Jurnal Teknologi

Full paper

g-Jitter Induced Mixed Convection Flow of Heat and Mass Transfer Past an Inclined Stretching Sheet

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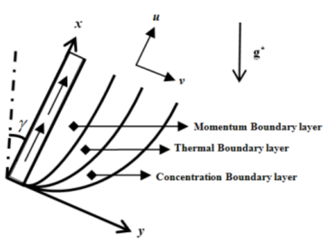
Article history

Received :2 February 2014
Received in revised form :
3 August 2014
Accepted :15 October 2014

Graphical abstract



Abstract

This paper studies unsteady mixed convection boundary layer flow of heat and mass transfer past an inclined stretching sheet associated with the effect of periodical gravity modulation or g-jitter. The temperature and concentration are assumed to vary linearly with x, where x is the distance along the plate. The governing partial differential equations are transformed to a set of coupled ordinary differential equations using non-similarity transformation and solved numerically by Keller-box method. Numerical results for velocity, temperature and concentration profiles as well as skin friction, Nusselt number and Sherwood number are presented and analyzed for different values of inclination angle parameter.

Keywords: g-Jitter; mixed convection; heat and mass transfer; keller-box; inclined stretching sheet

Abstrak

Kertas kerja ini mengkaji olakan campuran di dalam aliran lapisan sempadan bagi pemindahan haba dan jisim merentasi helaian regangan condong bersama kesan modulasi graviti berkala atau ketar-g. Suhu dan kepekatan permukaan diandaikan berubah secara linear terhadap x, dengan x merupakan jarak di sepanjang permukaan. Persamaan perbezaan separa menakluk diubah ke sistem persamaan perbezaan biasa menggunakan persamaan tak serupa dan diselesaikan secara kaedah berangka menggunakan kaedah kotak-Keller. Penyelesaian berangka untuk profil halaju, suhu dan kepekatan termasuk geseran kulit, nombor Nusselt, dan nombor Sherwood dipaparkan dan dianalisis untuk parameter sudut condongan yang berbeza.
Kata kunci: Ketar-g; olakan campuran; pemindahan haba dan jisim; kotak-Keller; helaian regangan condong

1.0 INTRODUCTION

The production of sheeting material which includes both metal and polymer sheets arises in a number of industrial manufacturing processes. The fluid dynamics due to a stretching surface is important in many extrusion processes. For many practical applications the stretching surfaces undergo cooling or heating that cause surface velocity and temperature variations. Problems involving the boundary layer flow due to a stretching surface in the vertical and inclined direction in a steady or unsteady, viscous and incompressible fluid when the buoyancy forces are taken into account have been considered by many researchers such as Sharidan *et al.*[1], Ali *et al.*[2] and Aurangzaib *et al.*[3]. Meanwhile, various studies have been conducted involving g-jitter forces associated with microgravity. g-Jitter or periodical gravity modulation can be defined as the inertia effects due to quasi-steady, oscillatory or transient accelerations arising from crew motions and machinery vibrations in parabolic aircrafts, space shuttles or other microgravity environment. The specific amplitude and frequency of the g-jitter accelerations depend on the dynamic behaviour of the spacecraft structure, the location of the body, and the type and location of the sources generating contributing forces [4]. On the other hand, Li[5] has found that the frequency and amplitude of the g-jitter play an important role in determining the convective flow behaviour of the system. Rees and Pop[6] have studied the effect of g-jitter on free convection embedded in a porous medium near a stagnation point.

A considerable amount of attention has been focused in recent years by various researchers to study problems involving the effect of g-jitter on heat and mass transfer [6-14]. These combined effects have many practical applications, such as binary alloy solidification systems, since the quality of the final products is strongly correlated to the concentration distribution in the melt during processing, the migration of moisture through the air contained in fibrous insulations and grain storage insulations, dispersion of chemical contaminants through water-saturated soil, etc. Li and Shu[15] have carried out a numerical study on double diffusive convection driven by g-jitter in a microgravity environment. They found that an increase of g-jitter force (amplitude) may cause the nonlinear convective effects become much more obvious, which is drastically change the concentration fields. After that, Shu *et al.*[16] extended the previous problem by describing a numerical study of g-jitter driven double-diffusive convective flows which includes the thermal and concentration distributions in binary alloy melt systems subject to an external magnetic field. On the other hand, Li *et al.*[17] have presented a finite element model for the g-jitter induced double-diffusive convection and solidification phenomena with and without the presence of magnetic fields in a Sn-doped Bi crystal growth system planned for space experiments. Then, Sharidan *et al.*[7] presented an exact analytical solution for the problem of laminar combined heat and mass transfer by mixed convection of a fully developed flow driven by a combination of g-jitter. They found that, the temperature and concentration fields are primarily controlled by diffusion and the free convection effect may be neglected.

