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The Homological Functors of Some Abelian Groups of Prime Power Order

Rosita Zainal,[a] Nor Muhainiah Mohd Ali,[a,] * Nor Haniza Sarmin,[a] Samad Rashid[b]

[a]*Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, Johor Malaysia*
[b]*Department of Mathematics, Faculty of Science, Firoozkooh Branch, Islamic Azad University, Tehran, Iran*

*Corresponding author: normuhainiah@utm.my

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Graphical abstract

$$G \otimes G \cong \begin{cases} \mathbb{Z}_{p^5} & ;\ \text{if } G \cong \mathbb{Z}_{p^5}, \\ \mathbb{Z}_{p^4} \times \mathbb{Z}_p^3 & ;\ \text{if } G \cong \mathbb{Z}_{p^4} \times \mathbb{Z}_p, \\ \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}^3 & ;\ \text{if } G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}, \\ \mathbb{Z}_{p^3} \times \mathbb{Z}_p^8 & ;\ \text{if } G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_p \times \mathbb{Z}_p, \\ \mathbb{Z}_{p^2}^4 \times \mathbb{Z}_p^5 & ;\ \text{if } G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_{p^2} \times \mathbb{Z}_p, \\ \mathbb{Z}_{p^2} \times \mathbb{Z}_p^{15} & ;\ \text{if } G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p, \\ \mathbb{Z}_p^{25} & ;\ \text{if } G \cong \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p. \end{cases}$$

Abstract

The homological functors of a group were first introduced in homotopy theory. Some of the homological functors including the nonabelian tensor square and the Schur multiplier of abelian groups of prime power order are determined in this paper. The nonabelian tensor square of a group G introduced by Brown and Loday in 1987 is a special case of the nonabelian tensor product. Meanwhile, the Schur multiplier of G is the second cohomology with integer coefficients is named after Issai Schur. The aims of this paper are to determine the nonabelian tensor square and the Schur multiplier of abelian groups of order p^5, where p is an odd prime.

Keywords: Nonabelian tensor square; Schur multiplier; abelian groups

Abstrak

Fungtor homologi bagi suatu kumpulan telah diperkenalkan pertama kali dalam teori homotopi. Beberapa fungtor homologi termasuk kuasa dua tensor tak abelan dan pendarab Schur untuk kumpulan abelan berperingkat kuasa perdana telah ditentukan di dalam kertas kerja ini. Kuasa dua tensor tak abelan suatu kumpulan G yang diperkenalkan oleh Brown dan Loday pada tahun 1987 adalah merupakan kes khas pendarabaan tensor tak abelan. Sementara itu, pendarab Schur kumpulan G merupakan kohomologi kedua dengan pekali nombor di mana ianya dinamai sempena Issai Schur. Matlamat kertas kerja ini adalah untuk mengira kuasa dua tensor tak abelan dan pendarab Schur untuk kumpulan abelan peringkat p^5, di mana p ialah nombor perdana ganjil.

Kata kunci: Kuasa dua tensor tak Abelan; pendarab Schur; kumpulan abelan

■1.0 INTRODUCTION

The homological functors of a group G including the nonabelian tensor square and the Schur multiplier were originated in homotopy theory as well as in algebraic K-theory. The nonabelian tensor square was first discussed by Brown *et al.*[1] denoted by $G \otimes G$. It is the group generated by the symbols $g \otimes h$ and defined by the relations $gg' \otimes h = \left({}^{g}g' \otimes {}^{g}h\right)\left(g \otimes h\right)$ and $g \otimes hh' = \left(g \otimes h\right)\left({}^{h}g \otimes {}^{h}h'\right)$ for all $g, g', h, h' \in G,$ where ${}^{h}g = hgh^{-1}$ denotes the conjugate of g by h. The Schur multiplier of G, $M(G)$, is the second cohomology group with integer coefficients. Let G be a group with a free presentation $1 \to R \to F \to G \to 1.$ Then the Schur multiplier of G is isomorphic to $R \cap F'/[R, F]$ where F is any free group and $G \cong F/R$ is any free presentation of G. In 1987, a book named The Schur Multiplier by Karpilovsky[2], collects the theory, computation, and application of the Schur multiplier for many type of groups.

Various methods were used in order to determine the nonabelian tensor square and the Schur multiplier of the groups. Beside, many researches on these two homological functors for various groups have been conducted over the years.

Recently, Zainal *et al.*[3] characterized the nonabelian tensor square for nonabelian groups of order p^4, where p is an odd prime. Meanwhile, Rashid *et al.*[4] computed the nonabelian tensor square of groups of order $8q$. The Schur multiplier of certain bieberbach groups with abelian point groups have been determined by Mat Hassim *et al.*[5] in 2013. Earlier, Zainal *et al.*[6] conducted a research on the Schur multiplier and nonabelian tensor square for abelian groups of order p^n, where p is an odd prime and n is equal to 3 and 4. Hence, by continuing those researches, the nonabelian tensor square and the Schur multiplier of the abelian groups of order p^5, where p is an odd prime are determined.

