_r}{b}$	q : Darcy velocity K : hydraulic conductivity S : specific storage h : head K_x, K_y and K_z : hydraulic conductivities in *x*, *y* and *z* axes R : source/sink, T : transmissivity Q : constant pumping rate $\delta(x)$: Dirac delta function l : distance between river and wells H : head value at $t = 0$ b : thickness of the streambed λ : stream bed leak coefficient K_r, w_r hydraulic conductivities and width of river
Groundwater drawdown in pumping wells 5) Thies equation [26] for homogenuous confined aquifer. 6) Dupuit-Thiem equation for unconfined flow.	5) $H - h = \frac{Q}{4\pi T}W(u)$ where $u = \frac{(x^2 + y^2)S}{4Tt}$ [25] 6) $\Delta s_d = \frac{2.3Q}{2\pi T}\log_{10}\frac{R}{r_w}$ [27]	x, y are the distances at any time after the start of pumping W : well function R : radius of influence r_w : radius of the main pumping well s_d : water drawdown
Chemical pollutants Transport 7) Mass flux Q and concentration gradient $\frac{\partial C}{\partial s}$ are related by Fick's Law [22]. 8) A 1D pollutants transport under advection dispersion process. 9) With adsorptions. 10) With adsorption and degradation.	7) Fick's Law: $Q_m = \varphi D\frac{\partial C}{\partial s}$ 8) $\frac{\partial C}{\partial t} = D_x\frac{\partial^2 C}{\partial x^2} - U_x\frac{\partial C}{\partial x}$ [28] 9) $\frac{\partial C}{\partial t} = D_x\frac{\partial^2 C}{\partial x^2} - U_x\frac{\partial C}{\partial x} - G$ [9, 29] where $G = K_d\frac{\rho_b}{\varphi}\frac{\partial C}{\partial t}$ 10) $R_d\frac{\partial C}{\partial t} = D_x\frac{\partial^2 C}{\partial x^2} - U_x\frac{\partial C}{\partial x} - kC$ [15] where	Q_m [M/L^{2}T] : mass flux s : distance C : solute concentration D : diffusivity of mass transport U_x, U_y and U_z : Darcy velocities in *x*, *y*, and *z* axis D_x, D_y and D_z : Diffusivity in *x*, *y* and *z* directions G [M/L^3/T] : source\sink term ρ_b : bulk density K_d : distribution coefficient φ : effective porosity k : degradation rate.

11) A 2D contaminant transport with degradation and adsorption 12) A 3D contaminant transport with degradation and adsorption	$R_d=(1+K_d\frac{\rho_b}{\varphi})$ 11) $R_d\frac{\partial C}{\partial t}=D_x\frac{\partial^2 C}{\partial x^2}+D_y\frac{\partial^2 C}{\partial y^2}-U_x\frac{\partial C}{\partial x}-U_y\frac{\partial C}{\partial y}-kC$ 12) $R_d\frac{\partial C}{\partial t}=D_x\frac{\partial^2 C}{\partial x^2}+D_y\frac{\partial^2 C}{\partial y^2}+D_z\frac{\partial^2 C}{\partial z^2}-U_x\frac{\partial C}{\partial x}-U_y\frac{\partial C}{\partial y}$ $-U_z\frac{\partial C}{\partial z}-kC$	R_d : retardation factor
Microorganisms Transport: 13) A 1D Transport under Advection dispersion only under kinetic attachment and detachment process. 14) Classical colloid filtration theory (CFT) is common approach in RBF [30]: In this process the K_{att} coefficient influenced by sticking α and collision η_0 factors which they depend on particle velocity and properties. **15)** Inactivation or death of microbes during transport and is commonly described by a first-order rate expression [31].	13) $\frac{\partial C}{\partial t}=D_x\frac{\partial^2 C}{\partial x^2}-(U_x+U_s)\frac{\partial C}{\partial x}$ [32] where $U_s=\frac{b+.67(\rho_s-\rho)gd_s^2}{b+(\frac{.98}{\varepsilon})18\mu}$ [33]. 14) $\frac{\rho b}{\varphi}\frac{\partial S}{\partial t}=K_{att}C-\frac{\rho b}{\varphi}K_{\det}S$ [34]. 15) $K_{att}=\frac{3(1-\varphi)U_x}{2d_c}\eta_0\alpha$ [34].	U_s : settling velocity of bacteria ρ_s : density of bacteria ρ : water density d_s diameter of bacteria b : ratio of average free sedimentation segment length to grain radius($b\approx 1$) ε : empirical correction factor considering the influences of grain surfaces ($0\le\varepsilon\le 1$) K_{att} microbial attachment rate $K_{\det}$ microbial detachment rate α and η_0 are sticking and collision factors respectively. d_c is the average grain size

