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Modelling of *Tudung Saji* Weaving Using Elements in Group Theory

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Graphical abstract

Abstract

This paper describes a relation between the practice of Malay food cover weaving and mathematics. The cover, known as *tudung saji*, is woven using a specific technique called triaxial or hexagonal weave. Its tessellated parallelograms form an illusion of three dimensional cubes that are found interesting in mathematical studies of symmetrical patterns using group theory. Some of the properties of triaxial template patterns of order n^3 are also discussed in this paper.

Keywords: Food cover weaving; *tudung saji*; triaxial template patterns

Abstrak

Kertas kerja ini menghuraikan hubungan antara amalan tenunan penutup makanan Melayu dan matematik. Penutup makanan, yang dikenali sebagai tudung saji, ditenun menggunakan teknik tertentu yang dipanggil teknik tenunan tiga paksi atau heksagon. Keselarian yang terdapat pada untaian daun membentuk ilusi kiub tiga dimensi didapati menarik untuk kajian corak simetri matematik menggunakan teori kumpulan. Beberapa ciri corak templat tiga paksi peringkat n^3 juga dibincangkan dalam kertas ini.

Kata kunci: Tenunan penutup makanan; tudung saji; corak template tiga paksi

1.0 INTRODUCTION

Art is an expression that gives impact to our life. Without realizing it, we live surrounded by many forms of arts. Some examples of arts that are normally attracting are named as wall paintings, space decorations, murals and framed paintings. In our point of view, such artistic expressions can be found in the crafts, paintings, weavings and ornaments, where their beauty is exhibited through their geometrical elements.

Other than words, people use mathematics as tools to convey messages or express things. However, mathematics today is now a diverse world of science which can benefit people in any field. In this article, we will focus on the elements of algebra and geometry that are related to the weaving of *tudung saji*.

A field of study that seeks the connection between mathematics and the arts is known as ethnomathematics. In other words, ethnomathematics is a research domain that highlights the relationship between mathematics and cultural arts. Many people have long been interested in the mathematical aspects of the arts. In fact, many studies have been conducted to investigate the connection. For instance, Fenton[1] claims that permutations can be taught through rhythm patterns. In his study, a circle diagram for "clapping music" was created based on Reich's method for creating variations, which is recognizable as a permutation. The study also leads to the notions of groups and cyclic groups. In another study, Harker[2] maintains that groups can represent art. Several examples of groups in art include a dihedral group of order eight, D_4 as compass directions, symmetric groups on four symbols, S_4 as hanging sculpture and rotational symmetry group of a cube through origami.

The focus of our discussion is on the art of *tudung saji* weaving. *Tudung saji* is a food cover mostly used by the Malays to cover their foods. However, since the food covers can be made in various sizes, people nowadays use these food covers as their home decorations. In this study, we restrict on the food cover that are produced in Melaka and Terengganu only. Some examples of the food covers are listed as follows (Figure 1).[3]

Figure 1 From left to right: Flock of Pigeons, Standing, Cape Flower, Bold Head, Sailboats and Five States patterns of *tudung saji* weaving

These covers are woven using a specific technique called triaxial weave, where the strands are plaited in three directions.[4] Basically, the weaving is started by building a cone-shaped framework of triaxial weaves. Five strands are plaited together to form a pentagonal opening. This is followed by interlacing another five strands at the vertices of the pentagon to form hexagonal openings. Every moment, five strands are interlaced to enlarge the structure, which gradually takes the form of a cone as more strands are added.[5] Once the framework is built, colored strands are inserted upward and across the openings to create patterns, resulting in hexagonal tessellations that at times give the illusion of three-dimensional cubes.

■2.0 SOME MATHEMATICAL ELEMENTS IN GROUP THEORY

In the context of this paper, mathematical ideas are perceived as embedded in the cultural practice of *tudung saji* weaving.

One of the elements investigated is related to group theory. The study of group theory involves the definition of a group, subgroups, cyclic groups, abelian groups, permutations and group presentations. A group $\langle G,*\rangle$ is a set G, closed under a binary operation $*$, such that the following axioms are satisfied:[6]

i. The binary operation $*$ is associative.

ii. There is an element e in G such that $e*x=x*e=x$, for all x in G (this element e is called identity element for $*$ on G).

iii. For each a in G, there is an element a' in G with the property that $a'*a=a*a'=e$ (this element is an inverse of a with respect to the operation $*$).

