

MIXED CONVECTION BOUNDARY LAYER FLOW OVER A VERTICAL PERMEABLE PLATE IN A POROUS MEDIUM: OPPOSING FLOW CASE

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Abstract. The opposing flow case of steady mixed convection boundary layer flow over a vertical permeable surface in a porous medium with uniform thermal wall conditions, i.e. when both the wall temperature and the wall heat flux are constants, is considered. The Rayleigh number for the porous medium is assumed to be large so that the boundary layer approximation can be considered. Under these assumptions, it is found that the problem depends both on the mass transfer parameters and the mixed convection parameters. It is shown that the governing equations can be reduced to a similarity form, i.e. to ordinary differential equations. These equations are solved numerically using a very efficient implicit finite difference scheme known as the Keller-box method. Numerical results for the reduced skin friction and non-dimensional wall temperature are presented and discussed in detail for various values of the parameters of interest. An important finding of this problem is that there are dual solutions of the governing boundary layer equations and the separation of boundary layer occurs for certain cases.

Keywords: Mixed convection, boundary layer, permeable plate, porous medium, opposing flow

Abstrak. Kajian aliran menentang bagi aliran lapisan sempadan olakan campuran mantap terhadap permukaan menegak telap dalam medium berliang dengan syarat-syarat permukaan terma yang seragam, iaitu apabila kedua-dua suhu dan fluks haba dinding adalah malar, dipertimbangkan. Nilai nombor Rayleigh bagi medium berliang diandaikan besar supaya penghampiran lapisan sempadan boleh dipertimbangkan. Dengan andaian ini, didapati masalah ini bergantung pada parameter pemindahan jisim dan parameter olakan campuran. Seterusnya, ditunjukkan bahawa persamaan-persamaan menakluk boleh diturunkan kepada bentuk keserupaan, iaitu bentuk persamaan pembezaan biasa. Persamaan-persamaan tersebut diselesaikan secara berangka menggunakan suatu skim beza sehingga tersirat yang efisien, dikenali sebagai kaedah kotak Keller. Keputusan berangka bagi geseran kulit terturun dan suhu dinding tanpa matra dibincangkan bagi nilai-nilai parameter yang dipertimbangkan. Penemuan penting dalam masalah ini adalah terdapatnya penyelesaian dual bagi persamaan menakluk lapisan sempadan dan pemisahan lapisan sempadan berlaku untuk beberapa kes.

Kata kunci: Olakan campuran, lapisan sempadan, plat telap, medium berliang, aliran menentang

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1.0 INTRODUCTION

Fluid flow and heat transfer in porous media have been of considerable interest, especially in the last several decades. This is primarily because of the numerous applications of flow through porous media, such as storage of radioactive nuclear waste materials, transpiration cooling, separation processes in chemical industries, filtration, transport processes in aquifers, groundwater pollution, etc. Theories and experiments of thermal convection in porous media, and the state-of-the-art reviews, with special emphasis on practical applications have been presented in the recent books by [1-7].

The motivation for the present study is the fact that both free and forced convection exist simultaneously in many of the practical applications mentioned above. This is particularly relevant in situations where the Rayleigh numbers are large, as in the extraction of crude oil, when free convection effects can become very important. Buoyancy effects induced by a heated body can cause significant changes in the fluid flow and heat transfer mechanism when compared to the basic forced convection flow. In contrast to a forced convection boundary layer flow where buoyancy effects are negligible, a mixed convection flow is influenced considerably by buoyancy effects, but they are not the only driving force of the flow [8]. In the case of a vertical or inclined plate, there is a non-vanishing component of the buoyancy force tangential to the plate affecting the boundary layer flow. This is called direct mixed convection [9].