2.2 Initial and Boundary Conditions:

Selecting an appropriate boundary and initial conditions for RBF system is an important issue for the accuracy of the solution. The initial condition is determined at $t=0$. Boundary conditions for RBF problems are of three types, Dirichlet, Neumann or Robin boundary conditions. Dirichlet boundary condition refers to constant boundary conditions which can be expressed mathematically as:

$$C(x,y,z,t)=f_1(x,y,z,t)$$

and

$$h(x,y,z,t)=g_1(x,y,z,t)$$

In RBF system, the concentration\head of the surface water body, e.g., river, lake, or sea could be treated as Dirichlet boundary conditions [29, 35, 36]. Neumann boundary condition specifies the value for flux of mass Q_n or water q_n normal to the boundary surface. Mathematically, these conditions can be expressed as [23, 29]:

$$Q_n(x,y,z,t)=f_2(x,y,z,t)$$

and

$$q_n(x,y,z,t)=g_2(x,y,z,t)$$

Neumann boundary condition can be used for aquifer and wells in RBF system [37]. Robin boundary condition is a specification of linear combination of Dirichlet and Neumann boundary conditions and can be mathematically expressed as [23, 29]

$$aC(x,y,z,t)+bQ_n(x,y,z,t)=f(x,y,z,t)$$

and,

$$ah(x,y,z,t)+bq_n(x,y,z,t)=g(x,y,z,t).$$

2.3 Analytical Methods

Analytical solutions are commonly used to verify numerical solutions because of its accuracy and simplicity. Generally analytical solutions require numerical integration and satisfy boundary value problems. Some of the exact solutions should be left in terms of integrals or infinite series. But some others do not. Laplace transform, Fourier Transform, Hankel Transform and Green's function are more commonly used to get the exact solution for RBF process which offers an easy and effective way in solving a variety of problems in RBF. In this section, all analytical approaches mentioned will be discussed in details.

2.3.1 Laplace Transformation

Laplace transform is often interpreted as a transformation from the time-domain, in which inputs and outputs are functions of time [38]. Many researches are conducted using Laplace transformation for problems in RBF. Laplace transforms are implemented by [19, 20] to present semi-analytic solutions for three-dimensional flow to horizontal wells in unconfined and leaky aquifers. Connell [39] presented a series of semi-analytical and analytical solutions to describe the movement of contaminants released from a surface vertically through the unsaturated zone to the water table and then within groundwater. The unsaturated zone transport is described by three different approaches that differ in how the plant root zone is represented. Since the model based on the assumption that no transverse dispersion of solute as it moves through the unsaturated zone, then the solution worked best for shallow water tables or for long times in deeper water tables. Huang [35] used Laplace transform to describe the head distribution in unconfined aquifer with single pumped horizontal well parallel to fully penetrating stream. The solution can simulate head distributions in confined aquifer and in an aquifer infinitely extending in horizontal direction if the well is located far away from the stream. Laplace transformation is used to solve advection-diffusion model equation to describe solute transfer in aquifer [29] and to measure the efficiency of RBF system in Cryptosporidium oocysts removal [30]. Chen [40] developed a novel method for solving multi-species advective–dispersive transport equations sequentially coupled with first-order decay reactions analytically. The method first performs Laplace transform with respect to time and the generalized integral transform technique with respect to the spatial coordinate to convert the set of partial differential equations into a system of algebraic equations. Subsequently, the system of algebraic equations is solved using simple algebraic manipulation, thus the concentrations in the transformed domain for each species can be independently obtained. Singh [28] produced analytical solution for the contaminant concentration in one-dimensional uniform groundwater flow in a homogenous semi-infinite porous aquifer. The model is adapted for space-time variation of contaminants concentration and time-dependent source contamination. The solution is investigated under two expressions for the temporally dependent dispersion distribution in the aquifer: sinusoidal varying function and exponentially increasing function of time. It is assumed that initially the aquifer is not solute free; i.e., the aquifer is not clean and the initial concentration is an exponentially decreasing function of the space variable and is tending to zero toward infinity. Also, the concept that dispersion is directly proportional to the seepage velocity is employed.