For every group, there must be at least one subgroup. A subgroup of G can be defined as a subset H of a group G which is a group itself under the operation of G.[7] Proper subgroup, indicated by $H<G$, means that H is a subgroup of G, but not equal to G itself. The subgroup e or is called the trivial subgroup of G whereas the subgroup that is not e or G is called a nontrivial proper subgroup of G. In this case, we also discuss the order of a group and the order of an element. The order of a group is the number of elements contained in a group. The word order can also be applied to an element, where the order of an element g is n if n is the smallest positive integer for which $g^n=e$, in which e is the identity element.[7]

Furthermore, some geometrical and symmetrical notions are also investigated in this study. Some simple symmetry operations and the conforming symmetry elements are shown (Table 1).[8]

Table 1 Simple symmetry operations and their conforming symmetry elements

Symmetry operation	Geometrical representation	Symmetry element
Rotation	Line (axis)	Rotation axis
Inversion	Point (center)	Center of inversion
Reflection	Plane	Mirror plane
Translation	Vector	Translation vector

According to Cromwell,[9] there exists a set of tiles that forms a modular system for creating geometric patterns. These tiles can be assembled to create many traditional Islamic patterns from Central Asia, which is known as Central Asian modular design system (CAMS) tiles (Figure 2).[9] The earliest Islamic examples of CAMS patterns are found among the Seljuk brickwork designs on the eleventh and twelfth century minarets.

Another example of analysis of geometrical study is shown by Cromwell in the theoretical analysis of the statistical distribution of the topological types of small knots[10], which is observed in interlaced ornament derived from plaitworks. He also discussed on some knot theory, where a knot is an embedding of a circle in three dimensional spaces. His study has revealed that the distribution is highly skewed, where a few knot types are very common and a large proportion is not found at all. Furthermore, a geometrical study of Gothic architecture is also shown by Huber.[11] In his research, the facade of the Cathedral of Notre Dame in Paris is approached for instructors of single variable integral calculus course in calculating the areas and volumes of Gothic structure.

The geometrical study is continued with the research done by Cromwell[12] in Hybrid 1-point and 2-point constructions for some Islamic geometric designs. The technique for blending the 1-point and 2-point applications of the 'polygons in contact' method of constructing Islamic geometric patterns is described. In his study, two special tiles provide the bridge and some hybrid structures are shown to underlie some traditional Turkish patterns. Interestingly, a design from Humayun's Tomb at Delhi, India uses the 2-point bow tie and decagon tiles, where these tiles also shows the design from panel 56 of the Topkapi Scroll and wooden door panels in Turkey. Besides that, the research depicts the symmetrical combination of tiles to form hybrid tiles in the construction of Topkapi Scroll[13] and other designs.

As mentioned before, the study of the Topkapi Scroll is an important documentary source for the study of Islamic geometric ornament. This research also gives a mathematical analysis of some exemplary star patterns that illustrate a variety of methods of construction. Besides that, it shows that the practice of producing a design by replicating a template using reflections in its sides restricts the range of symmetry types produced.

Based on the previous study of patterns using symmetry and algebra, this research is done in order to find the relations between the elements of triaxial template patterns with the elements in group theory. Since *tudung saji* is a traditional craft that are often used by the modern people, this is one of the best ways to promote the beauty of art in the *tudung saji* patterns by embedding their elements into groups.

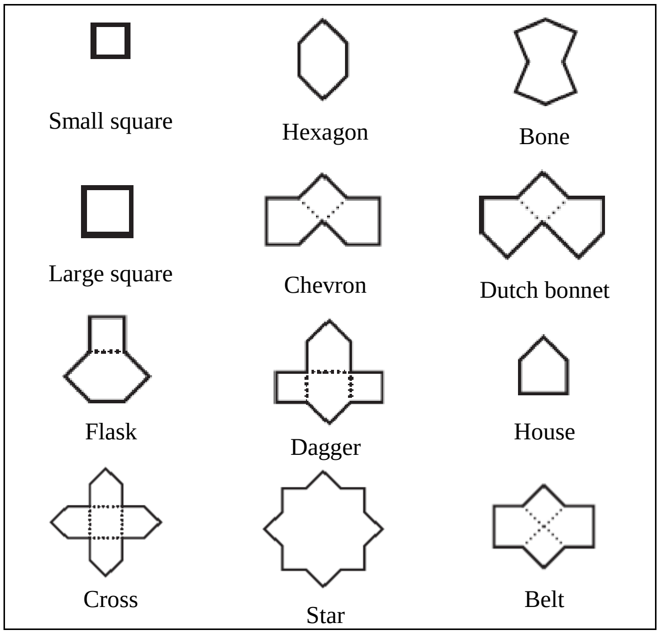


Figure 2 Central Asian modular design system (CAMS)