2.0 PRELIMINARIES

In this section, some preliminary results that are used in the computation of the nonabelian tensor square and the Schur multiplier of abelian groups of order p^5, where p is an odd prime are presented. Burnside[7] has constructed the classification of the groups as in the following theorem.

Theorem 1[7]
Let G be an abelian group of order p^5, where p is an odd prime. Then exactly one of the following holds:

$$G \cong \mathbb{Z}_{p^5}. \tag{1.1}$$

$$G \cong \mathbb{Z}_{p^4} \times \mathbb{Z}_p. \tag{1.2}$$

$$G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}. \tag{1.3}$$

$$G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_p \times \mathbb{Z}_p. \tag{1.4}$$

$$G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_{p^2} \times \mathbb{Z}_p. \tag{1.5}$$

$$G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_p^3. \tag{1.6}$$

$$G \cong \mathbb{Z}_p^5. \tag{1.7}$$

Theorem 2[2]

Let G be any finite group, then

i) $M(G)$ is a finite group, whose elements have order dividing the order of G.
ii) $M(G) = 1$ if G is cyclic.

Theorem 3[2]

If G_1 and G_2 are finite groups, then

$$M(G_1 \times G_2) = M(G_1) \times M(G_2) \times (G_1 \otimes G_2).$$

Theorem 4[8]

Let $G \cong \mathbb{Z}_m$ and $H \cong \mathbb{Z}_n$ be cyclic groups that act trivially on each other. Then $G \otimes H \cong \mathbb{Z}_{(m,n)}$.

Theorem 5[1]

Let A, B and C be groups with given actions of A on B and C, and of B and C on A. Suppose that the latter relations

i) ${}^{bc}a = {}^{cb}a$, so that $B \times C$ acts on A,
ii) induce the trivial action of B on $A \otimes C$: ${}^{b}(a \otimes c) = a \otimes c$,
iii) induce the trivial action of C on $A \otimes B$: ${}^{c}(a \otimes b) = a \otimes b$, for all $a \in A$, $b \in B$, $c \in C$. Then

$$A \otimes (B \times C) = (A \otimes B) \times (A \otimes C).$$

Remark 1[9]

The tensor product which maps two abelian groups to an abelian group is commutative, associative and distributive.

Theorem 6[1]

Let G and H be groups. Then

$$(G \times H) \otimes (G \times H) \cong (G \otimes G) \times (G \otimes H) \times (H \otimes G) \times (H \otimes H).$$

3.0 RESULTS

In this section, the nonabelian tensor square and the Schur multiplier of abelian groups of order where p^5, where p is an odd prime is stated in Theorem 7 and Theorem 8 respectively.

Theorem 7

Let G be an abelian group of order p^5, where p is an odd prime. Then exactly one of the following holds:

$$G \otimes G \cong \begin{cases} \mathbb{Z}_{p^5} & ; \text{ if } G \cong \mathbb{Z}_{p^5}, \\ \mathbb{Z}_{p^4} \times \mathbb{Z}_p^3 & ; \text{ if } G \cong \mathbb{Z}_{p^4} \times \mathbb{Z}_p, \\ \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}^3 & ; \text{ if } G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}, \\ \mathbb{Z}_{p^3} \times \mathbb{Z}_p^8 & ; \text{ if } G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_p \times \mathbb{Z}_p, \\ \mathbb{Z}_{p^2}^4 \times \mathbb{Z}_p^5 & ; \text{ if } G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_{p^2} \times \mathbb{Z}_p, \\ \mathbb{Z}_{p^2} \times \mathbb{Z}_p^{15} & ; \text{ if } G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p, \\ \mathbb{Z}_p^{25} & ; \text{ if } G \cong \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p. \end{cases}$$

Proof:
Let $G \cong \mathbb{Z}_{p^5}$. By Theorem 4, $G \otimes G \cong \mathbb{Z}_{p^5} \otimes \mathbb{Z}_{p^5} \cong \mathbb{Z}_{p^5}$. The nonabelian tensor square for $G \cong \mathbb{Z}_{p^4} \times \mathbb{Z}_p$, $G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}$ and $G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p$ are computed by using Theorem 4 and Theorem 6 as follows:

i) $G \cong \mathbb{Z}_{p^4} \times \mathbb{Z}_p$.