It is well-known that injection or withdrawal (suction) of fluid through a surface, as in mass transfer cooling, can significantly modify the flow field and affect the rate of heat transfer in forced or free convection [10]. Cheng [11] was the first to consider the effects that the lateral injection or suction of fluid through the wall has on the free convection boundary layer over a vertical permeable surface embedded in a porous medium when the wall temperature varies as x^m while the transpiration velocity varies as $x^{(m-1)/2}$, where m is a constant. Thus, he obtained similarity solutions for this problem. Cheng [11] applied the results obtained to the problem of the convective movement of water discharged from a geothermal power plant into groundwater of a different temperature and in the natural recharging of an aquifer by groundwater of a different temperature. The problem was considered by Merkin [12] in the case when the lateral velocity of injection or withdrawal is constant. Further, Chaudhary *et al.* [13, 14] considered similarity solutions for free convection boundary-layer flow along a vertical permeable surface in a porous medium for both variable wall temperature and variable wall heat flux. Recently, Magyari and Keller [15] presented closed form analytic solutions for the problem of free convection boundary layer on a heated permeable vertical flat plate embedded in a porous medium with the wall temperature distribution of the form considered by Cheng [11]. Exact analytical solutions for the cases $m = 1, -1/3$ and $-1/2$ were given in [11] and the characteristics of the corresponding boundary layers are discussed in detail as functions of the suction or injection parameter.

Cheng [16] was the first to study the mixed convection boundary layer flow about a vertical impermeable surface in a fluid-saturated porous medium when the surface is held at a constant temperature different from that of the ambient fluid. Later, Merkin [17] considered this problem for both cases of a surface prescribed with a constant wall temperature (CWT) and a constant wall heat flux (CHF). In addition, Merkin [18] studied dual solutions occurring in the problem of mixed convection flow over a vertical surface in a porous medium with constant surface temperature for the case of opposing flow. The aim of the present paper is to study the opposing flow case of the steady mixed convection boundary layer flow over a vertical permeable surface in a porous medium for the cases of constant wall temperature (CWT) and constant wall heat flux (CHF). To our best knowledge, this particular problem has not been studied before.

2.0 BASIC EQUATIONS

Consider the steady mixed convection flow over a semi-infinite vertical permeable flat plate, which is embedded in a fluid saturated porous medium and maintained at constant temperature T_w or constant heat flux q_w . The plate is aligned parallel to a free stream with velocity $u_e(x)$ and temperature T_∞ and the suction or injection rate of the mass flow is $v_w(x)$ as shown in Figures 1(a) and 1(b).

It is assumed that the temperature of the fluid is everywhere below the boiling point, the convective fluid and the porous medium are in local thermodynamic equilibrium, the viscous dissipation is neglected, the physical properties of the fluid except the density are constant and that the Boussinesq and boundary layer

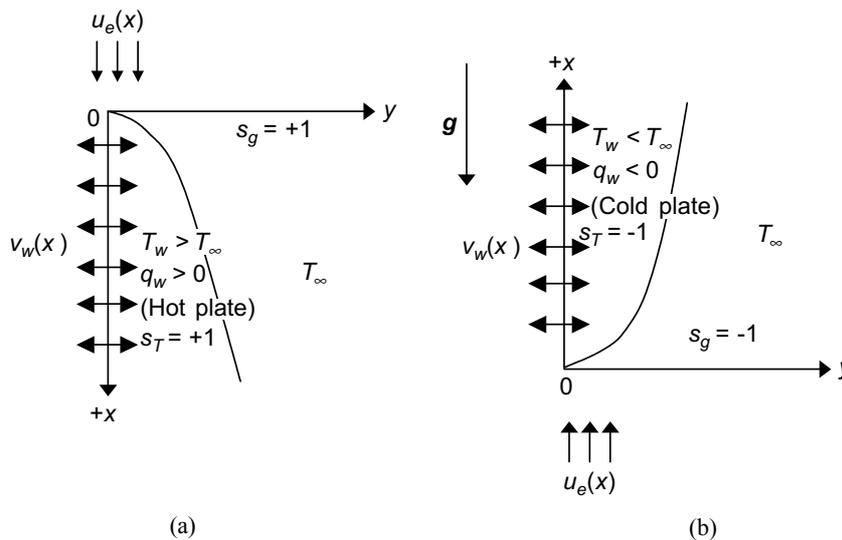


Figure 1 Physical model and coordinate system

approximations are valid. Under these assumptions, the governing boundary layer equations are [4],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u = u_e(x) - s_g \frac{gK\beta}{\nu} (T - T_\infty) \otimes \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \quad (3)$$

subject to the boundary conditions

$$v = v_0 x^m, \quad (\text{i}) \text{ CWT} : T = T_w$$

$$(\text{ii}) \text{ CHF} : \frac{\partial T}{\partial y} = -\frac{q_w}{k} \quad \text{on } y = 0 \quad (4)$$