In spite of all these studies, we have found that the effect of double diffusion has not been considered by other researchers in the case of inclined stretching sheet with the effect of g-jitter. The aim of this paper is to study the effect of g-jitter on mixed convection flow and mass transfer past an inclined stretching sheet. The governing boundary layer equations are transformed into the non-dimensional partial differential equations which are solved numerically using Keller-box method. The numerical results of the effect of frequency of the oscillation, Ω, amplitude of modulation, ε, Prandtl number, Pr, Schmidt number, *Sc*, mixed convection parameter, λ and the angle of inclination parameter γ on the skin friction, Nusselt and Sherwood number will be presented graphically and in tabular form.

Following Rees and Pop [6], and Sharidan *et al.*[1], we consider a simple model problem in which gravitational fields takes the form

$$\mathbf{g}^*(t) = g(t)\mathbf{k} = g_0\left[1+\varepsilon\cos(\pi\omega t)\right]\mathbf{k}, \tag{1}$$

where g_0 is the time-averaged value of the gravitational acceleration $\mathbf{g}^*(t)$ acting along the direction on the unit vector **k,** which is oriented in the upward direction, ε is a scaling parameter, which gives the magnitude of the gravity modulation relative to g_0, t is the time and ω is the frequency of oscillation of the g-jitter driven flow. If $\varepsilon \ll 1$, then the forcing may be seen as a perturbation of the mean gravity. Since the governing equations of this problem are non-linear, this kind of forcing leads to the phenomenon of streaming, where a time-periodic forcing with zero means produces a periodic response consisting of a steady-state solution with a non-zero mean and time-dependent fluctuations involving higher harmonics [1].

■2.0 BASIC EQUATION

Consider an unsteady, viscous, incompressible, double-diffusive mixed convection boundary layer flow over an inclined stretching sheet with the presence of g-jitter effect. The coordinate system is such that *x* measures the distance along the plate and *y* measures the distance normally into the fluid. It is assumed that the plate has a linear velocity $u_w(x)$ moves in *x*-direction of the flow. It is further assumed that the temperature and concentration of the plate varies linearly with the distance *x* along the plate, where $T_w(x) > T_\infty$ and $C_w(x) > C_\infty$ with $T_w(x)$ and T_∞ being the temperature of the plate and uniform temperature of ambient fluid while $C_w(x)$ and C_∞ being the concentration of plate and uniform concentration of the ambient fluid.

Meanwhile, the velocity, temperature and concentration of the continuous stretching surface are assumed to be in the form of $u_w(x) = cx,\ T_w(x) = T_\infty + ax$ and $C_w(x) = C_\infty + bx$ where *a*, *b*, and *c* are constants and $c > 0$. The flow configurations and coordinate system are shown in Figure 1.

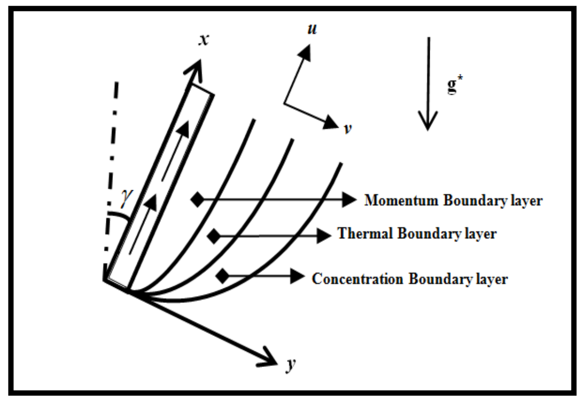


Figure 1 Physical model and coordinate system

Under the usual boundary layer approximation, along with Boussinesq approximations, the governing equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+ g(t)\left[\beta_T(T-T_\infty)+\beta_c(C-C_\infty)\right]\cos\gamma, \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}, \tag{5}$$

subject to the following initial and boundary conditions

$$\begin{gathered} t\le 0: u=v=0,\ T=T_\infty,\ C=C_\infty \text{ for any } x,y, \\ t>0: u_w(x)=cx, v=0, T_w(x)=T_\infty+ax, \\ C_w(x)=C_\infty+bx \text{ on } y=0, \\ u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty, \end{gathered} \tag{6}$$

where *u* and *v* are the velocity components along *x* and *y* axes, *a* and *c* are constants, *T* is the fluid temperature, *C* is the concentration of the fluid, α is the coefficient of thermal diffusivity, β_T and β_C is the thermal and concentration expansion coefficient respectively, *D* is mass diffusivity, γ is the angle of inclination and υ is kinematic viscosity, respectively.