2.3.2 Fourier Transformation

Fourier transformation is widely used in many applications in physics and engineering. The existence of Fourier transformation supposes $f(x)$ goes to zero when $|x|$ goes to infinity [41]. Allan and Elnajjar [36] used 2D convection-dispersion transport model to simulate the concentration distribution of pollutants in ground water. The applied equation is governed by several parameters including water velocity, permeability and diffusion rate and solved by using 2D Fourier transformation. The results obtained showed that it will take a very long time before the contaminated material disappears. Also, the authors indicated that as time goes on, the contaminated material will continue to spread over a larger area but with less concentration. Chen [42] presented an analytical model for describing the three-dimensional contaminant transport from an area source in a radial flow field in finite thickness aquifer. Series technique, Laplace transform and the two finite Fourier cosine transforms are used to solve the analytical model. This model is an efficient tool for describing nonuniform flow contaminant transport. Singh [43] conducted a comparative study between Laplace Transform Technique (LTT) and Fourier Transform Technique (FTT) in solving one-dimensional transport equation along unsteady groundwater flow in semi-infinite aquifer. The concentration at contaminants source is considered at the origin and goes to zero at the end of the aquifer. The aquifer is considered homogenous, semi-infinite and has initial uniform contaminants concentration and the velocity is considered to change with time. The result indicates that FTT is better than LTT in predicting contaminants concentration along groundwater flow in semi homogenous aquifer.

Many analytic solutions of RBF problems are based on a combination of Laplace and Fourier transforms where the primary difficulty lies in the analytic back transformation. Fourier and Laplace transformations are applied together by researchers [11, 24, 44], to estimate the stream depletion rate induced from the pumping well hydraulically connected to finite width stream. The head of stream is treated as constant in their analytical models. Intaraprasong and Zhan [45] developed their solution by using Fourier and Laplace transformations along with variable stream stage represented by a periodic function for seasonal variations or a function changed in space and time for flood wave. An overview of these solutions is given in Hunt [16].

2.3.3 Bessel's Functions and Hankel Transformation

Hankel transform has been found to be the most appropriate for problems formulated with equations expressed in polar and cylindrical coordinates. Both of Bessel function expansion and Hankel transforms are used to simulate a two-dimensional advection contaminant transport equation in cylindrical coordinate [42, 46, 47].

2.3.4 Green Functions

Solution of partial differential equations can be expressed in terms of an integral with a fundamental function, called Green's function, which is a part of a closed-form solution for linear models and it represents the solution as result of the influences of a system to a unit point load or source. Park and Zhan [48] developed an analytical model to solve 3D Advection Dispersion contaminant transport equations in finite thickness aquifer using Green's function. The result indicates that the upper and lower aquifer boundaries affect concentration distribution. Also, it is shown that the source geometry effects on the concentration at near field while it does not affect on the concentration at far field. These analytical solutions assumed simplified aquifer geometry with no slope, a uniform one-dimensional groundwater flow and mass constant dispersion coefficients at all scales.