3.0 EMBEDING THE TUDUNG SAJI PRACTICE INTO ELEMENTS IN GROUP THEORY

Due to its specific technique of weaving, the resulting weaves on the *tudung saji* resemble 'tessellated parallelograms' or 'repeating hexagons'. However, some patterns emerge neat and simple while other patterns seem more complicated and tricky. Furthermore, many of the created patterns have five-fold symmetry at the top (which is at the peak of the *tudung saji*) and six-fold symmetry elsewhere due to the way the strands are woven in the pentagonal and hexagonal openings of the framework.

Previously, Adam[3] developed a computer-generated weaving template as a tool to simulate the weaving technique of triaxial plane patterns. The template consists of three directions, A, B and C to represent the three directions of *tudung saji* weaving. The template generates several original food cover patterns and some fictitious ones, hence contributing towards the creation of new *tudung saji* patterns and designs. The ordering of two-colour and three-colour strands follows counter-clockwise directions, where gives rise to many beautiful patterns such as Pati Sekawan (Flock of Pigeons), *Bunga Tanjung*, *Kapal Layar* (Sailboats), *Corak Butang* (Buttons), *Lima Buah Negeri* (Five States) *Bunga Biskut* (Biscuit Flower) and many others. An example of a template pattern that was created using two-colour strands, red and yellow is shown (Figure 3). The generated pattern is called *Pati Sekawan* (Flock of Pigeons). Some characteristics of the triaxial template pattern that is done by Adam is shown (Table 2).[3]

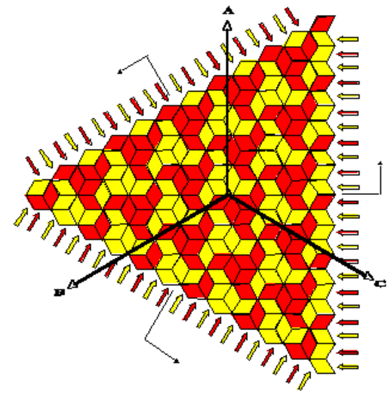


Figure 3 An example of *Pati Sekawan* (Flock of Pigeons) template

This template can be generalized by the *n* number of strands, where *n* is between two and six, i.e $2 \leq n \leq 6$. Now, we let the group of template patterns be denoted as G_T. This G_T group is a group of anti-clockwise rotational symmetry, just like the symmetric group of order six, which consist of elements 1, 2 and 3. However, in this case we have G_{T1} which consists of elements 0 and 1, G_{T2} consists of elements 0,1 and 2, G_{T3} consists of elements 0,1,2 and 3, and so on until G_{T6}, which consists of elements 0, 1, 2, 3, 4, 5 and 6. The order of the group G_{Tn} is represented by n^3. For example, the order of the group G_{T3} is 3^3 which are 27 elements.

Some examples of blocks of two-strand and three-strand template patterns that are produced from two-colour strands, red and yellow are shown (Table 3). There are two possible orientations of the same pattern for the two-strand blocks, whereas the three-strand blocks yield six possible orientations for the *Kapal Layar* (Flock of Pigeons) pattern, two possible orientations for the *Butang* (Buttons) pattern, and one orientation for the *Bunga Tanjung* pattern.

Table 4 depicts the analysis of triaxial patterns of *tudung saji* by using the elements in group theory. The analysis is done according to their patterns which resulted in the existence of isomorphism and other symmetrical characteristics of the patterns.