$$u = u_e(x), \quad T = T_\infty \quad \text{as } y \rightarrow \infty$$

where “-ve” sign in Equation (2) is for the opposing flow case. Here x and y are, respectively, the Cartesian coordinates along the plate and normal to it, u and v are respectively the velocity components along the x - and y -axes, T is the fluid temperature, g is the acceleration due to gravity, K is the permeability of the porous medium and k , α_m , β and ν are the thermal conductivity of the porous medium, equivalent thermal diffusivity, coefficient of thermal expansion and kinematic viscosity, respectively. Here s_g denotes the projection on the positive x -axis of $g/|g|$. Thus, $s_g = +1$ when the positive x -axis points in the direction of g (i.e. vertically downwards) and $s_g = -1$ when it points in the direction opposite to g (i.e. vertically upwards). m is a constant and $v_0 < 0$ corresponds to suction, while $v_0 > 0$ corresponds to injection.

According to the problem of mixed convection boundary layer flow over a vertical flat plate, we can introduce the following similarity variables [16, 17],

$$\eta = \left(\frac{u_e}{2\alpha_m x} \right)^{1/2} y, \quad \psi = (2\alpha_m u_e x)^{1/2} f(\eta)$$

$$(\text{i}) \text{ CWT} : T = T_\infty + s_T |T_w - T_\infty| \theta(\eta)$$

$$(\text{ii}) \text{ CHF} : T = T_\infty + s_T \left(\frac{2\alpha_m x}{u_e} \right)^{1/2} \frac{|q_w|}{k} \theta(\eta) \quad (5)$$

where $u_e(x) = U_\infty$ for case (i) CWT, and $u_e(x) = Ax^{1/3}$ for case (ii) CHF, with $A (> 0)$ being a constant. Here ψ is the stream function, which is defined in the usual way as $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$, while s_T denotes the sign of $(T_w - T_\infty)$ for the case of CWT and the sign of (q_w) for the case of CHF. Thus, the case $s_T = +1$ corresponds to the situation when the buoyancy force accelerates the motion, while the case $s_T = -1$ corresponds to the situation when the buoyancy force decelerates the flow. It is worth mentioning that the mathematical meaning of the similarity variables (5) is that, by employing the similarity transformation using the similarity variables, the partial differential equations are reduced to ordinary differential equations which are much easier to be solved analytically or numerically. On the other hand, from the physical point of view, the meaning of the similarity variables is that the flow is the same (similar) in each transversal section of the flat plate.

Substituting (5) into Equations (1) – (3) yields: for the CWT case,

$$f' = 1 - s_g s_T \lambda \theta \tag{6}$$

$$\theta'' + f\theta' = 0 \tag{7}$$

subject to boundary conditions

$$f(0) = f_w, \quad \theta(0) = 1, \quad \theta(\infty) = 0 \tag{8}$$

which can be reduced to

$$f''' + f f'' = 0 \tag{9}$$

$$f(0) = f_w, \quad f'(0) = 1 - \lambda, \quad f'(\infty) = 1 \tag{10}$$

where $\lambda (\geq 0)$ is the mixed convection parameter, f_w is the mass transfer parameter measuring the strength of mass transfer through the wall and primes denote differentiation with respect to η . The parameters λ and f_w are defined as (Merkin [18])

$$\lambda = \frac{gK \beta |T_w - T_\infty| \left(\frac{x}{v\alpha_m} \right)}{\left(\frac{U_\infty x}{\alpha_m} \right)} = \frac{gK \beta |T_w - T_\infty|}{U_\infty v} = \frac{Ra}{Pe} \tag{11}$$

$$f_w = -2(2\alpha_m U_\infty)^{\frac{1}{2}} v_0$$

where Ra and Pe being the Rayleigh and Péclet numbers, respectively. The special case of $\lambda = 0$ corresponds to pure forced convection. Note that f_w depends on the sign of v_0 . Thus, $f_w > 0$ corresponds to suction or withdrawal (i.e. fluid is removed from the porous medium through the wall), $f_w < 0$ corresponds to injection

(i.e. fluid is injected into the porous medium from the wall) and $f_w = 0$ corresponds to an impermeable wall, which was solved exactly by Merkin [17].