■3.0 SUBJECTS IN MATHEMATICAL MODELING OF RIVERBANK FILTRATION SYSTEM

Due to increasing interest of RBF technique as main source of high quality drinking water supply, various subjects concerning this approach attract scientists and engineers attentions. In this section, the main processes in RBF system that have been studied by analytical modelling methods are produced.

3.1 Groundwater Flow in RBF System

Most of analytical modelling studies on groundwater flow of RBF systems concern on the drawdown of water due to pumping test and the ratio of river water in production wells. Several analytical models have been widely used to describe stream depletion rate induced from vertical, horizontal wells and collector wells.

3.1.1 Vertical Wells:

Theis [8] was the first person to introduce a model for a pumping well which fully penetrates an unconfined aquifer near a stream. Theis conceptual model assumes no resistance (conductance) between the stream and the aquifer. Jenkins [10] applied the solution developed by Theis in water management problems. Such models assume a full hydraulic connection between the stream and the aquifer and do not consider the effect of shallow aquifer penetrating by a stream. This led to a development of this simple model by Hunt [11] considering narrow and shallow stream. Another analytical solutions have been developed for stream and aquifer with finite width by Hunt [16] and for 3 layered system with a well placed in the upper, unconfined aquifer by Christensen [49]. Zlotnik and Tartakovsky [12] used an approach similar to Hunt [11], neglecting the stream width, to investigate stream depletion problem in an aquifer with leakage from the bed. Another phenomena that have been analyzed analytically include pumping from a well in unconfined/confined aquifers for both line-width and finite-width streams [25]. The following Poisson's equation was used by Bakker [50] to model 2D steady state flow to wells in horizontal plane located in strip aquifer at RBF site:

$$\nabla^2 \varphi = -N_r ,$$

where N_r is the real recharge to the aquifer and considered as a constant. Fourier cosine series:

$$\varphi = \sum_{n=1}^{N} a_n \cos(n\pi X) e^{-n\pi Y}$$

is implemented in the solution. This article focuses on the hydraulic aspects of RBF system than on the water quality aspects. The boundary of both river and strip aquifers are taken to be curved and the variation of water levels in river was taken in consideration in this model. The model can be used to compute the fraction between the river and the strip aquifers.

3.1.2 Horizontal and Collector Wells

Recently, many mathematical modeling studies concern in horizontal systems (collector wells (RC), infiltration galleries, horizontally drilled wells) where these become more popularly used in RBF systems due to its larger capacity and its suitability for installing in thickness aquifer. Hantush and Papadopulos [51] presented analytical solutions to simulate drawdown distribution around RC. They assumed that the RC is discharged steadily from homogenous confined and unconfined aquifers with uniform hydraulic conductivity. In order to approximate the drawdown in RC, another analytical solutions of flow to RC was developed by Debrine [52] and a uniform flux distribution along the laterals was claimed. The results of this model study agreed rather well with the solutions of Hantush and Papadopulos [51]. It is concluded that these solutions give a good describe for the drawdown in a RC well. A new multilayer approach was presented by Bakker [14] for the modeling of ground water flow to radial collector wells. The horizontal groundwater flow inside one lateral was treated analytically. Anderson [53] developed an explicit analytic solution to represent a 2D flow groundwater to horizontal well in unconfined aquifer in riverbank system. In contract with negative and positive well discharges which have an exactly one solution, his result indicate that the well discharges between zero and critical discharge has two alternative solutions. The first lies on single valued physical planed but the other lies on multivalued physical plane. The study shows that the stability of free space is produced when the head is maintained at the top of the well screen. Generally, it is difficult to simulate the drawdown of water inside collector wells analytically due to its complex radial flow [54].