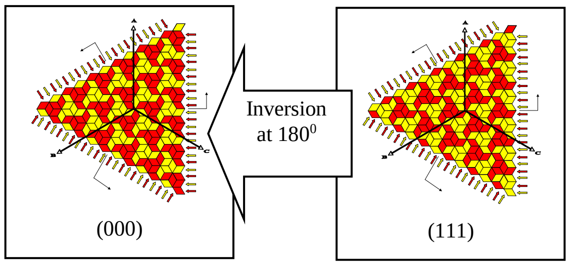
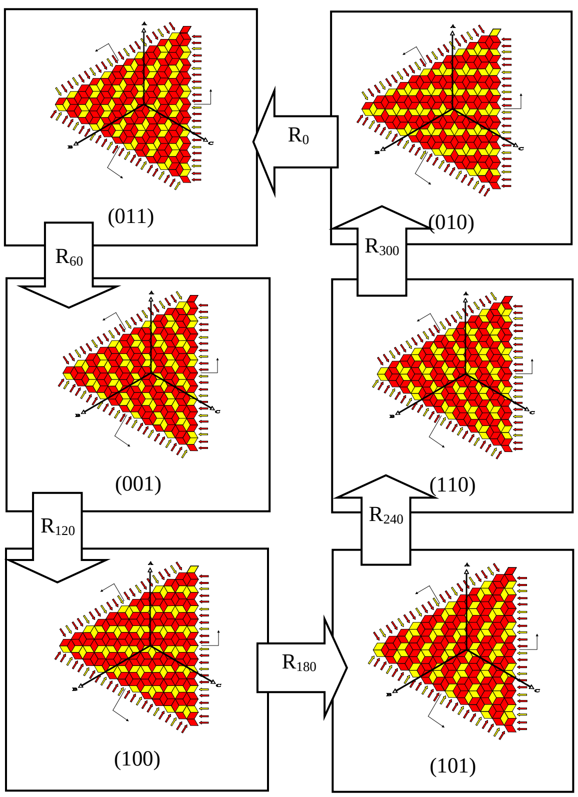
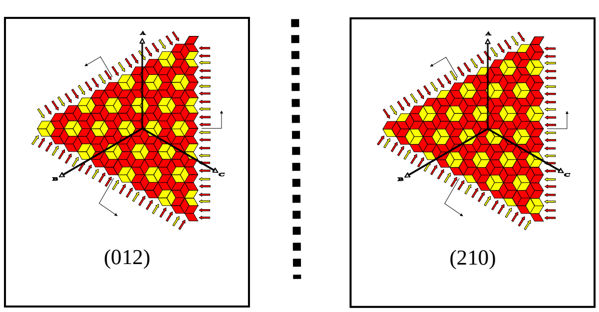
Table 2 Characteristic of template patterns

No.	Strand Label	No.	Pattern	(Group X element)	No.	Strand Label	No.	Pattern	(Group X element)
2	0 = RY, 1 = YR	8	A1 – A2	2X4=8	6	0=RRRRRY, 1=RRRRYR, 2=RRRYRR, 3=RRYRRR, 4=RYRRRR, 5=YRRRRR	216	A1-A2 B1-B6 C1-C6 D1-D2 E1-E2	2X12=24 6X12=72 6X12=72 2X12=24 2X12=24
3	0 = RRY, 1 = RYR, 2=YRR	27	A1-A6 B1-B2 C	6X3=18 2X3=6 1X3=3		0=RRRRYY, 1=RRRYYR, 2=RRYYRR, 3=RYYRRR, 4=YYRRRR, 5=YRRRRY		A B C1-C6 D1-D6 E1-E2 F1-F2	1X12=12 1X12=12 6X12=72 6X12=72 2X12=24 2X12=24
4	0=RRRY, 1=RRYR, 2=RYRR, 3=YRRR	64	A1-A2 B1-B2	2X16=32 2X16=32		0=RRRYYY, 1=RRYYYR, 2=RYYYRR, 3=YYYRRR, 4=YYRRRY, 5=YRRRYY		A1-A2 B1-B6 C1-C6 D1-D2 E1-E2	2X12=24 6X12=72 6X12=72 2X12=24 2X12=24
	0=RRYY, 1=RYYR, 2=YYRR,		A1-A2 B1-B2	2X16=32 2X16=32		0=RRYRYR, 1=RYRYRR, 2=YRYRRR, 3=RYRRRY, 4=YRRRYR, 5=RRRYRY		A1-A2 B1-B6 C1-C6 D1-D2 E1-E2	2X12=24 6X12=72 6X12=72 2X12=24 2X12=24
5	0=RRRRY, 1=RRRYR, 2=RRYRR, 3=RYRRR, 4=YRRRR	125	A1-A2 B1-B2 C	2X25=50 2X25=50 1X25=25					
	0=RRRYY, 1=RRYYR, 2=RYYRR, 3=YYRRR, 4=YRRRY		A1-A2 B1-B2 C	2X25=50 2X25=50 1X25=25					

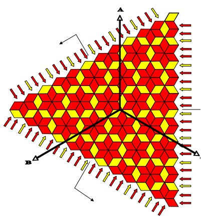
Table 3 The element and symmetrical characteristics of each pattern