$$\begin{aligned} G \otimes G &\cong \left(\mathbb{Z}_{p^4} \times \mathbb{Z}_p\right) \otimes \left(\mathbb{Z}_{p^4} \times \mathbb{Z}_p\right) \\ &\cong \left(\mathbb{Z}_{p^4} \otimes \mathbb{Z}_{p^4}\right) \times \left(\mathbb{Z}_{p^4} \otimes \mathbb{Z}_p\right) \times \left(\mathbb{Z}_p \otimes \mathbb{Z}_{p^4}\right) \times \left(\mathbb{Z}_p \otimes \mathbb{Z}_p\right) \\ &\cong \mathbb{Z}_{p^4} \times \mathbb{Z}_p^3. \end{aligned}$$

ii) $G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}$.

$$\begin{aligned} G \otimes G &\cong \left(\mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}\right) \otimes \left(\mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}\right) \\ &\cong \left(\mathbb{Z}_{p^3} \otimes \mathbb{Z}_{p^3}\right) \times \left(\mathbb{Z}_{p^3} \otimes \mathbb{Z}_{p^2}\right) \times \left(\mathbb{Z}_{p^2} \otimes \mathbb{Z}_{p^3}\right) \times \left(\mathbb{Z}_{p^2} \otimes \mathbb{Z}_{p^2}\right) \\ &\cong \mathbb{Z}_{p^3} \times \mathbb{Z}_{p^2}^3. \end{aligned}$$

iii) $G \cong \mathbb{Z}_{p^2} \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p$.

$$\begin{aligned} G \otimes G &\cong \left(\left(\mathbb{Z}_{p^2} \times \mathbb{Z}_p\right) \times \left(\mathbb{Z}_p \times \mathbb{Z}_p\right)\right) \otimes \left(\left(\mathbb{Z}_{p^2} \times \mathbb{Z}_p\right) \times \left(\mathbb{Z}_p \times \mathbb{Z}_p\right)\right) \\ &\cong \left(\left(\mathbb{Z}_{p^2} \times \mathbb{Z}_p\right) \otimes \left(\mathbb{Z}_{p^2} \otimes \mathbb{Z}_p\right)\right) \times \left(\left(\mathbb{Z}_{p^2} \times \mathbb{Z}_p\right) \otimes \left(\mathbb{Z}_p \times \mathbb{Z}_p\right)\right) \times \\ &\quad \left(\left(\mathbb{Z}_p \times \mathbb{Z}_p\right) \otimes \left(\mathbb{Z}_{p^2} \times \mathbb{Z}_p\right)\right) \times \left(\left(\mathbb{Z}_p \times \mathbb{Z}_p\right) \otimes \left(\mathbb{Z}_p \times \mathbb{Z}_p\right)\right) \\ &\cong \mathbb{Z}_{p^2} \times \mathbb{Z}_p^{15}. \end{aligned}$$

For the group G that is isomorphic to $\mathbb{Z}_{p^3} \times \mathbb{Z}_p \times \mathbb{Z}_p$, $\mathbb{Z}_{p^2} \times \mathbb{Z}_{p^2} \times \mathbb{Z}_p$ and $\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p$, by using Theorem 4 until Theorem 6 together with Remark 1, we have:

i) $G \cong \mathbb{Z}_{p^3} \times \mathbb{Z}_p \times \mathbb{Z}_p$.

$$\begin{aligned}G\otimes G &\cong \left(\left(\mathbb{Z}_{p^3}\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\otimes\left(\left(\mathbb{Z}_{p^3}\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\\ &\cong\left(\left(\mathbb{Z}_{p^3}\times\mathbb{Z}_p\right)\otimes\left(\mathbb{Z}_{p^3}\times\mathbb{Z}_p\right)\right)\times\left(\left(\mathbb{Z}_{p^3}\times\mathbb{Z}_p\right)\otimes\mathbb{Z}_p\right)\times\\ &\quad\left(\mathbb{Z}_p\otimes\left(\mathbb{Z}_{p^3}\times\mathbb{Z}_p\right)\right)\times\left(\mathbb{Z}_p\otimes\mathbb{Z}_p\right)\\ &\cong\mathbb{Z}_{p^3}\times\mathbb{Z}_p^8.\end{aligned}$$

ii) $G\cong\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\times\mathbb{Z}_p$.

$$\begin{aligned}G\otimes G &\cong \left(\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\times\mathbb{Z}_p\right)\otimes\left(\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\times\mathbb{Z}_p\right)\\ &\cong\left(\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\otimes\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\right)\times\left(\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\otimes\mathbb{Z}_p\right)\times\\ &\quad\left(\mathbb{Z}_p\otimes\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\right)\times\left(\mathbb{Z}_p\otimes\mathbb{Z}_p\right)\\ &\cong\mathbb{Z}_{p^2}^4\times\mathbb{Z}_p^5.\end{aligned}$$

iii) $G\cong\mathbb{Z}_p\times\mathbb{Z}_p\times\mathbb{Z}_p\times\mathbb{Z}_p\times\mathbb{Z}_p$.