Following Sharidan *et al.*[1], the complexity of the problem is reduced by introducing the following non-dimensional variables,

$$\tau = \omega t, \eta = (c/\upsilon)^{1/2} y, \psi = (c\upsilon)^{1/2} x f(\tau,\eta)$$
$$\theta(\tau,\eta) = \frac{(T-T_\infty)}{(T_w-T_\infty)}, \phi(\tau,\eta) = \frac{(C-C_\infty)}{(C_w-C_\infty)} \qquad (7)$$
$$g(\tau) = \frac{g(t)}{g_0}$$

where ψ is the stream function which is defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. It is noted that equation (2) is satisfied with the non-dimensional variables introduced. By using Equation (7), Equations (3), (4) and (5) become

$$\frac{\partial^3 f}{\partial\eta^3} + f\frac{\partial^2 f}{\partial\eta^2} - \left(\frac{\partial f}{\partial\eta}\right)^2 + \lambda\left[1+\varepsilon\cos(\pi\tau)\right](\theta + N\phi)\cos\gamma = \Omega\frac{\partial^2 f}{\partial\tau\,\partial\eta}, \qquad (8)$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2} + f\frac{\partial\theta}{\partial\eta} - \frac{\partial f}{\partial\eta}\theta = \Omega\frac{\partial\theta}{\partial\tau}, \qquad (9)$$

$$\frac{1}{Sc}\frac{\partial^2\phi}{\partial\eta^2} + f\frac{\partial\phi}{\partial\eta} - \frac{\partial f}{\partial\eta}\phi = \Omega\frac{\partial\phi}{\partial\tau}. \qquad (10)$$

The boundary conditions (6) become

$$f=0,\ \frac{\partial f}{\partial\eta}=1,\ \theta=1,\ \phi=1,\ \text{on } \eta=0,$$
$$\frac{\partial f}{\partial\eta}\to 0,\ \theta\to 0,\ \phi\to 0,\ as\ \eta\to\infty, \qquad (11)$$

where Pr is the Prandtl number, Ω is the non-dimensional frequency, N is the buoyancy ratio and λ is the mixed convection parameter, which are defined as

$$\Pr = \frac{\upsilon}{\alpha},\ \Omega = \frac{\omega}{c},\ N = \frac{\beta_c(C_w-C_\infty)}{\beta_T(T_w-T_\infty)},\ \lambda = \frac{Gr_x}{\mathrm{Re}_x^2}, \qquad (12)$$

where $Gr_x = g_0\beta_T\left[T_w(x)-T_\infty\right]\frac{x^3}{\upsilon^2}$ being the local Grashof number and $\mathrm{Re}_x = u_w(x)\frac{x}{\upsilon}$ is the local Reynolds number, respectively. We notice that $\lambda > 0$ corresponds to aiding flow and $\lambda < 0$ to opposing flow, respectively.

The physical quantities of interest include the skin friction coefficient, C_f, the local Nusselt number, Nu_x, and the Sherwood number, Sh_r which are defined as

$$C_f = \frac{\tau_w(x)}{\left(\frac{\rho u_w^2}{2}\right)},\quad Nu_x = \frac{q_w(x)x}{k(T_w-T_\infty)},$$
$$Sh_r = \frac{m_w(x)x}{D(C_w-C_\infty)}, \qquad (13)$$

where the $\tau_w(x)$, $q_w(x)$ and $m_w(x)$ are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$m_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}. \qquad (14)$$

Here, k is the thermal conductibility, μ is the dynamic viscosity, D is the mass diffusivity, τ_w is the shear stress at the wall, q_w is the average convective heat transfer coefficient and m_w is the average mass transfer coefficient. Using variables (6), we obtain

$$C_f\,\mathrm{Re}_x^{1/2} = 2\frac{\partial^2 f}{\partial\eta^2}(\tau,0),\ \frac{Nu_x}{\mathrm{Re}_x^{1/2}} = -\frac{\partial\theta}{\partial\eta}(\tau,0),$$
$$\frac{Sh_r}{\mathrm{Re}_x^{1/2}} = -\frac{\partial\phi}{\partial\eta}(\tau,0). \qquad (15)$$

■3.0 RESULTS AND DISCUSSION

The system of the unsteady governing equations (8) , (9) and (10) together with the boundary conditions (11) is nonlinear differential equations depending on the various values of the parameters such as frequency of the oscillation, Ω, amplitude of modulation, ε, Prandtl number, Pr, Schmidt number, Sc, mixed convection parameter, λ and the angle of inclination parameter γ. These equations are solved numerically by using finite difference method which is known as Keller-Box method. This method has been found to be very suitable in dealing with nonlinear parabolic problems.