Now for the CHF case, Equations (1) – (3) with (5) lead to the following similarity equations:

$$f' = 1 - s_g s_T \lambda^* \theta \quad (12)$$

$$\theta'' + \frac{2}{3} f \theta' - \frac{1}{3} f' \theta = 0 \quad (13)$$

subject to the boundary conditions

$$f(0) = f_w^*, \quad \theta'(0) = -1, \quad \theta(\infty) = 0 \quad (14)$$

where the mixed convection parameter, $\lambda^* (\geq 0)$, and mass transfer parameter, f_w^* , are now defined by (Merkin [18]),

$$\lambda^* = \frac{gK \beta \left(\frac{|q_w| x}{k} \right) \left(\frac{x}{v \alpha_m} \right)}{\left(\frac{u_e x}{\alpha_m} \right)^{3/2}} = \frac{gK \beta \left(\frac{|q_w|}{k} \right) \left(\frac{1}{v \alpha_m} \right)}{\left(\frac{A}{\alpha_m} \right)} = \frac{Ra^*}{Pe^{3/2}} \quad (15)$$

$$f_w^* = - \left(\frac{2}{3} \right) (2\alpha_m A)^{-1/2} v_0$$

It is seen from Figure 1 that in both CWT and CHF cases, we have $s_g s_T = +1$ (i.e. opposing flow).

The physical quantities of interest for this problem is the skin friction coefficient C_f for the case of CWT and the wall temperature T_w for the case of CHF, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2}, \quad T_w = T_\infty + s_T \left(\frac{2\alpha_m x}{u_e} \right)^{1/2} \frac{|q_w|}{k} \theta(0) \quad (16)$$

where the skin friction is given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (17)$$

Using Equations (5), (16) and (17) we get

$$C_f (2 \text{Re}_x / \text{Pr})^{1/2} = f''(0), \quad \theta(0) = \frac{k(T_w - T_\infty)}{|q_w|(2\alpha_m x / u_e)^{1/2}} \quad (18)$$

where $f''(0)$ is the reduced skin friction and $\theta(0)$ is the non-dimensional wall temperature which are considered in this problem. Here, Re_x is the local Reynolds number and Pr is the modified Prandtl number for a porous medium.

3.0 RESULTS AND DISCUSSION

The two problems given by Equations (9) – (10) and (12) – (14) are solved numerically using the Keller-box method (see Cebeci and Bradshaw [19]) for the reduced skin friction and wall temperature for several values of the mixed convection, suction and injection parameters.

3.1 CWT Case

The results for the reduced skin friction $f''(0)$ for $f_w = 0$ and various values of λ are given in Table 1. The exact numerical results reported by Merkin [17] are also included in this table. Comparison shows very good agreement and it supports very well the validity of the present computations. It can also be seen from this table that the problem

Table 1 Values of $f_1''(0)$ and $f_2''(0)$ for various λ in the impermeable wall ($f_w = 0$) and CWT cases

λ	$f_1''(0)$ present	$f_1''(0)$ Merkin [17]	$f_2''(0)$ present	$f_2''(0)$ Merkin [17]
0	0.00000			
0.1	0.07721			
0.2	0.14905			
0.3	0.21519			
0.4	0.27525			
0.5	0.32875			
0.6	0.37511			
0.7	0.41358			
0.8	0.44317			
0.9	0.46253			
1.0	0.46961	0.46960		
1.05	0.46759	0.46758	0.00002	0.00004
1.10	0.46106	0.46105	0.00192	0.00194
1.15	0.44909	0.44907	0.00865	0.00866
1.20	0.43017	0.43015	0.02218	0.02219
1.25	0.40156	0.40152	0.04538	0.04539
1.30	0.35684	0.35664	0.08491	0.08497
1.35	0.26174	0.25758	0.17851	0.17856
$\lambda_c = 1.354$	0.24112	0.22428	0.24112	0.22428