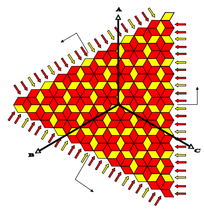
No. of strand	Name of patterns	Elements	No. of elements	Symmetry operation	Geometrical representation	Symmetry element
2	Flock of Pigeons	{(000),(111)}	2	Inversion at a point	Point(center)	Center of inversion
3	Sailboats	{(010),(110),(100),(101),(001),(011)}	6	Rotation	Line(axis)	Rotation axis
3	Buttons	{(012),(021)}	2	Reflection	Plane	Mirror plane
3	*Bunga Tanjung*	{(111),(000),(222)}	3	Rotation	Line(axis)	Rotation axis

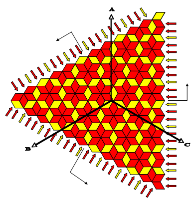
Table 4 Analysis on the characteristics of each pattern

Patterns	**Templates**	**Characteristics**
Pati Sekawan or Flock of Pigeons		i. *Pati Sekawan* (PS) pattern is seen to have the inversion at a point *i* with 180 degree rotation (from (111) to (000)), where PG=0, GP=1. ii. This pattern is said to be isomorphic to S_2, symmetric group of two letters. iii. Show isomorphism: To show the isomorphism, PS and S_2 is given as PS=$\{R_0, R_{180}\}$ and $S_2=\{(1),(12)\}$. Next, let $x \in$ PS and let $y \in S_2$. By definition of function, the mapping is one to one and onto. By definition of isomorphism, it preserves the operation. Therefore, PS is isomorphic to S_2, i.e. PS $\cong S_2$
Kapal Layar or Sailboats		i. *Kapal Layar* (KL) pattern is seen to have a six fold rotational symmetry, where RRY=0, RYR=1, YRR=2. ii. This pattern is said to be isomorphic to D_3, dihedral group of order six. iii. Show isomorphism: To show their isomorphism, KL and D_3 are given as KL=$\{R_0, R_{60}, R_{120}, R_{180}, R_{240}, R_{300}\}$ and $D_3=\{R_0, R_{120}, R_{240}, L_1, L_2, L_3\}$. Here, since KL is generated by R_{60}, i.e. KL $\cong \langle R_{60} \rangle$, which is a cyclic group. Therefore, by definition of isomorphism, KL is isomorphic to the cyclic group of order six, C_6, or KL $\cong C_6$.
Corak Butang or Buttons		i. *Corak Butang* (CB) pattern is seen to have a reflection. ii. The elements of CB = {(012),(210)}. iii. Since the CB pattern does not have the identity element, thus the mapping of elements of CB to any finite groups cannot be done. iv. The only possible properties that CB hold is the reflection at a mirror plane.

Bunga Tanjung


(111)


(000)


(222)

i. *Bunga Tanjung* (BT) pattern is seen to have 3 elements, which are the identity, and inverse of each other.
ii. BT= {(111),(000),(222)}, which the inverse of (000) = (222) and the trivial element is (111), where PPG=0, PGP=1, GPP=2.
iii. This pattern preserves a 180 degree rotational symmetry at A and B axes.
iv. This pattern is said to be isomorphic with some of elements in D_3, dihedral group of order 6.
v. Show isomorphic:
Let BT={R_0, R_A, R_B} and
A= {R_0, L_1, L_2}, where A in D_3.

From the analysis in Table 4, it indicates that there are three patterns that possessed the isomorphism with the elements in group theory namely Flock of Pigeons, Sailboats and Cape Flower. However, since Buttons pattern does not fulfill the requirement of a group, we still can define the symmetrical properties of its elements which is the reflection at a mirror plane.

■4.0 CONCLUSION

In this research, it appears that the practice of *tudung saji* weaving is relevant to the study of group theory. This is proven by the findings of connections between the elements in triaxial patterns of *tudung saji* with elements in groups.

From the analysis, we can see that most of the patterns preserve symmetrical operation, namely anticlockwise rotational symmetry as shown in the Sailboats pattern, which have six-fold anticlockwise rotational symmetry. This pattern can also be defined as the group of template pattern which have order six. Generally, the order of group of triaxial template pattern, G_{Tn} can be defined by n^3, for n number of strands. Therefore, higher number of strands will gives higher possibility for the making of *tudung saji* weaving patterns.

Furthermore, this study also shows that every element in the specific *tudung saji* patterns can be mapped to some of the elements of finite groups.

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