Table 1 Comparison of the mean skin friction, $F''(0)$ and mean heat transfer rate, $\Theta'(0)$ for $\lambda_c = -0.13$, $\varepsilon = 1$ and Pr=0.72

	$F''(0)$		
	$\Omega = 0.2$	$\Omega = 1$	$\Omega = 6.8$
Present	1.0803	1.0792	1.0792
Sharidan *et al.*[1]	1.0802	1.0793	1.0793
	$\Theta'(0)$		
Present	0.7858	0.7868	0.7868
Sharidan *et al.*[1]	0.7857	0.7868	0.7867

Table 1 represents the comparison of the result between Sharidan *et al.*[1] for vertical stretching sheet with the present result by neglecting the buoyancy ratio, (N=0), inclination angle, $(\gamma = 0)$ and Schmidt number, (Sc=0). The mean skin friction and heat transfer rate are the averages of the set of values of $f''(\tau,0)$ and $\theta'(\tau,0)$ respectively in the range of $0 < \tau < 2$ and obtained by using trapezoidal rule. It can be seen from the Table 1 that very good agreement between the results exists. This agreeable comparison lends confidence in the numerical results obtained in this paper.

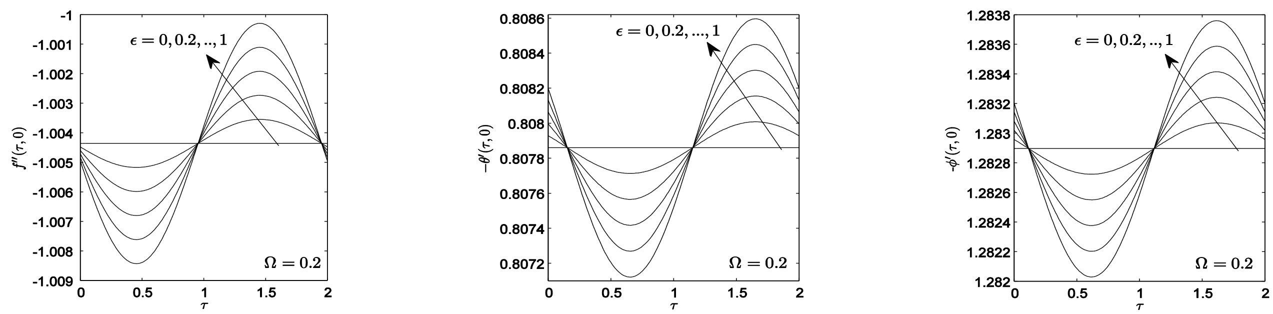
Following Sharidan *et al.*[1], in all result we vary ε from 0 to 1 since values of ε above 1 is equivalent to having the perceived gravity reverse its direction over part of the g-jitter cycle. The computation for the appropriate steady solution were always started with $\varepsilon = 0$ and convergence to a steady periodic state was demanded to have taken place when

$$\max\left|\theta'(\tau,0)-\theta'(\tau-2,0)\right|<10^{-6} \qquad (16)$$

over the whole period.