$$\begin{aligned}G\otimes G &\cong \left(\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\times\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)\otimes\\ &\quad\left(\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\times\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)\\ &\cong\left(\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\otimes\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\right)\times\\ &\quad\left(\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\otimes\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)\times\\ &\quad\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\otimes\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\right)\times\\ &\quad\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\otimes\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)\\ &\cong\mathbb{Z}_p^{25}.\end{aligned}$$

Theorem 8

Let G be an abelian group of order p^5, where p is an odd prime. Then exactly one of the following holds:

$$M(G)\cong\begin{cases}1 & ;\text{ if } G\cong\mathbb{Z}_{p^5},\\ \mathbb{Z}_p & ;\text{ if } G\cong\mathbb{Z}_{p^4}\times\mathbb{Z}_p,\\ \mathbb{Z}_{p^2} & ;\text{ if } G\cong\mathbb{Z}_{p^3}\times\mathbb{Z}_{p^2},\\ \mathbb{Z}_p^3 & ;\text{ if } G\cong\mathbb{Z}_{p^3}\times\mathbb{Z}_p\times\mathbb{Z}_p,\\ \mathbb{Z}_{p^2}\times\mathbb{Z}_p^2 & ;\text{ if } G\cong\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\times\mathbb{Z}_p,\\ \mathbb{Z}_p^6 & ;\text{ if } G\cong\mathbb{Z}_{p^2}\times\mathbb{Z}_p^3,\\ \mathbb{Z}_p^{10} & ;\text{ if } G\cong\mathbb{Z}_p^5.\end{cases}$$

Proof:

For the first group in Theorem 1, that is $G\cong\mathbb{Z}_{p^5}$ is a cyclic group, then by Theorem 2, we have $M(G)$ = 1. By Theorem 2 until Theorem 4, we have

$$M(G)=M\left(\mathbb{Z}_{p^4}\right)\times M\left(\mathbb{Z}_p\right)\times\left(\mathbb{Z}_{p^4}\otimes\mathbb{Z}_p\right)=\mathbb{Z}_p \text{ and}$$

$$M(G)=M\left(\mathbb{Z}_{p^3}\right)\times M\left(\mathbb{Z}_{p^2}\right)\times\left(\mathbb{Z}_{p^3}\otimes\mathbb{Z}_{p^2}\right)=\mathbb{Z}_{p^2}$$ for $G\cong\mathbb{Z}_{p^4}\times\mathbb{Z}_p$ and $G\cong\mathbb{Z}_{p^3}\times\mathbb{Z}_{p^2}$, respectively. Next, by using Theorem 2 until Theorem 5,

$$M(G)=M\left(\mathbb{Z}_{p^3}\right)\times M\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\left(\mathbb{Z}_{p^3}\otimes\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)=\mathbb{Z}_p^3$$

for $G\cong\mathbb{Z}_{p^3}\times\mathbb{Z}_p\times\mathbb{Z}_p$ and

$$\begin{aligned}M(G)&=M\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\times M\left(\mathbb{Z}_p\right)\times\left(\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\right)\otimes\mathbb{Z}_p\right)\\ &=\mathbb{Z}_{p^2}\times\mathbb{Z}_p^2\end{aligned}$$

for $G\cong\mathbb{Z}_{p^2}\times\mathbb{Z}_{p^2}\times\mathbb{Z}_p$. For $G\cong\mathbb{Z}_{p^2}\times\mathbb{Z}_p^3$, Theorem 2 until Theorem 4 together with Theorem 6 are used. The Schur multiplier for this group is given as follows;

$$\begin{aligned}M(G)&=M\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_p\right)\times M\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\\ &\quad\left(\left(\mathbb{Z}_{p^2}\times\mathbb{Z}_p\right)\otimes\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)\\ &=\mathbb{Z}_p^6.\end{aligned}$$

For the last case, that is $G\cong\mathbb{Z}_p^5$, by using Theorem 2 until Theorem 6,

$$\begin{aligned}M(G)&=M\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\right)\times M\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\\ &\quad\left(\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\times\mathbb{Z}_p\otimes\left(\mathbb{Z}_p\times\mathbb{Z}_p\right)\right)\\ &=\mathbb{Z}_p^{10}.\end{aligned}$$

4.0 CONCLUSION

In this paper, the nonabelian tensor square and the Schur multipler for all abelian groups of order p^5, where p is an odd prime has been determined. The results obtained show that all the nonabelian tensor square and the Schur multiplier are elementary abelian.

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