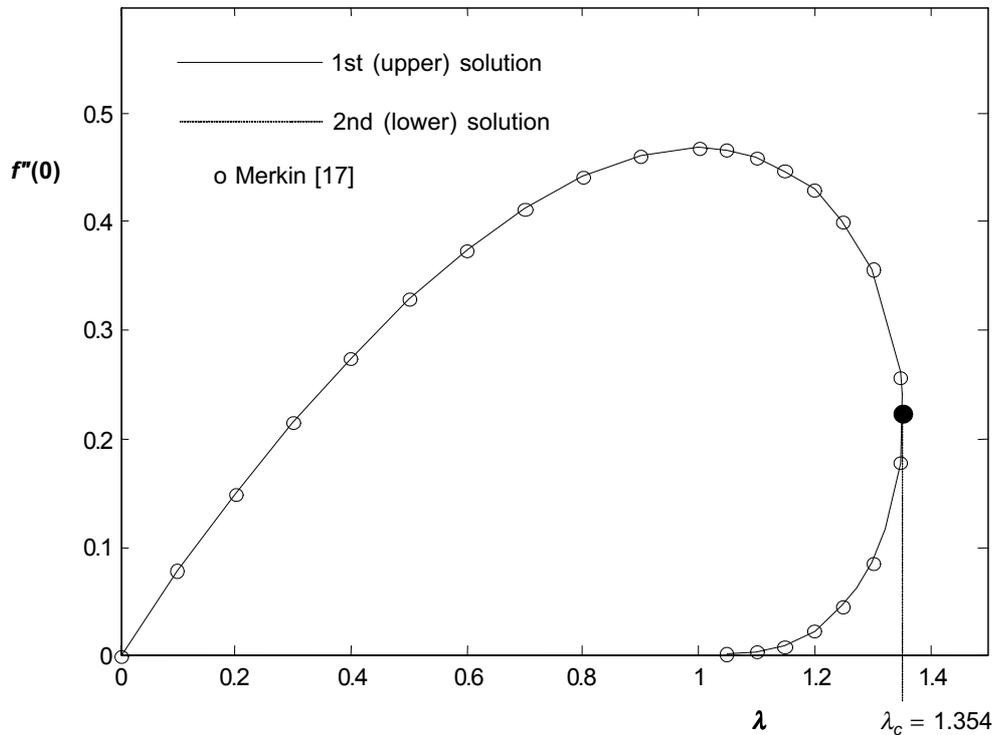


Figure 2 Variation of the reduced skin friction $f''(0)$ with λ for the impermeable wall ($f_w = 0$) and CWT cases

defined by Equations (9) and (10) has solutions only in the range of $0 < \lambda \leq \lambda_c = 1.354$, where λ_c is a critical value of λ . For λ in the range of $1 < \lambda < 1.354$, the solution is not unique, there being two (dual) solutions $f_1''(0)$ and $f_2''(0)$ for a given value of λ . The corresponding results of Table 1 are also presented in graphical form in Figure 2.

The variation of the reduced skin friction $f''(0)$ with λ is shown in Figure 3 for the suction case with $f_w = 1$ and 2. This figure shows that the behaviour of the reduced skin friction $f''(0)$ for the case of suction is similar to the case of an impermeable wall, as shown in Figure 2. However, as the suction increases, the values of $f''(0)$ increase and the range of admissible values of λ also increases. We notice again the existence of dual solutions in the range of $0 < \lambda < \lambda_c$, where λ_c is the critical value of λ depending on $f_w > 0$. We also notice from Figure 3 that these two solutions will meet at the final point or the final physically relevant value of λ , namely λ_c . The values of λ_c for each f_w are shown in Figure 3 and Table 2, and the corresponding numerical values of $f''(0)$ for $f_w = 0, 1$ and 2 are also shown in Table 2. There is no solution beyond this point due to the boundary layer separation. It can be concluded from Figure 3 that increasing suction delays boundary layer separation.