3.2 Efficiency of RBF System

Assessment of raw water quality plays an important role in planning and operating a RBF well field. Many variables affect the quality of water produced in RBF system such as: site hydrogeology, well type and location, biogeochemical reactions, stream depletion rate and source water quality. The evaluation of RBF systems based on the quality of water produced and the efficacy of the natural treatment (contaminant removal). Changes in water chemistry according natural process in the first few meters in RBF and the thickness of the biological layer formed in riverbed-aquifer interface are important elements in filtration process. If this layer is too thick it will impact the system performance by reducing the infiltration rate [7]. Moreover, if the compounds are poorly degradable or the travelling time from surface water to wells is too short then contaminants can reach drinking water wells with high concentration affecting the quality of water [15]. Thus different analytical solutions are provided to estimate the best transient time to get water that achieves quality requirements [14, 55, 56]. Advection - dispersion transport model usually used to simulate the contaminant transportation from river to wells. In addition the highly confining layers such as clay tills are common in many RBF systems around the world (e.g. Canada, northern Europe and the northern United States).The seasonal variation of water due to temperature changes influence the groundwater velocity and stream depletion rate. Kumar and Kumar [17] generated numerical and analytical solutions for contaminants transport in unsteady groundwater flow, considering the seasonal variation of temperature. Recently the effect of seasonal variation is studied analytically by Singh [28]. Dillon [13] examined the best locations for the well in RBF site established in brackish aquifers of semi-arid land such that the salinity of drinking water supply is acceptable and in the same time the removal of cyanobacteria toxins is adequate. Two main parameters are considered in this model: minimum travel time between the river and the wells and the stream depletion rate. For simple flow geometry the following analytical expression is presented

$$t_{\min} = \frac{2\pi b\varphi a^2}{3Q},$$

where b is the average saturated thickness of the aquifer, φ is the effective porosity of aquifer, a is the distance between well and river, and Q is the discharge rate of the pumping well and

$$\frac{q}{Q} = erfc\left(\frac{a}{\sqrt{4\alpha t}}\right),$$

where q is the rate of induced infiltration from the river, α is the aquifer diffusivity and t is the time since the beginning of pumping.

The maximum concentration of contaminants in the well under adsorption and degradation process is given by the simple equation

$$\frac{C_w}{C_r} = \frac{q}{Q} e^{-\lambda R t_{\min}}$$

where C_w is the concentration of pollutant in the well, C_r the concentration of pollutant in the river, λ is the decay factor and R is retardation factor.

3.3 Contaminants Transport in RBF Aquifer

In fact, two types of contaminants can be found in groundwater: microorganism and chemical pollutants. For chemical contaminants, most of RBF studies concern on nitrate compound, dissolved organic carbon (DOC) and total organic compound (TOC) e.g.[57-60]. Analytical modelling for contaminants transport in RBF system are rare and few. Advection- dispersion transport equation usually used to simulate the contaminant transport process. Singh [43] constructed one-dimensional transport analytic model to simulate contaminants transport from rivers along unsteady groundwater flow in semi-infinite aquifer. Generally, mathematical modelling of contaminants transport in RBF system is similar to those conducted to simulate the fate and transport of contaminants in porous media under adsorption and degradation process. Table 2 summarizes some models constructed to simulate solute transport in aquifer for cases that have some similarity with RBF. Also, the table illustrates the similarities and differences between these cases and RBF system.

Table 2 Summary of the similarities and differences between analytical models for the fate and transport of contaminants in aquifer and RBF systems

