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Reynolds Number–Strouhal Number Relationship for Cylindrical Bluff Body with Variation of Aspect Ratio in High Reynolds Number

Nor Azwadi Che Sidik*, Tey Wah Yen

Faculty of Mechanical Engineering, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor, Malaysia

*Corresponding author: azwadi@fkm.utm.my

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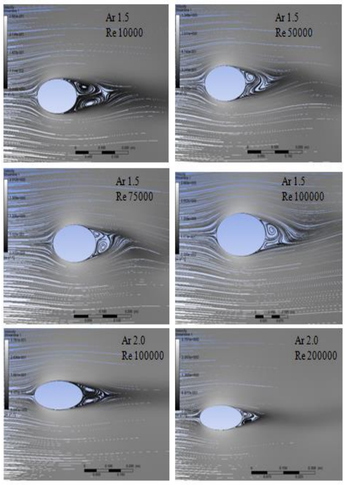
Graphical abstract



Abstract

The effect between Reynolds number and bluff body aspect ratio to the flow parameters such as Strouhal number and drag coefficient are studied. The range of Reynolds number applied is within 10000 and 200000 while three aspect ratio (Ar) where Ar = 1.0, 1.5 and 2.0 are implemented. Finite volume method with the aid of ANSYS CFX codes is deployed using the turbulence SST model. Equations of Re-St relationship for Ar 1.0 and 1.5 are then hypothesized as well in this paper for the range of 10000<Re<100000.

Keywords: Bluff bodies; Reynolds number; Strouhal number; drag coefficient; aspect ratio; ANSYS CFX codes

■1.0 INTRODUCTION

The external flow across the varied geometry of bodies will produce varied structure of eddies that produced at the wake of those structure are responsible for mixing and dissipative properties of turbulence [1]. The extensive study on the topics has been explored by many researchers before. This could be ascribable to the instrumental importance in the industrial applications such as the study of fluid motion across the submarines, ships, airplanes and cars associated with the analysis in energy conversion and structural design. The resulting eddies structure many include separation, wake flow, vortex shedding and curved free shear layers [2].

The previous research may include the study of flow characteristics across the bluff body in three types of flow regions: the low, intermediate and high Reynolds number region flow. It could be classified that for Re less than 500, it is low Reynolds number flow [3], for Re between 500 and 1000, it is intermediate Reynolds number flow [4], and for Re larger than 1000, it can be categorized as high Reynolds number flow [5]. Most of the researches before focused on the flow around the simple structure such as cylindrical bluff body, square, or airfoil due to their simplicity in geometry and wide application in engineering.

As the flow passes by, the flow pattern formed behind the bluff bodies will be the flow separation and a pair of steady symmetric vortices. As Re increases, the length of the re-circulation region behind the body will expand [2]. At a critical onset Reynolds number [6, 7], the vortices become unstable, and then the separated vortices are form to alternately shed in crest and trough pattern. The periodic phenomenon is referred to as vortex shedding, whereas the anti-symmetric wake flow pattern is referred to as the von Karman vortex street [2]. According to Tamai *et al.* [8], a higher Reynolds number will produce lower lift and drag coefficient, bringing the connotation that the completion length has an effect on the development of the wake.

Zdravkovich [9] have concluded the previous research outcome on the varied modes of vortex shedding for Re lower than 300. Although discontinuity did exist for the relationship between Reynolds number (Re) and Strouhal number (St), the researches before generally show the vague idea that increment in Re will result in the rise of St. Moreover the shedding frequency is proportional to the freestream velocity, and the value of St remains constant if the width between shear layers remains unchanged. Williamson [10] has consolidated the compilation above by summarizing the equations that relate Re and St for flow of cylindrical wake with Reynolds number lower than 1200. Most of them are expressed in 3-term tradition, 2-term tradition, 3-term expansion in "1/" √("Re") and 2-term √("Re")-formula.