From Figure 3, it can also be seen that the first (upper) solution curve represents a stable flow (laminar flow), i.e. physically relevant solution, whilst the second (lower)

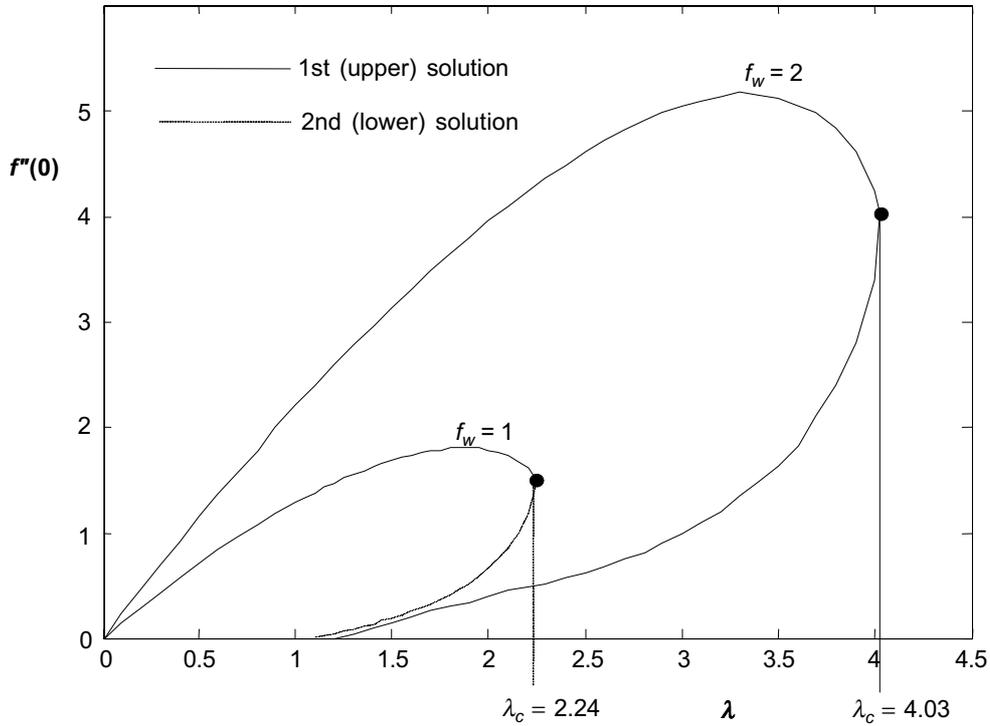


Figure 3 Variation of the reduced skin friction $f''(0)$ with λ for the suction ($f_w = 1$ and 2) and CWT cases

Table 2 The critical values of λ (λ_c) and the corresponding values of $f''(0)$ for $f_w = 0.1$ and 2 (case of CWT)

λ_c	$f''(0)$ ($f_1''(0) = f_2''(0)$)		
	$f_w = 0$	$f_w = 1$	$f_w = 2$
1.354	0.24112		
2.24		1.5157	
4.03			4.0381

solution curve corresponds to an unstable flow (the flow becomes turbulent), which is a physically unrealistic situation. Basically, the second solutions have no physical meaning or significance. Although such solutions are deprived of physical significance, they are nevertheless of interest in so far as the differential equations are concerned. It is, however, important to mention that, in general, the boundary layer equations are not applicable after the point where it separates (see the review paper by Brown and

Stewartson [20]). To describe the flow and heat transfer in the present problem after the point of the boundary layer separation, the full Darcy and energy equations have to be solved.

Further, Equation (9) subjected to the boundary conditions (10) has been solved numerically for several values of $f_w < 0$, namely $f_w = -0.5, -1$ and -1.5 . The results are shown in Figure 4. It is noticed that as the magnitude of injection increases, the reduced skin friction $f''(0)$ decreases, and as λ increases, the reduced skin friction also increases until it reaches a maximum, and then it decreases again. It is also shown in Figure 4 that as the magnitude of injection increases, the reduced skin friction becomes smaller and it vanishes at smaller values of λ . On the other hand, it is found that as λ increases, eventually there will not be any reduced skin friction, i.e. the reduced skin friction vanishes, and boundary layer separation occurs. This behaviour is different from that of an impermeable wall or the suction case described above. It is also found that for the injection case, there are no dual solutions.