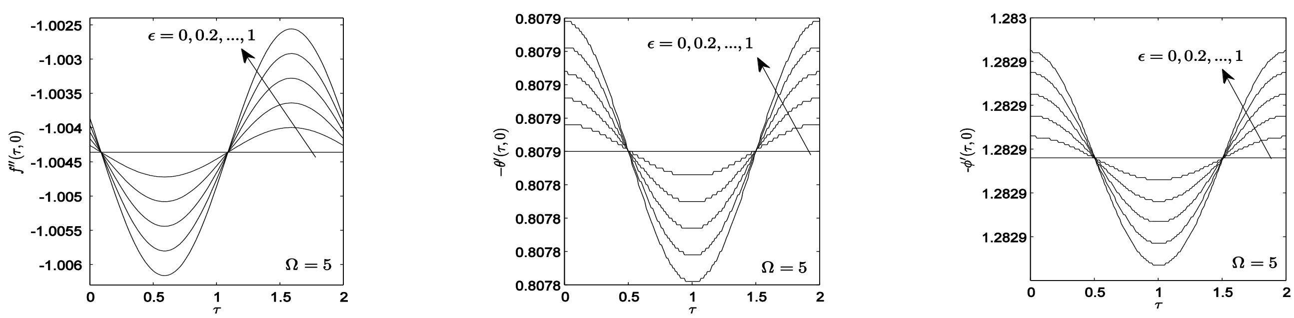
Figure 2 to 5 illustrate the effect of amplitude of modulation, frequency of oscillation and inclination angle parameter for fixed values, $\lambda_c=-0.005$, Pr=0.72, Sc=1.5, N=1, $\Omega=0.2$ and 5. The results obtained show the variation of skin friction, $f''(\tau,0)$, rate of heat and mass transfer, $-\theta'(\tau,0)$ and $-\phi'(\tau,0)$. From these figures, we can see that, the effect of increasing ε give an almost proportional increase or decrease in the skin friction and rate of heat and mass transfer. We can also observe that, skin friction, heat and mass transfer coefficients increase as the inclination angle parameter increase. This is towards the fact that as the plate is inclined from the vertical the buoyancy force effect due to the thermal and mass diffusions decrease as $\cos\gamma$ decreases. However, when the values Ω increase, the corresponding curves show the different trends as can be seen from Figure 2 to 5 particularly for the variation of rate of heat and mass transfer. Also, it is observed from the Figure 3 and 5, as the values of frequency of oscillation is larger, the rate of heat and mass transfer are changed fairly small.

Figure 6 depicts the effect of amplitude of modulation, ε on the velocity, temperature and concentration profile. Three different values of amplitude $(\varepsilon=0,0.5,1)$ are chosen. It is observed that the temperature and concentration profile are decrease with the increase of amplitude of modulation, whereas the velocity profile increases. These graphical behaviour of profiles can be verified from the boundary conditions shown in Equation (11).



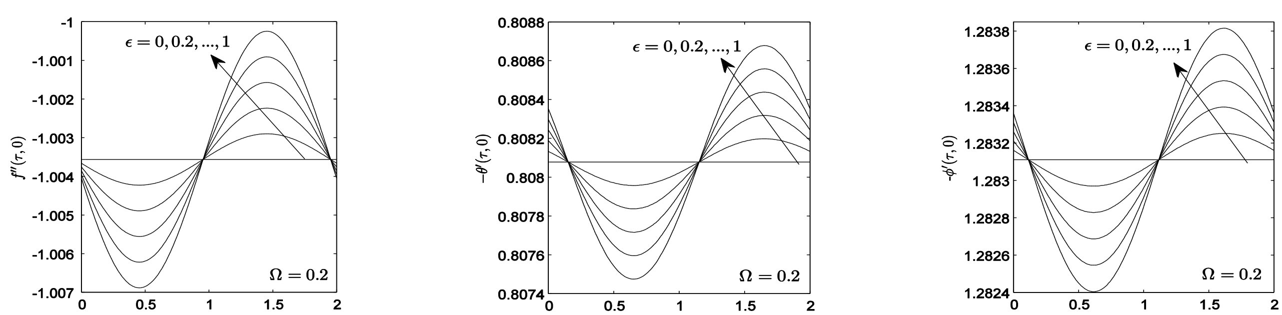
a) b) c)

Figure 2 Variation of skin friction, $f''(\tau,0)$ and rate of heat and mass transfer, $-\theta'(\tau,0)$ and $-\phi'(\tau,0)$ with τ for $\lambda_c=-0.005$, $\Pr=0.72, Sc=1.5$, $\gamma=\pi/6$, $\Omega=0.2$ and different values of ε for N=1