Sungsu [11] have focused on the study of wake formation due to domain of various size of a sphere at and below Reynolds number 600. He concluded that the constant periodic vortex shedding is initiated at Re=450, and at Re=500, irregular shaped vortices will be formed. In the sphere of experimental work today, digital particle image velocimetry (DPIV) has been deployed to study the vortex shedding processes occurring at the end of a stack of parallel plates due to an oscillating flow induced by an acoustic standing wave. The results shown by Lei Shi [12] have illustrated a good compromise with the Re-St equations as modeled by Rokho [13] with the range of Reynolds number below 5000.

The research conclusion of Muammer [14] can be considered as one of the cornerstone for the study on Re-St relationship. He pledged that values of Strouhal number, as well as the wake patterns, are functions of the cross-section of the cylinders and Reynolds numbers.

The other researches on Re-St relationship for the range of low or moderate Reynolds number such as the work of Lei [15], Ploumhans *et al.* [16] and Provansal [17] and a great deal of findings have been grasped. For the Re-St study on the range of high Reynolds number, although some of the corresponding researches have been made such as the work of Albarède *et al.* [18], Sampaio *et al*. [19] and Sohankar *et al.* [20], the clear insight into the specific Re-St relationship with the association of obstacles cross sectional shape remains as a big room for further improvement.

This paper will concentrate on the Re-St relationship for high Reynolds number, and accordingly their drag and lift coefficients too, associated with the effect cylindrical cross sectional aspect ratio. The equation of Strouhal number in the function of Reynolds number and aspect ratio for cylindrical body in high Reynolds number will be proposed in the study.

■2.0 METHODOLOGY

The structure to be tested will have the horizontal length of 50mm and vertical length of 50 mm as well, bounded with the rectangular block with the thickness 1 mm with following dimensions as shown in Figure 1. The horizontal length of the structure will be enlarged with the aspect ratio 1.0, 1.5 and 2.0. The aspect ratio of the cylindrical-like structure can be defined as the ratio of structural length in parallel with flow direction to the structural length in tangent with flow direction. The meshing process created the total nodes of around 11355 and elements of 39677. The fluid domain is water in isothermal condition of 25^{0}C. The turbulence model applied is the SST model with the initial boundary condition of 0 m/s throughout the domain.

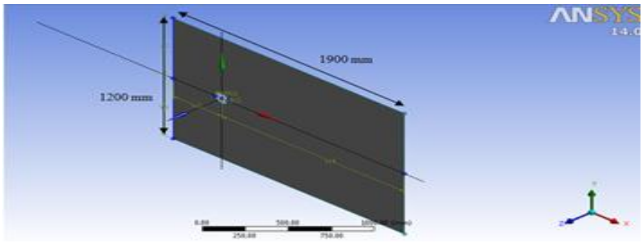


Figure 1 Geometrical setup for the computation domain

The Reynolds Averaged Navier-Stokes (RANS) equations will be applied in this study. There are quite a number of RANS model existed such as the Spalart-Allmaras (SA) model, Realizable k-epsilon (RKE) model, Wilcox k-omega (WKO) model and Shear Stress Transport (SST) model. Among the models, according to the study done by Ugur *et al*., the SST model displayed better overall predictive capabilities among them in terms of velocity, vorticity and shear stress. Since SST is able to capture the thickness of the shear layer more accurately, it will be a robust model among the variation among RANS equations to model the effect of turbulence flow of the near wake of bluff body [21]. The governing equations include the RANS continuity equation and momentum equation as shown as follows.

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \overline{u_i}}{\partial t} + \overline{u_k}\frac{\partial \overline{u_i}}{\partial u_k}\right) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\mu\frac{\partial \overline{u_i}}{\partial x_j}\right) + \frac{\partial R_{ij}}{\partial x} \tag{2}$$

To close the above equations, WKO is one of the method. WKO k-ω model is a two equations model introduced by Wilkox in 1993 and it is effective to predict the flow parameter near boundaries without a requirement for wall function. The model can be expressed as [22]:

$$\partial_t k + U_j \partial_j k = 2\nu_T |S|^2 - C_\mu k\omega + \partial_j\left\{\left(\nu + \frac{\nu_T}{\sigma_k}\right)\partial_j k\right\} \tag{3}$$

$$\partial_t \omega + U_j \partial_j \omega = 2C_{\omega 1}|S|^2 - C_\omega \omega^1 + \partial_j\left\{\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\partial_j \omega\right\} \tag{4}$$