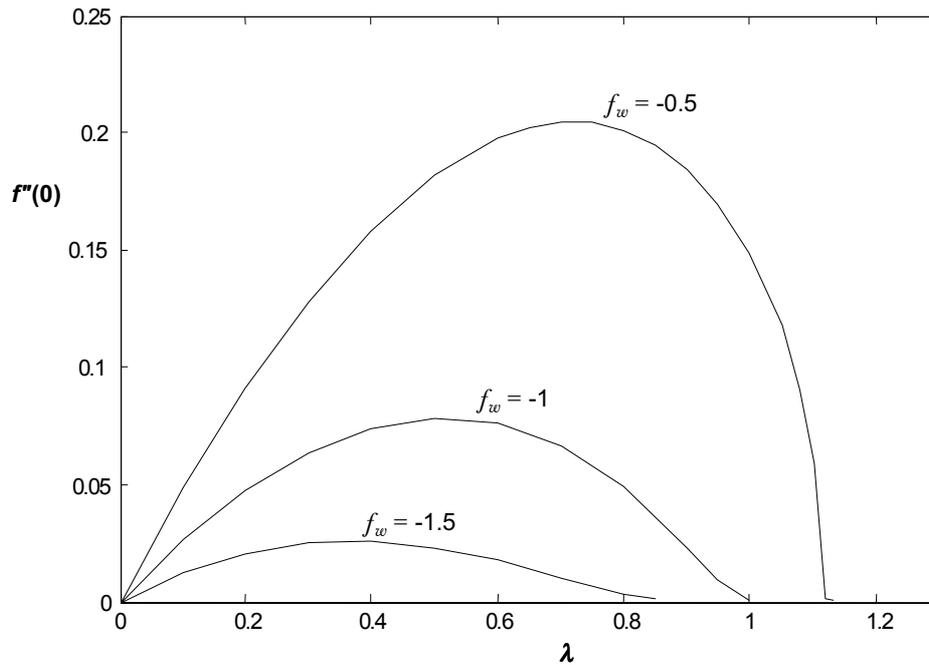


Figure 4 Variation of the reduced skin friction $f''(0)$ with λ for the injection ($f_w^* = -0.5, -1$ and -1.5) and CWT cases

3.2 CHF Case

In this section, we solve Equations (12) – (14), i.e. the CHF case, for wall temperature $\theta(0)$. Figure 5 shows the variation of the wall temperature $\theta(0)$ with λ^* for $f_w^* = -1, -0.5, 0, 1$ and 1.5 , corresponding to injection, impermeable and suction cases. For the

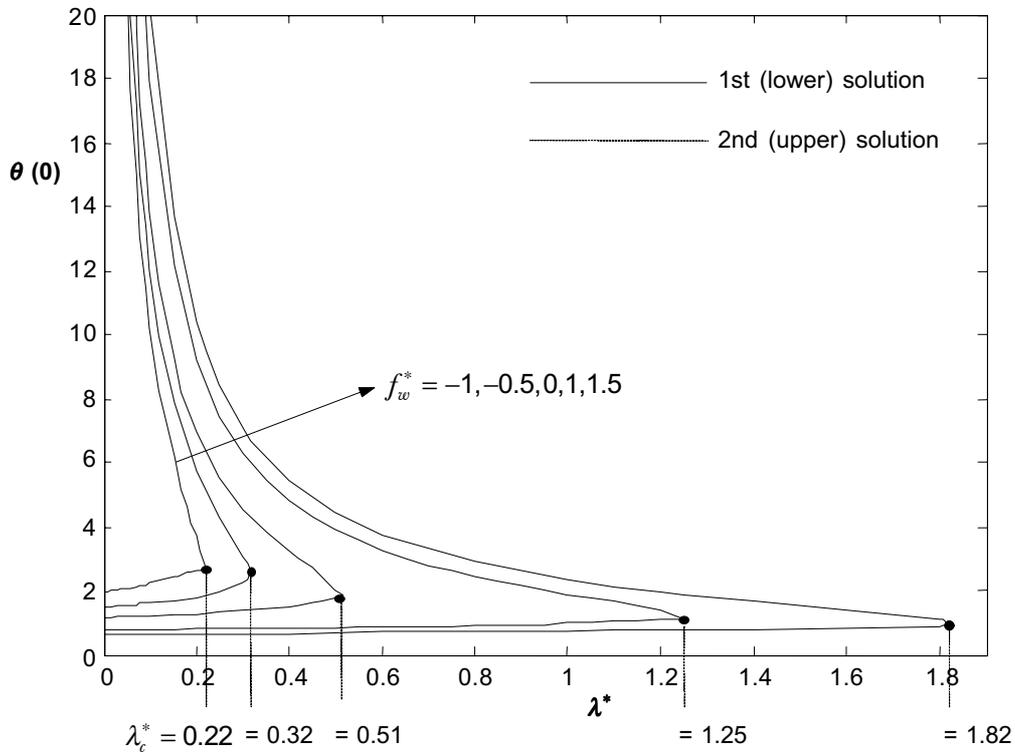


Figure 5 Variation of the wall temperature $\theta(0)$ with λ^* for $f_w^* = -1, -0.5, 0, 1$ and 1.5 (CHF cases)