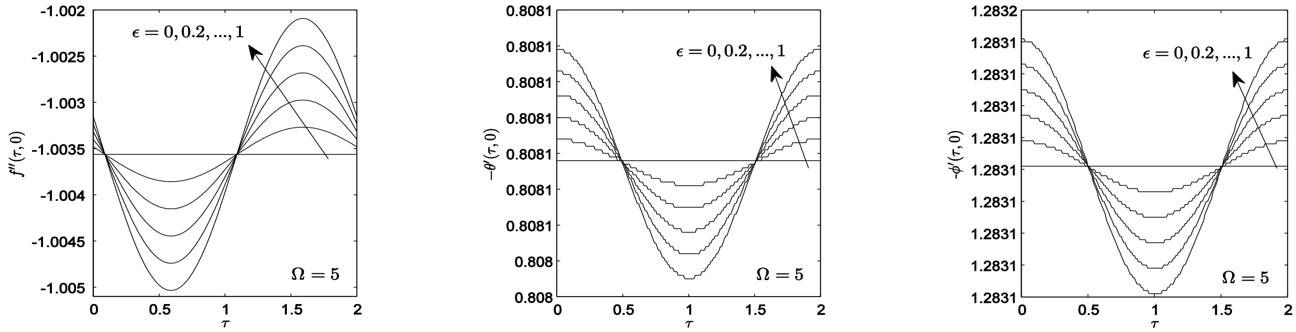
a) b) c)

Figure 3 Variation of skin friction, $f''(\tau,0)$ and rate of heat and mass transfer, $-\theta'(\tau,0)$ and $-\phi'(\tau,0)$ with τ for $\lambda_c=-0.005$, $\Pr=0.72, Sc=1.5$, $\gamma=\pi/6$, $\Omega=5$ and different values of ε for N=1



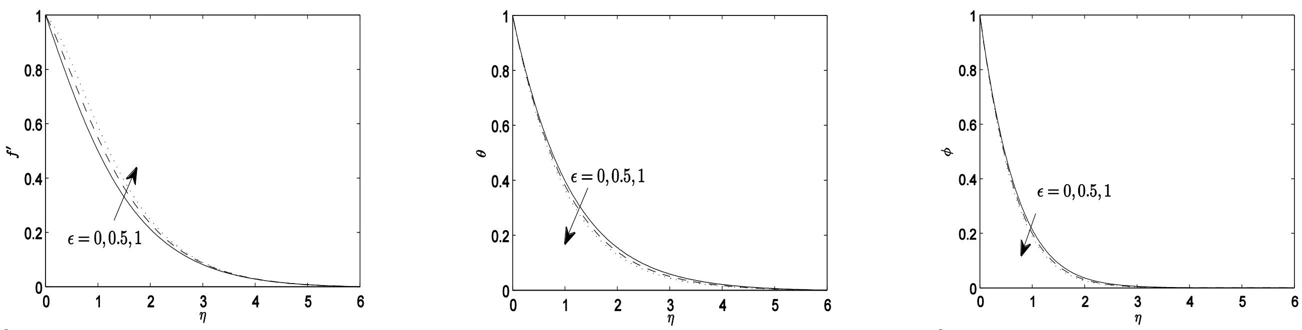
a) b) c)

Figure 4 Variation of skin friction, $f''(\tau,0)$ and rate of heat and mass transfer, $-\theta'(\tau,0)$ and $-\phi'(\tau,0)$ with τ for $\lambda_c=-0.005$, $\Pr=0.72, Sc=1.5$, $\gamma=\pi/4$, $\Omega=0.2$ and different values of ε for N=1



a) b) c)

Figure 5 Variation of skin friction, $f''(\tau,0)$ and rate of heat and mass transfer, $-\theta'(\tau,0)$ and $-\phi'(\tau,0)$ with τ for $\lambda_c = -0.005$, $\Pr = 0.72, Sc = 1.5$, $\gamma = \pi/4$, $\Omega = 5$ and different values of ε for N=1



a) b) c)

Figure 6 Velocity (a), temperature (b) and concentration (c) profiles for different values of ε when $\Pr = 0.72, Sc = 1.5$, $\gamma = \pi/3$, $\Omega = 0.2$ and $\lambda = 1$

■4.0 CONCLUSION

In this paper, the effect of g-jitter induced mixed convection flow past an inclined stretching sheet has been numerically investigated. The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations is solved numerically using the Keller-box method. The effects of the frequency of the oscillation, Ω, amplitude of modulation, ε, mixed convection parameter, λ and the angle of inclination parameter γ for Prandtl number Pr = 0.72 and Schmidt number, Sc = 1.5 on the skin friction coefficient and rate of heat and mass transfer have been examined in detail. Furthermore, the numerical results for velocity, temperature and concentration fields also graphically displayed. Comparison of the results shows that the obtained results agree very well with the previous results for vertical stretching plate $(\gamma = 0)$ reported by Sharidan *et al.*[1]

Acknowledgement

The authors would like to acknowledge MOE and Research Management Centre–UTM for the financial support through vote numbers 09j71 and 06h67 for this research. The first author also would like to express many thanks to UTM for Zamalah scholarship.

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