No	References	Purpose	Similarity with RBF	Differences with RBF
1	[36] [61, 62]	Presented mathematical models to simulate pollutants transport contaminants due to tank leak from waste disposal site, industrial wastewater and domestic wastewater.	The model includes migration and natural attenuation of pollutants into soil.	Usually RBF aquifer is confined and both of the seepage velocity and initial concentration of river dependent on time due to its dynamic flow.
2	[39]	Developed an integrated modelling approach to estimate fate and transport of contaminants in the subsurface system.	The solute is assumed to be moved vertically in 1D advection–dispersion equation in unsaturated zone, and then coupled to the 3D advection–dispersion equation in saturated zone.	Usually, vertical flow of contaminants in RBF system is neglected in most sides [63].
3	[48, 64-66]	Developed analytical model for contaminant transport from one- two- and three- finite sources in finite thickness aquifer using Green's function method.	The contaminants transported in aquifer under advection-dispersion process with degradation process.	The source geometry is finite and has constant boundary conditions while the river has finite width and deep and infinite long with dynamic velocity.
4	[42]	Describing the 3D contaminant transport from an area source in a radial flow field in finite thickness aquifer.	The model includes transport process under advection-dispersion process.	The model does not include adsorption and degradation process. The model accounts for non-uniform flow.
5	[47] [28]	Developed an analytical model to simulate pollutants transportation in homogenous aquifer with time dependence source.	The model accounts for time dependent source. Velocity and dispersion are assumed to vary with time.	The model does not include adsorption and degradation process.

4.0 DISCUSSION

Several studies conducted from 2001 until now for one, two and three dimensional equations dealt with several assumptions. For example, some of these researches consider steady or unsteady flow and others consider homogenous aquifers. Also, confined and unconfined aquifers, besides the leaky aquifer are discussed in these studies. For contaminant transport, some studies consider zero initial condition of contaminants in the aquifer and others considered it dependent with time. Most of these studies, except for few studies, assumed constant concentration of pollutants in the river. In Table 3, a review of different assumptions correlates to the aquifer, river, well and water flow is carried out against the articles of riverbank filtration that has published from year 2001 to 2013.

Table 3 Summary assumption assumed in several of different researches from 2001 to 2013

			Park and Zhan [48]	Dillon [13]	Fox [24]	Kim [5]	Zhan and Park [20]	Bakker [14]	Massabó [46]	Zlotnik and Tartakovsky [12]	Hunt [16]	Chesnaux and Allen [55]	Hunt [44]	Intaraprasong and Zhan [45]	Christensen [49]	Faulkner [30]	Chen [67]	Chen [42]	Singh [47]	Bakker [50]	Wang [68]	Singh [43]	Huang [35]	Baalousha [25]	Singh [28]	Chen [40]	Anderson [53]
Purpose	Contaminant transport		×	×		×			×							×	×	×	×		×	×			×	×	
	Drawdown/Stream depletion rate			×	×		×	×		×	×	×	×	×	×					×			×	×			×
Aquifer	Homogenous-anisotropic		×			×	×	×	×	×	×	×			×		×		×	×		×	×	×	×		
	Nonhomogeneous																										
	Confined						×							×						×				×			
	Unconfined			×				×			×	×	×				×			×			×	×			×
	Leaky aquifer						×			×			×		×												
	Strip aquifer																			×							
	Initially free of concentration		×	×		×			×							×	×	×	×							×	
	Initially non free of contaminants								×								×				×	×			×		
	Horizontally infinite		×	×			×			×			×	×	×						×	×	×	×	×		×
	Horizontally finite				×						×	×						×	×							×	
	Vertically infinite						×		×																		×
	Vertically finite		×	×	×					×	×	×	×	×	×		×	×	×	×			×				
River/ Lake/ bond	Constant Source concentration		×	×		×										×		×			×					×	
	Time dependent source concentration								×								×		×			×			×		
	Fully penetrating on the Stream			×						×				×		×		×		×			×				
	Partially penetrating on the stream				×								×											×			
	Finite width stream		×		×						×									×				×			×
	Line stream									×			×		×								×	×			
Well	Well Type	Horizontal					×																×				×
		Vertical		×	×					×			×	×	×			×		×				×			
		Collector well						×																			×
	Well location	Infinite/semi-infinite distance			×		×	×																			
		Near the river		×						×	×		×	×	×					×			×	×			×
	Pumping rate	Constant		×	×		×			×	×		×	×	×			×		×			×				×
		Dependent on time																									
Water flow	Steady flow		×	×	×	×	×	×	×	×	×	×		×	×	×				×	×		×	×	×	×	×
	Non steady flow																	×	×			×					
Equation	One dimensional			×		×						×				×	×				×	×			×	×	
	Two dimensional				×					×	×		×	×	×				×	×				×			×
	Three dimensional		×				×	×	×									×					×				