CHF case, there exists dual solutions for both cases of injection and suction. It can also be seen from this figure that the problem defined by Equations (12) - (14) has solutions only in the range of $0 < \lambda^* \leq \lambda_c^*$, and for λ^* in this range, the solution is not unique, there being two (dual) solutions, $\theta_1(0)$ and $\theta_2(0)$ for a given value of λ^* . For each value of f_w^* , there is a unique solution at the point $\lambda^* = \lambda_c^*$, which is the critical value of λ^* where both solutions meet at the final point. There is no solution beyond this point, due to the boundary layer separation. The critical values of λ^* for each value of f_w^* and the corresponding values of $\theta(0)$ are shown in Table 3. It can be seen from Figure 5 that the behaviour of the wall temperature $\theta(0)$ for the cases of injection and suction are similar to the case of an impermeable wall.

It is also shown in Figure 5 that as the magnitude of injection decreases, the wall temperature $\theta_1(0)$ (first solution) decreases, while the wall temperature $\theta_2(0)$ (second solution) increases. It is also noticed that as λ^* increases, $\theta_1(0)$ increases, while $\theta_2(0)$ decreases. It can also be seen that for the injection case, as the magnitude of injection increases, the solution stops and the boundary layer starts to separate at smaller value of λ^* , while for the suction case, as f_w^* increases, the boundary layer starts to separate at larger value of λ^* . This physically means that suction delays boundary layer separation compared to injection and impermeable cases for the CHF cases.

Table 3 The critical values of $\lambda^*(\lambda_c^*)$ and the corresponding values of $\theta(0)$ for $f_w^* = -1, -0.5, 0, 1$ and 1.5 (case of CHF)

λ_c^*	$\theta(0) (\theta_1(0) = \theta_2(0))$				
	$f_w^* = -1$	$f_w^* = -0.5$	$f_w^* = 0$	$f_w^* = 1$	$f_w^* = 1.5$
0.22	2.6822				
0.32		2.6433			
0.51			1.7672		
1.25				1.1017	
1.82					0.9074

4.0 CONCLUSIONS

In this paper, we have investigated the problem of mixed convection boundary flow over a vertical permeable semi-infinite flat plate of constant wall temperature (CWT) and constant wall heat flux (CHF), which is embedded in a fluid saturated porous medium for the opposing flow case. The problem depends on two parameters, the mass transfer parameters, f_w (CWT) and f_w^* (CHF), as well as the corresponding mixed convection parameters, $\lambda(\geq 0)$ and $\lambda^*(\geq 0)$. For some situations that we have considered, we made the following conclusions for the case of CWT: (1) For an impermeable plate ($f_w = 0$), we have obtained numerical solutions for $f''(0)$, which are in very good agreement with those of Merkin [17]; (2) Dual solutions exist for the case of suction ($f_w > 0$) and they are similar to those for an impermeable wall ($f_w = 0$). It is also found that increasing suction delays boundary layer separation. The upper (first) solution is likely to be physically relevant and stable solution. However, for the case of $f_w < 0$ (injection) there is no such dual solution and the flow behaviour is different from that of the impermeable and suction cases. For the case of CHF: (1) Dual solutions exist for the cases of injection ($f_w^* < 0$), impermeable wall ($f_w^* = 0$) and suction ($f_w^* > 0$) and the pattern of figures are similar for all the three cases; (2) In contrast to the CWT case, the lower (first) solution is likely to be physically relevant and stable solution. These two (dual) solutions will meet at the final point or the final physically relevant value of λ^* , namely λ_c^* . There is no solution beyond this point due to the boundary layer separation. It can also be concluded that suction delays boundary layer separation compared to injection and impermeable cases.

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