In the equations above, k is the turbulent kinetic energy, ω is the turbulent frequency, while S is the mean velocity strain tensor. The k equation is altered only by changing ε to $C_\mu k\omega$ and the ω equation is analog to the ε equation and the constants are: $C_{\omega 1}=5/9$, $C_{\omega 2} = 3/40$, $\sigma_k=\sigma_\omega=2$ and $C_\mu=0.09$.

However the model has two problems: the spurious sensitivity to free stream condition and not reliable in flows with detached shear layer. As a result, SST model is developed from the WKO in order to improve predictions in adverse pressure gradient boundary layers and to solve the problem of free stream sensitivity. A bound equation is introduced by Menter (1994). The model is further developed into some functions and they have been coded in the ANSYS CFX software.

The drag coefficient, lift coefficient, and the Strouhal number are among the parameters in interest to be tested. These parameters can be defined as [15, 19]:

$$C_D = \frac{2F_D}{pu^2 A} \tag{5}$$

$$St = \frac{fD}{U} \tag{6}$$

in which f is the vortex shedding frequency, A is the reference area of the structure and U is the velocity of the approach flow at the center of the cylinder.

■3.0 RESULT AND DISCUSSION

3.1 Re-St Relation for Different Aspect Ratio

Strouhal number can be physically interpreted as the weight ratio between the wake effect and the average ambient velocity effect, and it could represent the ratio of the unsteadiness of the flow due to the inertia forces of the flow.

At aspect ratio 1.0, as the Reynolds number increases, the wake of the structure shows an increase for the frequency of vortex shedding with decrease in Strouhal number. In other words, when Strouhal number can be physically interpreted as the weight ratio between the wake effect and the average ambient velocity effect, and it could represent the ratio of the unsteadiness of the flow due to the inertia forces of the flow.

At aspect ratio 1.0, as the Reynolds number increases, the wake of the structure shows an increase for the frequency of vortex shedding with decrease in Strouhal number as per Figure 2. In other words, when the flow inertia overrides, the wake tends to go stable and vortex oscillation will be faded. In accordance with the diminishing of wake unsteadiness which will create power lost, the drag coefficients decreases as well when the Reynolds number goes higher. Using statistical analysis, the Re-St relationship of the flow can be computed as the followed proposed equation:

$$\mathrm{St} = 2.345 \mathrm{~x~Re} - 0.27 \tag{7}$$

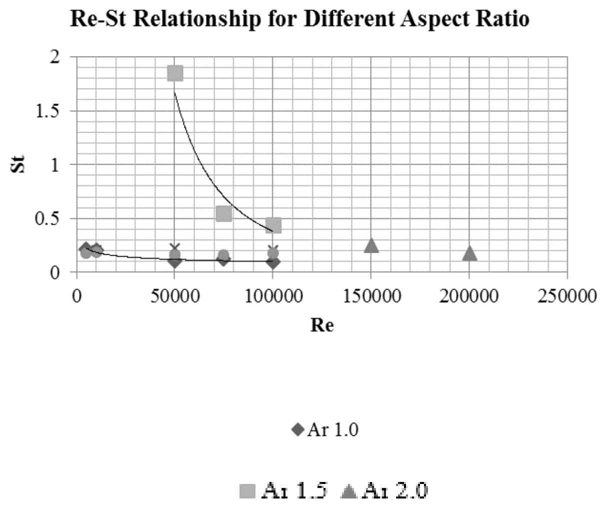


Figure 2 The graph for Re-St Relationship

The coefficient of determination of the equation above is 0.919. From the equation above, the results computed differs from what the results computed by the Sampaio *et al.* (2000) when the Reynolds number goes beyond 20000. This is because the model and coarse near wall meshes applied by Sampaio *et al*. are unable to capture the drag crisis and unsuitable to model the turbulent boundary layer and its transition [5]. In this region of aspect ratio, the results generally shows compromising matching with the work done by P Albarede (1991) [23] and Sampaio *et al.* (2000), especially within in range of lower Re region (Re < 50000).