Based on Table 3 we can divide the analytical models developed in literature in two main groups according to its purpose. The first one is related to water flow and stream depletion rate while the second one related to attenuation of contaminants. Also, different assumptions adopted in this model can be divided into five categories: Assumptions related to (i) aquifer, (ii) river/lake or bonds, (iii) wells, (iv) water flow and (vi) equations.

Aquifers

Although most kinds of aquifer are studied analytically in literature, there are some types of aquifer needs further researches. Basing on Table 3 we can conclude the following remarks

1- All studies concerned in homogenous aquifer where it is noticed that there is no analytical model conducted for nonhomogenous one.
2- Confined and unconfined aquifer, besides the leaky and multiplayer aquifers have a satisfactory number of

analytical models. However only one study conducted for water flow in strip aquifer (water flow in horizontal plane)

3- Most studies consider zero initial concentration in aquifers to simplify the calculations. However in several cases it is more realistic to consider non free contaminant aquifer.

4- Finite width aquifer that is horizontally extended is more common in various study rather than horizontally finite or vertically infinite.

River

River is the source of pollutants and filtered water in RBF systems. Most of bacteria accumulated in riverbed sediment where most of degradation process is occurred [9]. From Table 3, it is found that rivers are dealt as both constant head/concentration boundary or time dependent boundaries, finite width or line source, and fully penetrating or partially penetrating into aquifer, however, fully penetrating is more abundant.

Wells

From Table 3, vertical wells are discussed analytically in detail not only from 2001 but since 1941 [26]. In the last decades, horizontal well began to be taken in consideration especially horizontal well in one lateral. Analytical models for collector wells are few due to its complex flow. Also it is noticed that all studies assumed constant rate of pumping well rather that pumping rate dependent on time.

Water Flow and Equations

One, two and three dimensional water flow/contaminants equations under steady state conditions are solved analytically enough in literature as seen in Table 3. However, few studies only consider unsteady case and most of them conducted for contaminant transport rather than water flow.

■5.0 CONCLUSION AND FUTURE RECOMMENDATIONS

The main objective of this review is to summarize the previous analytical modelling studies on RBF system. From this study we can divide the mathematical models in RBF systems into two main groups: the first group involved several and various models that describe the flow of groundwater induced from pumping wells near the river. Most of these models concern in determining the stream depletion rate and drawdown of water in the wells. Second group includes all mathematical models that concerned in pollutants transportation from river to wells. Such models are few and usually researchers use models that simulate the solute transport in porous media to deal with RBF case despite of the differences of the source kind. The lack of modelling efforts is partly explained by the difficulty of checking the models with real experimental data values.

To the best of our knowledge, there is no analytical model being conducted to simulate the potential of microorganisms in RBF system and furthermore there are few numerical models. The limitations of mathematical modelling for pathogens transport are reviewed by Tufenkji [34].

Most studies consider initial conditions to be zero and constant boundary conditions. Few researches investigate on nonzero initial conditions [68] or boundary conditions dependent on time [28]. Moreover, there are few researches being conducted to study the influence of pumping wells and stream depletion rate on contaminants degradation and adsorption.

Furthermore, only one study exists [50] that deals with river as curved boundary in simulating the groundwater flow. However, this study focuses on the hydraulic aspects of RBF system than on the water quality aspects.

Generally, the analytical models of RBF system are still few and rare and further future research is needed.

Acknowledgement

The research is partly funded by UTM- CIAM flagship grant, vot no. Q.J130000.2409.02G00 and Q.J130000.2426.01G22 Shaymaa is thankful to UTM for the PhD scholarship.

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