From Figure 3, the drag coefficient reduces when the momentum of flow goes higher. However at Re = 75000, the drag coefficient undergoes sudden rise. From the work of P. Albared [23], deduction can made that the flow may enter the transition region, where by in this region, the flow parameters might have abnormal abrupt trend change, in both drag coefficient and Strouhal number.

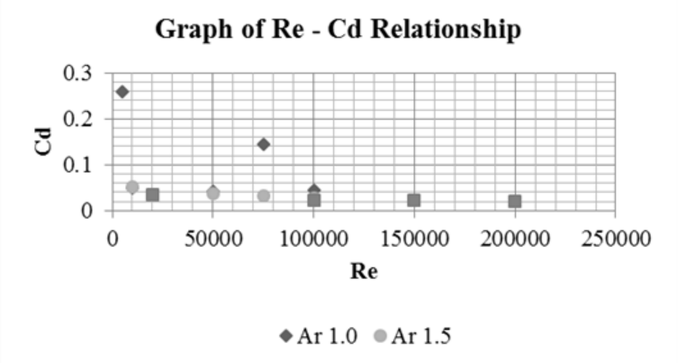


Figure 3 The drag coefficient for different Re at different cylindrical aspect ratio

When the bluff body reaches the aspect ratio 1.5, the minimum Reynolds number required to form vortex shedding will be increased. According to Albarede [23], at aspect ratio 1.0, the minimum Re required will be 3000. This can see from the computation that Strouhal number does not exist at Re = 10000. Both Strouhal number and drag coefficient decreases with the increment of Re. The St-Re relationship for this case can be hypothesized in equation with coefficient of determination 0.92.

$$\mathrm{St} = 2 \times 10^{10} \times \mathrm{Re}^{-2.121} \tag{7}$$

Figure 4 demonstrate that, when the body structure is extended to Ar = 2.0, no vortex shedding can be observed below Re = 100000. This supports the idea that the increasing aspect ratio will enlarge the minimum Re to form flow instability. Drag coefficient reduces as well with the Re accretion.

One phenomenon that can be observed as well will be the region of separation will decrease when the Reynolds number goes higher and the structure aspect ratio increases. The region of separation connotes the abrupt changing of velocity, and the smaller the region of separation, the lower the drag coefficient.

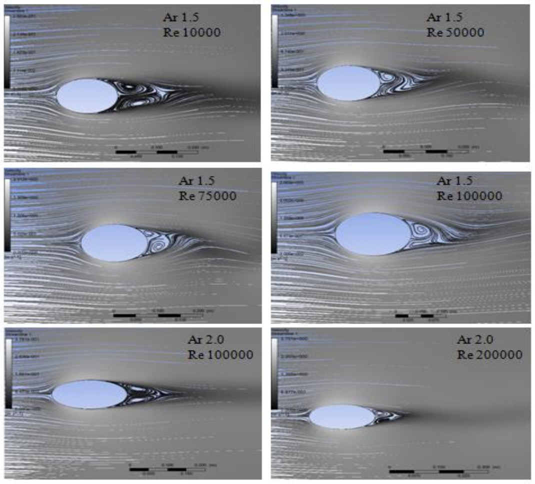


Figure 4 The velocity streamline for comparison of region of flow separation for different Re at different cylindrical aspect ratio

■7.0 SUMMARY

The increment in Re will generally decrease the Strouhal number and drag coefficient. In other words, in order to keep the flow to be free of wake instability, the domain Re must be below its minimum vortex-shedding-forming Reynolds number, and to reduce the wake instability, Re shall be controlled in a higher range; meanwhile to decrease wake power lost, the aspect ratio shall be extended. The Re-St equations for Ar 1.0 and 1.5 are modeled within the range of 10000 and 100000 which complement the inexistence of Re-St equation for bluff body at high Reynolds number. The area of flow separation region decreases when the fluid flows at higher Re and the aspect ratio extends.

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